Data Structures and Algorithms Design (SEZG519/SSZG519)

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S2 Characterizing Time Complexity, Asymptotic Notation, Recurrence Relation, Master Theorem

Content of S2

1. Characterizing Time Complexity
    1. Use of Asymptotic Notation
    2. Big-Oh, Big-Omega, Theta Notations
2. Analyzing Recursive Algorithm
    1. Recurrence Relation
    2. Runtime of Recursive Algorithm
    3. Master Theorem

Analyzing Algorithm

→Used to mean the prediction of resource consumption

→But, what is the resource?

Analyzing Algorithm

→Used to mean the prediction of resource consumption

→But, what is the resource?

Primarily i) memory, ii) communication bandwidth, iii) computer hardware

But, most often we are interested in **computational time**

Which computer should be taken as a base case or standard?

Random Access Machine (RAM) model of a computer

Random Access Machine Model

Instructions in RAM that takes one unit of time

1) Arithmetic: Add, Sub, Mul, Div, Rem, Floor, Ceil

2) Data movement: Load, Store, Copy

3) Control: Subroutine call, Return, Conditional and Unconditional Branch

Data Types in RAM (fixed size, like 8 bit or 16 bit or 32 bit)

1) Integer

2) Float

RAM model: What is not an instruction?

1) “Sort” – even if in some computer sort can be done in one struction
2) “exponentiation” – x^y

→ there may be many algorithms to compute x^y, but it is not a single instruction if y is a variable or a large integer

→ But, x^k is a single instruction, where k is a constant and very small

RAM model: memory hierarchy

We do not consider any complex memory hierarchy, like having cache or virtual memory.

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We do not consider any complex memory hierarchy, like having cache or virtual memory.

Simplicity of RAM model

→ Though simple, but an excellent predictor of performance on actual computer

→Though simple, exact prediction can be challenging

→Often, it would require tools like combinatorics, probability theory, algebraic dexterity and the ability to identify the most significant terms in a formula

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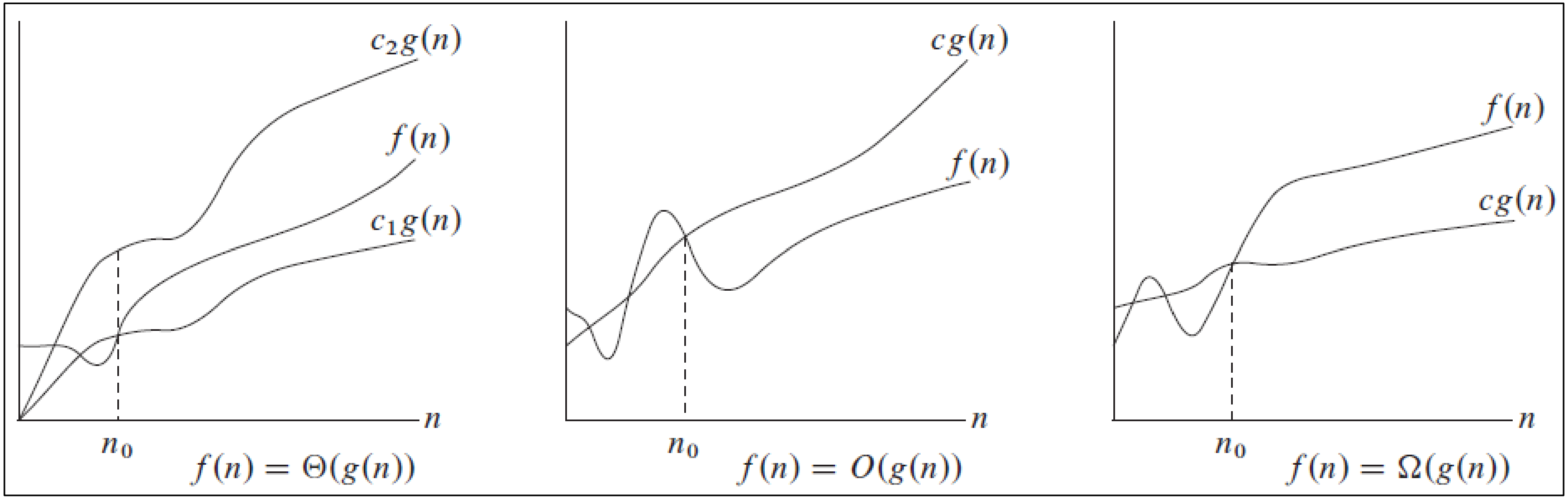
Characterizing Time Complexity

