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52 (2) MATH (II) 2·1

2016

MATHEMATICS-II

Paper : 2·1

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Give very short answer : ***(any eight)*** $1\times8=8$

   (i) What are Predicates ?

   (ii) Give an example of partially ordered relation.

   (iii) Give an example of a function which is neither one-one nor onto.

   (iv) Define a planner graph.

   (v) Define symmetric and skew-symmetric matrices.

Contd.

(iv) Define $f:\mathbb{R}\to\mathbb{R}$; $g:\mathbb{R}\to\mathbb{R}$ and $h:\mathbb{R}\to\mathbb{R}$ by $f(x)=x^4$;

$g(x)=x^3-4x$; $h(x)=\dfrac{1}{x^2+1}$.

Determine the following :

(a) $(f\circ h)(x)$ (b) $(g\circ h)(x)$

(v) Define linear dependence and linear independence of vectors in a vector space. Find the dimension of $Q\left(\sqrt{2},\sqrt{3}\right)$ over Q.

(vi) Write the Breadth First Search algorithm.

4. (i) Out of 250 students in a school 128 like tea, 87 like coffee and 134 like milk. Moreover 31 like both tea and coffee, 54 like both milk and tea and 30 like both milk and coffee. Determine the number of students who like none of the beverages. 5

Or

(ii) Show that the relation R defined by $(x,y)R(a,b)\Leftrightarrow x^2+y^2=a^2+b^2$ is an equivalence relation on a plane. Describe the equivalence class.

5. (i) Whether the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=x^2+3x-4$ is invertible ? If so, fined the inverse. 4

Or

(ii) Solve the following system of equations by Gaussian elimination method

$$x+2y+3z=14$$

$$2x+3y+4z=15$$

$$3x+4y+z=12$$

6. (i) Define basis of a vector space. Show that the vectors $(1,1,-1)$, $(2,-3,5)$ and $(-2,1,4)$ of R^3 are linearly independent. 5

(vi) Give an example to illustrate cyclic permutation.

(vii) Define tautology.

(viii) What is the number of edges in a simple graph with 10 vertices?

(ix) State the pigeonhole principle.

(x) Write the basis of R^3.

2. Solve ***any six :*** $3\times6=18$

(i) Prove that ${}^{n}C_k = {}^{n-1}C_{k-1} + {}^{n-1}C_k$

(ii) Verify Caylay-Hamilton theorem for the matrix $\begin{pmatrix} 3 & 2 \\ -2 & 4 \end{pmatrix}$

(iii) Obtain the CNF of the following expression $[q \vee (p \wedge q)] \wedge \sim [(p \vee r) \wedge q]$.

(iv) Define a binary tree and prove that in a binary tree, the number of vertices is always odd.

(v) For two sets A and B; prove that

$$A \setminus B = B^C \setminus A^C$$

(vi) If $n \geq 0$; prove that $\sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}$

(vii) Determine the number of sides of a regular polygon with 27 diagonals.

(viii) Simplify the following Boolean expression

$$xw + \overline{\overline{x}z} + (y + \overline{z})$$

3. Do ***any five :*** $4\times5=20$

(i) Find the rank of the following matrix

$$\begin{pmatrix} 2 & 4 & -2 & 2 \\ 1 & 2 & -3 & 0 \\ 3 & 6 & -4 & 3 \\ 2 & 4 & -6 & 0 \end{pmatrix}$$

(ii) Prove the following by mathematical induction: $n^2 > 2n+1;\ \forall n \geq 3$

(iii) Test the consistency of the following system of equations and solve if exist

$6y - 18z + 1 = 0$; $2x + 6y + 11 = 0$;

$6x + 2y - 6z = -3$