Federal Reserve Bank of Dallas
Globalization and Monetary Policy Institute
Working Paper No. 165
http://www.dallasfed.org/assets/documents/institute/wpapers/2013/0165.pdf

Monitoring Housing Markets for Episodes of Exuberance: An Application of the Phillips et al. (2012, 2013) GSADF Test on the Dallas Fed International House Price Database*

Efthymios Pavlidis
Lancaster University

Alisa Yusupova
Lancaster University

Ivan Paya
Lancaster University

David Peel
Lancaster University

Enrique Martínez-García
Federal Reserve Bank of Dallas

Adrienne Mack
Federal Reserve Bank of Dallas

Valerie Grossman
Federal Reserve Bank of Dallas

December 2013

Abstract

The detection of explosive behavior in house prices and the implementation of early warning diagnosis tests are of great importance for policy-making. This paper applies the GSADF test developed by Phillips et al. (2012) and Phillips et al. (2013), a novel procedure for testing, detection and date-stamping of explosive behavior, to the data from the Dallas Fed International House Price Database documented in Mack and Martínez-García (2011). We discuss the use of the GSADF test to monitor international housing markets. We assess the international boom and bust cycle experienced during the past 15 years through this lens—with special attention to the United States, the United Kingdom, and Spain. Our empirical results suggest that these three countries experienced a period of exuberance in housing prices during the late 90s and the first half of the 2000s that cannot be attributed solely to the behavior of fundamentals. Looking at all 22 countries covered in the International House Price Database, we detect a pattern of synchronized explosive behavior during the last international house boom-bust episode not seen before.

JEL codes: C22, G12, R31

* Efthymios G. Pavlidis, Lancaster University Management School, Lancaster, LA1 4YX, UK. e.pavlidis@lancaster.ac.uk. Alisa Yusupova, Lancaster University Management School. a.yusupova@lancaster.ac.uk. Ivan Paya, Lancaster University Management School. i.paya@lancaster.ac.uk. David Peel, Lancaster University Management School. d.peel@lancaster.ac.uk. Enrique Martínez-García, Federal Reserve Bank of Dallas. 2200 N. Pearl Street, Dallas, TX 75201. 214-922-5262. enrique.martinez-garcia@dal.frb.org. Adrienne Mack, Federal Reserve Bank of Dallas. 214-922-5793. adrienne.mack@dal.frb.org. Valerie Grossman, Federal Reserve Bank of Dallas. 214-922-6367. valerie.grossman@dal.frb.org. The International House Price Database can be accessed online at http://www.dallasfed.org/institute/houseprice/index.cfm. We acknowledge the support of the Federal Reserve Bank of Dallas. All remaining errors are ours alone. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Dallas Fed or the Federal Reserve System.

1 Introduction

The latest international boom and bust in housing markets has generated increased interest in the dynamics of house prices as well as house price to economic fundamental ratios—namely the house price to income ratio and the house price to rent ratio. In this context, time series methods that can detect and date periods of explosive behavior are particularly relevant. *Ex post*, these methods can provide useful information for improving our understanding of the housing market—setting light also on the effectiveness of monetary policy and macro-prudential regulation. Moreover, these methods produce in *real-time* warning signals that can be used by central banks and regulators to monitor the macroeconomic developments of housing.

In a recent series of papers, Phillips et al. (2012) and Phillips et al. (2013) propose a new recursive flexible-window testing procedure for identifying periods of explosiveness. We refrain from using the term bubbles in connection with these methods, although bubbles can lead to explosive behavior, because bubble-detection requires the specification and estimation of an economic model for the housing market (see Himmelberg et al. (2005)) which is out of the scope of our project. The procedure of Phillips et al. (2012) and Phillips et al. (2013) performs better to detect explosive behavior in time series data than standard methods such as integration/cointegration tests (e.g., Diba and Grossman (1988)), variance bound tests (e.g., LeRoy and Porter (1981), Shiller (1981)), specification tests (e.g., West (1987)) as well as Chow and CUSUM-type tests (e.g., Homm and Breitung (2012)).

Since the second quarter of 2013, the Federal Reserve Bank of Dallas' International House Price Database in partnership with the Department of Economics at Lancaster University Management School publishes indicators of exuberance in real house prices and house-prices-to-income ratios based on the method of Phillips et al. (2012) and Phillips et al. (2013). In this paper, we describe these indicators and provide technical details for their computation. We also provide an illustration of the procedure and its economic interpretation using data from the International House Price Database database documented in Mack and Martínez-García (2011) and the OECD with special emphasis on the experiences of the United States (U.S.), the United Kingdom (U.K.), and Spain.

The remainder of the paper proceeds as follows: Section 2 outlines the standard asset pricing equation on housing and describes how explosive behavior in house prices may arise. In section 3 we provide extensive discussion and further details on the $GSADF$ test procedure of Phillips et al. (2012) and Phillips et al. (2013) that we implement with the Dallas Fed's International House Price Database. Then we present novel quantitative findings in section 4 based on the implementation of the $GSADF$ test procedure to the experiences of the U.S., the U.K., and Spain. We also note the strong synchronization of periods of explosive behavior in the last boom and bust episode across all countries covered in the International House Price Database. Section 5 provides some additional discussion and concludes.

2 Rational Bubbles in Housing Markets

Assuming risk neutrality and a constant expected return on an alternative investment opportunity (i.e., a risk-free free rate r_t such that $\mathbb{E}_t(r_t) = r$ for all t), the price of an asset can be derived from the following no-arbitrage condition,

$$\underbrace{r}_{\text{constant risk-free rate}} = \underbrace{\mathbb{E}_t(R_t)}_{\text{expected asset return}}, \tag{1}$$

where the asset return is defined as $R_t \equiv \frac{P_{t+1}+X_{t+1}}{P_t} - 1$ and the expectations operator $\mathbb{E}_t$ is based on all information available up to time t.

Re-arranging the condition in (1), the asset price can easily be expressed as follows,

$$P_t = \mathbb{E}_t\left[\frac{P_{t+1} + X_{t+1}}{1+r}\right], \tag{2}$$

where P_t denotes the price of the asset at time t, X_{t+1} is the stream of payoffs derived from the asset at time $t+1$, and r is the constant risk-free interest rate. For further details on the present value model, see Gordon and Shapiro (1956) on the standard dividend discount model, and Blanchard and Watson (1982) and Campbell et al. (1997) for more general processes for the payoff stream $\{X_t\}_{t=1}^{\infty}$.

The expected return on housing must be equal (up to a constant) to the risk-free rate by arbitrage, so similarly the simple present value model in (2) can be used to study house prices (see, e.g., Clayton (1996)). The payoff stream $\{X_t\}_{t=1}^{\infty}$ in this case is given by the economic rents of housing, including housing services. The asset pricing equation in (2) applied to the housing case implies that house prices must be equal to the discounted present value of its expected future rents plus its re-sale value.

Solving equation (2) recursively T periods forward, we obtain an expression for the house price as a function of the expected discounted flow of all future payoffs up to time T plus a terminal condition that determines the present discounted of the time T re-sale value of the house, i.e.,

$$P_t = \mathbb{E}_t\left[\sum_{i=1}^{T}\left(\frac{1}{1+r}\right)^i X_{t+i}\right] + \mathbb{E}_t\left[\left(\frac{1}{1+r}\right)^T P_{t+T}\right]. \tag{3}$$

Letting T go to infinity and imposing the transversality condition,

$$\lim_{T\to\infty} \mathbb{E}_t\left[\left(\frac{1}{1+r}\right)^T P_{t+T}\right] < \infty, \tag{4}$$

the (unique) no-bubbles solution to the expectational difference equation that characterizes house prices in (2) yields,

$$P_t^* = \mathbb{E}_t\left[\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i X_{t+i}\right]. \tag{5}$$

P_t^* is referred to as the fundamental value of housing. Equation (5) states that the fundamental value of housing should be equal to the present discounted value of all future rents from housing, so the only fundamentals determining the price are the expected rents $\mathbb{E}_t[X_{t+i}]$ for all $i \geq 1$ and the constant risk-free rate r. Imposing the transversality condition in (4) rules out non-fundamental behavior (bubbles), and this implies that the housing price corresponds to its fundamental value (i.e., $P_t = P_t^*$).

However, without imposing the transversality condition in (4), the forward solution to the expectational difference equation in (2) for the house price P_t is not unique. It includes the fundamental price determined in (5)—that is, the no-bubbles solution P_t^*—plus a nonstationary component (see, e.g., Sargent (1987) and LeRoy (2004)) in the following form,

$$P_t = P_t^* + (1+r)^t c_t, \tag{6}$$

where $\{c_t\}_{t=1}^{\infty}$ is a martingale—that is, a stochastic process that satisfies $\mathbb{E}_t c_{t+1} = c_t$. If the nonstationary (or bubble) component cannot be ruled out, it introduces explosive behavior as shown in Diba and Grossman (1988) that affects the time series of house prices. Moreover, there are infinitely many solutions of the form presented in (6) that solve equation (2).