Big-Oh Notation, Omega and Theta Notations:

- Asymptotic notation primarily describes the running times of algorithms, i.e., time complexity



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Characterizing Time Complexity

Big-Oh Notation: f(n) = O(g(n)).

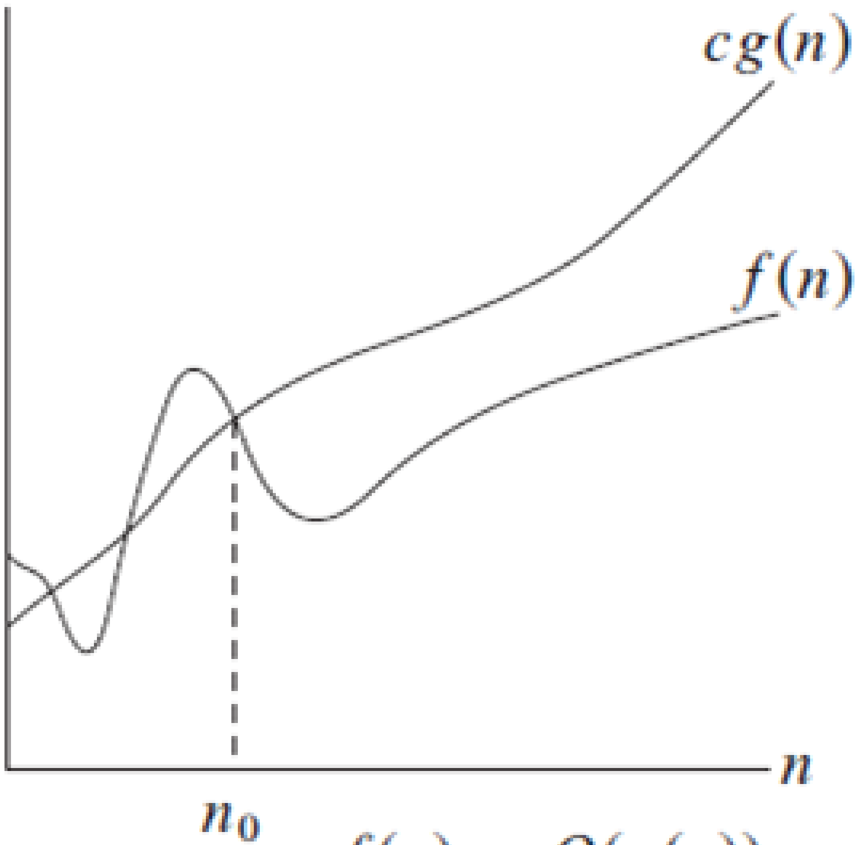
- g(n) is an asymptotically upper bound for f(n).

$$O(g(n)) = \{f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \le f(n) \le cg(n) \text{ for all } n \ge n_0\}.$$

- f(n) = Θ(g(n)) implies that f(n) = O(g(n)),

  i.e., Θ(g(n)) ϵ O(g(n))

Graphical representation of Big-O



$$f(n) = O(g(n))$$

Example: Time Complexity Big-O

Ex-1 $f(n) = 2n+2$

$2n+2 \leq \underline{10n}$, where $n \geq 1$

Here, $c = 10$, **$g(n) = n$**

$f(n) = O(g(n)) = O(n)$.

Ex-2 $f(n) = 2n+2$

$2n+2 \leq \underline{10n^2}$, where $n \geq 1$

Here, $c = 10$, **$g(n) = n^2$**

$f(n) = O(g(n)) = O(n^2)$.

