KOMUTATIVNA ALGEBRA, 2019/20

4. DN/ 4nd HW : 7. 4. 2020

Rok za oddajo/ Deadline: 23:59, 31. 3. 2020

(1) Naj bo S multiplikativna podmnožica v kolobarju R.

Pokaži, da je $\sqrt{(}S^{-1}I) = S^{-1}\sqrt{I}$ za vsak ideal $I \lhd R$.

Pokaži, da je $\operatorname{ann}(S^{-1}M) = S^{-1}(\operatorname{ann} M)$ za poljuben končno generiran R-modul. (Opomba: $\operatorname{ann} M \lhd R$ in $\operatorname{ann}(S^{-1}M) \lhd S^{-1}M$.)

(2) Naj bo $l(M)$ dolžina modula. Pokaži, da je $l(X \otimes Y) \leq l(X)l(Y)$. Upoštevamo $\infty \cdot 0 = 0$.

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(1) Let S be multiplicatively closed subset of a ring R and N be R-modules. We extend scalars with $\phi\colon R \to S$.

Show that $\sqrt{(}S^{-1}I) = S^{-1}\sqrt{I}$ for every ideal $I \lhd R$.

Show that $\operatorname{ann}(S^{-1}M) = S^{-1}(\operatorname{ann} M)$ for every finitely generated R-module. (Remark: $\operatorname{ann} M \lhd R$ in $\operatorname{ann}(S^{-1}M) \lhd S^{-1}M$.)

(2) Let $l(M)$ be the length of a module. Show that $l(X \otimes Y) \leq l(X)l(Y)$. We identify $\infty \cdot 0 = 0$.