We can alternatively define the nonstationary component of the solution in (6) as $B_t = (1+r)^t c_t$. With this characterization, the rational bubble can simply be expressed as the difference between the housing price, P_t, and its fundamental-based value, P_t^*, i.e.,

$$B_t = P_t - P_t^*. \tag{7}$$

This bubble component $\{B_t\}_{t=1}^{\infty}$ has an explosive behavior if $r > 0$ since it satisfies that,

$$\mathbb{E}_t (B_{t+1}) = (1+r)B_t, \tag{8}$$

given that the underlying component c_t follows a martingale process. With this notation, we define bubbles in house prices, $\{B_t\}_{t=1}^{\infty}$, as departures from the fundamental value of housing. If $B_t = 0$, there is no bubble and prices are determined only the the expected future discounted rents. In turn, if $B_t > 0$ there is a bubble that induces explosiveness into the time series of house prices P_t.

If house prices include a nonstationary (bubble) component $\{B_t\}_{t=1}^{\infty}$ that satisfies condition (8), then it is because investors in the housing market are expecting the non-fundamental component of the price of houses (the bubble) to keep growing at a rate that equals the constant risk-free rate r. The theory of rational bubbles under the expectational difference equation in (2) can be understood from that observation. For simplicity, let us assume that B_t is strictly positive in order to illustrate the point. An investor is willing to pay today B_t units more than its fundamental value P_t^* for a house, if the investor believes that he will be sufficiently compensated through future price increases for that higher payment today. If enough investors share the same belief about housing, then they will buy houses driving the price up and confirming the expectation of future price increases anticipated by the investors in what is referred as a self-fulfilling prophecy.

2.1 House Prices with Rational Bubbles

According to the present value model of housing in (2), house prices can be expressed as,

$$\begin{aligned} P_t &= P_t^* + B_t, \text{ where} \\ P_t^* &= \mathbb{E}_t\left[\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i X_{t+i}\right], \\ B_t &= \frac{1}{1+r}\mathbb{E}_t\left(B_{t+1}\right). \end{aligned} \tag{9}$$

If a rational bubble is a large part of the total price of houses, would become disconnected from the fundamentals of the housing market in some way. In order to detect a bubble in time series, therefore, one logical path is to test for the possibility of such a disconnect.

Adapting the alternative representation proposed by Campbell and Shiller (1987), house prices can be expressed as,

$$P_t - \frac{1}{r}X_t = \left(\frac{1+r}{r}\right)\mathbb{E}_t\left[\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i \Delta X_{t+i}\right] + B_t, \tag{10}$$

where Δ is the difference operator (i.e., $\Delta X_t = X_t - X_{t-1}$). A plausible assumption would be that the payoff stream for housing, X_t, follows a random walk with drift, i.e.,

$$X_t = \mu + X_{t-1} + \epsilon_t,\ \epsilon_t \sim WN\left(0, \sigma_\epsilon^2\right), \tag{11}$$

where ϵ_t is white noise. This fundamental process has a unit root, and it is said to be integrated of order 1 (or $I(1)$). In the absence of bubbles (if $B_t = 0$), equation (10) shows that the house price $P_t = P_t^*$ is given by,

$$P_t^* = \frac{1+r}{r^2}\mu + \frac{1}{r}X_t. \tag{12}$$

The house price P_t has also a unit root and is cointegrated with X_t such that $P_t - \frac{1}{r}X_t$ is stationary. In other words, house prices and fundamentals—housing rents—should be integrated of order 1 (i.e., $I(1)$) if $B_t = 0$.

Diba and Grossman (1988) observe that fundamental prices, the fundamental house prices P_t^* in our context, are integrated of the same order as the payoff process X_t in the absence of bubbles (as illustrated for the $I(1)$ case here). In the presence of a bubble ($B_t > 0$), the house price in (9) contains the explosive root from B_t and so does the linear combination $P_t - \frac{1}{R}X_t$ in (10). This differentiates the fundamental value P_t^* from the bubble process B_t underlying the house price P_t, as the fundamental value is an integrated process of the same order as the fundamentals while the bubble component is characterized by an explosive autoregressive process implied by condition (8) instead.

Hence, if house prices P_t and fundamentals X_t are integrated of the same order (i.e., $I(1)$), we could exclude the presence of bubbles. If house prices exhibit explosive autoregressive behavior or shifts from $I(1)$ to explosive, then we would argue that a rational bubble has formed in housing. When the process for house price loses its explosiveness, then we could claim that the bubble has popped.

We use a new recursive procedure based on the augmented Dickey-Fuller (ADF) test which allows the testing, *ex post* identification and date stamping of explosive behavior in economic time series to house price data. This econometric method has been developed in a series of papers by Phillips et al. (2011), Phillips et al. (2012) and Phillips et al. (2013) with tests that deal with the structural change from a random walk ($I(1)$ process) to explosive behavior. These tests exploit the disconnect that emerges between the integrated process for the fundamental value and the explosiveness of the bubble, and can also be applied in real-time to provide warning signals of explosiveness in the time series of house prices.

2.2 Long-Run House Price Anchors

The house-price-to-rent ratio is often used as an indicator of over or undervaluation of housing relative to the expenses of renting. The house price solution presented in (10) under the Campbell and Shiller (1987) representation can be rewritten in terms of the house-price-to-rent ratio easily as follows,

$$\frac{P_t}{X_t} = \frac{1}{r} + \frac{1}{X_t}\left\{\left(\frac{1+r}{r}\right)\mathbb{E}_t\left[\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i \Delta X_{t+i}\right] + B_t\right\}, \tag{13}$$

which under the assumption of an $I(1)$ process for housing rents becomes,[1]

$$\frac{P_t}{X_t} = \frac{1}{r} + \frac{1}{X_t}\left\{\frac{1+r}{r^2}\mu + B_t\right\}. \tag{14}$$

Since the fundamental price of housing and rents follow a random walk with drift, detrending the time series we remove the drift component μ implying a stable house-price-to-rent ratio in the absence of bubbles. If this ratio deviates from its long-run average, it can be an indication that house prices have become misaligned from fundamentals.

Working with housing rents is not without its problems, as housing rents are often measured with great error. The present-value model in (2) can be complemented with other conditions to express house prices in terms of a set of macroeconomic variables (fundamentals). In that spirit, we focus our attention on real disposable income and—in particular—on the house-price-to-income ratio. The price-to-income ratio is commonly used in the literature as an alternative to the house-price-to-rent ratio to assess whether house prices are sustainable in the sense of being consistent with the economic fundamentals of the housing market. This alternative ratio provides a metric of house prices relative to the ability of households to pay (see, e.g., Himmelberg et al. (2005) and Girouard et al. (2006)). In that way, it incorporates one the key determinants of the demand for housing.

We have discussed so far how rational bubbles can emerge when fundamentals are otherwise integrated processes. Since the asset pricing model in (2) can also generate explosive price dynamics due to the explosive behavior of the fundamentals of housing, we follow Phillips et al. (2012)'s suggestion to apply the same econometric tests used for real house prices to the long-run anchors for house price determination—the price-to-income ratio, and the price-to-rent ratio.[2] The idea is that controlling for the possibility of explosive behavior in the fundamentals with these two ratios, house prices that display patterns of explosiveness are

[1] For a discussion of a more general solution with log-linear approximation methods, see the work of Engsted et al. (2012).

[2] We note that, apart from income and rent, there are other fundamental drivers of housing prices, such as the cost of foregone interest, the cost of property taxes and maintenance costs (see, e.g., the discussion in Himmelberg et al. (2005)). Since explosive behavior in prices may be induced by these other fundamental drivers, the presence of explosive dynamics in house-price-to-income and price-to-rent ratios cannot be considered conclusive but indicative for the presence of bubbles in the housing market.

more likely to be driven by the non-fundamental (bubble) component of the solution.

3 Testing for Explosive Behavior: The $GSADF$ Procedure

The time series econometric method used for testing and detecting explosive behavior is based on the following Augmented Dickey-Fuller (ADF) regression equation,

$$\Delta y_t = a_{r_1,r_2} + \beta_{r1,r2} y_{t-1} + \sum_{i=1}^{k} \psi^i_{r_1,r_2} \Delta y_{t-i} + \epsilon_t, \tag{15}$$

where y_t denotes a time series process (in our case, real house prices, the price-to-income ratio, or the price-to-rent ratio), $\epsilon_t \overset{iid}{\sim} N(0, \sigma^2_{r_1,r_2})$, and r_1 and r_2 denote fractions of the total sample size that specify the starting and ending points of a subsample period.

