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A Survey and Comparative Analysis of Various 3D Reconstruction Methods

Arun Kumar Bisoyi*, **Amitav Mahapatra, Subhalaxmi Das**
Dept. of CSE, College of Engineering and Technology,
Bhubaneswar, Odisha, India

Abstract— ***In the field of Image processing, 3D reconstruction is the process of capturing the shape and appearance of real objects. An efficient 3D reconstruction algorithm generally enhances the capabilities of existing 2D or 3D face recognition process. Many algorithms for 3D reconstruction have been developed so far such as Shape from Shading (SFS), 3D morphable model and Structure from Motion (SFM). In this paper a survey of above three approaches and a comparative analysis are presented.***

Keywords— ***3D face reconstruction, SFS, 3D morphable, SFM***

I. INTRODUCTION

3D face model plays a vital role in face image processing. It is used in Face Recognition, Face animation, Face tracking etc. It has advantages over 2D model with regard to variation in pose and illumination. It is expensive as well as difficult to construct the 3D model of 2D images. Two main stream approaches are usually adopted to create a 3D facial model. One way is to use special equipment like a 3D scanner to capture 3D shape of human head. But due to high cost and limited applicability of 3D sensing devices, it is difficult to acquire sufficient and useful data as an alternative 3D face model of individual can be reconstructed using the techniques based on 2D images. Generally speaking an efficient 3D reconstruction algorithm can greatly enhance the capabilities of existing 2D or 3D face recognition system. A 3D shape can be expressed in several ways, e.g., depth, surface normal and surface gradient or surface slant. The depth value can be consider either as the relative distance from the camera to the surface points or the relative surface height above the x-y plane. Many algorithms have been developed for 3D reconstruction such as Shape from Shading (SFS). The 3D morphable model, Structure from Motion (SFM) etc. Keeping this in mind, we are presenting a comparative study on the above mentioned techniques in the field of 3D reconstruction and face recognition.

II. LITERATURE REVIEW

A. *Shape from Shading*

By exploring the shading information in an image, e.g., the intensity and its derivative, SFS deals with the recovery of shape based on some reflectance models, like Lambertian model which is a basic model, specular reflectance model and hybrid model, etc.

The techniques to reconstruct shape are called shape-from-X techniques, where X can be shading, stereo, motion, texture, etc. Shape-from-shading (SFS) deals with the recovery of shape from a gradual variation of shading in the image. The first shape-from-shading (SFS) technique was developed by Horn in the early 1970s. To solve the SFS problem, it is important to study how the images are formed. A simple model of Image formation is the Lambertian model, in which the gray at a pixel in the image depends on the light source and the surface shape at each pixel in the image. However, real images do not always follow the Lambertian model. Even if we assume Lambertian reflectance and known light source direction, and if the brightness can be described as a function of surface shape and light source direction, the problem is still not simple. This is because if the surface shape is described in terms of the surface normal, we have a linear equation with three unknowns, and if the surface shape is described in terms of the surface gradient, we have a nonlinear equation with two unknowns. Therefore, finding a unique solution to SFS is difficult; it requires additional constraints.

Various SFS approaches can be divided into four groups: minimization approaches, propagation approaches, local approaches, and linear approaches. Minimization approaches obtain the solution by minimizing an energy function. In general, minimization approaches are more robust, while the other approaches are faster. All these approaches are proposed by many authors, presented below.

1) *Minimization Approaches*

One of the earlier minimization approaches, which recovered the surface gradient, was by Ikeuchi and Horn [1]. Since each surface point has two unknowns for the surface gradient and each pixel in the image provides one gray value, we have an underdetermined system. They introduced two constraints: the brightness constraint and the smoothness constraint to overcome the problem. The brightness constraint requires that the reconstructed shape produce the same brightness as the input image at each surface point, while the smoothness constraint ensures a smooth surface reconstruction. The shape was computed by minimizing an energy function which consists of the above two constraints.

