KOMUTATIVNA ALGEBRA 2019/20
1. DN / 1st HW

Rok za oddajo / Deadline: 18. 3. 2020

1. Element $e \in R$ je idempotent, če je $e^2 = e$. Ideal I je idempotenten, če je $I^2 = I$. Dokaži:

   (a) Glavni ideal je idempotenten natanko tedaj, ko je generiran z idempotentom.

   (b) Ideal generiran s končno idempotenti je glavni in idempotenten.

(Namig) Pokaži: Če sta e in f idempotenta, je tudi $e + f - ef$ idempotent.

2. Naj bo ideal $\sqrt{I}$ končno generiran. Pokaži, da obstaja število n, za katerega je $\sqrt{I}^n \subset I$. Pokaži, da je predpostavka o končni generiranosti potrebna.

3. Pokaži, da sta množici $X_1 = \{(t^3, t^4, t^5) \mid t \in k\} \subset \mathbb{A}^3_k$ in $X_2 = \{(\cos x, \sin x) \mid x \in \mathbb{R}\} \subset \mathbb{A}^2_{\mathbb{R}}$ variateti (algebraični množici).

   Pokaži, da $X_3 = \{(\cos x, x) \mid x \in \mathbb{R}\} \subset \mathbb{A}^2_{\mathbb{R}}$ in $X_4 = \{(e^x, x) \mid x \in \mathbb{C}\} \subset \mathbb{A}^2_{\mathbb{C}}$ nista variateti (algebraični množici).

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1. An element $e \in R$ is an idempotent, if $e^2 = e$. An ideal I is idempotent, if $I^2 = I$. Prove:

   (a) A principal ideal is idempotent if and only if it is generated by an idempotent.

   (b) An ideal generated by finitely many idempotents is principal and idempotenten.

(Hint) Show: If e and f are idempotent, then $e + f - ef$ is idempotent.

2. Let $\sqrt{I}$ be a finitly generated ideal. Show that $\sqrt{I}^n \subset I$ for some $n \in \mathbb{N}$. Show that the statemant does not hold if $\sqrt{I}$ is not finitely generated.

3. Show that the sets $X_1 = \{(t^3, t^4, t^5) \mid t \in k\} \subset \mathbb{A}^3_k$ and $X_2 = \{(\cos x, \sin x) \mid x \in \mathbb{R}\} \subset \mathbb{A}^2_{\mathbb{R}}$ are varieties (algebraic sets).

   Show that the sets $X_3 = \{(\cos x, x) \mid x \in \mathbb{R}\} \subset \mathbb{A}^2_{\mathbb{R}}$ and $X_4 = \{(e^x, x) \mid x \in \mathbb{C}\} \subset \mathbb{A}^2_{\mathbb{C}}$ are not varieries (algebraic sets).