Sets: A.2.1, A.2.2, A.2.3

MA 403 - Advanced Calculus - Brandiece Berry

A.2.1a) Show that $\mathbf{A \cup B = B \iff A \subset B}$

To show sets $A \cup B = B$, show $A \cup B \subset B$ and $B \subset A \cup B$
Try to prove $\iff$ statements both ways

Set Equality Proof Frame

1. Let $x \in A \cup B$
2. Show $A \cup B \subset B$
   Subset Proof Frame
   1. Let $x \in A \cup B$
   2. Show $x \in B$
   3. Then $A \cup B \subset B$ $\blacksquare$
3. Let $y \in B$
4. Show $B \subset A \cup B$
   Subset Proof Frame
   1. Let $y \in B$
   2. Show $y \in A \cup B$
   3. Then $B \subset A \cup B$ $\blacksquare$
5. If $A \cup B \subset B$ and $B \subset A \cup B$, then $A \cup B = B$ $\blacksquare$

Proof:

$\Leftarrow$ Given $A \subset B$
To show $A \cup B = B$, show $A \cup B \subset B$ and $B \subset A \cup B$

Let $x \in A \cup B$ and given that $A \subset B$,
Implications

1. If $x \in A \cup B$, then $x \in A$, $x \in B$, or $x \in A \cap B$
2. If $x \in B$, or $x \in A \cap B$, it follows that $x \in B$
3. If $x \in A$, and $A \subset B$ it follows $x \in B$ and $A \cup B \subset B$

Since $x \in A \cup B$ and $A \subset B$, it can be concluded that $x \in B$ and $A \cup B \subset B$ by definition of subsets.

Let $y \in B$
Implications

1. $y \in B$ if follows that $y \in A \cup B$ by definition of union of sets.

Since $y \in B$ and $A \subset B$, it can be concluded that $y \in A \cup B$, and $B \subset A \cup B$ by definition of subsets.

Therefore, given that $A \subset B$, and it is shown that $A \cup B \subset B$ and $B \subset A \cup B$, then $A \cup B = B$.

$\Rightarrow$ Given $A \cup B = B$
Show $A \subset B$
Let $x \in A$
Implications

1. If $x \in A \cup B$, by the definition of the union of sets $x \in A$ or $x \in B$
2. However by assumption, $A \cup B = B$, then it follows that $x \in B$.
   Since $x \in A$ and it is shown that $x \in B$, $A \subset B$

Therefore, $A \cup B = B \iff A \subset B$
$\blacksquare$

#sets #subsets #union #set_equality #proof

A.2.1b) Show that $A \cap B = A \iff A \subset B$

To show $A \cap B = A$, *show* $A \cap B \subset A$ *and* $A \subset A \cap B$

Set Equality Proof Frame

1. Let $x \in A \cap B$
2. Show $A \cap B \subset A$
   Subset Proof Frame
   1. Let $x \in A \cap B$
   2. Show $x \in A$
   3. Then $A \cap B \subset A$ $\blacksquare$
3. Let $y \in A$
4. Show $A \subset A \cap B$
   Subset Proof Frame
   1. Let $y \in A$
   2. Show $y \in A \cap B$
   3. Then $A \subset A \cap B$ $\blacksquare$
5. If $A \cap B \subset A$ and $A \subset A \cap B$, then $A \cap B = A$ $\blacksquare$

Proof:

$\Leftarrow$ Given $A \subset B$
Show $A \cap B = A$

Let $x \in A \cap B$ and given that $A \subset B$
Implications

1. If $x \in A \cap B$, $x \in A$ and $x \in B$ by definition of intersection of sets.

Since $x \in A \cap B$ and $x \in A$, it follows that $A \cap B \subset A$

Let $y \in A$ and given that $A \subset B$
Implications

1. If $y \in A$, and $A \subset B$, it follows that $y \in B$

2. If $y \in A$ and $y \in B$, by definition of intersection, $y \in A \cap B$
   Since $y \in A$ and $A \subset B$ and it is shown that $y \in A \cap B$ and $A \subset A \cap B$

Therefore, given that $A \subset B$, and it is shown that $A \cap B \subset A$ and $A \subset A \cap B$, then $A \cap B = A$.

