KOMUTATIVNA ALGEBRA, 2019/20
3. DN/ 3nd HW : 25. 3. 2020

Rok za oddajo/ Deadline: 23:59, 31. 3. 2020

(1) Naj bosta M in N R-modula. S homomorfizmom $\phi\colon R \to S$ razširimo skalarje. Pokaži, da je
$$M_S \otimes_S N_S \cong (M \otimes_R N)_S.$$
Ali lahko kaj podobnega poveš za omejitev skalarjev?

(2) Naj bo $I \lhd R$ nilpotenten ideal (obstaja $n \in \mathbb{N}$, da je $I^n = 0$). Naj bosta M in N poljubna R-modula.

Pokaži, da iz $IM = M$ sledi $M = 0$.

Pokaži, da je homomorfizem $\phi\colon N \to M$ surjektiven natanko tedaj, ko je $\bar{\phi}\colon N/IM \to M/IM$ surjektiven.

Pokaži, da $\{m_\lambda \mid \lambda \in \Lambda\}$ generira M kot R-modul natanko tedaj, ko $\{\bar{m}_\lambda \mid \lambda \in \Lambda\}$ generira M/IM kot R/I-modul.

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(1) Let M and N be R-modules. We extend scalars with $\phi\colon R \to S$.

Show that
$$M_S \otimes_S N_S \cong (M \otimes_R N)_S.$$
Can you tell something similar for the restriction of scalars.

(2) Let $I \lhd R$ be an nilpotent ideal (there exists $n \in \mathbb{N}$ s.t. $I^n = 0$). Let M in N be any R-modules.

Show, if $IM = M$, then $M = 0$.

Show that the homomorphism $\phi\colon N \to M$ is surjective if and only if $\bar{\phi}\colon N/IM \to M/IM$ is surjective.

Show that $\{m_\lambda \mid \lambda \in \Lambda\}$ generates M as a R-module if and only if $\{\bar{m}_\lambda \mid \lambda \in \Lambda\}$ generates M/IM as a R/I-module.