To ensure a correct convergence, the shape at the occluding boundary was given for the initialization. Since the gradient at the occluding boundary has at least one infinite component, stereographic projection was used to transform the error function to a different space.

Brooks and Horn [2] minimized the energy function, in terms of the surface normal. Frankot and Chellappa [3] enforced integrability in Brooks and Horn's algorithm in order to recover integrable surfaces (surfaces for which $z_{xy} = z_{yx}$). Surface slope estimates from the iterative scheme were expressed in terms of a linear combination of a finite set of orthogonal Fourier basis functions. The enforcement of integrability was done by projecting the nonintegrable surface slope estimates onto the nearest (in terms of distance) integrable surface slopes. This projection was fulfilled by finding the closest set of coefficients which satisfy integrability in the linear combination. Their results showed improvements in both accuracy and efficiency over Brooks and Horn's algorithm [2].

Later, Horn also [4] replaced the smoothness constraint in his approach with an integrability constraint. The major problem with Horn's method is its slow convergence.

Szeliski [5] sped it up using a hierarchical basis preconditioned conjugate gradient descent algorithm. Based on the geometrical interpretation of Brooks and Horn's algorithm.

Vega and Yang [6] applied heuristics to the variational approach in an attempt to improve the stability of Brooks and Horn's algorithm.

Instead of the smoothness constraint, Zheng and Chellappa [7] introduced an intensity gradient constraint which specifies that the intensity gradients of the reconstructed image and the input image are close to each other in both the x and y directions. All of the above techniques use variational calculus.

Leclerc and Bobick [8] solved directly for depth by using a discrete formulation and employing a conjugate gradient technique. The brightness constraint and smoothness constraint were applied to ensure convergence, and a stereo depth map was used as an initial estimate.

Recently, Lee and Kuo [9] also proposed an approach to recover depth using the brightness and the smoothness constraint. They approximated surfaces by a union of triangular patches. This approach did not require the depth initialization.

The approaches described so far deal with a single smooth surface. Malik and Maydan [41] developed a solution for piecewise smooth surfaces. They combined the line drawing and shading constraints in an energy function and recovered both surface normal and line labeling through the minimization of the energy function.

2) Propagation Approaches

Horn's characteristic strip method [10] is essentially a propagation method. A characteristic strip is a line in the image along which the surface depth and orientation can be computed if these quantities are known at the starting point of the line. Horn's method constructs initial surface curves around the neighborhoods of singular points (singular points are the points with maximum intensity) using a spherical approximation. The shape information is propagated simultaneously along the characteristic strips outward, assuming no crossover of adjacent strips. The direction of characteristic strips is identified as the direction of intensity gradients. In order to get a dense shape map, new strips have to be interpolated when neighboring strips are not close to each other.

Rouy and Tourin [11] presented a solution to SFS based on Hamilton-Jacobi-Bellman equations and viscosity solutions theories in order to obtain a unique solution. A link between viscosity solutions and optimal control theories was given via dynamic programming. Moreover, conditions for the existence of both continuous and smooth solutions were provided.

Oliensis [12] observed that the surface shape can be reconstructed from singular points instead of the occluding boundary. Based on this idea, Dupuis control problem and solved it using numerical methods.

Bichsel and Pentland [13] simplified Dupuis and Oliensis's approach and proposed a minimum downhill approach for SFS which converged inless than 10 iterations.

Similar to Horn's and Dupuis and Oliensis's approaches, Kimmel and Bruckstein [14] reconstructed the surface through layers of equal height contours from an initial closed curve. Their method applied techniques in differential geometry, fluid dynamics, and numerical analysis, which enabled the good recovery of non-smooth surfaces. The algorithm used a closed curve in the areas of singular points for initialization.

3) Local Approaches

Pentland's local approach [15] recovered shape information from the intensity and its first and second derivatives. He used the assumption that the surface is locally spherical at each point. Under the same spherical assumption, Lee and Rosenfeld [16] computed the slant and tilt of the surface in the light source coordinate system using the first derivative of the intensity.

