Prove the following assertion by contraposition: If x is irrational, then $x + r$ is irrational for all rational numbers r.

If Then statement: If x is irrational then x+r is irrational for all rational numbers.
The Contrapositive: If $x + r$ is rational for all rational numbers r, then x is rational.

Then there exists $\frac{a}{b}$ such that $x + r = \frac{a}{b}$. It is given that r is already a rational number, let it be given by $\frac{p}{q}$, and therefore x can be written as:
$x = \frac{a}{b} - \frac{p}{q}$
$x = \frac{aq-bp}{bq} = \frac{w}{z}$ where $a, b, p, q, w, z \in \mathbb{Z}$
$\therefore$ by proof by contrapositive, if x is irrational, $x + r$ is irrational for all rational numbers r.
■

#irrational #proof #contrapositive #if_then