As noted in the previous section, the emergence and popping of a bubble process $\{B_t\}_{t=1}^{\infty}$ that satisfies condition (8) is indicated by a shift from a random walk—under the assumption that fundamentals are $I(1)$—to an explosive autoregressive process. Therefore, we are interested in testing with equation (15) the null hypothesis of a unit root, $H_0 : \beta_{r_1,r_2} = 0$, against the alternative of explosive behavior in y_t, $H_1 : \beta_{r_1,r_2} > 0$. Let

$$ADF^{r_2}_{r_1} = \frac{\widehat{\beta}_{r1,r2}}{\text{s.e.}(\widehat{\beta}_{r1,r2})} \tag{16}$$

denote the test statistic corresponding to this null hypothesis. It is easy to see that setting $r_1 = 0$ and $r_2 = 1$ yields the standard ADF test statistic, ADF^1_0. The limit distribution of ADF^1_0 is given by,

$$\frac{\int_0^1 W dW}{\int_0^1 W^2}, \tag{17}$$

where W is a Wiener process. The ADF test compares the ADF^1_0 statistic with the right tail critical value from its limit distribution. When the test statistic exceeds the critical value, the unit root hypothesis is rejected in favor of explosive behavior.

Although widely employed, the standard ADF test has extremely low power in detecting episodes of explosive behavior when these episodes end with a large drop in prices, i.e. in the presence of boom-bust dynamics. As a matter of fact, nonlinear dynamics, such as those displayed by periodically collapsing bubbles, frequently lead to finding spurious stationarity even though the process under examination is inherently explosive as noted by Evans (1991).

In order to deal with the effect of a price collapse on the performance of the test, Phillips et al. (2011) proposed a recursive procedure which is based on the estimation of the ADF regression on subsamples of the available data. Bubble detection under this test is reduced to testing for a change from $I(1)$ to explosive in a univariate time series, where the change point is unknown. In particular, the authors propose estimating (15) using a forward expanding sample with the end of the sample period r_2 increasing from r_0 (the minimum window size) to one (the last available observation). In this procedure, the beginning of the sample is held constant at $r_1 = 0$. The test statistic, called sup ADF ($SADF$), is defined as follows,

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2}. \tag{18}$$

Under the null hypothesis, the limit distribution of the $SADF$ statistic is given by,

$$\sup_{r_2 \in [r_0, 1]} \frac{\int_0^{r_2} W dW}{\int_0^{r_2} W^2}. \tag{19}$$

Similarly to the standard ADF test, when the $SADF$ statistic exceeds the right tailed critical value from its limit distribution, the unit root hypothesis is rejected in favor of explosive behavior.

The $SADF$ test performs well when there is a single boom-bust episode in the time series. Simulation experiments in Homm and Breitung (2012) reveal that the $SADF$ outperforms alternative methods, such as the modified Bhargava (1986), the modified Busetti and Taylor (2004), and the modified Kim (2000) (with the corrections of Kim et al. (2002)), in terms of power. However, the test may perform poorly when there are more than one boom-bust episodes in the sample and may also be inconsistent.[3]

More recently, Phillips et al. (2012) and Phillips et al. (2013) derived a new unit root test, the Generalized $SADF$ ($GSADF$), that covers a larger number of subsamples than the $SADF$ by allowing both the ending point, r_2, and the starting point, r_1, to change. This extra flexibility results in substantial power gains in comparison to the $SADF$. Moreover, the test is consistent with multiple boom-bust episodes within a given time series. The $GSADF$ statistic is defined by,

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2}. \tag{20}$$

Under the null hypothesis, the limit distribution of the $GSADF$ statistic is

$$\sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} \left\{ \frac{\frac{1}{2} r_w [W(r_2)^2 - W(r_1)^2 - r_w] - \int_{r_1}^{r_2} W(r) dr [W(r_2) - W(r_1)]}{r_w^{1/2} \{ r_w \int_{r_1}^{r_2} W(r)^2 dr - [\int_{r_1}^{r_2} W(r) dr]^2 \}^{1/2}} \right\}, \tag{21}$$

where $r_w = r_2 - r_1$. Again, rejection of the unit root hypothesis in favor of explosive behavior requires that the test statistic exceeds the right tailed critical value from its limit distribution.

[3] These test procedures were proposed to test for a change in persistence between $I(0)$ and $I(1)$. In the simulations of Homm and Breitung (2012), Chow-type break test is also considered. The Chow-type test often exhibits the highest power in their estimations and its estimators for the unknown break date tend to be most reliable in finite samples.

3.1 The Date-Stamping Strategy

The first step of the Phillips et al. (2012) procedure is to test the unit root hypothesis by comparing the $GSADF(r_0)$ to the $1-\alpha$ critical value, where α is the nominal significance level. Suppose that the $GSADF$ test rejects the null hypothesis of a unit root. In many cases, it is of prime interest to detect the period(s) that the time series under examination displayed explosive dynamics. Moreover, it is important for policy formation and monitoring purposes to examine whether the time series is currently in an explosive regime or not. Hence, the second step of the procedure is to identify period(s) of explosive behavior if $GSADF$ test rejects the null.

Phillips et al. (2012) recommend a dating strategy based on the Backward sup ADF statistic, i.e.,

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} BADF_{r_1}^{r_2}. \tag{22}$$

The authors define the origination date of the period of exuberance as the first observation that the $BSADF$ statistic exceeds its critical value

$$\widehat{r}_e = \inf_{r_2 \in [r_0, 1]} \{ r_2 : BSADF_{r_2}(r_0) > scu_{r_2}^{\beta_T} \}, \tag{23}$$

and the termination date as the first observation after $\widehat{r}_e$ for which the $BSADF$ falls below its critical value

$$\widehat{r}_f = \inf_{r_2 \in [\widehat{r}_e, 1]} \{ r_2 : BSADF_{r_2}(r_0) < scu_{r_2}^{\beta_T} \}, \tag{24}$$

where $scu_{r_2}^{\beta_T}$ is the $100\beta_T\%$ critical value of the sup ADF based on $\lfloor r_2 T \rfloor$ observations and β_T is the chosen significance level. The researcher may choose to neglect very short periods of exuberance by setting a minimum duration period.

When the $BSADF$ statistic exceeds the finite-sample critical values of the $SADF$, we argue that the empirical evidence suggests that the time series displays explosive behavior. Because the distributions of the $SADF(r_0)$ and $GSADF(r_0)$ are non-standard, critical values have to be obtained through Monte Carlo simulations. The consistency of the above dating strategy in the presence of one or two periodically collapsing bubbles is established in Phillips et al. (2012).

3.2 Other Technical Details

The computation of the $SADF$, $BSADF$, and $GSADF$ test statistics necessitates the selection of the minimum window size r_0 and the autoregressive lag length k. Regarding the minimum window size, this has to be large enough to allow initial estimation but it should not be too large in order to avoid missing short episodes of exuberance. We follow Phillips et al. (2012) and set the minimum size equal to 36 observations.

With respect to the autoregressive lag length, we provide online through the International House Price Database results for two cases: k equal to 1 and 4. Our results in the remainder of the paper will be reported only for the case of lag length set at $k = 4$, unless otherwise noted, to save space. The choice of a fixed lag length is appealing because it allows us to employ a recursive least squares approach which reduces substantially the computational cost of estimation.

More sophisticated lag length selection procedures in ADF-type tests based on information criteria (such as the Modified Information Criteria of Ng and Perron (2001)) and sequential hypothesis testing (see, e.g., Ng and Perron (1995)) could, in principle, be applied but with a huge computational cost. Furthermore, Phillips et al. (2012) show that sequential hypothesis testing results in severe size distortions and a reduction in power of both the $SADF$ and $GSADF$ tests.

The implementation of the unit root tests also requires the limit distributions of the $SADF$, $BSADF$, and $GSADF$ test statistics. These distributions are non-standard and depend on the minimum window size. Hence, critical values have to be obtained through Monte Carlo simulations. We obtain finite sample critical values by generating 2,000 random walk processes with $N(0, 1)$ errors. Asymptotic $SADF$ and $GSADF$ critical values are provided in Phillips et al. (2012) Table 1.

4 Empirical Evidence on International House Prices

Since the second quarter of 2013, the International House Price Database of the Federal Reserve Bank of Dallas includes $SADF$, $GSADF$, and $BSADF$ test-statistics for real house prices and house-price-to-income ratios for all available countries together with the corresponding critical values. The statistics are updated quarterly, once data for house prices and economic fundamentals—personal disposable income—become available.

In this section, we provide an illustration of the procedure of Phillips et al. (2012) and Phillips et al. (2013) by describing the implementation of the tests described in the previous section to a subset of the countries included in the International House Price Database—namely the U.S., the U.K., and Spain—for the period between the first quarter of 1975 and the second quarter of 2013. For these three countries in particular, we investigate the explosive behavior of house prices and complement the price-to-income ratio data from the International House Price Database with price-to-rent data reported for these three countries from the OECD.