$\Rightarrow$ Given $A \cap B = A$
Show $A \subset B$

Let $x \in A$
Implications

1. Given that $A \cap B = A$, if $x \in A$, then $x \in B$ by definition of intersection.
2. Since $x \in A$, and it is shown that $x \in B$, then $A \subset B$

Therefore, $A \cap B = A \iff A \subset B$
■

#sets #subsets #intersection #set_equality #proof

A.2.1c) Show that $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

To show two sets are equal, show they are subsets of each other
Show $(A \cup B) \cap C \subset (A \cap C) \cup (B \cap C)$ and $(A \cap C) \cup (B \cap C) \subset (A \cup B) \cap C$

Set Equality Proof Frame

1. Let $x \in (A \cup B) \cap C$
2. Show $(A \cup B) \cap C \subset (A \cap C) \cup (B \cap C)$
   Subset Proof Frame
   1. Let $x \in (A \cup B) \cap C$
   2. Show $x \in (A \cap C) \cup (B \cap C)$
   3. Then $(A \cup B) \cap C \subset (A \cap C) \cup (B \cap C)$
      ■
3. Let $y \in (A \cap C) \cup (B \cap C)$
4. Show $(A \cap C) \cup (B \cap C) \subset (A \cup B) \cap C$
   Subset Proof Frame
   1. Let $y \in (A \cap C) \cup (B \cap C)$
   2. Show $y \in (A \cup B) \cap C$
   3. Then $(A \cap C) \cup (B \cap C) \subset (A \cup B) \cap C$
      ■
5. If $(A \cup B) \cap C \subset (A \cap C) \cup (B \cap C)$ and $(A \cap C) \cup (B \cap C) \subset (A \cup B) \cap C$, then $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
   ■

Proof:

Let $x \in (A \cup B) \cap C$
Implications

1. If $x \in (A \cup B) \cap C$, then the following must be true:

        1. $x \in (A \cup B) \cap C$, $x \in A \cup B$ and $x \in C$ by definition of the intersection of sets.
        2. $x \in A \cup B$, and therefore $x \in A$ or $x \in B$ by definition of the union of sets.
    2. Since $x \in (A \cup B) \cap C$, $x \in A$ or $x \in B$ and $x \in C$, it follows that if
        1. $x \in A$ and $x \in C$, then $x \in A \cap C$
        2. $x \in B$ and $x \in C$, then $x \in B \cap C$
    3. Because $x \in A \cup B$ and $x \in C$, it follows that $x \in A \cap C$ or $x \in B \cap C$
       By definition of the union of sets it follows that $x \in (A \cap C) \cup (B \cap C)$

It follows that $(A \cup B) \cap C \subset (A \cap C) \cup (B \cap C)$

Let $y \in (A \cap C) \cup (B \cap C)$
Implications

1. If $y \in (A \cap C) \cup (B \cap C)$, $y \in A \cap C$ or $y \in B \cap C$, by definition of the union of sets.
2. Since $y \in A \cap C$ or $y \in B \cap C$
    1. $y \in A$ and $y \in C$
    2. $y \in B$ and $y \in C$
3. Since it is shown that $y \in C$, and $y \in A$ or $y \in B$, by definition $y \in (A \cup B) \cap C$

It follows that $(A \cap C) \cup (B \cap C) \subset (A \cup B) \cap C$
Therefore,
$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
■

#sets #subsets #set_equality #intersection #union #proof

A.2.1d)

A.2.2a) Describe the sets $\bigcup_{n=1}^{N}\left(-\frac{1}{n}, \frac{1}{n}\right)$ and $\bigcap_{n=1}^{N}\left(-\frac{1}{n}, \frac{1}{n}\right)$

The union of $-\frac{1}{n}$ and $\frac{1}{n}$, starting with n=1, where $n \in \mathbb{N}$, would be the set of all rational numbers $[-1, 1]$.