Ex-3 $f(n) = 2n+2$

$2n+2 \leq \underline{10n^3}$, where $n \geq 1$

Here, $c = 10$, $g(n) = n^3$

$f(n) = O(g(n)) = O(n^3)$.

Ex-4 $f(n) = 2n^2+5$

$2n^2+5 \leq \underline{2n^2+5n^2} = 7n^2$, where $n \geq 1$

Here, $c = 7$, $g(n) = n^2$

$f(n) = O(g(n)) = O(n^2)$.

$$O(g(n)) = \{f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq f(n) \leq cg(n) \text{ for all } n \geq n_0\} .$$

Example: Time Complexity Big-O

Ex-5 f(n) =7n-2

Here, c = 7, n >=1

→ 7n - 2 ≤ cn, **g(n) = n**

f(n) = O(g(n)) = O(n).

Ex-6 f(n) = $20n^3$ + 10nlogn + 5

Here, c = 35, **g(n) = n^3**

f(n) = O(g(n)) = O(n^3).

Ex-7 f(n) = 3logn + loglogn

Here, c = 4, **g(n) = logn**

f(n) = O(g(n)) = O(logn).

Ex-8 f(n) = $\mathbf{2^{100}}$

Here, c = $\mathbf{2^{100}}$, **g(n) = 1**

f(n) = O(g(n)) = O(1).

Ex-9 f(n) = 5/n

Here, c = **5**, **g(n) =** 1/n

f(n) = O(g(n)) = O(1/n).

$$O(g(n)) = \{f(n) : \text{ there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \le f(n) \le cg(n) \text{ for all } n \ge n_0\}.$$

Time Complexity: Big-Omega

Omega Notation: f(n) = Ω(g(n)).

- g(n) is an asymptotically lower bound for f(n).

$$\Omega(g(n)) = \{ f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \le c g(n) \le f(n) \text{ for all } n \ge n_0 \} .$$

Example: Omega Notation

Ex-1 f(n) =2n+2

$2n+2 \geq \underline{2n}$, where $n \geq 1$

Here, c = 2, **g(n) = n**

$f(n) = \Omega(g(n)) = \Omega(n)$

Ex-2 f(n) =2n+2

$2n+2 \geq \underline{\sqrt{n}}$, where $n \geq 1$

Here, c = 1, **$g(n) = \underline{\sqrt{n}}$**

$f(n) = \Omega(g(n)) = \Omega(\underline{\sqrt{n}})$

Ex-3 f(n) =2n+2

$2n+2 \geq \underline{\log n}$, where $n \geq 1$

Here, c = 1, **g(n) = log n**

$f(n) = \Omega(g(n)) = \Omega(\log n)$

Ex-4 $f(n) = 2n^2+5$

$2n^2+5 \geq \underline{2n^2}$, where $n \geq 1$

Here, c = 2, **$g(n) = n^2$**

$f(n) = \Omega(g(n)) = \Omega(n^2)$.

$$\Omega(g(n)) = \{f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq cg(n) \leq f(n) \text{ for all } n \geq n_0\}.$$

Characterizing Run Time

Theta Notation: f(n) = Θ(g(n)).

- g(n) is an asymptotically tight bound for f(n).

$$\Theta(g(n)) = \{ f(n) : \text{there exist positive constants } c_1, c_2, \text{ and } n_0 \text{ such that } 0 \le c_1 g(n) \le f(n) \le c_2 g(n) \text{ for all } n \ge n_0 \}.$$ [1]

Example: Theta Notation

Ex-1 f(n) = $\frac{n^2}{2} - \frac{n}{2}$

$\frac{n^2}{4} \le \frac{n^2}{2} - \frac{n}{2} \le \frac{n^2}{2}$, where **n≥2**

$c_1 = \frac{1}{4}$, $c_2 = \frac{1}{2}$, **g(n) = n^2**

f(n) = Θ(g(n)) = Θ(n^2).