The quarterly time series for real house prices of the U.S., the U.K., and Spain are displayed jointly in Figure 4. The real house price appreciation has been very significant for these three countries since the mid 1980s as can be seen in the figure. However, the evidence shows that the real house price appreciation experienced by the U.K. and Spain has been notably larger than that of the U.S. This seems to set Spain and the U.K. apart, but as our results would show later it does not mean that the evidence of explosive behavior is somehow weaker for the U.S.

The ratio of real house prices to real personal disposable income and the OECD's price-to-rent ratio are two long-run anchors in the determination of house prices. These ratios are plotted in Figure 4 showing a similar boom period during the late 1990s and the first half of the 2000s followed by a severe correction in all three countries. Interestingly, the differences between the U.K. and Spain become more noticeable when we look at these anchors ratios. Both ratios reverted during the 1990s back to their pre-1985 average for the U.K., but remained elevated for Spain. Spain's correction since around 2006 has been more severe than that of the U.K., whose ratios have remain elevated since the early 2000s in spite of the housing bust experienced. In spite of these differences, our test results based on the procedure of Phillips et al. (2012) and Phillips et al. (2013) will show that both countries (as well as the U.S.) went through a period of so-called exuberance almost at the same time since the late-1990s until the mid-2000s.

Figure 1: Real House Prices: The Cases of the U.S., the U.K. and Spain

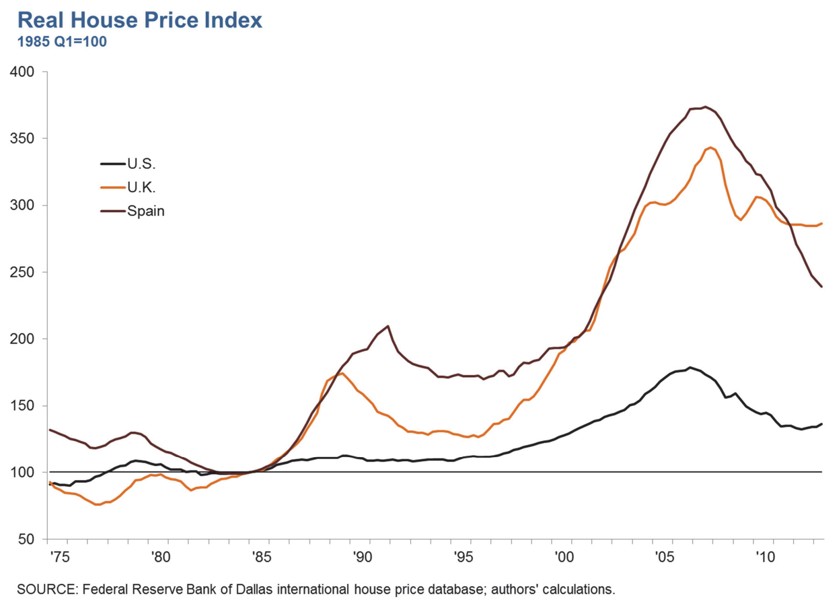
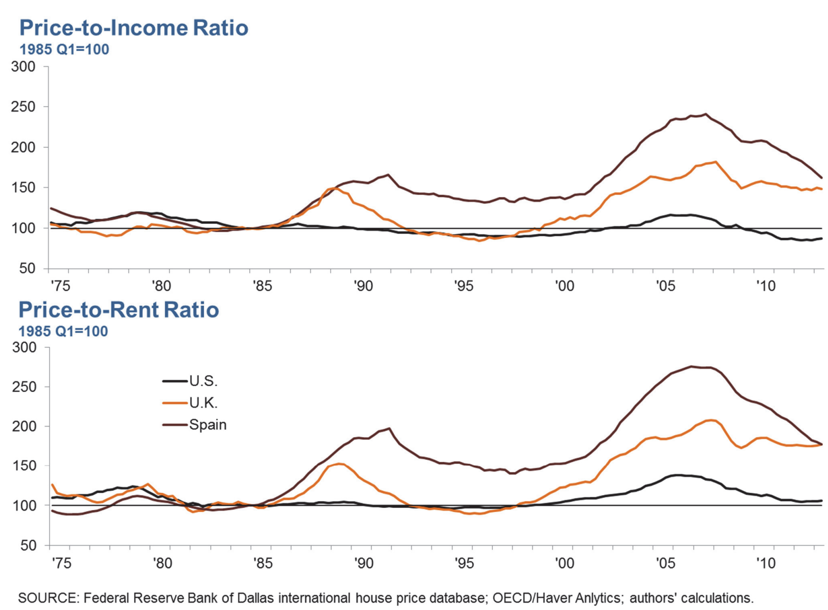


Figure 2: Real House Prices: The Long-Run Anchors



The top panel of Table 4 reports the estimated $SADF$ and $GSADF$ test statistics for the U.S., the U.K., and Spain on three variables: real house prices as well as the price-to-income and price-to-rent ratios. All results are for autoregressive lag length, k, four. The bottom panel of Table 4 reports the 90%, 95% and 99% critical values for the $SADF$ and $GSADF$ statistics.

Focusing on real house prices, we observe that the $SADF$ test statistics are greater than the 95% critical values for the U.S. and the U.K. but not for Spain. This implies that we can reject the null hypothesis at the 5% level in all cases, except for Spain. Further, the $GSADF$ statistics for all three countries are greater than the 99% critical values in accordance with the higher power of the $GSADF$ test. Overall, there is strong evidence that real house prices have exhibited periods of explosive behavior in the given time series of these three countries.

Table 4, as indicated before, also reports results for price-to-income ratios from the International House Price Database and price-to-rent ratios from the OECD for the same set of countries. What we observe is similar to the evidence on real house prices. The $GSADF$ statistics for house-price-to-income and house-price-to-rent ratios is above the 99% critical values for the U.S. and the U.K., and above the 95% critical values for Spain. Hence, the evidence from these two ratios seems to corroborate that these three countries have experienced periods of explosive behavior since the first quarter of 1975 until the second quarter of 2013.

Table 1: Evidence of Explosive Behavior in the Housing Markets

Panel A: Test Statistics						
	Real House Prices		**Price-to-Income Ratio**		**Price-to-Rent Ratio**	
Country	$SADF$	$GSADF$	$SADF$	$GSADF$	$SADF$	$GSADF$
United States	1.52^{**}	3.81^{***}	-0.78	3.47^{***}	1.20^{*}	6.06^{***}
United Kingdom	1.83^{**}	3.34^{***}	1.50^{**}	2.65^{***}	0.11	2.44^{***}
Spain	0.39	3.34^{***}	0.01	1.84^{**}	0.25	2.33^{**}
Panel B: Critical Values						
90%	0.98	1.54	0.98	1.54	0.98	1.54
95%	1.25	1.80	1.25	1.80	1.25	1.80
99%	1.89	2.39	1.89	2.39	1.89	2.39

Notes: *, **, and ***, denote statistical significance at the 10, 5 and 1 percent significance level respectively.

Having established that there is strong empirical support for explosiveness, the next step of the procedure of Phillips et al. (2012) and Phillips et al. (2013) is to identify the actual period(s) of explosive behavior in the time series—which we do next.

United States Figures 4, 4 and 4 display our main results for the U.S. The bottom panel shows the time evolution of the $BSADF$ statistics for real house prices, house-price-to-income ratios and house-price-to-rent ratios respectively—together with the 95% $SADF$ critical value sequence in finite-samples. In the top panel, we observe the respective time series of all three variables examined (real house prices, house-price-to-income ratios and house-price-to-rent ratios).

We see that real house prices for the U.S. entered an explosive regime around the second half of the 1990s which lasted until around 2006. With the two long-run anchor ratios, we observe a period of exuberance that tends to be shorter—specially with the house-price-to-income ratio—but otherwise all three variables overlap to signal a period of non-fundamental explosive behavior occurring in the first half of the 2000s.

