NEKA, 2020/21
3. DN/ 3rd HW : 20. 11. 2020
Rok za oddajo/ Due date: 23:59, 4. 12. 2020

(1) Naj bo A centralna enostavna k-algebra. Naj vse $x, y, z, w \in A$ velja

$$[x, y][z, w] + [z, w][x, y] \in k.$$

Pokaži, da je $\deg A = 1$ ali $\deg A = 2$.

(2) Naj bo A enostavna $\mathbb{R}$-algebra lihe dimenzije. Pokaži, da je $A \cong M_n(\mathbb{R})$ za nek lih n.

(3) Naj bodo A,B in C končno dimenzionalne centralno enostavne k algebre, za katere velja

$$A \otimes B \cong A \otimes C.$$

Pokaži, da je $B \cong C$.

(4) Naj bo A centralno enostavna k-algebra stopnje n. Pokaži, da je A razcepljena natanko tedaj, ko vsebuje podalgebro izomorfno $k^n = k \times k \times \cdots \times k$.

(5) Naj bo S podalgebra algebre A. Pokaži:

(a) Če je S komutativna, potem je tudi $C_A(C_A(S))$ komutativna.

(b) Če je $S = C_A(U)$ za neko množico $U \subset A$, potem je $C_A(C_A(S)) = S$.

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(1) Let A be a central simple k-algebra. Assume that for any $x, y, z, w \in A$ we have

$$[x, y][z, w] + [z, w][x, y] \in k.$$

Show that $\deg A = 1$ or $\deg A = 2$.

(2) Let A be a simple $\mathbb{R}$-algebra of odd dimension. Show that $A \cong M_n(\mathbb{R})$ for some odd n.

(3) Let A,B and C be finitely central simple k algebras such that

$$A \otimes B \cong A \otimes C$$

Show that $B \cong C$.

(4) Let A be a central simple k-algebra of degree n. Show that A is split, if and only if it containes a subalgebra isomprphic $k^n = k \times k \times \cdots \times k$.

(5) Let S be a subalgebra of algebra A. Show:

(a) If S is commutative, then $C_A(C_A(S))$ is commutative and

(b) If $S = C_A(U)$ for some subset $U \subset A$, then $C_A(C_A(S)) = S$.