Total number of printed pages–6

52 (IT–2) 2·1

2019

MATHEMATICS–II

Paper : IT-2·1

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any ten*** of the following : $1 \times 10 = 10$

   (a) How many edges does a graph have with 12 vertices?

   (b) Give an example of 1-place predicate.

   (c) Give an example of antisymmetric relation.

   (d) State the Cayley-Hamilton theorem.

   (e) State the generalized pigeonhole principle.

Contd.

(f) Write the Euler's formula for a planner graph.

(g) Give an example of partially ordered relation.

(h) Determine the power set of $A = \{a, \{a\}\}$.

(i) For propositions p and q write the truth table for $p \to q$.

(j) Give an example of tautology.

(k) State the absorption law of Boolean algebra.

(l) What is the rank of the matrix $\begin{pmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$?

(m) How many ways 5 persons can sit around a circular table?

2. Answer ***any five*** of the following : 2×5=10

(a) Determine the eigenvalues for the matrix $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$.

(b) Prove that maximum number of edges in any simple graph with n-vertices is given by $\frac{n(n-1)}{2}$.

(c) Find the number of triangle that can be obtained by joining 12 points on a plane out of which 5 points are collinear.

(d) Using algebra of proposition show that $(p \wedge q) \vee (p \wedge \sim q) \equiv p$.

(e) Express $E(x, y, z) = x(y'z)'$ in its complete sum of products form.

(f) Define many one-into function with an example.

(g) Negate the following statements :

(i) If tomorrow is holiday then we will go either to cinema or market.

(ii) A is divisible of 7 iff the sum of the digits is divisible by 7.

3. Answer ***any five*** of the following : 3×5=15

(a) Illustrate with an example that every relation is not necessarily a function.

(b) Prove by mathematical induction that the sum of the cubes of three consecutive natural numbers is divisible by 9.

(c) Express the following matrix as sum of symmetric and skew-symmetric matrices.

$$\begin{pmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{pmatrix}$$

(d) Find the number of sides of a polygon whose number of diagonal is 27.

(e) For real-valued functions $f: A \to B$, $g: B \to C$ and $h: C \to D$ prove that $h \circ (g \circ f) = (h \circ g) \circ f$.

(f) Obtain the principal conjunctive normal form of $(\sim p \to q) \wedge (q \leftrightarrow p)$ without using truth table.

(g) Prove : ${}^{n}C_k = {}^{n-1}C_{k-1} + {}^{n-1}C_k$.

4. Answer ***any five*** of the following : 5×5=25

(a) Prove that if R is an equivalence relation on a set A, then R^{-1} is also an equivalence relation on A.

(b) Among the first 500 positive integers determine the integer which are not divisible by 2, nor by 3 nor by 5.

(c) Using truth table show that the proposition $\sim (q \to r) \wedge r \wedge (p \to q)$ is a contradiction.

(d) Show that composition of relation is associative.

(e) Define basis and dimension of a vector space. Under what condition the union of two subspaces of a vector space is a subspace? Justify your answer.

(f) Define rank of a matrix. Find the rank of the following matrix by reducing to echelon form.

$$\begin{pmatrix} 2 & 4 & -2 & 2 \\ 1 & 2 & -3 & 0 \\ 3 & 6 & -4 & 3 \\ 1 & 2 & -1 & 1 \end{pmatrix}$$

(g) State Cayley-Hamilton theorem and verify for the matrix $\begin{pmatrix} 2 & -2 \\ 3 & 4 \end{pmatrix}$.

(h) Define the following : ***(any five)***

Binary tree, simple graph, complete graph, regular graph, circuit, path, bi-partite graph, Hamiltonian cycle.