Figure 3: Date-Stamping with U.S. Real House Prices

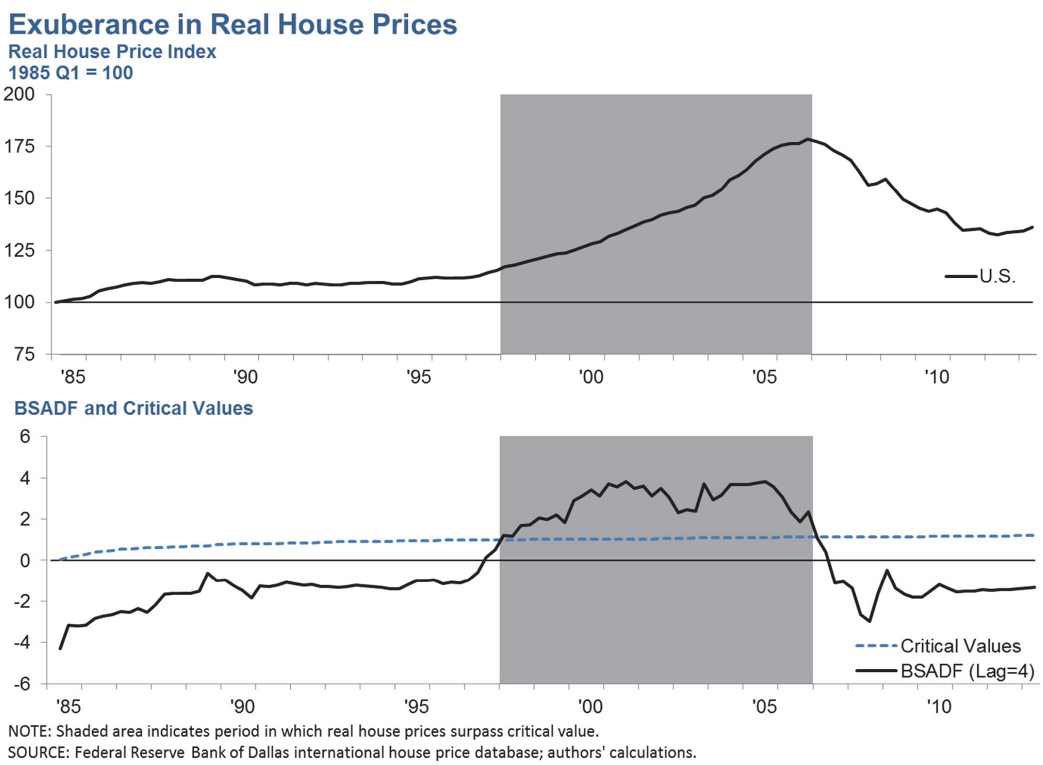


Figure 4: Alternative Date-Stamping with U.S. Price-to-Income Ratio

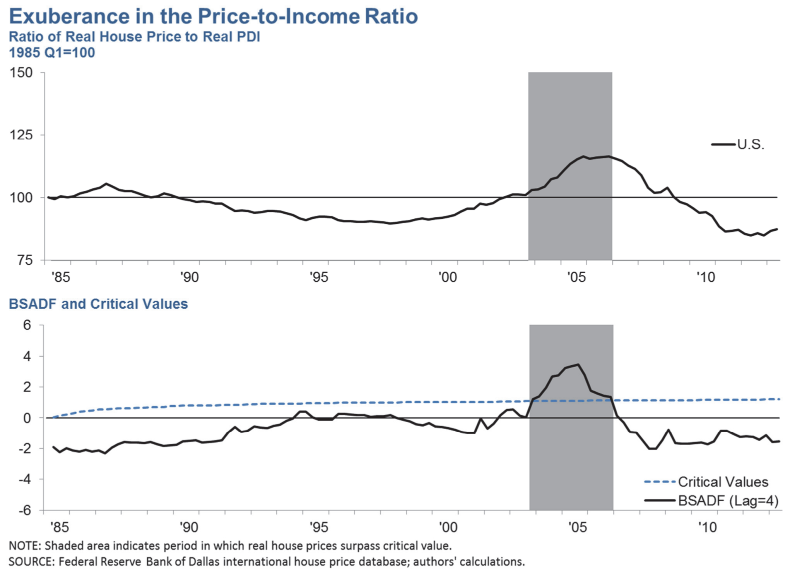
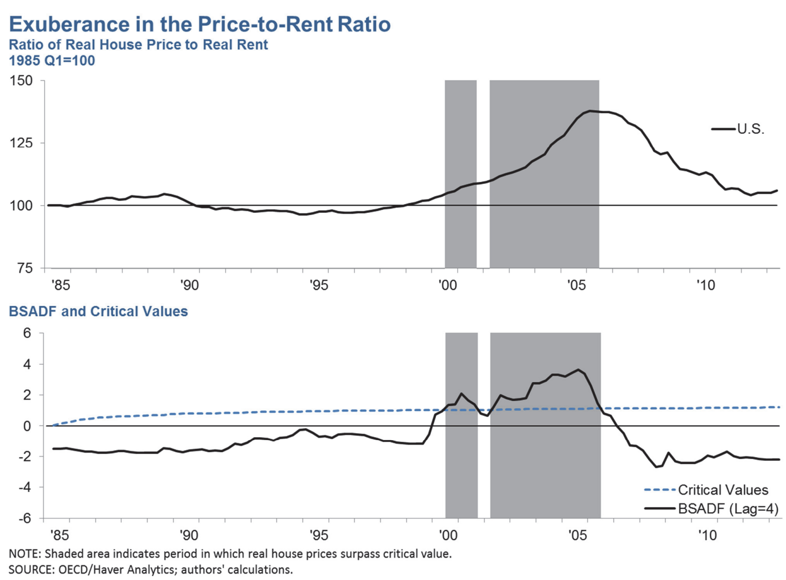


Figure 5: Alternative Date-Stamping with U.S. Price-to-Rent Ratio



United Kingdom Figures 4, 4 and 4 display our main results for the U.K. The interpretation of the figures is analogous to those of the U.S. discussed earlier. The U.K. displays an additional episode of explosive behavior in the late 1980s when looking at real house prices and the house-price-to-income ratio, although that is not present in the data for the house-price-to-rent ratio. We observe that the periods of exuberance are aligned whether we look at real house prices or the house-price-to-income ratio. All variables including the house-price-to-rent ratio point towards a common period of exuberance in the first half of the 2000s with a re-emergence around 2006. This coincides largely with the period of rapid house price appreciation that preceded the bust in 2007-08.

