KOMUTATIVNA ALGEBRA, 2019/20
7. DN/ 7nd HW : 22. 4. 2020

Rok za oddajo/ Deadline: 23:59, 29. 4. 2020

(1) Naj R cel Noetherski kolobar. Pokaži:

a) R je kolobar z enolično faktorizacijo natanko tedaj ko so vsi minimalni praideali nad glavnimi ideali spet glavni.

b) Če je R kolobar z enolično faktorizacijo, potem je vsak minimalni neničelni praideal glavni.
(Minimalni neničelni praideali so minimali elementi množice $\text{Spec}(R)\backslash\{0\}$.)

Namig: Uporabiš lahko, da ima cel kolobar enolično faktorizacijo natanko tedaj, ko vsak praideal vsebuje pra-element.

(2) Za praideal P s $S_P(0)$ označimo jedro homomorfizma $R \to R_P$. Pokaži:

a) $S_P(0)$ je vsebovan v vsakem P-primarnem idealu.

b) $S_P(0)$ je P-primaren ideal natanko tedaj, ko je P minimalen praideal.

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(1) Let R be a Noetherian integral domain. Show:

a) R is a unique factorization domain if and only if every minimal prime ideal over a principal ideal is principal.

b) If R is a unique factorization domain then every minimal nonzero prime ideal of R is principal.
(Minimal nonzero prime ideals are minimal elements of $\text{Spec}(R)\backslash\{0\}$.)

Hint: You can use the fact that an integral domain is UFD if and only if every nonzero prime ideal contains a prime element.

(2) For a prime ideal P denote kernel of homomorphism $R \to R_P$ with $S_P(0)$ Show:

a) $S_P(0)$ is contained in every P-primary ideal.

b) $S_P(0)$ is P-primary ideal if and only if P is a minimal prime ideal.