Computational Astrophysics N-Body Project

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Overview

Verification of Density function rho(r)

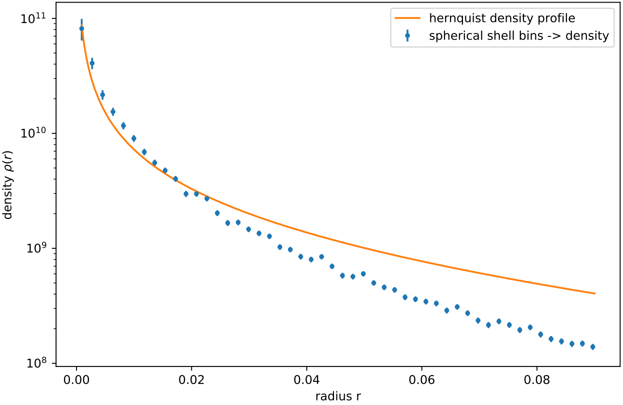

Figure: hernquist density, scale parameter a $\approx$ 0.3017

Unit Discussion

assuming	G=1	l = 1 parsec	m = 1 jupiter mass
SI	6.6741e-11 $m^3 kg^{-1} s^{-2}$	3.086e16 m	1.898e27 kg
factors	α =6.6741e-11	β =3.24e-17	γ =5.27e-28

From

$$\alpha \frac{m^3}{s^2 kg} = G = 1 \tag{1}$$

unit of time follows $1s \hat{=} \sqrt{\alpha m^3 kg^{-1}} = \sqrt{\alpha \beta^3 \gamma^{-1}}\ parsec^{3/2} M_j^{-1/2}$.
Calculating back to SI units, velocity and time are:

$$[t] = \sqrt{\alpha^{-1}\beta^{-3}\gamma} sec \quad [v] = \sqrt{\alpha\beta\gamma^{-1}} \frac{m}{sec} \tag{2}$$

Direct Force Calculation

To find the analytical solution the following equation was used:

$$a_{analytical}(r) = \frac{G}{r^2} \int_0^r 4\pi\rho(r) r^2 dr \tag{3}$$

To solve the integral numerically with N particles:

$$M(r) = \sum_{i, r_i < r}^{N} m_i \tag{4}$$

Direct Force Calculation

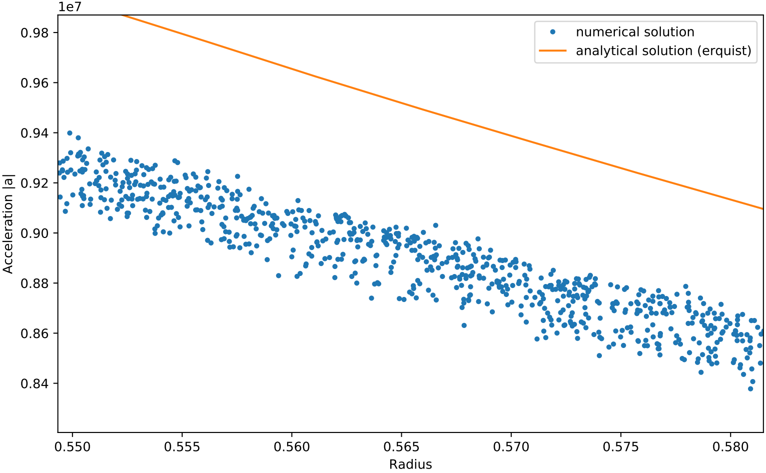

Figure: Softening $\epsilon = 0.1$

Direct Force Calculation

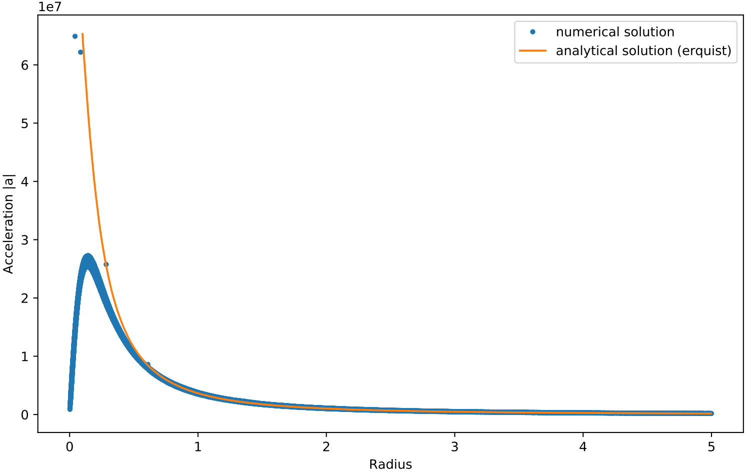

Figure: Softening $\epsilon = 0.1$

Direct Force Calculation

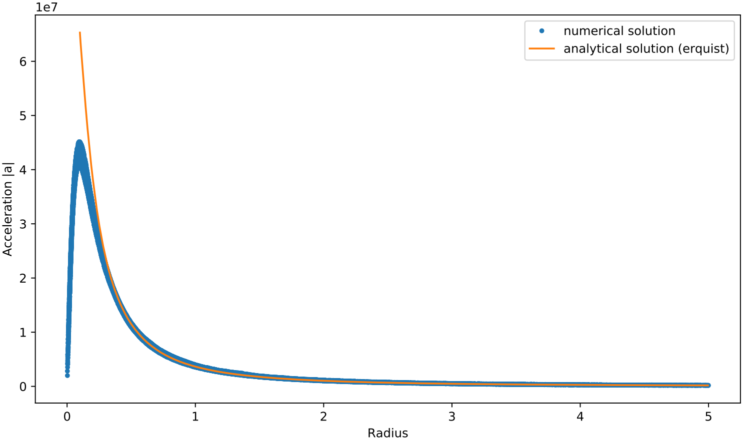

Figure: Softening $\epsilon = 0.05$

Time to relax

From lecture 4 and task description:

$$t_{relax} = \frac{N}{8 \log N} t_{cross} \tag{5}$$

$$v_c = \sqrt{\frac{G \cdot M(r < R_{hm})}{R_{hm}}} \tag{6}$$

To estimate t_{cross}:

$$v_c \approx \frac{R_{hm}}{t_{cross}} \rightarrow t_{cross} = \frac{R_{hm}}{v_c} \tag{7}$$

$$t_{relax} = \frac{N}{8 \log N} \sqrt{\frac{R_{hm}^3}{G \cdot M(r < R_{hm})}} \tag{8}$$

With N = 50010 from data we get $t_{relax} \approx 0.6905$. Which is about 1.05e16 sec, 3.33e8 yrs! A higher softening leads to lower velocities, which leads to higher relaxation time.

Direct force with softening

- ok
- this
- is a test