MAT 271 Project
Brianna Harte and Khalid Yahia

Dimensions of Cans

Type of can	Circumference	Height	Radius	Volume	Surface Area
Tomato Paste (Brianna Harte)	$16.5cm$	$8.1cm$	$8.25/\pi\ cm$	$V = 551.31/\pi$	$A = 133.65 + 136.125/\pi$
Morton Salt (Khalid Yahia)	$28cm$	$14cm$	$14/\pi\ cm$	$V = 2744/\pi$	$A = 392 + 392/\pi$

Photos of Cans

Tomato Paste Can Calculations

Constraint Function:

$$\frac{136.125}{\pi} + 133.65 = 2\pi r^2 + 2\pi rh$$

Objective Function:

$$V = \pi r^2 h$$

Calculations:

$$\frac{136.125}{\pi} + 133.65 - 2\pi r^2 = 2\pi rh$$

$$\frac{136.125}{2\pi^2 r} + \frac{133.65}{2\pi r} - r = h$$

First, we solve for one of the variables of the constraint function to plug into the objective function. I solved for h

$$V = \pi r^2 \left(\frac{136.125}{2\pi^2 r} + \frac{133.65}{2\pi r} - r \right)$$

Now, I plugged in h into the objective function and simplified.

$$V = \frac{136.125r}{2\pi} + \frac{133.65r}{2} - \pi r^3$$

Derivative Funciton

$$V' = \frac{272.25}{4\pi} + \frac{267.3}{4} - 3\pi r^2$$

$$V'' = -6\pi r$$

Critical Point:

$$0 = \frac{272.25}{4\pi} + \frac{267.3}{4} - 3\pi r^2$$

$$3\pi r^2 = \frac{272.25}{4\pi} + \frac{267.3}{4}$$

I set the derivative = to 0 to see where the graph has a max value

$$r^2 = \frac{272.25}{12\pi^2} + \frac{267.3}{12\pi}$$

$$r = +/- \sqrt{\frac{272.25}{12\pi^2} + \frac{267.3}{12\pi}}$$

$$\approx 3.064160092$$

I chose the positive number because a radius can't be negative.

Less than $r = \sqrt{\frac{272.25}{12\pi^2} + \frac{267.3}{12\pi}}$	$r = \sqrt{\frac{272.25}{12\pi^2} + \frac{267.3}{12\pi}}$	Greater than $r = \sqrt{\frac{272.25}{12\pi^2} + \frac{267.3}{12\pi}}$
+	MAX	-
	$V'' = -6\pi r$ <0 Concave Down= MAX	

Here, I made a chart and checked if the function was >0 or < 0 to see where the function increased and decreased. I also checked V'' to see if it concaved up or down at the critical point.

Finding the Height:

$$h = \frac{136.125}{2\pi^2 r} + \frac{133.65}{2\pi r} - r$$

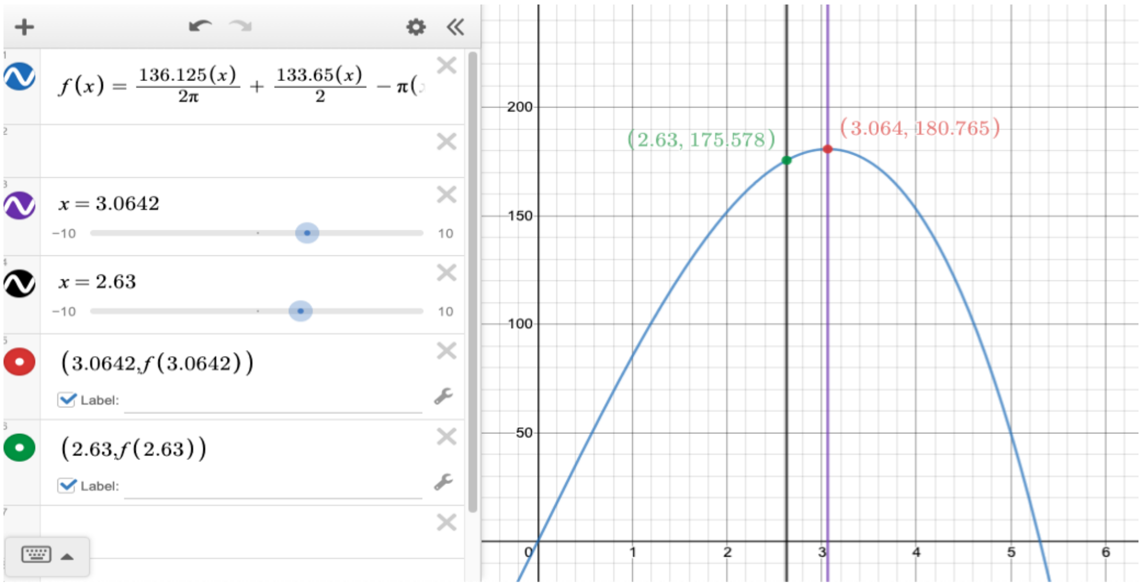
$$h = \frac{136.125}{2\pi^2 \left(\sqrt{\frac{272.25}{12\pi^2} + \frac{267.3}{12\pi}} \right)} + \frac{133.65}{2\pi \left(\sqrt{\frac{272.25}{12\pi^2} + \frac{267.3}{12\pi}} \right)} - \sqrt{\frac{272.25}{12\pi^2} + \frac{267.3}{12\pi}}$$

