Show that $A \cap B = A \iff A \subset B$

To show $A \cap B = A$, show $A \cap B \subset A$ and $A \subset A \cap B$

Set Equality Proof Frame

1. Let $x \in A \cap B$
2. Show $A \cap B \subset A$

   Subset Proof Frame

   1. Let $x \in A \cap B$
   2. Show $x \in A$
   3. Then $A \cap B \subset A$ ■
3. Let $y \in A$
4. Show $A \subset A \cap B$

   Subset Proof Frame

   1. Let $y \in A$
   2. Show $y \in A \cap B$
   3. Then $A \subset A \cap B$ ■
5. If $A \cap B \subset A$ and $A \subset A \cap B$, then $A \cap B = A$ ■

Proof:

$\Leftarrow$ Given $A \subset B$

Show $A \cap B = A$

Let $x \in A \cap B$ and given that $A \subset B$

Implications

    1. If $x \in A \cap B$, $x \in A$ and $x \in B$ by definition of intersection of sets.

Since $x \in A \cap B$ and $x \in A$, it follows that $A \cap B \subset A$

Let $y \in A$ and given that $A \subset B$

Implications

    1. If $y \in A$, and $A \subset B$, it follows that $y \in B$
    2. If $y \in A$ and $y \in B$, by definition of intersection, $y \in A \cap B$
       Since $y \in A$ and $A \subset B$ and it is shown that $y \in A \cap B$ and $A \subset A \cap B$

Therefore, given that $A \subset B$, and it is shown that $A \cap B \subset A$ and $A \subset A \cap B$, then $A \cap B = A$.

$\Rightarrow$ Given $A \cap B = A$

Show $A \subset B$

Let $x \in A$

Implications

    1. Given that $A \cap B = A$, if $x \in A$, then $x \in B$ by definition of intersection.
    2. Since $x \in A$, and it is shown that $x \in B$, then $A \subset B$

Therefore, $A \cap B = A \Longleftrightarrow A \subset B$

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