Figure 6: Date-Stamping with U.K. Real House Prices

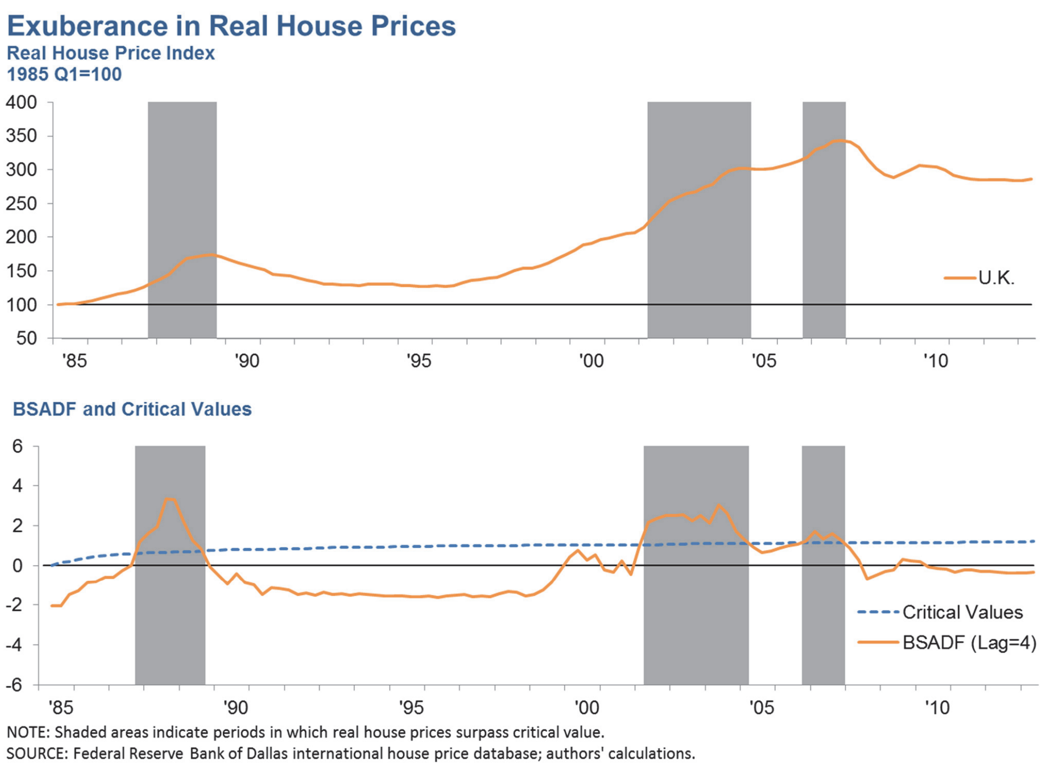


Figure 7: Alternative Date-Stamping with U.K. Price-to-Income Ratio

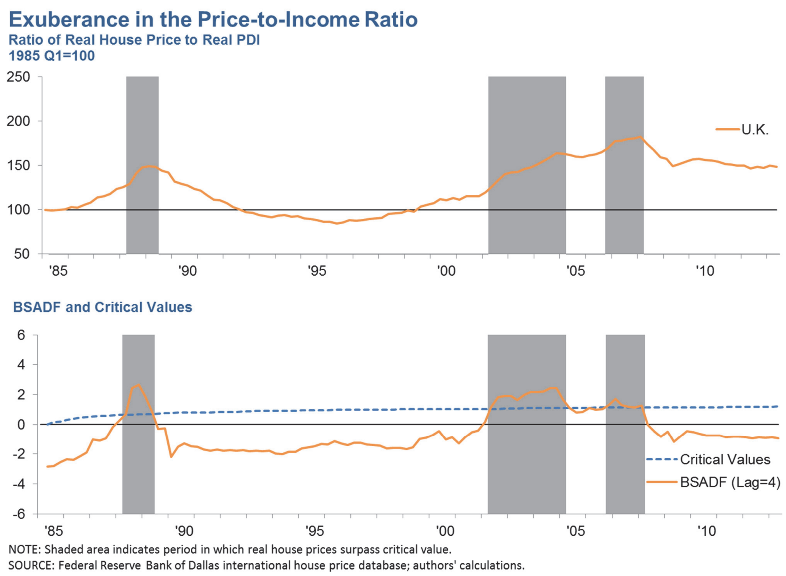
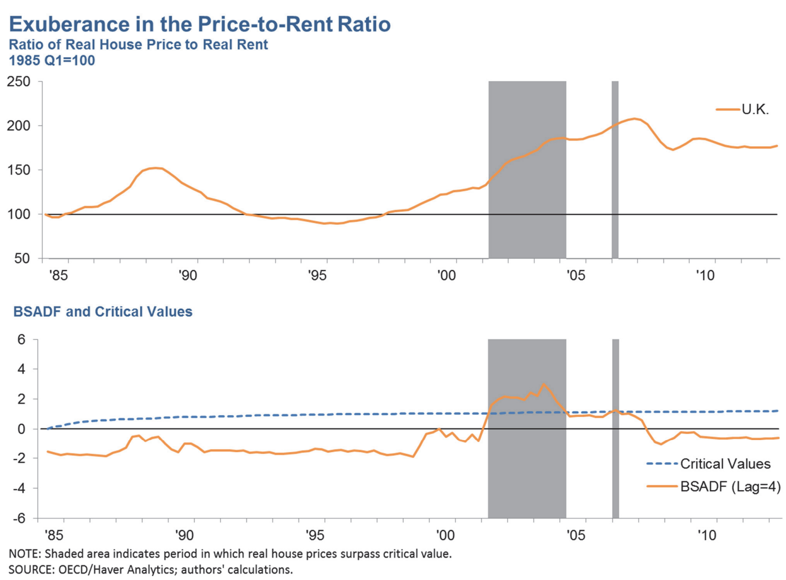


Figure 8: Alternative Date-Stamping with U.K. Price-to-Rent Ratio



Spain Figures 4, 4 and 4 display our main results for Spain. Once again, the interpretation of the figures is the same as for the U.S. before. In this case, our evidence suggests that the rapid acceleration of house prices was not driven by non-fundamental explosive behavior except for a period of exuberance in the first half of the 2000s. The house-price-to-income ratio and the house-price-to-rent ratio tend to give us a shorter period of exuberance for the latest boom-bust episode in the Spanish housing market than real house prices, suggesting that part of the appreciation could be due to the behavior of the fundamentals (and possibly due to nonstationarity in the fundamentals).

We also observe that there is some evidence of explosive behavior in the second half of the 1980s—a period that coincided with Spain being admitted into what now is the European Union but before the collapse of the European Monetary System in 1992-93 and the severe recession that affected the country in the early 1990s. However, we only find marginal evidence for explosive behavior at that time with the house-price-to-rent ratio and no evidence with the house-price-to-income ratio (or with real house prices).

Figure 9: Date-Stamping with Spain Real House Prices

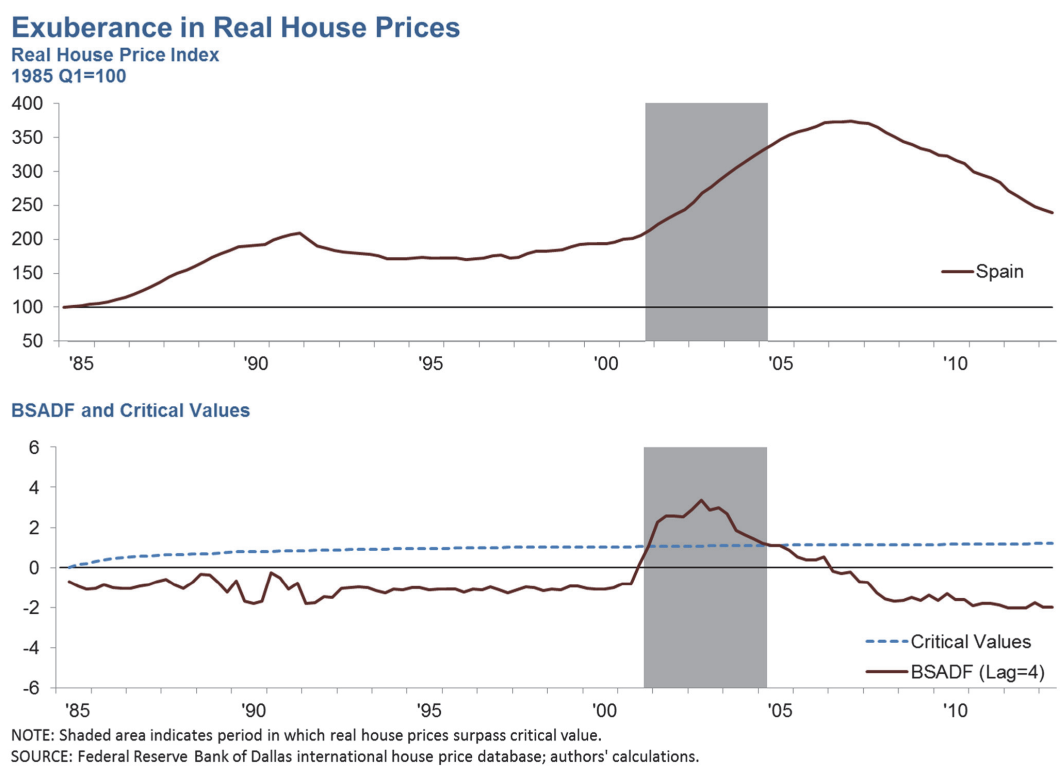


Figure 10: Alternative Date-Stamping with Spain Price-to-Income Ratio

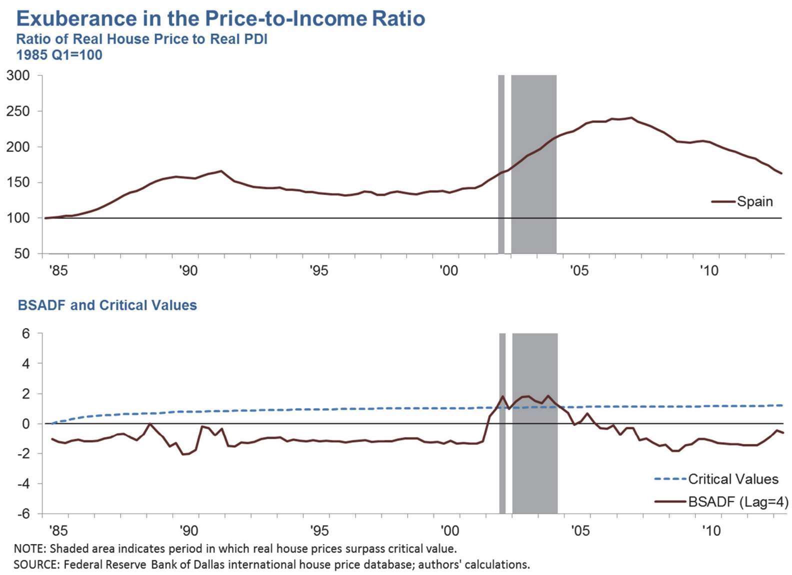
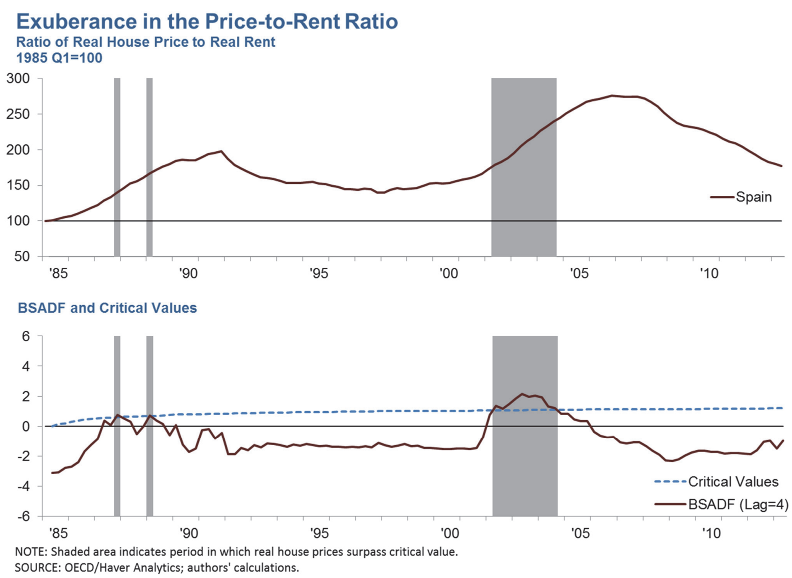


Figure 11: Alternative Date-Stamping with Spain Price-to-Rent Ratio



Synchronization Across Countries Table A, included in the Appendix, reports results for real house prices and price-to-income ratios for each of the 22 countries covered in the International House Price Database.[4] Our findings are consistent with those for the U.S., the U.K., and Spain. With the $SADF$ test statistics, we cannot reject the null in many cases at conventional significance levels. However, the $GSADF$ statistics offer strong evidence of explosive behavior for all countries in the database except Finland, Italy, South Korea and Norway (although Norway only with the house-price-to-income ratio). Hence, our results indicate that periods of explosive behavior were widespread across a large number of countries given our available time series since the first quarter of 1975 until the second quarter of 2013.