$$h = \frac{136.125 + 133.64\pi - 2\pi^2 \left(\sqrt{\frac{272.25}{12\pi^2} + \frac{267.3}{12\pi}} \right)^2}{2\pi^2 \left(\sqrt{\frac{272.25}{12\pi^2} + \frac{267.3}{12\pi}} \right)}$$

$$h \approx 6.127641151$$

I plugged r into the equation we created earlier to solve for the height when r is at the max.

Graph of Max Volume and Actual Volume:



I found the max on the graph of the optimized function.

Volume Check:

$$V = \pi\left(\sqrt{\frac{272.25}{12\pi^2} + \frac{267.3}{12\pi}}\right)^2\left(\frac{136.125 + 133.64\pi - 2\pi^2\left(\sqrt{\frac{272.25}{12\pi^2} + \frac{267.3}{12\pi}}\right)^2}{2\pi^2\left(\sqrt{\frac{272.25}{12\pi^2} + \frac{267.3}{12\pi}}\right)}\right)$$

$V \approx$ 180.765 (also found in Desmos)

I checked to make sure my volume added up to the one on Desmos to check my height.

How well optimized is the can:

$$\frac{|175.578 - 180.765|}{175.578} = \frac{5.187}{175.578} = .02954$$

The tomato paste can is 2.954% away from the volume being maximized.

Minimizing Surface Area with Original Volume:

Objective Function:

$$SA = 2\pi r^2 + 2\pi rh$$

Constraint Function:

$$\frac{551.31}{\pi} = \pi r^2 h$$

Calculations:

$$\frac{551.31}{\pi^2 r^2} = h$$

$$SA = 2\pi r^2 + 2\pi r\left(\frac{551.31}{\pi^2 r^2}\right)$$

First, I solved for a variable in the constraint function- I solved for h. Then, I plugged the h into the objective function to have only one variable in the equation.

$$SA = 2\pi r^2 + \frac{1102.62}{\pi r}$$

$$\textit{Derivatives}:$$

$$SA' = 4\pi r - \frac{1102.62\pi}{\pi^2 r^2}$$

$$SA'' = 4\pi + \frac{2205.24}{\pi r^3}$$

I calculated the derivatives.

$$\textit{Critical Point}:$$

$$0 = 4\pi r - \frac{1102.62\pi}{\pi^2 r^2}$$

$$4\pi r = \frac{1102.62\pi}{\pi^2 r^2}$$

$$4\pi^3 r^3 = 1102.62\pi$$

$$r^3 = \frac{1102.62}{4\pi^2}$$

$$r = \left(\frac{1102.62}{4\pi^2}\right)^{\frac{1}{3}}$$

$$\approx 3.034045176$$

I set the S' equation = 0 to solve for the critical points.

I put the critical point in the graph to make sure the function is a minimum by checking if the equation was – or + where r<0 and r>0. I found where it decreased and increased and found that it was a minimum.

