4) $\lambda(A)=\sum_{n\in A}\frac{e^{in^2}}{3^{|n|}}$ kompleksna mera, $\mu(A)=\sum_{n\in A\cap 2\mathbb{Z}}\frac{1}{(n+1)^2}$ pozitivna mera.

• $|\lambda|$:

$$|\lambda|(A)=\sup\Big\{\sum_{n=1}^{\infty}|\lambda(A_n)| \,;\, (A_n)_n \text{ particija za } A\Big\} \underset{\substack{\text{NAJVEČ,}\\ \text{ČE RAZBIJEMO}\\ \text{PO TOČKAH}}}{\overset{\text{CBS}}{=}} \sum_{n\in A}\left|\frac{e^{in^2}}{3^{|n|}}\right| = \sum_{n\in A}\frac{1}{3^{|n|}}$$

• Lebesguev razcep:

$\mu(A)=0 \Rightarrow A\subseteq \mathbb{Z}\setminus 2\mathbb{Z}$ LIHA ŠTEVILA

Domneva: $$\lambda(A)=\sum_{n\in A}\frac{e^{in^2}}{3^{|n|}}=\underbrace{\sum_{n\in A\cap 2\mathbb{Z}}\frac{e^{in^2}}{3^{|n|}}}_{\lambda_a}+\underbrace{\sum_{n\in A\cap(\mathbb{Z}\setminus 2\mathbb{Z})}\frac{e^{in^2}}{3^{|n|}}}_{\lambda_s}$$

$\lambda_a \ll \mu$ po definiciji, $\lambda_s \perp \mu$, ker λ_s skoncentrirana na $\mathbb{Z}\setminus 2\mathbb{Z}$.

• Radon-Nikodymov odvod: $\exists f\in L^1(\mu)$, da je $\lambda_a(A)=\int_A f\,d\mu$.

Poskusimo $$\frac{d\lambda_a}{d\mu}=\sum_{n\in A\cap 2\mathbb{Z}}\frac{e^{in^2}(n+1)^2}{3^{|n|}}$$

$$\int_A\sum_{n\in A\cap 2\mathbb{Z}}\frac{e^{in^2}(n+1)^2}{3^{|n|}}\,d\mu \underset{\cancel{\text{TONELLI}}\ \text{FUBINI}}{=} \sum_{n\in A\cap 2\mathbb{Z}}\frac{e^{in^2}(n+1)^2}{3^{|n|}\underbrace{(n+1)^2}_{\mu(\{*\})}}=\lambda_a(A).$$

$$\bullet\ |\lambda|(\mathbb{Z})=\sum_{n\in\mathbb{Z}}\frac{1}{3^{|n|}}=2\sum_{n=0}^{\infty}\frac{1}{3^n}-\underbrace{1}_{\frac{1}{3^0}}=2\frac{1}{1-\frac{1}{3}}-1=\frac{2}{\frac{2}{3}}-1=2$$

5) $f:[0,1]^3\to[0,\infty]$, $f(x,y,z)=\begin{cases}\frac{1}{\sqrt{|y-z|}}, & \text{če } y\neq z\\ \infty, & \text{sicer}\end{cases}$ merljiva, ker zvezna.

$f\in L^1(m_3)$:

$$\int_{[0,1]^3}|f|\,dm_3 = \cancel{\iint_{\{y\neq z\}}|f|\,dm_3+\iint_{\{y=z\}}|f|\,dm_3=\iint_{\{y\neq z\}}} \overset{\cancel{\text{TONELLI}}}{=} \int_{\{y\neq z\}}|f|\,dm_3+\underbrace{\int_{\{y=z\}}|f|\,dm_3}_{\text{MERA } 0}=$$

$$=\int_{\{y\neq z\}}\frac{1}{\sqrt{|y-z|}}\,dm_3 \overset{\text{TONELLI}}{=} \int_{[0,1]_x}dm\int_{[0,1]_y}dm\int_{[0,y]\cup[y,1]_z}\frac{1}{\sqrt{|y-z|}}\,dm \overset{R=L}{=} \underbrace{\int_0^1 dt\int_0^1 dy\int_0^y\frac{dz}{\sqrt{y-z}}}_{I_1}+\underbrace{\int_0^1 dt\int_0^1 dy\int_y^1\frac{dz}{\sqrt{z-y}}}_{I_2}=$$

$$I_1=1\cdot\int_0^1 dy\,\frac{-1}{\frac{1}{2}}\sqrt{y-z}\,\Big|_{z=0}^{z=y}=2\int_0^1\sqrt{y}\,dy=2\,\frac{1}{\frac{3}{2}}\,y^{\frac{3}{2}}\Big|_{y=0}^{y=1}=\frac{4}{3}$$

$$I_2=1\cdot\int_0^1 dy\,\frac{1}{\frac{1}{2}}\sqrt{z-y}\,\Big|_{z=y}^{z=1}=2\cdot\int_0^1\sqrt{1-y}\,dy=2\cdot\frac{-1}{\frac{3}{2}}(1-y)^{\frac{3}{2}}\Big|_{y=0}^{y=1}=\frac{4}{3}$$

$$\Rightarrow \int_{[0,1]^3}|f|\,dm_3=\frac{8}{3}<\infty \quad ✓$$