



**Q: How does the performance vary when we increase  $d$  and  $w$ ?**

Answer:

➤ Fixed  $d$

When  $d = 1$  or  $2$  or  $4$ , if increase  $w$ , we can see accuracy will also increase;

When  $d = 4$ , accuracy will keep at 10% for all  $w$ ;

When  $d = 0$ , run four times, since weight initialization will change, accuracy will also not same for these four times according to figure left; Figure right only run  $d = 0$  one time.

➤ Fixed  $w$

When  $w = 1$ , except  $d = 0$ , all other  $d$  has same accuracy;

When  $w = 2$ , depth 0 > depth1 > depth 2 = depth3 = depth4 for accuracy;

When  $w = 10$ , depth 2 has higher accuracy than others;

➤ Conclusion

Fixed  $d$ , if increase  $w$ , accuracy will also increase except  $d = 4$  and  $d = 0$ , which means increasing width will improve performance for reasonable  $d$ .

Fixed  $w$ , if increase  $d$ , accuracy will not increase monotonously,  $d = 2$  has highest accuracy when  $w = 10$ ;  $d = 4$  has lowest accuracy for all  $w$ ; we can get the conclusion that increasing depth doesn't help for performance, this maybe because increasing the depth of a neural network will let the approximate functions with increased non-linearity, increasing the chance of over-fitting.

```

# Code
import numpy as np
import sys
import math
from operator import itemgetter
import random
import matplotlib.pyplot as plt

eta = 0.1

# Reading Data Files in arff format, return a 2D matrix which store the data
def read_file(filename):
    file_data = []
    with open(filename, 'r') as f:
        for line in f:
            if line[0].isdigit():
                features = []
                for data in line.split(','):
                    try:
                        features.append(int(data))
                    except:
                        features.append(data)
                file_data.append(features)
    return file_data

# figure number of labels and output units to be used
def split_data(train_data):
    d_label = dict()
    ### not finished
    for tr in train_data:
        label = tr[-1]
        if label in d_label.keys():
            d_label[label] += 1
        else:
            d_label[label] = 1
    n = len(d_label.keys())
    y = np.zeros((n, n))
    for j in range(n):
        y[j][j] = 1
    return len(d_label), y

```

```

def initial_weights(w, d, feature):
    weights = []
    d += 1
    for k in range(d):
        if d == 1:
            weight = np.array([random.uniform(-0.1,0.1) for i in
range(feature*10)])
            weight = weight.reshape(10, feature)
            weights.append(weight)
        else:
            if k == 0:
                weight = np.array([random.uniform(-0.1,0.1) for i in
range(feature*w)])
                weight = weight.reshape(w, feature)
                weights.append(weight)
            elif k == (d-1):
                # number of output layer is 10
                weight = np.array([random.uniform(-0.1,0.1) for i in
range(w * 10)])
                weight = weight.reshape(10,w)
                weights.append(weight)
            else:
                weight = np.array([random.uniform(-0.1,0.1) for i in
range(w * w)])
                weight = weight.reshape(w,w)
                weights.append(weight)

    return weights

# compute X using sigmoid function
def sigmoid(s):
    res = []
    for a in s:
        if a > 50:
            res.append(1 - 10**(-50))
        elif a < -50:
            res.append(10**(-50))
        else:
            res.append(1 / (1 + np.exp(-a)))

    res = np.array(res)
    return res

# compute x
def compute_x(weights, d, tr):

```

```

X = []
X.append(tr[:-1])
# forwards compute X
for di in range(d):
    s_hidden = np.dot(weights[di], X[-1])
    x_hidden = sigmoid(s_hidden)
    X.append(x_hidden)
return X

# compute DELTA
def compute_delta(L, X, weights):
    depth = len(weights)
    d = depth
    DELTA = []
    while d > 0 :
        if d == depth:
            x = X[d]
            n = len(x)
            delta = -(L - x) * x * (np.ones(n)-x)
            DELTA.append(delta)
            d -= 1
        else:
            last = DELTA[-1]
            x = X[d]
            n = len(x)
            weight = weights[d]
            delta = x * (np.ones(n) - x) * np.dot(np.transpose(weight), last)
            DELTA.append(delta)
            d -= 1
    return DELTA

# backpropagation algorithm
def learn(w, d, train_data, test_data, y):
    global eta
    # construct network with w, d and initialize weights
    feature = len(train_data[0]) - 1
    weights = initial_weights(w, d, feature)
    # Repeat 200 times
    d += 1
    for i in range(200):
        for tr in train_data:
            X = compute_x(weights, d, tr)
            # backwards compute DELTA
            index = tr[-1]

```

```

L = y[:,index] # L is vector
DELTA = compute_delta(L, X, weights)

# update weights
for di in range(d):
    x = X[di]
    delta = np.matrix(DELTA[d-di-1]).T
    g = eta * delta * x
    g = np.array(g)
    weights[di] -= g

```

```

# test data
accu = 0
for te in test_data:
    X = compute_x(weights, d, te)
    y = X[-1]
    if np.argmax(y) == te[-1]:
        accu += 1
te_len = len(test_data)
accuracy = float(accu) / te_len
return accuracy

```

```
def main():
```

```

    file = ['optdigits_train.arff.txt', 'optdigits_test.arff.txt']
    train_data = read_file(file[0])
    test_data = read_file(file[1])
    depth = [1,2,3,4]
    width = [1,2,5,10]

    # number of lables(d) and output units (y)
    d_label, y = split_data(train_data)
    accu_list = []
    accuracy = learn(0, 0, train_data, test_data, y)
    acc = []
    acc.append(accuracy)
    accu_list.append(acc * 4)

    for d in depth:
        acc = []
        for w in width:
            accuracy = learn(w, d, train_data, test_data, y)
            acc.append(accuracy)

```

```
        accu_list.append(acc)

print len(accu_list)
print accu_list

# plot figure
print "start to plot:"
plt.plot(width, accu_list[0], 'r', marker = '*')
plt.plot(width, accu_list[1], 'y', marker = '*')
plt.plot(width, accu_list[2], 'g', marker = '*')
plt.plot(width, accu_list[3], 'c', marker = '*')
plt.plot(width, accu_list[4], 'b', marker = '*')
plt.title('neural network')
plt.xlabel('width')
plt.ylabel('accuracy')
plt.legend(['depth = 0', 'depth = 1', 'depth = 2', 'depth = 3', 'depth = 4'], loc = 0)
plt.savefig("neural.png")
plt.clf()

main()
```