2	$\left(\frac{1102.62}{4\pi^2}\right)^{\frac{1}{3}}$	4
-	0- MIN	+
Decreasing	$SA'' = 4\pi + \frac{2205.24}{\pi r^3}$ SA''>0 Concave up- MIN	Increasing

I plugged the critical point into SA'' function and saw that it was concaving up, so it was a minimum.

$$\textit{Finding the Height}:$$

$$h = \frac{551.31}{\pi^2 r^2}$$

I plugged r into the equation created in the beginning to solve for h.

$$h = \frac{551.31}{\pi^2\left(\left(\frac{1102.62}{4\pi^2}\right)^{\frac{1}{3}}\right)^2}$$

$$h \approx 6.068091056$$

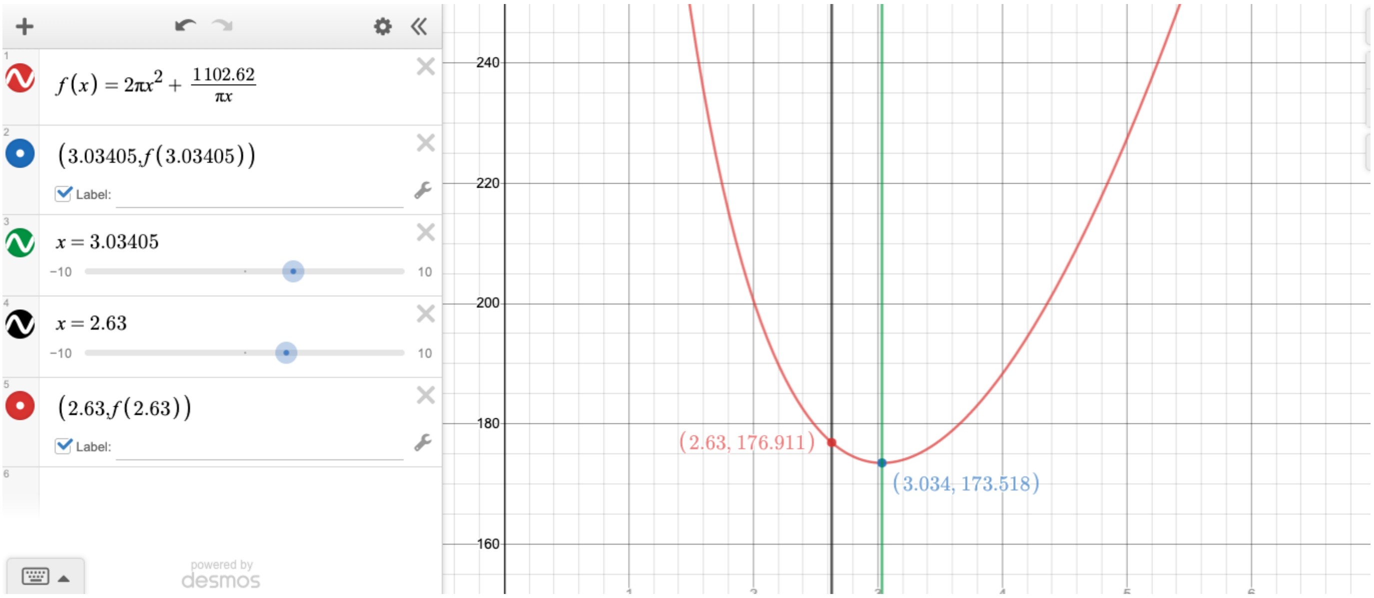
$$\textit{Volume Check}:$$

To make sure I had the right height, I plugged all the numbers in for surface area on my calculator and made sure they came out to the same surface area on Desmos.

$$V = 2\pi\left(\frac{1102.62}{4\pi^2}\right)^{\frac{1}{3}2} + 2\pi\left(\frac{1102.62}{4\pi^2}\right)^{\frac{1}{3}}\left(\frac{551.31}{\pi^2\left(\left(\frac{1102.62}{4\pi^2}\right)^{\frac{1}{3}}\right)^2}\right)$$

$$V = 173.5182$$

$Graph$:



$How\ well\ optimized\ is\ the\ can$:

$$\left|\frac{176.911 - 173.518}{176.911}\right| = .019179$$

The can is 1.9179% away from the surface area being minimized.

