KOMUTATIVNA ALGEBRA, 2019/20

5. DN/ 5nd HW : 8. 4. 2020

Rok za oddajo/ Deadline: 23:59, 14. 4. 2020

(1) Naj bo R_2 podkolobar kolobarja R_1.

1.1. Predpostavimo, da ima R_2 samo en praideal P_2. Pokaži, da obstaja minimalni praideal $P_1 \lhd R_1$ za katerega je $R_2 \cap P_1 = P_2$

1.2. Pokaži, da za vsak minimalni praideal $P_2 \lhd R_2$, obstaja minimalni praideal $P_1 \lhd R_1$, za katerega je $R_2 \cap P_1 = P_2$.
(Namig: koliko praidealov ima R_P, če je P minimalni praideal.)

(2) Naj bo M R-modul in N_1, N_2 podmodula, za katera sta M/N_1 in M/N_2 Noetherska. Pokaži, da je tudi $M/(N_1 \cap N_2)$ Noetherski.

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(1) Let R_2 be a subring of R_1.

1.1. Assume that R_2 has exactly one prime ideal P_2. Show that there exists prime ideal $P_1 \lhd R_1$ such that $R_2 \cap P_1 = P_2$.

1.2. Show that for any minimal prime ideal $P_2 \lhd R_2$ there exists a minimal prime ideal $P_1 \lhd R_1$, such that $R_2 \cap P_1 = P_2$.
(Hint: How many prime ideals does R_P have, if P is a minimal prime ideal.)

(2) Let M be a R-module and N_1, N_2 submodules such that M/N_1 and M/N_2 are Noetherian. Show that $M/(N_1 \cap N_2)$ is Noetherian.