The intersection of $-\frac{1}{n}$ and $\frac{1}{n}$, starting with n=1, where $n \in \mathbb{N}$, would be the empty set $\emptyset$ by definition of an empty set. This is because every element in the first set, $-\frac{1}{n}$, would always be negative; yet $n = n$ for both sets. Thus these sets would never overlap because $\frac{1}{n}$ would never be negative.

#sets #intersection #union #Naturals

A.2.2b) Describe the sets $\bigcup_{n=1}^{N}(-n, n)$ and $\bigcap_{n=1}^{N}(-n, n)$

The union of $-n$ and n, starting with n=1, where $n \in \mathbb{N}$, would represent the set of $\mathbb{Z}$ by definition of the set of integers.

The intersection of $-n$ and n, starting with n=1, where $n \in \mathbb{N}$, would be the empty set $\emptyset$ by definition of an empty set. This is because every element in the first set, $-n$, would always be negative; yet $n = n$ for both sets. Thus these sets would never overlap because n would never be negative.

#sets #intersection #union #Naturals #Integers

A.2.2c) Describe the sets $\bigcup_{n=1}^{N}[n, n+1]$ and $\bigcap_{n=1}^{N}[n, n+1]$

The union of n and $n+1$, starting with n=1, where $n \in \mathbb{N}$, would represent the set of all positive integers $\mathbb{Z}^+$.

The intersection of n and $n+1$, starting with n=1, where $n \in \mathbb{N}$, would represent the set of all positive integers $\mathbb{Z}^+$. This is because every element successor in set n, is also an element within set $n+1$, which also includes n.

#sets #intersection #union #Naturals #Integers #consecutive

A.2.3a) Describe the sets $\bigcup_{n=1}^{\infty}\left(-\frac{1}{n}, \frac{1}{n}\right)$ and $\bigcap_{n=1}^{\infty}\left(-\frac{1}{n}, \frac{1}{n}\right)$

The union of $-\frac{1}{n}$ and $\frac{1}{n}$, starting with n=1, where $n \in \mathbb{N}$, would be the set of all real numbers $(-\infty, \infty)$.

The intersection of $-\frac{1}{n}$ and $\frac{1}{n}$, starting with n=1, would be the empty set $\emptyset$ by definition of an empty set. This is because every element in the first set, $-\frac{1}{n}$, would always be negative; yet $n = n$ for both sets. Thus these sets would never overlap because $\frac{1}{n}$ would never be negative.

#sets #intersection #union #Naturals

A.2.3b) Describe the sets $\bigcup_{n=1}^{\infty}(-n. n)$ and $\bigcap_{n=1}^{\infty}(-n, n)$

The union of $-n$ and n, starting with n=1, where $n \in \mathbb{N}$, would be the set of all real numbers $(-\infty, \infty)$.

The intersection of $-n$ and n, starting with n=1, would be the empty set $\emptyset$ by definition of an empty set. This is because every element in the first set, $-n$, would always be negative; yet $n = n$ for both sets. Thus these sets would never overlap because n would never be negative.

#sets #intersection #union #Naturals

A.2.3c) Describe the sets $\bigcup_{n=1}^{\infty}[n. n+1]$ and $\bigcap_{n=1}^{\infty}[n. n+1]$

The union of n and $n+1$, starting with n=1, would be the set of positive real numbers $[1, \infty)$

The intersection of n and $n+1$, starting with n=1, would be the empty set $\emptyset$ by definition of an empty set. This is because every element in the second set, $n+1$, would always be 1 greater than items in the first set, n; thus these sets would never overlap because n and $n+1$ would never equal.

#sets #intersection #union #Naturals