MORTON SALT Can's Calculations

Constraint Function:

$$\frac{392}{\pi}+392=2\pi r^2+2\pi rh$$

Objective Function:

$$V=\pi r^2 h$$

Calculations:

$$\frac{392}{\pi}+392-2\pi r^2=2\pi rh$$

$$\frac{392}{2\pi^2 r}+\frac{392}{2\pi r}-r=h$$

$$V=\pi r^2\left(\frac{392}{2\pi^2 r}+\frac{392}{2\pi r}-r\right)$$

$$V=\frac{196r}{\pi}+196r-\pi r^3$$

Derivative Funciton:

$$V'=\frac{196}{\pi}+196-3\pi r^2$$

$$V''=-6\pi r$$

Critical Point:

$$0=\frac{196}{\pi}+196-3\pi r^2$$

$$3\pi r^2=\frac{196}{\pi}+196$$

$$r^2=\frac{196}{3\pi^2}+\frac{196}{3\pi}$$

$$r=+/-\sqrt{\frac{196}{3\pi^2}+\frac{196}{3\pi}}$$

$$\approx 5.236019152$$

Less than	$r=\sqrt{\frac{196}{3\pi^2}+\frac{196}{3\pi}}$	Greater than
+	MAX	-
Increasing	V''= -6πr <0 Concave Down= MAX	Decreasing

Finding the Height:

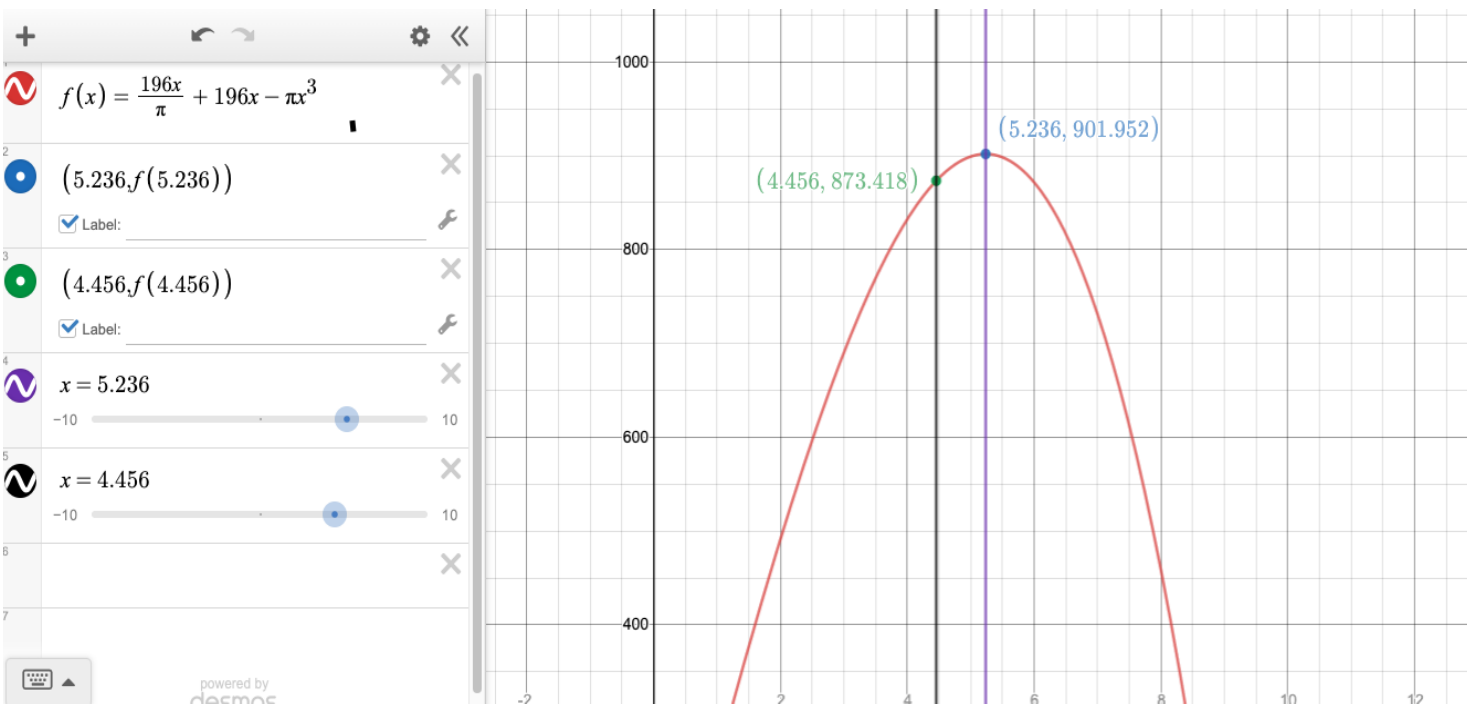
$$h = \frac{196}{\pi^2 r} + \frac{196}{\pi r} - r$$