Ex-2 f(n) =$6n^3$ **{≠ Θ(n^2), why?}**

$c_1 n^2 \le 6n^3 \le c_2 n^2$, where n≥1

There exists no c_2 that implies $6n^3 \le c_2 n^2$

$$\Theta(g(n)) = \{f(n) : \text{there exist positive constants } c_1, c_2, \text{ and } n_0 \text{ such that } 0 \le c_1 g(n) \le f(n) \le c_2 g(n) \text{ for all } n \ge n_0\} .$$[1]

Time Complexity: Little-Oh, Little-omega

o-notation:

$$o(g(n)) = \{f(n) : \text{ for any positive constant } c > 0, \text{ there exists a constant } n_0 > 0 \text{ such that } 0 \le f(n) < cg(n) \text{ for all } n \ge n_0\} .$$

ω-notation:

$$\omega(g(n)) = \{f(n) : \text{ for any positive constant } c > 0, \text{ there exists a constant } n_0 > 0 \text{ such that } 0 \le cg(n) < f(n) \text{ for all } n \ge n_0\} .$$

Notation Summary

Because these properties hold for asymptotic notations, we can draw an analogy between the asymptotic comparison of two functions f and g and the comparison of two real numbers a and b:

$$
\begin{array}{lll}
f(n) = O(g(n)) & \text{is like} & a \leq b\,, \\
f(n) = \Omega(g(n)) & \text{is like} & a \geq b\,, \\
f(n) = \Theta(g(n)) & \text{is like} & a = b\,, \\
f(n) = o(g(n)) & \text{is like} & a < b\,, \\
f(n) = \omega(g(n)) & \text{is like} & a > b\,.
\end{array}
$$

Properties of Time Complexity

- Comparison

Transitivity:

$$f(n) = \Theta(g(n)) \text{ and } g(n) = \Theta(h(n)) \quad \text{imply} \quad f(n) = \Theta(h(n)) ,$$
$$f(n) = O(g(n)) \text{ and } g(n) = O(h(n)) \quad \text{imply} \quad f(n) = O(h(n)) ,$$
$$f(n) = \Omega(g(n)) \text{ and } g(n) = \Omega(h(n)) \quad \text{imply} \quad f(n) = \Omega(h(n)) ,$$
$$f(n) = o(g(n)) \text{ and } g(n) = o(h(n)) \quad \text{imply} \quad f(n) = o(h(n)) ,$$
$$f(n) = \omega(g(n)) \text{ and } g(n) = \omega(h(n)) \quad \text{imply} \quad f(n) = \omega(h(n)) .$$

Reflexivity:

$$f(n) = \Theta(f(n)) ,$$
$$f(n) = O(f(n)) ,$$
$$f(n) = \Omega(f(n)) .$$

Summary of Properties

- Comparison

Symmetry:

$$f(n) = \Theta(g(n)) \text{ if and only if } g(n) = \Theta(f(n)) .$$

Transpose symmetry:

$$f(n) = O(g(n)) \text{ if and only if } g(n) = \Omega(f(n)) ,$$

$$f(n) = o(g(n)) \text{ if and only if } g(n) = \omega(f(n)) .$$

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    3. Master Theorem

Analyzing Recursive Algorithms

Algorithm recursiveMax(A, n):
  Input: An array A storing $n \geq 1$ integers.
  Output: The maximum element in A.

if $n = 1$ **then**
  return $A[0]$
return $\max\{\text{recursiveMax}(A, n-1), A[n-1]\}$

Analyzing Recursive Algorithms

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if $n = 1$ **then**

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$$T(n) = \begin{cases} 2, if\ n = 1 \\ T(n-1) + 4, otherwise \end{cases}$$

Analyzing Recursive Algorithms

Algorithm recursiveMax(A, n):

Input: An array A storing $n \geq 1$ integers.

Output: The maximum element in A.

1 **if** $n = 1$ **then**

1 **return** $A[0]$

return max{recursiveMax($A, n-1$), $A[n-1]$}

1+ 1+ T(n-1) +1

$$T(n) = \begin{cases} 2, if\ n = 1 \\ T(n-1) + 4, otherwise \end{cases}$$

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Recurrence Relation

Def^n: A recurrence relation is an equation that defines a sequence based on a rule that gives the next term as a function of the previous term(s).

