CMPUT-340
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Assignment 4

Exercise 1

$$f(x,y)=2x^3-3x^2-6xy(x-y-1)$$

$$\nabla f=\begin{bmatrix} 6x^2-6x-6y(2x-y-1) \\ -6x(x-2y-1) \end{bmatrix}$$

$$H_f(x,y)=\begin{bmatrix} 12x-6-12y & -6(2x-2y-1) \\ -6(2x-2y-1) & 12x \end{bmatrix}$$

The critical points of the gradient(*f*) are [0, 0] and [1, 0].

If $H=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then eigenvalues are equal to λ such that,

$$(a-\lambda)(d-\lambda)-bc=0$$

$$H_f(0,0)=\begin{bmatrix} -6 & 6 \\ 6 & 0 \end{bmatrix}$$

$$(-6-\lambda)(0-\lambda)-36=0$$

so λ = -9.708, 3.708. Therefore (0,0) is a saddle point.

$$H_f(1,0)=\begin{bmatrix} 6 & -6 \\ -6 & 12 \end{bmatrix}$$

$$(6-\lambda)(12-\lambda)-36=0$$

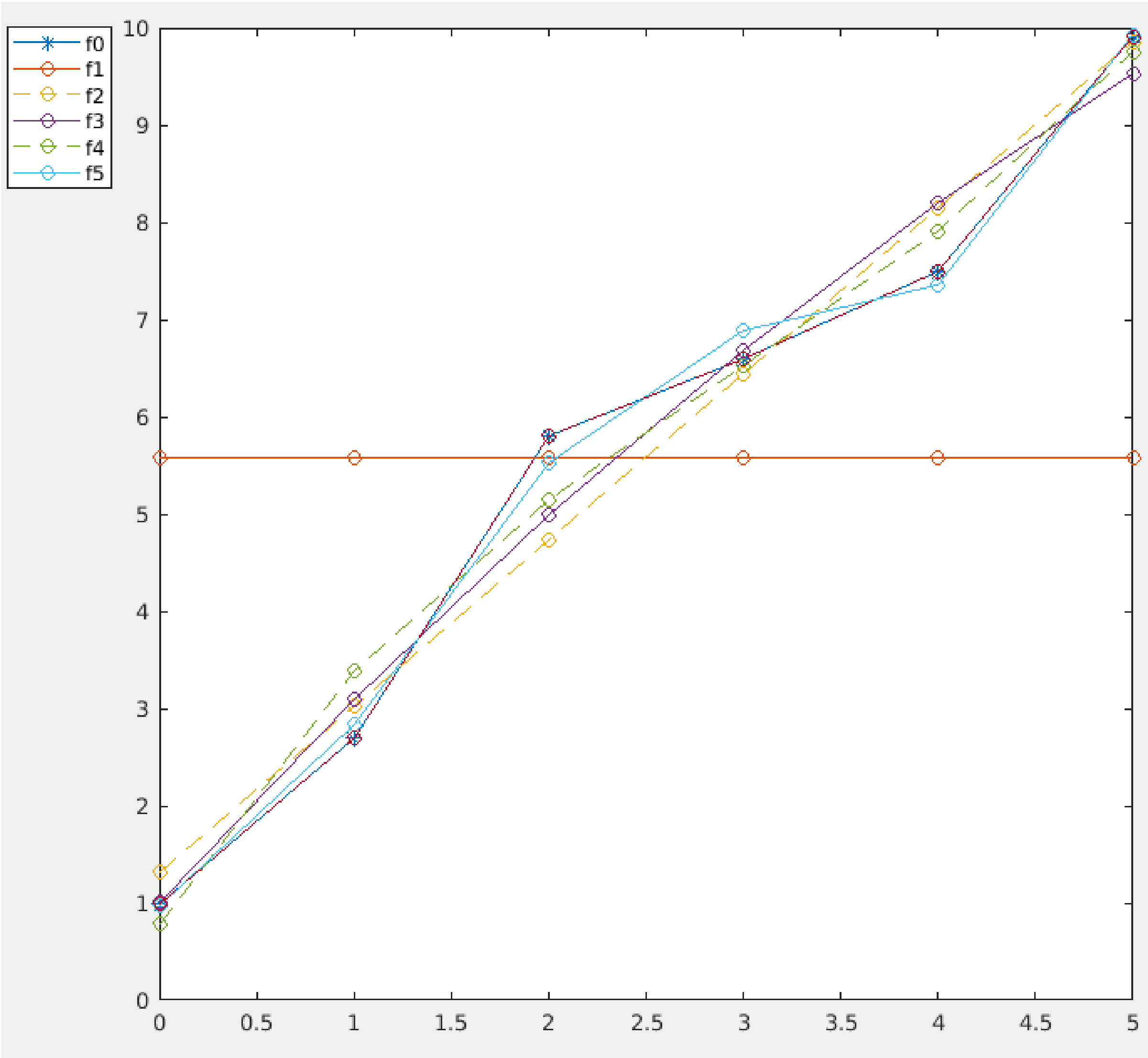
so λ = 2.292, 15.708. Therefore (1,0) is a minimum.