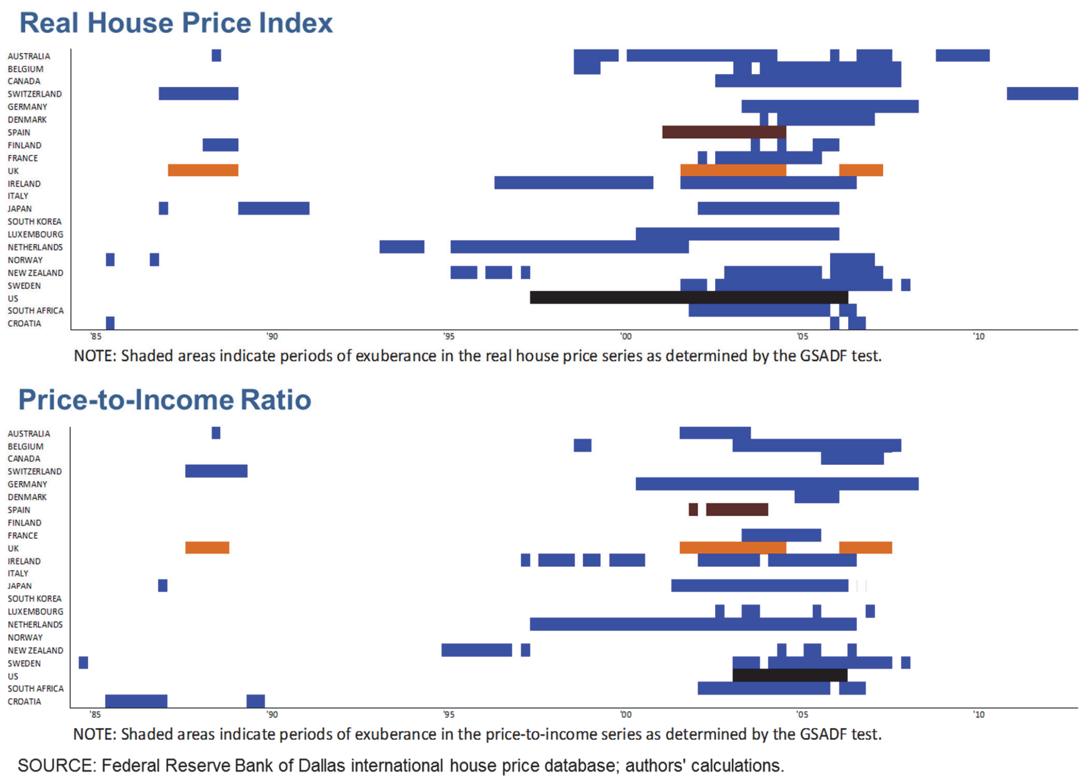
Figure 4 completes the description of the results derived with the procedure of Phillips et al. (2012) and Phillips et al. (2013) applied to data on house prices from the International House Price Database by plotting the periods of exuberance (explosive behavior) for all countries in the database. As before, we identify periods of explosiveness by comparing the time evolution of the $BSADF$ statistics for real house prices and the house-price-to-income ratio respectively against the 95% $SADF$ critical value sequence in finite-samples.

We include in Figure 4 two subplots for real house prices (top) and the house-price-to-income ratio (bottom). We observe that for the most part there is concordance between the periods of exuberance detected looking at either one of these two variables. However, it is worth pointing out, that periods of explosiveness based on non-fundamental behavior detected by the house-price-to-income ratio tend to be somewhat shorter than those we see with the real house price data.

Furthermore, it is interesting to point out that there is an unusual synchronization in the episodes of explosive observed since the late 1990s till the mid-2000s. This period of near simultaneous exuberance was pervasive across very different housing markets around the world whose fundamentals where not necessarily aligned, and it is unprecedented with the sample covered in the International House Price Database. We leave the exploration of synchronization for future research.

[4] See the Appendix also for a description of the data sources on house prices.

Figure 12: Date-Stamping with Real House Prices for All Countries in the International House Price Database



5 Concluding Remarks

In this paper we describe a novel test proposed by Phillips et al. (2012) and Phillips et al. (2013) applied to house price data from the International House Price Database of the Federal Reserve Bank of Dallas documented in Mack and Martínez-García (2011). We show that these tests are useful to detect and date-stamping periods of explosive behavior in housing markets, and more broadly to monitor the macroeconomic developments in housing.

Our work shows how this methodology can be implemented to detect the presence of explosive behavior that are not identifiable with other existing methodologies. As a result, we view this as a valuable tool for policy analysis as well as scholarly work in helping us detect explosiveness in international housing markets.

We illustrate the tests of Phillips et al. (2012) and Phillips et al. (2013) with data from the International House Price Database with a special focus on the cases of the U.S., the U.K., and Spain, which were greatly affected by the latest boom and bust in the housing markets. Our evidence suggests that even controlling for fundamental behavior, there is strong evidence that explosive behavior appeared in all three countries during the first half of the 2000s. This evidence contrasts with existing work that employs other more conventional methods of bubble-detection which produce less clear readings.

Furthermore, our results also show that the cases of the U.S., the U.K. and Spain were far from isolated. In fact, they were fairly typical of a period of exuberance that affected most of the countries covered currently in the International House Price Database since the late 1990s till the mid-2000s. The correction of such a widespread period of non-fundamental explosive behavior resulted in the most severe recession of the post-WWII period for most of these countries.

Appendix

A *SADF* and *GSADF* Statistics for All Countries in the Database

Table 2: Evidence of Explosive Behavior in the Housing Markets

Panel A: Test Statistics

	Real House Prices		**Price-to-Income Ratio**	
Country	*SADF*	*GSADF*	*SADF*	*GSADF*
Australia	2.23***	6.18***	1.08*	2.57***
Belgium	0.97	2.98***	−0.25	2.92***
Canada	0.32	3.76***	−1.13	2.16**
Switzerland	1.64**	2.3**	1.2*	2.08**
Germany	−0.59	2.1**	0.57	2.55***
Denmark	1.31**	2.83	−0.03	1.76*
Spain	0.39	3.34***	0.01	1.84**
Finland	0.94	1.45	−0.89	0.96
France	1.35**	2.21**	−0.03	2.46***
United Kingdom	1.83**	3.34***	1.50**	2.65***
Ireland	2.59***	3.71***	2.01***	2.19**
Italy	−1.28	−0.38	−1.52	0.85
Japan	1.66**	3.76***	0.88	4.63***
South Korea	−1.11	−0.32	0.49	0.49
Luxembourg	1.65**	3.89***	−0.27	1.59*
Netherlands	−0.43	4.13***	−0.17	3.13***
Norway	0.85	1.75*	0.22	0.31
New Zealand	1.77**	2.35**	0.43	3.1***
Sweden	0.18	3.79***	0.23	3.34***
United States	1.52**	3.81***	−0.78	3.47***
South Africa	−0.92	3.93***	−1.35	3.44***
Croatia	0.03	1.64*	0.87	2.23**
Panel B: Critical Values				
90%	0.98	1.54	0.98	1.54
95%	1.25	1.80	1.25	1.80
99%	1.89	2.39	1.89	2.39

Notes: *, **, and ***, denote statistical significance at the 10, 5 and 1 percent significance level respectively.