Mathematically, $x_{n+1} = f(x_n)$: a simple recurrence relation, also called as first order recurrence relation.

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Mathematically, $x_{n+1} = f(x_n)$: a simple recurrence relation, also called as first order recurrence relation.

Example of first order recurrence relation:

1) $x_{n+1}=2-x_{n/2}$

Recurrence Relation

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Mathematically, $x_{n+1} = f(x_n)$: a simple recurrence relation, also called as first order recurrence relation.

Example of first order recurrence relation:

1) $x_{n+1}=2-x_{n/2}$

A second order recurrence relation depends just on x_n and x_{n-1} and is of the form $x_{n+1}=f(x_n,x_{n-1})$

Example: $x_{n+1}=x_n+x_{n-1}$

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    3. **Master Theorem**

Analyzing Recursive Algorithms

Solving recurrence equations

1. Master Theorem for Dividing Functions

$$T(n) = aT\left(\frac{n}{b}\right) + g(n)$$

where g(n) is $O(n^k \log^p n)$, where p and k are integers.

a) $a < b^k$: if $p < 0$, then $T(n) = O(n^k)$

Analyzing Recursive Algorithms

Solving recurrence equations

1. Master Theorem for Dividing Functions

$$T(n) = aT(\frac{n}{b}) + g(n)$$

where g(n) is O($n^k \log^p$ n), where p and k are integers.

a) $a < b^k$: if $p < 0$, then $T(n) = O(n^k)$

if $p \geq 0$, then $T(n) = O(n^k \log^p n)$

Analyzing Recursive Algorithms

Solving recurrence equations

1. Master Theorem for Dividing Functions

$$T(n) = aT(\frac{n}{b}) + g(n)$$

where g(n) is O($n^k \, log^p$ n)

a) a < b^k : if p < 0, then T(n) = O(n^k)

if p ≥ 0, then T(n) = O($n^k \, log^p$ n)

b) a = b^k : if p > -1, then T(n) = O($n^k \, log^{p+1}$ n)

if p = -1, then T(n) = O(n^klog log n)

if p < -1, then T(n) = O(n^k)

Analyzing Recursive Algorithms

Solving recurrence equations

1. Master Theorem for Dividing Functions

$$T(n) = aT(\frac{n}{b}) + g(n)$$

where g(n) is $O(n^k \, log^p \, n)$

a) $a < b^k$: if $p < 0$, then $T(n) = O(n^k)$

if $p \geq 0$, then $T(n) = O(n^k \, log^p \, n)$

b) $a = b^k$: if $p > -1$, then $T(n) = O(n^k \, log^{p+1} \, n)$

if $p = -1$, then $T(n) = O(n^k \log \log n)$

if $p < -1$, then $T(n) = O(n^k)$

c) $a > b^k$: $T(n) = O(n^{log_b a})$

Solution using Master Theorem

g(n) is O($n^k \, log^p$ n)

Ex-1 T(n) = 4T($\frac{n}{2}$)+n,

a = 4, b = 2, k = 1, p = 0.

a = 4, b^k = 2 ➔ a > b^k

T(n) = O($n^{log_2 4}$) = O(n^2)

Ex-2 T(n) = 8T($\frac{n}{2}$)+n^2,

a = 8, b = 2, k = 2, p = 0.

a = 8, b^k = 4 ➔ a > b^k

T(n) = O($n^{log_2 8}$) = O(n^3)

Ex-3 T(n) = 8T($\frac{n}{2}$)+n log n,

a = 8, b = 2, k = 1, p = 1.

a = 8, b^k = 2 ➔ a > b^k

T(n) = O($n^{log_2 8}$) = O(n^3)

