NEKA, 2020/21
1. DN/ 1st HW : 23. 10. 2020

Rok za oddajo/ Due date: 23:59, 6. 11. 2020

(1) Naj bo k polje in A končno dimenzionalna k-algebra. Pokaži, da je vsak element $x \in A$ obrnljiv ali delitelj niča.

(2) Naj bo M artiniski in noetherski R-modul in $\varphi \in \mathrm{End}_R(M)$ poljuben endomorfizem. Pokaži, da obstaja $n \in \mathbb{N}$ za katero je $M = \mathrm{im}(\varphi^n) \oplus \ker(\varphi^n)$.

(3) Pokaži, da je modul M polenostaven natanko tedaj, ko je vsak cikličen podmodul polenostaven.

(4) Naj bo R poljuben kolobar z enoto. Pokaži, da je Izračunaj še Jacobsonov radikal zgornje trikotnih $n \times n$ matrik nad R. (Razultat izrazi z $\mathrm{rad}\, R$.)

(5) Naj bo R artiniski kolobar in G končna grupa. Pokaži, da je grupni kolobar RG polenostaven kolobar natanko tedaj, ko je R polenostaven in $|G|$ obrnljiva v R.

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(1) Let k be a field and A a finitely dimensional k-algebra. Show that every element $x \in A$ is either a unit or a zero-divisor.

(2) Let M be artinian and noetherian R-module and $\varphi \in \mathrm{End}_R(M)$ an endomorphism. Show that there exists $n \in \mathbb{N}$ such that $M = \mathrm{im}(\varphi^n) \oplus \ker(\varphi^n)$.

(3) Show that a module M is semisimple if and only if every cyclic submodule is semisimple.

(4) Let R be a ring with unity. Compute the Jacobson radical of the upper triangular $n \times n$ matrices over R. (Express the result using $\mathrm{rad}\, R$.)

(5) Let R be an artinian ring and G a finite group. Show that the group ring RG is semisimple ring if and only if R is semisimple and $|G|$ invertible in R.