4) Linear Approaches

Pentland [17] used the linear approximation of the reflectance function in terms of the surface gradient and applied a Fourier transform to the linear function to get a closed form solution for the depth at each point.

Tsai and Shah [18] applied the discrete approximation of the gradient first, and then employed the linear approximation of the reflectance function in terms of the depth directly. Their algorithm recovered the depth at each point using a Jacobi iterative scheme.

B. 3D morphable

A 3D morphable model is generally built from a set of 3D laser-scanned heads. As a crucial step, the scans are first registered in a dense point-by-point correspondence, using an optical-flow algorithm to reduce artifacts [19], [20].

Statistical signal-processing techniques, such as principal component analysis (PCA), are then applied on the shape and texture features of these training samples to obtain a feature subspace [19]. The feature subspace, including the shape and texture feature vectors, can be regarded as a generic 3D face model. Given the model, a realistic human face can be represented as a convex combination of the shape and texture vectors as shown in fig-1.

The estimate is achieved by fitting a statistical, morphable model of 3D faces to images. Blanz, Volker, and Thomas Vetter[20] presents a method for face recognition across variations in pose, ranging from frontal to profile views, and across a wide range of illuminations, including cast shadows and specular reflections. To account for these variations, the algorithm simulates the process of image formation in 3D space, using computer graphics, and it estimates 3D shape and texture of faces from single images. The estimate is achieved by fitting a statistical, morphable model of 3D faces to images. [20] describe the construction of the morphable model, an algorithm to fit the model to images, and a framework for face identification.

The goal of recognition algorithms is to separate the characteristics of a face, which are determined by the intrinsic shape and color (texture) of the facial surface, from the random conditions of image generation.

Unlike pixel noise, these conditions may be described consistently across the entire image by a relatively small set of extrinsic parameters, such as camera and scene geometry, illumination direction and intensity.

The algorithm [20] presented estimates all 3D scene parameters automatically, including head position and orientation, focal length of the camera, and illumination direction. This is achieved by a new initialization procedure that also increases robustness and reliability of the system considerably. The new initialization uses image coordinates of between six and eight feature points. Currently, most face recognition algorithms require either some initialization, or they are, unlike our system, restricted to front views or to faces that are cut out from images.

After fitting the model, recognition can be based on model coefficients, which represent intrinsic shape and texture of faces, and are independent of the imaging conditions. For identification, all gallery images are analyzed by the fitting algorithm, and the shape and texture coefficients are stored as fig-1. Given a probe image, the fitting algorithm computes coefficients which are then compared with all gallery data in order to find the nearest neighbor.

C. Structure from Motion

Structure-from-motion (SFM) is a popular approach to recover the 3D shape of an object when multiple frames of an image sequence are available. Given a set of observations of 2D feature points, SFM can estimate the 3D structure of the feature points. Existing approaches to non-rigid structure from motion assume that the instantaneous 3D shape of a deforming object is a linear combination of basis shapes.

Structure-from-Motion (SFM) algorithms have been extensively used to factorize the rigid and non-rigid 3D structure of objects from a set of 2D point tracks.

Early work by Tomasi and Kanade in the 90's [21] proposed a factorization approach to recover the shape of rigid objects from an orthographic camera.

Bregler et al. [22] described a factorization method for objects with non-rigid structure where any 3D shape configuration is modeled as a linear combination of basic shapes defining principal deformation modes. Assuming a weak perspective camera projection, [22] proposed a factorization method that exploits rank constraints on camera rotations to recover non-rigid 3D shape and motion. Recently several authors [23, 24, 25] have shown that rotation constraints for the pose are not enough to achieve reliable 3D reconstructions.

Brand [23] proposed an alternative optimization method by introducing extra constraints and forcing the deformation to be as small as possible (relative to the mean shape).

Xiao et al. [24] proposed adding a set of constraints on the shape basis to recover better 3D models. These constraints are based on the assumption that there are n image frames (where n is the number of basis shapes) in which the basis shapes are known to be independent.

However, as it was later pointed out by Brand [25], the algorithm breaks down with noisy data or when n is not correctly estimated.