B National Sources of House Price Data

	House Price Definition	Source and Time Coverage
Australia	Weighted average of 8 capital cities, new and existing detached house price index, per dwelling	Australia Bureau of Statistics 1986Q3-present
	Weighted average of 6 capital cities, new and existing dwelling price index, per dwelling	Australian Treasury 1960Q3-present
Belgium	Nationwide existing single-family house price index, per dwelling	Statistics Belgium 1973Q1-present
Canada	10 metropolitan areas, "fair" price of existing detached bungalows and two story executive dwellings, per dwelling	Royal Le Page 1993Q1-present
	10 metropolitan areas, "fair" price of existing detached bungalows and two story executive dwellings, per dwelling	University of British Columbia 1975Q1-2012Q1
Switzerland	Nationwide new and existing single-family house price index, per dwelling	Swiss National Bank 1970Q1-present
Germany	Nationwide existing terraced house price index, per dwelling	Deutsche Bundesbank 1995-present (annual)
	W. Germany existing terraced house price index, per dwelling	Deutsche Bundesbank 1990-2010 (annual)
	W. Germany new terraced house price index, per dwelling	Deutsche Bundesbank 1975-2010 (annual)
Denmark	Nationwide new and existing single-family house price index, per dwelling	Statistics Denmark 1992Q1-present
	Nationwide new and existing single-family house price index, per dwelling	Danmarks Nationalbank 1971Q1-present
Spain	Nationwide average price of existing dwellings, per square meter	Ministerio de Fomento 1995Q1-present
	Nationwide average price of new and existing dwellings, per square meter	Ministerio de la Vivienda 1987Q1-2004Q4
	Madrid average price of new dwellings, per square meter	Tecnigrama 1976-1986 (annual)
Finland	Nationwide existing single-family house price index, per square meter	Statistics Finland 1985Q1-present
	Nationwide existing apartment price index, per square meter	Statistics Finland 1970Q1-2009Q4
France	Nationwide existing detached house and apartment price index, per dwelling	INSEE 1996Q1-present
	Nationwide existing apartment price index, per dwelling	CEGDD - Ministère de l'Écologie 1936-2009 (annual)

	House Price Definition	Source and Time Coverage
United Kingdom	Nationwide new and existing dwelling price index, per dwelling	Department of Communities and Local Government 1968Q2-present
Croatia	Nationwide new and existing dwelling price index, pure price	Croatian National Bank 1997Q1-present
	Nationwide average price of new dwellings, per square meter	Croatian Bureau of National Statistics, 1965-2011 (annual)
Ireland	Nationwide average price of existing dwellings, per dwelling	Central Statistics Office 2005Q1-present
	Nationwide average price of existing dwellings, per dwelling	Department of Environment, Community & Local Government 1978Q1-present
	Nationwide average price of existing dwellings, per dwelling	Department of Environment, Community & Local Government 1974-2009 (annual)
Italy	13 main metropolitan area average price of new and existing dwellings, per square meter	Nomisma 1988S1-present
	13 main metropolitan area average price of new dwellings, per square meter	Il Consulente Immobiliare 1967-2001 (bi-annual)
Japan	Nationwide urban residential land price index, per square meter	Japan Real Estate Institute 1955S1-present
South Korea	Nationwide new and existing dwelling price index, per dwelling	Kookmin Bank 1986M1-present
	Kyung-Hwan Kim (1993) index:	
	- Nationwide quoted transactions and estimations of real estate agents	Korea Housing Bank 1982-1990 (annual)
	- Nationwide standard construction costs (excluding land)	Korea Housing Bank 1978-1981 (annual)
	- Nationwide weighted average of total factor costs single-family house and apartment construction	Korea Housing Bank 1974-1977 (annual)
Luxembourg	Nationwide new and existing house price index, per dwelling	L'Observatoire de l'Habitat 2005Q1-present
	Nationwide new and existing dwelling price index, per dwelling	Banque centrale du Luxembourg 1974-2009 (annual)
Netherlands	Nationwide existing single-family house price index, per dwelling	Statistics Netherlands 1995M1-present
	Nationwide average price of existing dwellings, per dwelling	Kadaster 1976M1-2010M12

Norway	Nationwide new and existing detached house price index, per dwelling	Statistics Norway 1992Q1-present
	Norges Bank forecasting model index:	
	- Nationwide sales reports of Norges Eindomsmeglerforbund real estate agents	Norges Eindomsmeglerforbund 1987Q1-2003Q4
	- Dwelling price based on national property register	GAB Register 1984Q1-1986Q4
	- Nationwide building cost index	Statistics Norway 1979Q1-1983Q4
	- Housing rent component of the Consumer Price Index	Statistics Norway 1972Q1-1978Q4
New Zealand	Nationwide new and existing detached house price index, per dwelling	Reserve Bank of New Zealand 1962Q2-present
Sweden	Nationwide new and existing one- and two-family house price index, per dwelling	Statistics Sweden 1986Q1-present
	Nationwide new and existing one- and two-family house price index, per dwelling	Statistics Sweden 1975-2010 (annual)
United States	Nationwide existing single-family house price index, per dwelling	FHFA 1975Q1-present
South Africa	Nationwide new and existing single-family house price index, per dwelling	ABSA 1966M1-present

Note: Time series backcasting is used to extend the house price indexes of Spain and the Netherlands from the first quarter of 1976 back to the first quarter of 1975. Time series nowcasting is used for Italy, Germany and Japan in order to complete the quarterly dataset and avoid long lags in its public release. Nowcasting are subsequently replaced with actual data from the national sources, as it becomes available.

References

Bhargava, A. (1986). On the theory of testing for unit roots in observed time series. *Review of Economic Studies 53*, 369–384.

Blanchard, O. and M. Watson (1982). *Crisis in the Economics Financial Structure*, Chapter Bubbles, Rational Expectations, and Financial Markets, pp. 295–315. Lexington Books, Lexington, Mass.

Busetti, F. and A. Taylor (2004). Tests of stationarity against a change in persistence. *Journal of Econometrics 123*, 33–66.

Campbell, J., A. Lo, and A. MacKinlay (1997). *The Econometrics of Financial Markets.* Princeton, NJ: Princeton University Press.

Campbell, J. and R. Shiller (1987). Cointegration and tests of present value models. *Journal of Political Economy 95*, 1062–1088.

Clayton, J. (1996). Rational expectations, market fundamentals and housing price volatility. *Real Estate Economics 24*, 441–470.

Diba, B. and H. Grossman (1988). Explosive rational bubbles in stock prices. *American Economic Review 78*, 520–530.

Engsted, T., T. Pedersen, and C. Tanggaard (2012). The log-linear return approximation, bubbles, and predictability. *Journal of Financial and Quantitative Analysis 47*, 643–665.

Evans, G. W. (1991). Pitfllas in testing for explosive bubbles in asset prices. *American Economic Review 81*(4), 922–930.

Girouard, N., M. Kennedy, P. van den Noord, and A. C. Christophe (2006). Recent house price developments: The role of fundamentals. *OECD Economics Department Working Papers No. 475*. OECD Publishing.

Gordon, M. and E. Shapiro (1956). Capital equipment analysis: The required rate of profit. *Management Science 3*(1), 102–110.

Himmelberg, C., C. Mayer, and T. Sinai (2005). Assessing high house prices: Bubbles, fundamentals and misperceptions. *Journal of Economic Perspectives 19*(4), 67–92.

Homm, U. and J. Breitung (2012, Winter). Testing for speculative bubbles in stock markets: A comparison of alternative methods. *Journal of Financial Econometrics 10*(1), 198–231.

Kim, J.-Y. (2000). Detection of change in persistence of a linear time series. *Journal of Econometrics 95*, 97–116.

Kim, J.-Y., J. Belaire-Franch, and R. Amador (2002). Corrigendum to 'detection of change in persistence of a linear time series'. *Journal of Econometrics 109*, 389–392.

LeRoy, S. (2004). Rational exuberance. *Journal of Economic Literature XLII*, 783–804.

LeRoy, S. and R. Porter (1981). The present-value relation: Tests based on implied variance bounds. *Econometrica 49*, 555–577.

Mack, A. and E. Martínez-García (2011, December). A cross-country quarterly database of real house prices: A methodological note. *Globalization and Monetary Policy Institute Working Paper no. 99*. Federal Reserve Bank of Dallas.

Ng, S. and P. Perron (1995). Unit root tests in arma models with data-dependent methods for the selection of the truncation lag. *Journal of the American Statistical Association 90*, 268–281.

Ng, S. and P. Perron (2001). Lag length selection and the construction of unit root tests with good size and power. *Econometrica 69*, 1519–1554.

Phillips, P. C., S.-P. Shi, and J. Yu (2012). Testing for multiple bubbles. *Cowles Foundation Discussion Papers 1843*. Cowles Foundation for Research in Economics, Yale University.

Phillips, P. C., S.-P. Shi, and J. Yu (2013). Testing for multiple bubbles: Historical episodes of exuberance and collapse in the s&p 500. *Cowles Foundation Discussion Papers 1914*. FIRN Research Paper.

Phillips, P. C., Y. Wu, and J. Yu (2011). Explosive behavior in the 1990's nasdaq: When did exuberance escalate asset values? *International Economic Review 52*(1), 201–226.

Sargent, T. (1987). *Macroeconomic Theory*. Boston: Academic Press.

Shiller, R. J. (1981). Do stock prices move too much to be justified by subsequent changes in dividends? *American Economic Review 71*, 421–436.

West, K. D. (1987). A specification test for speculative bubbles. *Quarterly Journal of Economics 102*(3), 553–580.