Supplementary Material

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I. CLOCK-COVARIANT WAVE EQUATIONS

In the main text, we introduce the clock-covariant derivative,

$$D_t \;\equiv\; \frac{\partial}{\partial t} \;-\; i\,\frac{\partial \theta}{\partial t},$$

which modifies standard quantum wave equations by adding a time-dependent phase shift proportional to $\partial_t \theta$. Here, we provide short derivations of the non-relativistic and relativistic cases.

A. Non-Relativistic Schrödinger Equation

Consider a non-relativistic particle with Hamiltonian H. The usual Schrödinger equation is

$$i\hbar\,\frac{\partial \psi}{\partial t} \;=\; H\,\psi.$$

In the clock-covariant framework, we replace $\partial/\partial t$ by D_t. Hence,

$$i\hbar\,D_t\psi \;=\; i\hbar\left(\frac{\partial \psi}{\partial t} \;-\; i\,\frac{\partial \theta}{\partial t}\,\psi\right) \;=\; H\,\psi.$$

Rearranging,

$$i\hbar\,\frac{\partial \psi}{\partial t} \;=\; H\,\psi \;+\; \hbar\,\frac{\partial \theta}{\partial t}\,\psi.$$

Thus, the term $\hbar\,(\partial_t \theta)\,\psi$ represents a time-dependent phase shift. When $\theta(x)$ fluctuates randomly, these phase shifts accumulate and lead to intrinsic decoherence (see Sec. II below).

B. Relativistic Klein–Gordon Equation

For a relativistic scalar field $\phi(x)$ of mass m, the usual Klein–Gordon equation in flat spacetime is

$$\left(\frac{\partial^2}{\partial t^2} \;-\; \nabla^2 \;+\; m^2\right)\phi(x) \;=\; 0.$$

In UCFT, we replace ∂_t by D_t. This yields extra terms proportional to $\partial_t \theta$ and $(\partial_t \theta)^2$. Explicitly,

$$\frac{\partial^2}{\partial t^2} \;\to\; D_t^2 \;=\; \left(\frac{\partial}{\partial t} - i\,\frac{\partial \theta}{\partial t}\right)^2 \;=\; \frac{\partial^2}{\partial t^2} \;-\; 2\,i\,\frac{\partial \theta}{\partial t}\,\frac{\partial}{\partial t} \;-\; \left(\frac{\partial \theta}{\partial t}\right)^2 \;-\; i\,\frac{\partial^2 \theta}{\partial t^2}.$$

Hence, the clock field $\theta(x)$ introduces phase-dependent corrections to the relativistic field equation, again driving an intrinsic decoherence mechanism.

II. CUMULANT EXPANSION FOR DECOHERENCE

We now show how random phase shifts from $\theta(x)$ suppress off-diagonal density matrix elements in a cumulant expansion approach.

A. Phase Shift and Density Matrix

A system with wavefunction $\psi(x)$ experiences a random phase shift

$$\phi \;=\; \int dt\, \frac{\partial \theta}{\partial t},$$

imparted by $\theta(x)$. Over two spatially separated regions, the relative phase difference is $\Delta\phi$. The off-diagonal element of the density matrix becomes

$$\rho_{\text{off-diag}} \;=\; \left\langle e^{\,i\,\Delta\phi} \right\rangle.$$

B. Gaussian Fluctuations and Cumulant Expansion

Assuming $\Delta\phi$ follows Gaussian statistics, the cumulant expansion gives

$$\langle \exp(i\,\Delta\phi) \rangle \;=\; \exp\!\left(-\frac{1}{2}\,\left\langle (\Delta\phi)^2 \right\rangle \right).$$

The variance $\langle (\Delta\phi)^2 \rangle$ is set by the phase correlation function $G(x-y)$. For a correlation length ξ, we typically have

$$G(x-y) \;\sim\; \exp\left(\frac{-|x-y|}{\xi} \right),$$

so that the typical fluctuation over distance ℓ scales as

$$\left\langle (\Delta\phi)^2 \right\rangle \;\sim\; \frac{\ell}{\xi}.$$

If the system has N independent degrees of freedom, these phase fluctuations add incoherently,

$$\left\langle (\Delta\Phi)^2 \right\rangle \;\sim\; N\,\frac{\ell}{\xi}.$$

Hence,

$$\rho_{\text{off-diag}} \sim \exp\left(-\frac{1}{2} N \frac{\ell}{\xi} \right).$$

Identifying the decoherence rate Γ_{decoh} with the inverse timescale for coherence loss, and introducing a microscopic rate Γ_0, we obtain

$$\Gamma_{\text{decoh}} \;\sim\; N\,\Gamma_0 \left(\frac{\ell}{\xi}\right)^{\alpha}, \quad (\alpha \approx 1).$$

This confirms the exponential suppression of off-diagonal terms for macroscopic systems ($\ell \gg \xi,\; N \gg 1$), thereby enforcing classicality in the limit of large system size.