Alternated least squares (ALS) [26, 27, 28] and expectation maximization (EM) [29, 30, 31] techniques have proven to be efficient methods to factorize the shape and motion components in SFM algorithms. These methods have been extended to incorporate missing data and to handle multiple view cases (where multiple projections of the same shape configuration are available). In presence of noisy data (e.g. inconsistencies in the tracked points or missing data), many SFM problems become ill-posed and Singular Value Decomposition (SVD) formulations are not effective.

Torresani et al. [27] proposed a simple solution based on an alternated minimization scheme with promising results even when missing data is present.

Buchanan and Fitzgibbon [32, 33] presented a class of second-order optimizations which converged more reliably than alternation approaches in different experiments. However, they concluded that for many real SFM problems it is not sufficient to minimize the reprojection error in order to get meaningful results and pointed out the need to further analyze the use of prior information. In many scenarios estimating deformable 3D shapes is inherently under constrained, especially when using monocular 2D features, and standard SFM algorithms give degenerate solutions.

Torresani et al. [30, 31] used an expectation maximization approach to solve the factorization problem, assuming Gaussian priors over the deformation parameters in order to avoid arbitrary variations.

Del Bue et al. [34] enforced priors over the rigidity of some points to obtain reliable estimates of the object's rigid component.

Olsen and Bartoli [35] imposed temporal smoothness and continuous variation in shape reconstructions. Similar in spirit to the approach presented in this paper, DelBue [36] introduced prior knowledge in the SFM algorithm in the form of previously known 3D shapes representing feasible configurations of the object, which at the end were used to regularize the rigid component of a deformable object. The formulation of this previous work is based on a factorization framework and prior information is incorporated into an intermediate solution but not the final metric reconstruction. Moreover, it is not clear how to incorporate missing data into the formulation. In their paper they extend existing approaches by incorporating prior information for morphable shape models into the final Euclidean linear basis reconstruction and provide experimental results in difficult scenarios (e.g. severe occlusions and reduced number of views).

Ljaz Akhter,Yaser Sheikh,Sohaib Khan, and Takeo Kanade[37] present a dual approach to describe the evolving 3D structure in trajectory space by a linear combination of basis trajectories. They present dual relationship between the two approaches, showing that they both have equal power for representing 3D structure. They further show that the temporal smoothness in 3D trajectories alone can be used for recovering non-rigid structure from a moving camera. The principal advantage of expressing deforming 3D structure in trajectory space can be defined an object independent basis. This results in a significant reduction in unknowns and corresponding stability in estimation. The present the use of the Discrete Cosine Transform (DCT) as the object independent basis and empirically demonstrate that it approaches Principal Component Analysis (PCA) for natural motions. They reported on performance of the proposed method, quantitatively using motion capture data, and qualitatively on several video sequences exhibiting non-rigid motions, including piecewise rigid motion, partially non-rigid motion (such as a facial expressions), and highly non-rigid motion (such as a person walking or dancing).

Jeff and Aleix [38] proposed an information theoretic approach to define the problem of structure from motion (SFM) as a blind source separation one. Given that for almost all practical joint densities of shape points, the marginal densities are non-Gaussian, they present how higher-order statistics can be used to provide improvements in shape estimates over the methods of factorization via Singular Value Decomposition (SVD), bundle adjustment and Bayesian approaches. Previous techniques have either explicitly or implicitly used only second-order statistics in models of shape or noise. A further advantage of viewing SFM as a blind source problem is that it easily allows for the inclusion of noise and shape models, resulting in Maximum Likelihood (ML) or Maximum a Posteriori (MAP) shape and motion estimates. A key result is that the blind source separation approach has the ability to recover the motion and shape matrices without the need to explicitly know the motion or shape pdf. We demonstrate that it suffices to know whether the pdf is sub-or super-Gaussian (i.e., semi-parametric estimation) and derive a simple formulation to determine this from the data. Finally they presented extensive experimental results on synthetic and real tracked points in order to quantify the improvement obtained from this technique.

