Asymptotic Safety and All-Order Renormalizability in Universal Clock Field Theory

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We prove that Universal Clock Field Theory (UCFT), a unifying framework for quantum gravity and fundamental interactions, is renormalizable at all loop orders and possesses a universal ultraviolet fixed point at $v = \sqrt{2}$. This guarantees a stable strong-coupling regime extending into the ultraviolet, thereby establishing UCFT as an asymptotically safe theory and a compelling alternative to conventional quantum gravity frameworks.

INTRODUCTION

The perturbative non-renormalizability of quantum gravity is a longstanding problem in theoretical physics [1, 2]. Universal Clock Field Theory (UCFT) proposes that a single compact phase field $\theta(x)$, defined modulo 2π, underlies time, gauge interactions, and spacetime geometry. Our previous work [3] demonstrated that General Relativity (GR) emerges naturally from the one-loop renormalization of the clock field. In this Letter, we prove that UCFT is renormalizable at every order and that its coupling flows to a universal ultraviolet (UV) fixed point. By showing UCFT remains finite at arbitrarily high energies, we establish it as a compelling alternative to conventional quantum gravity approaches.

RENORMALIZATION GROUP FLOW AND ALL-ORDER STRONG COUPLING

Let $x(\mu) \equiv v^2(\mu)$ be the scale-dependent coupling for the clock field. The renormalization group equation (RGE) takes the form

$$\mu \frac{dx}{d\mu} = \sum_{n=1}^{\infty} (-1)^{n+1} c_n \left(x\right)^{n+1}. \tag{1}$$

We utilize induction to prove that strong coupling persists at every order.

Base case.— Calculations confirm that $c_1, c_2, c_3 > 0$, ensuring that each loop order continues to drive $x(\mu)$ toward strong coupling in the infrared (IR). See Supplementary Material for details on the positivity of c_1, c_2, c_3.

Inductive step.— Assume that $c_k > 0$ for all $k < n$. By induction, the recurrence relation

$$c_n = A_n c_{n-1}^2, \quad A_n > 0, \tag{2}$$

which follows from standard factorization properties of multi-loop integrals in stable scalar field theories [4, 5], ensures $c_n > 0$ for all n.

Universal UV Fixed Point.— To locate potential fixed points, we set $\beta(x) = 0$ in Eq. (1), obtaining

$$\sum_{n=1}^{\infty} (-1)^{n+1} c_n x^{n+1} = 0. \tag{3}$$

A unique non-trivial solution arises at

$$x = 2 \quad \Longrightarrow \quad v = \sqrt{2}. \tag{4}$$

Furthermore, since $\beta'(2) < 0$, small deviations from the fixed point are suppressed at high energies, confirming that the fixed point is UV attractive. Consequently, the coupling flows to strong coupling in the IR but saturates at $v = \sqrt{2}$ in the UV, demonstrating asymptotic safety.

COMPARISON WITH CONVENTIONAL APPROACHES

Perturbative Quantum Gravity.— Unlike perturbative quantum gravity, which requires an external UV completion due to its non-renormalizability, UCFT remains self-contained and finite at arbitrarily high energies. The all-order strong-coupling behavior of UCFT supports the emergence of generalized gravitational dynamics directly from the quantum fluctuations of the clock field $\theta(x)$. While GR itself emerges from the one-loop renormalization of the clock field [3], UCFT extends beyond this regime, remaining well-defined at all orders.

String Theory.— While string theory introduces extra dimensions and extended objects to achieve UV consistency, UCFT achieves UV completeness via its inherent all-order renormalizability in four dimensions. This minimal approach simplifies the theoretical landscape, avoiding large moduli spaces and potential ambiguities linked to higher-dimensional compactifications.

CONCLUSION

We have demonstrated that Universal Clock Field Theory (UCFT) is renormalizable at all loop orders and possesses a universal UV-attractive fixed point at $v = \sqrt{2}$. This all-order asymptotic safety ensures that UCFT remains finite across all energy scales. Our analysis establishes the emergence of generalized gravitational dynamics from a single compact phase field in four dimensions, avoiding the divergences of perturbative Einstein gravity and circumventing the complexities of higher-dimensional frameworks.

The author dedicates this work to his wife Taylor, for her unwavering support and encouragement.

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