Exercise 4

a)

Degree	Fitted Polynomial
0	5.5833
1	$1.3190+1.7057x$
2	$1.0036+2.1789x-0.0946x^2$
3	$0.7897+3.1557x-0.6294x^2+0.0713x^3$
4	$0.9718+0.1200x+2.6340x^2-0.9912x^3+0.1063x^4$
5	$1-3.1783x+8.3042x^2-4.2125x^3+0.8458x^4-0.0592x^5$

Here is a plot of the fitted polynomials at the given values of t:



Polynomial	Derivative
$f_0(x)$	0
$f_1(x)$	1.7057
$f_2(x)$	$2.1789-0.1892x$
$f_3(x)$	$3.1557-1.2588x+0.2139x^2$
$f_4(x)$	$0.12+5.268x-2.9736x^2+0.4252x^3$
$f_5(x)$	$-3.1783+16.6084x-12.6375x^2+3.3832x^3-0.296x^4$

The higher the degree of the fitted polynomial the better the fit as seen from the plotted data. When comparing the slopes of the tangent lines given at each point from the derived derivatives from the above table and the approximate values from forward, backward, and central difference methods, the central difference method always had the most accurate results.

When the forward difference was off it was always below the actual derivative value by some difference say *p*. Interestingly enough when the forward difference method was not exact neither was the backwards difference method and that value was above the actual derivative value by the same difference *p*.

Exercise 5

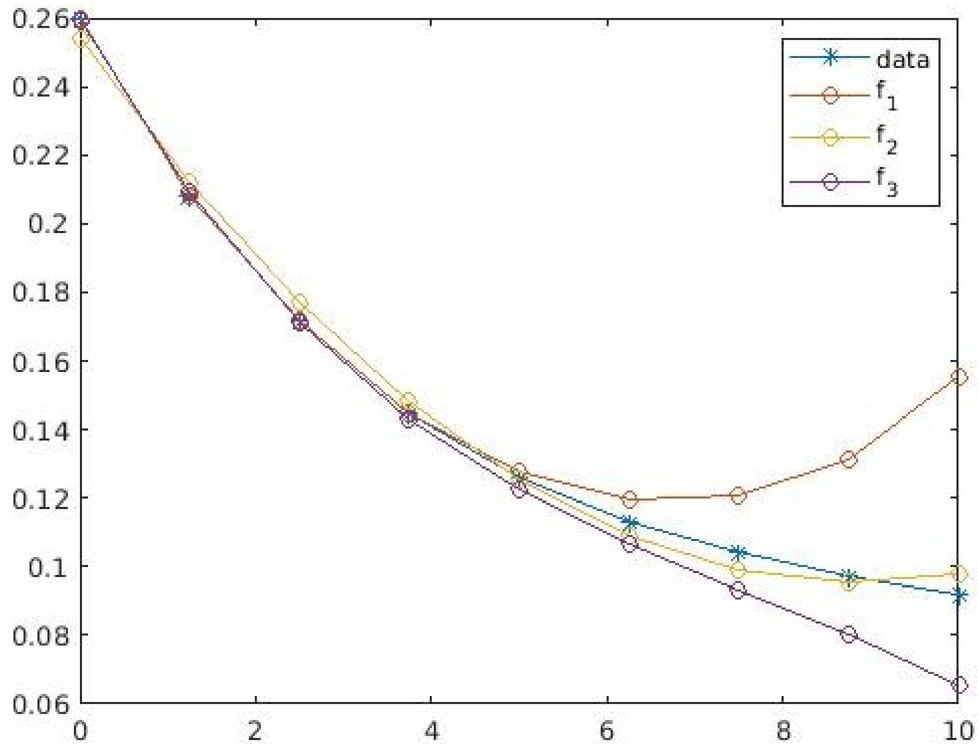
Using the following grouping intervals between 0km and 10km: [0, 1.25, 2.5], [3.75,5,6.25], and [7.5, 8.75, 10]. I used midpoint, trapezoid and Simpson's rule to approximate the integral value.

First approximate the function *f* using linear least squares and the Vandermonde matrix. Here are some of the approximation functions I derived and their comparison to the data given.

$$f_1 = 0.26 - 0.0555x + 0.0174x^2 - 0.0072x^3 + 0.0021x^4 - 0.00036727x^5 + 3.7865\text{e-}5x^6 - 2.1221\text{e-}6x^7 + 4.9932\text{e-}8x^8$$

$$f_2 = 0.2536 - 0.0356x + 0.002x^2$$

$$f_3 = 0.2593 - 0.0454x + 0.0046x^2 - 0.0002x^3$$



Using the second approximation function for the integral approximation methods I obtained the following values;

Integral Approx	Midpoint	Trapezoid	Simpson
1.4227	**1.1617**	**1.4279**	**1.1829**

Error Estimation

$$M(f) = (10-0)(0.1256) = 1.256$$

$$T(f) = \frac{10-0}{2}(0.2536 + 0.0976) = 1.756$$

$$E(f) \approx \frac{1.756 - 1.256}{3} = 0.166\ldots$$

So error in midpoint is 0.166… and error in trapezoid is about -0.08.

$$S(f) = \frac{2}{3}M(f) + \frac{1}{3}T(f) = \frac{2}{3}1.256 + \frac{1}{3}1.756 = 1.4226\ldots$$

In order to find the amount of fuel for 8 kilometers we would use the same approximation techniques with b = 8. If the approximate function is fitted enough the accuracy of the approximate integrations should also be close. Using my second function the integral at 8km would be approximately 1.2309.