Solution using Master Theorem

Ex-4 $T(n) = 2T(\frac{n}{2})+n,$

$a = 2, b = 2, k = 1, p = 0.$

$a = 2, b^k = 2 \rightarrow a = b^k$

$T(n) = O(n^k \, log^{p+1} n)$

$= O(n \log n)$

Ex-5 $T(n) = 4T(\frac{n}{2})+n^2,$

$a = 4, b = 2, k = 2, p = 0.$

$a = 4, b^k = 4 \rightarrow a = b^k$

$T(n) = O(n^k \, log^{p+1} n)$

$= O(n^2 \log n)$

Ex-6 $T(n) = 4T(\frac{n}{2})+n^2 \log n,$

$a = 4, b = 2, k = 2, p = 1.$

$a = 4, b^k = 4 \rightarrow a = b^k$

$T(n) = O(n^k \, log^{p+1} n)$

$= O(n^2 \log^2 n)$

Solution using Master Theorem

Ex-7 $T(n) = 2T(\frac{n}{2}) + \frac{n}{\log n}$,

$a = 2, b = 2, k = 1, p = -1.$

$a = 2, b^k = 2 \Rightarrow a = b^k$

$T(n) = O(n^k \log \log n)$

$= O(n \log \log n)$

Ex-8 $T(n) = T(\frac{n}{2}) + n^2$,

$a = 1, b = 2, k = 2, p = 0.$

$a = 1, b^k = 4 \Rightarrow a < b^k$

$T(n) = O(n^k \; log^p \; n)$

$= O(n^2)$

Ex-9 $T(n) = 2T(\frac{n}{2}) + n^2 \log^2 n$,

$a = 2, b = 2, k = 2, p = 2.$

$a = 2, b^k = 4 \Rightarrow a < b^k$

$T(n) = O(n^k \; log^p \; n)$

$= O(n^2 \; log^2 \; n)$

Master Theorem for Decreasing Functions

$$T(n) = aT(n-b) + g(n)$$

where g(n) is $O(n^k)$

a) $a < 1 : T(n) = O(n^k)$

b) $a = 1 : T(n) = O(n^{k+1})$

c) $a > 1 : T(n) = O(n^k a^{n/b})$

Solution using Master Theorem

Ex-1 T(n) = T(n-1)+1,
a = 1, b = 1, k = 0.

T(n) = O(n^{k+1}) = O(n)

Ex-3 T(n) = 2T(n-1)+1,
a = 2, b = 1, k = 0.

T(n) = O($n^k a^{n/b}$)
= O(2^n)

Ex-2 T(n) = T(n-1)+n,
a = 1, b = 1, k = 1.

T(n) = O(n^{k+1}) = O(n^2)

Ex-4 T(n) = 2T(n-1)+n,
a = 2, b = 1, k = 1.

T(n) = O($n^k a^{n/b}$)
= O($n2^n$)

Correctness of Algorithms

- An algorithm is said to be correct
  - if, for every input instance, it halts with the correct output.
- We say that a correct algorithm
  - solves the given computational problem.
- An incorrect algorithm
  - might not halt at all on some input instances, or
  - it might halt with an incorrect answer.

Some Mathematics

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Ordering Functions by Their Growth Rates

n	$\log n$	$\sqrt{n}$	n	$n \log n$	n^2	n^3	2^n
2	1	1.4	2	2	4	8	4
4	2	2	4	8	16	64	16
8	3	2.8	8	24	64	512	256
16	4	4	16	64	256	4,096	65,536
32	5	5.7	32	160	1,024	32,768	4,294,967,296
64	6	8	64	384	4,096	262,144	1.84×10^{19}
128	7	11	128	896	16,384	2,097,152	3.40×10^{38}
256	8	16	256	2,048	65,536	16,777,216	1.15×10^{77}
512	9	23	512	4,608	262,144	134,217,728	1.34×10^{154}
1,024	10	32	1,024	10,240	1,048,576	1,073,741,824	1.79×10^{308}

$1 < \log n < \mathrm{sqrt}(n) < n < n \log n < n^2 < n^3 < \ldots < 2^n < 3^n < n^n$