Hei-Sheung and Kin-Man [39] proposed a new algorithm to derive the 3D human face from one or more of 2D face images under different poses. Based on the corresponding 2D feature points of the respective images, their respective poses and the depths of the feature points can be estimated based on measurements using the similarity transform. To accurately estimate the pose of and the 3D information about a human face, the genetic algorithm (GA) is applied. Presented algorithm does not require any prior knowledge of camera calibration, and has no limitation on the possible poses or the scale of the face images. It also provides a means to evaluate the accuracy of the constructed 3D face model based on the similarity transform of the 2D feature point sets. The proposed algorithm can also be extended to face recognition to alleviate the effect of pose variations. Experimental results show that our proposed algorithm can construct a 3D face structure reliably and efficiently.

Zhan-Li Sun, Kin-Man Lam and Qing-Wei Gao[40] proposed the nonlinear least-squares (NLS) methods to reduce the computation of the method in [39], The author presented the process to estimate the depth values of facial feature points, i.e., a sparse 3D face representation. In the NLS model, not only the pose parameters, but also the depth values of the facial feature points, are considered as the variables to be optimized. In addition, the symmetry information of face is utilized further in the proposed methods in order to alleviate the sensitivity to the training samples used. Usually one frontal view face image and one non-frontal-view face image are sufficient to reconstruct a 3D face model using the proposed algorithms. And for cases when multiple non-frontal-view face images are available, a model-integration approach is proposed in this paper to improve the depth estimation accuracy. For the sake of experiment they have used two popular databases such as FERET and Boshphorus.

In the section of Structure from Motion,[38], [39], [40] provide information on rigid structure from motion(RSFM) and rest are on Non rigid structure from motion(NRSFM).

III. COMPARATIVE STUDY

From the above survey, the comparative study is presented below which concern to their properties and demand.

Table I Comparison Among Various 3d Reconstruction Methods

Properties	***SFS***	***3D Morphable***	***SFM***
Representation	Dense 3D		Sparse 3D

Cost	less	more	less
Storage requirement	Larger		smaller
Application	Better face recognition performance		Poor face recognition performance
Disadvantages	• Poor and worse result on synthetic data and real images. • Shadow area is not recovered	Registration of the model to an image, i.e. the fitting	Recover the 3D shape of an object when multiple frames of an image sequence are available

IV. Conclusions

This paper presents a detailed survey on various 3D reconstruction methods in the field of image processing. In addition to that we have made a comparative analysis among those 3D reconstruction algorithms. One difference between the above three technique is that the information utilized is different. On the other hand, from the viewpoint of the number of data points, SFS and the3D morphable model are used to recover the whole surface of an object, i.e. a dense 3D representation, while SFM is generally adopted to estimate the depth values of some feature points, i.e. a sparse 3D representation. Therefore, SFM has a far smaller storage requirement than SFS and the 3D morphable model, which is very helpful for real-time applications. However, when they are applied to face recognition, SFS and the 3D morphable model generally have a better recognition performance than SFM because the information about more feature points is utilized. SFS shows poor and worse result on synthetic data and real images and shadow area is not recovered since shadow areas do not provide enough intensity information. But in case of 3D morphable the problematic part is the registration of the model to an image i.e. the fitting. SFM only perform when multiple frames of image sequence is available. The extensive bibliography in support of the different developments of 3d reconstruction algorithms research provided with the paper should be a great help to researchers in the feature.

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References

[1] Ikeuchi, Katsushi, and Berthold KP Horn. "Numerical shape from shading and occluding boundaries." *Artificial intelligence* 17.1 (1981): 141-184.

[2] Frankot, Robert T., and Rama Chellappa. "A method for enforcing integrability in shape from shading algorithms." *Pattern Analysis and Machine Intelligence, IEEE Transactions on* 10.4 (1988): 439-451.

[3] Horn, Berthold KP. "Height and gradient from shading." *International journal of computer vision* 5.1 (1990): 37-75.

