

Supplementary Material

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I. CLOCK-COVARIANT WAVE EQUATIONS

In the main text, we introduce the clock-covariant derivative,

$$D_t \equiv \frac{\partial}{\partial t} - i \frac{\partial \theta}{\partial t},$$

which modifies standard quantum wave equations by adding a time-dependent phase shift proportional to $\partial_t \theta$. Here, we provide short derivations of the non-relativistic and relativistic cases.

A. Non-Relativistic Schrödinger Equation

Consider a non-relativistic particle with Hamiltonian H . The usual Schrödinger equation is

$$i\hbar \frac{\partial \psi}{\partial t} = H \psi.$$

In the clock-covariant framework, we replace $\partial/\partial t$ by D_t . Hence,

$$i\hbar D_t \psi = i\hbar \left(\frac{\partial \psi}{\partial t} - i \frac{\partial \theta}{\partial t} \psi \right) = H \psi.$$

Rearranging,

$$i\hbar \frac{\partial \psi}{\partial t} = H \psi + \hbar \frac{\partial \theta}{\partial t} \psi.$$

Thus, the term $\hbar (\partial_t \theta) \psi$ represents a time-dependent phase shift. When $\theta(x)$ fluctuates randomly, these phase shifts accumulate and lead to intrinsic decoherence (see Sec. II below).

B. Relativistic Klein–Gordon Equation

For a relativistic scalar field $\phi(x)$ of mass m , the usual Klein–Gordon equation in flat space-time is

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2 \right) \phi(x) = 0.$$

In UCFT, we replace ∂_t by D_t . This yields extra terms proportional to $\partial_t \theta$ and $(\partial_t \theta)^2$. Explicitly,

$$\frac{\partial^2}{\partial t^2} \rightarrow D_t^2 = \left(\frac{\partial}{\partial t} - i \frac{\partial \theta}{\partial t} \right)^2 = \frac{\partial^2}{\partial t^2} - 2i \frac{\partial \theta}{\partial t} \frac{\partial}{\partial t} - \left(\frac{\partial \theta}{\partial t} \right)^2 - i \frac{\partial^2 \theta}{\partial t^2}.$$

Hence, the clock field $\theta(x)$ introduces phase-dependent corrections to the relativistic field equation, again driving an intrinsic decoherence mechanism.

II. CUMULANT EXPANSION FOR DECOHERENCE

We now show how random phase shifts from $\theta(x)$ suppress off-diagonal density matrix elements in a cumulant expansion approach.

A. Phase Shift and Density Matrix

A system with wavefunction $\psi(x)$ experiences a random phase shift

$$\phi = \int dt \frac{\partial \theta}{\partial t},$$

imparted by $\theta(x)$. Over two spatially separated regions, the relative phase difference is $\Delta\phi$. The off-diagonal element of the density matrix becomes

$$\rho_{\text{off-diag}} = \langle e^{i\Delta\phi} \rangle.$$

B. Gaussian Fluctuations and Cumulant Expansion

Assuming $\Delta\phi$ follows Gaussian statistics, the cumulant expansion gives

$$\langle \exp(i\Delta\phi) \rangle = \exp\left(-\frac{1}{2} \langle (\Delta\phi)^2 \rangle\right).$$

The variance $\langle (\Delta\phi)^2 \rangle$ is set by the phase correlation function $G(x-y)$. For a correlation length ξ , we typically have

$$G(x-y) \sim \exp\left(\frac{-|x-y|}{\xi}\right),$$

so that the typical fluctuation over distance ℓ scales as

$$\langle (\Delta\phi)^2 \rangle \sim \frac{\ell}{\xi}.$$

If the system has N independent degrees of freedom, these phase fluctuations add incoherently,

$$\langle (\Delta\Phi)^2 \rangle \sim N \frac{\ell}{\xi}.$$

Hence,

$$\rho_{\text{off-diag}} \sim \exp\left(-\frac{1}{2} N \frac{\ell}{\xi}\right).$$

Identifying the decoherence rate Γ_{decoh} with the inverse timescale for coherence loss, and introducing a microscopic rate Γ_0 , we obtain

$$\Gamma_{\text{decoh}} \sim N \Gamma_0 \left(\frac{\ell}{\xi} \right)^\alpha, \quad (\alpha \approx 1).$$

This confirms the exponential suppression of off-diagonal terms for macroscopic systems ($\ell \gg \xi$, $N \gg 1$), thereby enforcing classicality in the limit of large system size.