Some Mathematics

- $\sum_{i=0}^{n} a^i = 1 + a + \cdots + a^n = \frac{1-a^{n+1}}{1-a}$
- $log_b a = c \; if \; a = b^c$
- $log_b ac = log_b a + log_b c$
- $log_b(a/c) = log_b a - log_b c$
- $log_b a^c = c log_b a$
- $log_b a = log_c a / log_c b$
- $b^{log_c a} = a^{log_c b}$

Case Studies: Analyzing Algorithms

```
Ex-1
#include <stdio.h>
void main(){
    int n=10;
    int a[n];
    a[3]=5;
    printf("%d",a[3]);
}
```

$T(n) = 1+(1+1) + (1+1) \rightarrow T(n) = O(1)$

```
Ex-2
#include <stdio.h>
void main(){
    int n; scanf(“%d”,&n);
    int a[n];
    for(int i=0;i<n;i++)
        scanf("%d",&a[i]);
    for(int i=0;i<n;i++)
        printf("%d",a[i]);
}
```

$T(n) = 2+(1+ (n+1) + 2(n)) + 2n + (1+ (n+1) + 2(n)) + 2n = 10n + 6$
$\rightarrow T(n) = O(n)$

Case Studies: Analyzing Algorithms

```
Ex-3
#include <stdio.h>
void main(){
 int n; scanf("%d",&n);
    int a[n];
    for(int i=0;i<n;i++)
        scanf("%d",&a[i]);
    for(int i=0;i<n;i++)
        for(int j=0;j<n;j++)
            printf("%d",a[i]);
}
```

T(n) = 2+(1+ (n+1) + 2(n)) + 2n + (1+ (n+1) + 2(n)) + n (1+ (n+1) + 2(n))
= $3n^2 + 10n + 6$
→ T(n) = O(n^2)

```
Ex-4
#include <stdio.h>
void main(){
 int n; scanf("%d",&n);
    int a[n];
    for(int i=0;i<n;i++)
        scanf("%d",&a[i]);
    for(int i=0;i<n;i++)
        for(int j=0;j<n/2;j++)
            printf("%d",a[i]);
}
```

T(n) = 2+(1+ (n+1) + 2(n)) + 2n + (1+ (n+1) + 2(n)) + n (1+ (n+1)/2 + 2(n/2))
→ T(n) = O(n^2)

Case Studies: Analyzing Algorithms

```
Ex-5
int findMinimum(int array[]) {
    int min = array[0];
    for(int i = 1; i < n; i++){
        if (array[i] < min) {
            min = array[i];
        }
    }
    return min;
}
```

T(n) = O(n)

Case Studies: Analyzing Algorithms

```
Ex-6
void fun(int n){
        if(n<=0)
                return;
        printf("%d",n);
        fun(n-1);
}
```

$T(n) = T(n-1) + 2 \rightarrow T(n) = O(n)$
Master Theorem for Decreasing Functions

```
Ex-7
void fun(int n){
        if(n<=0)
                return;
        printf("%d",n);
        fun(n/2);
}
```

$T(n) = T(n/2) + 2 \rightarrow T(n) = O(\log n)$
Master Theorem for Dividing Functions

Case Studies: Analyzing Algorithms

```
Ex-8
void fun(int n){
    if(n<=0)____1
        return;
    for(int i=0;i<k';i++) ____(k'+1)
        fun(n-1); ____k'*T(n-1)
}
```

(T(n) = 1 + (k'+1) + (k'(T(n-1)) = k'*T(n-1) + (k' + 2) ➔ T(n) depends on value of k' (Master Theorem for Decreasing Functions))*

```
Ex-9
void fun(int n){
    if(n>1){ ____1
        for(int i=0;i<n;i++) ____(n+1)
            printf("%d",i); ____n
        fun(n/2); ____T(n/2)
        fun(n/2); ____T(n/2)
    }
}
```

$T(n) = 1 + (n+1) + n + 2T(n/2) = 2T(n/2) + (2n + 2)$
$a = 2$, $b= 2$, $k = 1$, $p = 0$. $O(n \log n)$ as per Master Theorem for Dividing Functions

References

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Any Question!!


Thank you!!
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