KOMUTATIVNA ALGEBRA, 2019/20
11. DN/ 11th HW : 20. 5. 2020

Rok za oddajo/ Deadline: 23:59, 29. 5. 2020

(1) Naj bosta $R \subset R'$ cela kolobarja in R' končno generirana R-algebra. Pokaži, da obstajajo $y_1, \dots, y_n \in R'$, ki so algebraično neodvisni nad R in $0 \neq s \in R$, da je R'_s celosten nad $R[y_1, \dots, y_n]_s$.

(2) Naj bosta R in R' domeni, R celostno zaprt (v polju ulomkov), R' celosten nad R in $\mathfrak{m} \lhd R'$ maksimalen ideal. Pokaži, da je $\mathfrak{n} = R \cap \mathfrak{m} \lhd R$ maksimalen in $\dim(R_{\mathfrak{n}}) = \dim(R'_{\mathfrak{m}})$.

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(1) Let $R \subset R'$ be domains and R' finitely generated R-algebra. Show that there exist $y_1, \dots, y_n \in R'$ that are algebraically independent over R and $0 \neq s \in R$ such that R'_s is integral over $R[y_1, \dots, y_n]_s$.

(2) Let R and R' be domains, R normal, R' integral over R and $\mathfrak{m} \lhd R'$ a maximal ideal. Show that $\mathfrak{n} = R \cap \mathfrak{m} \lhd R$ is a maximal ideal and $\dim(R_{\mathfrak{n}}) = \dim(R'_{\mathfrak{m}})$.