$$h = \frac{196}{\pi^2\left(\sqrt{\frac{196}{3\pi^2} + \frac{196}{3\pi}}\right)} + \frac{196}{\pi\left(\sqrt{\frac{196}{3\pi^2} + \frac{196}{3\pi}}\right)} - \sqrt{\frac{196}{3\pi^2} + \frac{196}{3\pi}}$$

$$h = \frac{196 + 196\pi - 2\pi^2\left(\sqrt{\frac{196}{3\pi^2} + \frac{196}{3\pi}}\right)^2}{\pi^2\left(\sqrt{\frac{196}{3\pi^2} + \frac{196}{3\pi}}\right)}$$

$$h \approx 10.47211491$$

Graph:



Volume Check:

$$V = \pi\left(\sqrt{\frac{196}{3\pi^2} + \frac{196}{3\pi}}\right)^2\left(\frac{196 + 196\pi - 2\pi^2\left(\sqrt{\frac{196}{3\pi^2} + \frac{196}{3\pi}}\right)^2}{\pi^2\left(\sqrt{\frac{196}{3\pi^2} + \frac{196}{3\pi}}\right)}\right)$$

$V \approx 901.952$ (also found in Desmos)

How well optimized is the can:

$$\left|\frac{873.418-901.952}{873.418}\right| = 0.0627$$

The can is 6.27 % away from the volume being maximized.

Minimizing Surface Area with Original Volume:

$$Objective\ Function:$$
$$SA = 2\pi r^2 + 2\pi rh$$
$$Constraint\ Function:$$
$$\frac{2744}{\pi} = \pi r^2 h$$
$$Calculations:$$
$$\frac{2744}{\pi^2 r^2} = h$$
$$SA = 2\pi r^2 + 2\pi r\left(\frac{2744}{\pi^2 r^2}\right)$$
$$SA = 2\pi r^2 + \frac{5488}{\pi r}$$
$$Derivatives:$$
$$SA' = 4\pi r - \frac{5488\pi}{\pi^2 r^2}$$
$$SA'' = 4\pi + \frac{10976}{\pi r^3}$$
$$Critical\ Point:$$
$$0 = 4\pi r - \frac{5488\pi}{\pi^2 r^2}$$
$$4\pi r = \frac{5488\pi}{\pi^2 r^2}$$
$$4\pi^3 r^3 = 5488\pi$$
$$r^3 = \frac{5488\pi}{4\pi^3}$$
$$r = \left(\frac{5488}{4\pi^2}\right)^{\frac{1}{3}}$$
$$\approx 5.180258778$$

1	$(\frac{5488}{4\pi^2})^{\frac{1}{3}}$	7
-	0- MIN	+
Decreasing	$SA'' = 4\pi + \frac{10976}{\pi r^3}$ SA''>0 Concave up- MIN	Increasing

$\mathit{Finding\ the\ Height}$:

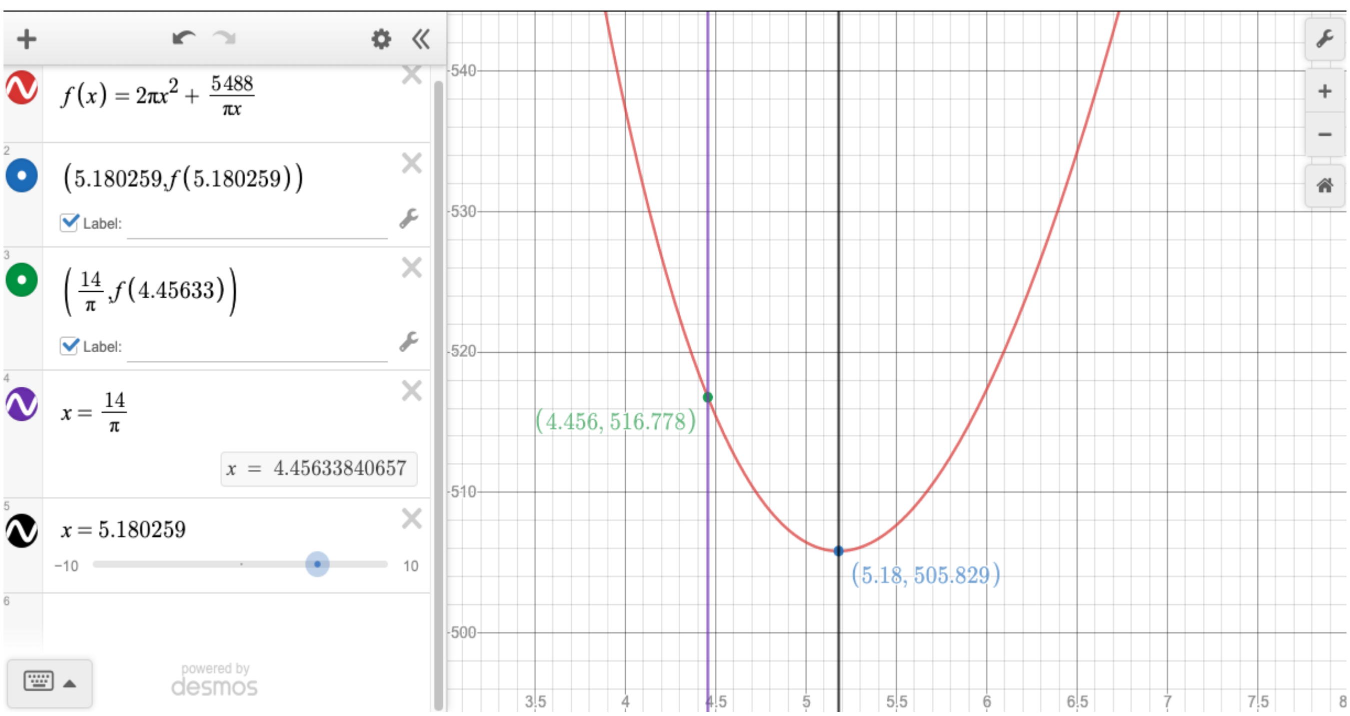
$$h = \frac{2744}{\pi^2 r^2}$$

$$h = \frac{2744}{\pi^2 (5.180258778)^2}$$

h =10.36052155

$\mathit{Volume\ Check}$:

$$V = 2\pi\,(5.180258778)^2 + 2\pi(5.180258778)(10.36052155)$$
$$V = 505.8294901$$



How well optimized is the can:

$$\left|\frac{516.778 - 505.829}{516.778}\right| = .02118704744$$

The can is 2.11870 % away from the surface area being minimized.

$Paragraph$:

Our best guess as to why companies do not maximize the volume and minimize the surface area of their can is because shipping costs are really high. When we do this project, we are only looking at the surface area and the volume of the can itself, not the trucks or the boxes they will be placed in. Each shipping truck is different in length, width and height, so maybe the cans being a little bit bigger and having a little less volume actually makes them fit better in the boxes they are shipping in. If the companies that make these cans have the businesses they are working for (tomato paste and salt company) best interest in mind, they want to save them money, and shipping is a massive cost. Spending a little more on the cans tin may actually be the most cost-effective way to ship the product. Imagine having a truck that is 10 feet high, and the boxes of cans are 6 feet tall, the truck can’t stack two boxes on each other because they would need 5-foot-tall boxes or smaller to do that. In other words, the company would have just paid to ship a smaller number of cans because the cans would fit into a 6-foot-tall box the best instead of a 5-foot box. Maybe having the cans the size they are would make them fit into the truck better.