A.2.1) Show that $A \cup B = B \iff A \subset B$

To show sets $A \cup B = B$, show $A \cup B \subset B$ and $B \subset A \cup B$
Try to prove $\iff$ statements both ways

Proof:
$\Leftarrow$ Given $A \subset B$
To show $A \cup B = B$, show $A \cup B \subset B$ and $B \subset A \cup B$

Let $x \in A \cup B$ and given that $A \subset B$,
Implications

1. If $x \in A \cup B$, then $x \in A$, $x \in B$, or $x \in A \cap B$
2. If $x \in B$, or $x \in A \cap B$, it follows that $x \in B$
3. Since $x \in A$, and $A \subset B$ it follows $x \in B$ and $A \cup B \subset B$

Since $x \in A \cup B$ and $A \subset B$, it can be concluded that $x \in B$ and $A \cup B \subset B$ by definition of subsets.

Let $y \in B$
Implications

1. $y \in B$ if follows that $y \in A \cup B$ by definition of union of sets.

Since $y \in B$ and $A \subset B$, it can be concluded that $y \in A \cup B$, and $B \subset A \cup B$ by definition of subsets.

Therefore, given that $A \subset B$, and it is shown that $A \cup B \subset B$ and $B \subset A \cup B$, then $A \cup B = B$.

$\Rightarrow$ Given $A \cup B = B$
Show $A \subset B$
Let $x \in A$
Implications

1. Let $x \in A \cup B$, by the definition of the union of sets
2. By assumption, $A \cup B = B$, then it follows that $x \in B$.
   Since $x \in A$ and it is shown that $x \in B$, $A \subset B$

Therefore, $A \cup B = B \iff A \subset B$
■

A.2.1) Show that $A \cap B = A \iff A \subset B$

To show $A \cap B = A$, show $A \cap B \subset A$ and $A \subset A \cap B$

$\Leftarrow$
Proof: Given $A \subset B$
Show $A \cap B = A$

Let $x \in A \cap B$ and given that $A \subset B$
Implications

1. If $x \in A \cap B$, $x \in A$ and $x \in B$

Since $x \in A \cap B$ and $x \in A$, it follows that $A \cap B \subset A$
$\Rightarrow$
Let $y \in A$ and given that $A \subset B$
Implications

1. If $y \in A$, and $A \subset B$, it follows that $y \in B$
2. If $y \in A$ and $y \in B$, by definition of intersection, $y \in A \cap B$

Since $y \in A$ and $A \subset B$ and it is shown that $y \in A \cap B$ and $A \subset A \cap B$

Therefore, $A \cap B = A \iff A \subset B$
$\blacksquare$