KOMUTATIVNA ALGEBRA, 2019/20
6. DN/ 6nd HW : 15. 4. 2020

Rok za oddajo/ Deadline: 23:59, 21. 4. 2020

(1) Naj bo M R-modul in $\{m_\lambda \mid \lambda \in \Lambda\} \subset M$ neka podmnožica.

Pokaži, $\{m_\lambda \mid \lambda \in \Lambda\}$ generira M natanko tedaj, ko za vsak maksimalni ideal $\mathfrak{m} \lhd R$ množica $\{\frac{m_\lambda}{1} \in M_\mathfrak{m} \mid \lambda \in \Lambda\}$ generira $R_\mathfrak{m}$-modul $M_\mathfrak{m}$.

(2) Naj bo M Noetherski R-modul. Pokaži, da so naslednje trditve ekvivalentne.

a.) M ima končno dolžino

b.) Obstaja končni produkt maksimalnih idealov, ki je pod annihilatorjem modul M

c.) Vsak pra ideal P, ki vsebuje $\operatorname{ann}(M)$ je maksimalen.

d.) $R/\operatorname{ann}(M)$ je Artiniski kolobar.

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(1) Let M be a R-module and $\{m_\lambda \mid \lambda \in \Lambda\} \subset M$ a subset.

Show that, $\{m_\lambda \mid \lambda \in \Lambda\}$ generates M if and only if the set $\{\frac{m_\lambda}{1} \in M_\mathfrak{m} \mid \lambda \in \Lambda\}$ generaes $R_\mathfrak{m}$-modul $M_\mathfrak{m}$ for every maximal ideal $\mathfrak{m} \lhd R$.

(2) Let M be a Noetherian R-module. Show that the following conditions are eqivalent

a.) M has finite length

b.) M is annihilated by some finite product of maximal ideals.

c.) Every prime ideal P s.t $\operatorname{ann}(M) \subset P$ is maximal.

d.) $R/\operatorname{ann}(M)$ is Artinian.