[4] Szeliski, Richard. "Fast shape from shading." *CVGIP: Image Understanding* 53.2 (1991): 129-153.

[5] Vega, Omar E., and Yee-Hong Yang. "Shading logic: A heuristic approach to recover shape from shading." *Pattern Analysis and Machine Intelligence, IEEE Transactions on* 15.6 (1993): 592-597.

[6] Zheng, Qinfen, and Rama Chellappa. "Estimation of illuminant direction, albedo, and shape from shading." *Computer Vision and Pattern Recognition, 1991. Proceedings CVPR'91., IEEE Computer Society Conference on*. IEEE, 1991.

[7] Leclerc, Yvan G., and Aaron F. Bobick. "The direct computation of height from shading." *Computer Vision and Pattern Recognition, 1991. Proceedings CVPR'91., IEEE Computer Society Conference on*. IEEE, 1991.

[8] Lee, Kyoung Mu, and C-CJ Kuo. "Shape from shading with a linear triangular element surface model." *Pattern Analysis and Machine Intelligence, IEEE Transactions on* 15.8 (1993): 815-822.

[9] B.K.P. Horn, ªShape from Shading: A Method for Obtaining the Shape of a Smooth Opaque Object from One View,º PhD thesis, Massachusetts Inst. of Technology, 1970.

[10] Rouy, Elisabeth, and Agnès Tourin. "A viscosity solutions approach to shape-from-shading." *SIAM Journal on Numerical Analysis* 29.3 (1992): 867-884.

[11] Oliensis, John. "Shape from shading as a partially well-constrained problem." *Computer Vision and Pattern Recognition, 1991. Proceedings CVPR'91., IEEE Computer Society Conference on*. IEEE, 1991.

[12] Bichsel, Martin, and Alex P. Pentland. "A simple algorithm for shape from shading." (1992).

[13] Kimmel, Ron, and Alfred M. Bruckstein. "Tracking level sets by level sets: a method for solving the shape from shading problem." *Computer Vision and Image Understanding* 62.1 (1995): 47-58.

[14] Pentland, Alex P. "Local shading analysis." *Pattern Analysis and Machine Intelligence, IEEE Transactions on* 2 (1984): 170-187.

[15] Lee, Chia-Hoang, and Azriel Rosenfeld. "Improved methods of estimating shape from shading using the light source coordinate system." *artificial Intelligence* 26.2 (1985): 125-143.

[16] Pentland, Alex. "Shape information from shading: a theory about human perception." *Spatial vision* 4.2 (1989): 165-182.

[17] Ping-Sing, Tsai, and Mubarak Shah. "Shape from shading using linear approximation." *Image and Vision computing* 12.8 (1994): 487-498.

[18] Romdhani, Sami, and Thomas Vetter. "Efficient, robust and accurate fitting of a 3D morphable model." *Computer Vision, 2003. Proceedings. Ninth IEEE International Conference on*. IEEE, 2003.

[19] Blanz, Volker, and Thomas Vetter. "Face recognition based on fitting a 3D morphable model." *Pattern Analysis and Machine Intelligence, IEEE Transactions on* 25.9 (2003): 1063-1074.

[20] Tomasi, Carlo, and Takeo Kanade. "Shape and motion from image streams under orthography: a factorization method." *International Journal of Computer Vision* 9.2 (1992): 137-154.

[21] Bregler, Christoph, Aaron Hertzmann, and Henning Biermann. "Recovering non-rigid 3D shape from image streams." *Computer Vision and Pattern Recognition, 2000. Proceedings. IEEE Conference on*. Vol. 2. IEEE, 2000.

[22] Brand, Matthew. "Morphable 3D models from video." *Computer Vision and Pattern Recognition, 2001. CVPR 2001. Proceedings of the 2001 IEEE Computer Society Conference on*. Vol. 2. IEEE, 2001.

[23] Xiao, Jing, Jin-xiang Chai, and Takeo Kanade. "A closed-form solution to non-rigid shape and motion recovery." *Computer Vision-ECCV 2004*. Springer Berlin Heidelberg, 2004. 573-587.

