Show that $\mathbf{A} \cup \mathbf{B} = \mathbf{B} \iff \mathbf{A} \subset \mathbf{B}$

To show sets $A \cup B = B$, show $A \cup B \subset B$ and $B \subset A \cup B$

Try to prove $\iff$ statements both ways

Set Equality Proof Frame

1. Let $x \in A \cup B$
2. Show $A \cup B \subset B$

   Subset Proof Frame

   1. Let $x \in A \cup B$
   2. Show $x \in B$
   3. Then $A \cup B \subset B$ $\blacksquare$
3. Let $y \in B$
4. Show $B \subset A \cup B$

   Subset Proof Frame

   1. Let $y \in B$
   2. Show $y \in A \cup B$
   3. Then $B \subset A \cup B$ $\blacksquare$
5. If $A \cup B \subset B$ and $B \subset A \cup B$, then $A \cup B = B$ $\blacksquare$

Proof:

$\Leftarrow$ Given $A \subset B$
To show $A \cup B = B$, show $A \cup B \subset B$ and $B \subset A \cup B$

Let $x \in A \cup B$ and given that $A \subset B$,
Implications

1. If $x \in A \cup B$, then $x \in A$, $x \in B$, or $x \in A \cap B$
2. If $x \in B$, or $x \in A \cap B$, it follows that $x \in B$
3. If $x \in A$, and $A \subset B$ it follows $x \in B$ and $A \cup B \subset B$

Since $x \in A \cup B$ and $A \subset B$, it can be concluded that $x \in B$ and $A \cup B \subset B$ by definition of subsets.

Let $y \in B$
Implications

1. $y \in B$ if follows that $y \in A \cup B$ by definition of union of sets.

Since $y \in B$ and $A \subset B$, it can be concluded that $y \in A \cup B$, and $B \subset A \cup B$ by definition of subsets.

Therefore, given that $A \subset B$, and it is shown that $A \cup B \subset B$ and $B \subset A \cup B$, then $A \cup B = B$.

$\Rightarrow$ Given $A \cup B = B$
Show $A \subset B$
Let $x \in A$
Implications

1. If $x \in A \cup B$, by the definition of the union of sets $x \in A$ or $x \in B$

2. However by assumption, $A \cup B = B$, then it follows that $x \in B$. Since $x \in A$ and it is shown that $x \in B$, $A \subset B$

Therefore, $A \cup B = B \iff A \subset B$

■

#subsets #union #set_equality