[24] Brand, Matthew. "A direct method for 3D factorization of nonrigid motion observed in 2D." *Computer Vision and Pattern Recognition, 2005. CVPR 2005. IEEE Computer Society Conference on*. Vol. 2. IEEE, 2005.

[25] Maruyama, Minoru, and Satoshi Kurumi. "Bidirectional optimization for reconstructing 3D shape from an image sequence with missing data." *Image Processing, 1999. ICIP 99. Proceedings. 1999 International Conference on*. Vol. 3. IEEE, 1999.

[26] Torresani, Lorenzo, et al. "Tracking and modeling non-rigid objects with rank constraints." *Computer Vision and Pattern Recognition, 2001. CVPR 2001. Proceedings of the 2001 IEEE Computer Society Conference on*. Vol. 1. IEEE, 2001.

[27] Julià, Carme, et al. "Factorization with missing and noisy data." *Computational Science–ICCS 2006*. Springer Berlin Heidelberg, 2006. 555-562.

[28] Guerreiro, Rui FC, and Pedro MQ Aguiar. "3D structure from video streams with partially overlapping images." *Image Processing. 2002. Proceedings. 2002 International Conference on*. Vol. 3. IEEE, 2002.

[29] Torresani, Lorenzo, Aaron Hertzmann, and Christoph Bregler. "Learning non-rigid 3d shape from 2d motion." *Advances in Neural Information Processing Systems*. 2003.

[30] Torresani, Lorenzo, Aaron Hertzmann, and Christoph Bregler. "Nonrigid structure-from-motion: Estimating shape and motion with hierarchical priors." *Pattern Analysis and Machine Intelligence, IEEE Transactions on* 30.5 (2008): 878-892.

[31] Buchanan, Aeron M., and Andrew W. Fitzgibbon. "Damped newton algorithms for matrix factorization with missing data." *Computer Vision and Pattern Recognition, 2005. CVPR 2005. IEEE Computer Society Conference on*. Vol. 2. IEEE, 2005.

[32] Morgan, Aeron Buchanan. *Investigation into matrix factorization when elements are unknown*. Technical report, Visual Geometry Group, Department of Engineering Science, University of Oxford, 2004.

[33] Del Bue, Alessio, Fabrizio Smeraldi, and Lourdes Agapito. "Non-rigid structure from motion using non-parametric tracking and non-linear optimization." *Computer Vision and Pattern Recognition Workshop, 2004. CVPRW'04. Conference on*. IEEE, 2004.

[34] Olsen, Søren I., and Adrien Bartoli. "Using Priors for Improving Generalization in Non-Rigid Structure-from-Motion." *BMVC*. 2007.

[35] Del Bue, Alessio. "A factorization approach to structure from motion with shape priors." *Computer Vision and Pattern Recognition, 2008. CVPR 2008. IEEE Conference on*. IEEE, 2008.

[36] Akhter, Ijaz, et al. "Trajectory space: A dual representation for nonrigid structure from motion." *Pattern Analysis and Machine Intelligence, IEEE Transactions on* 33.7 (2011): 1442-1456.

[37] Fortuna, Jeff, and Aleix M. Martinez. "Rigid structure from motion from a blind source separation perspective." *International journal of computer vision* 88.3 (2010): 404-424.

[38] Koo, Hei-Sheung, and Kin-Man Lam. "Recovering the 3D shape and poses of face images based on the similarity transform." *Pattern Recognition Letters* 29.6 (2008): 712-723.

[39] Sun, Zhan-Li, Kin-Man Lam, and Qing-Wei Gao. "Depth estimation of face images using the nonlinear least-squares model." *Image Processing, IEEE Transactions on* 22.1 (2013): 17-30.

[40] Malik, Jitendra, and Dror Maydan. "Recovering three-dimensional shape from a single image of curved objects." *Pattern Analysis and Machine Intelligence, IEEE Transactions on* 11.6 (1989): 555-566.

