all variables.

Theorem: Let α be a boolean expression in the variables $x_1, x_2, \ldots x_n$. Suppose that $\alpha_1, \alpha_2, \ldots \alpha_m$ are mean terms in the variables $x_1, x_2, \ldots x_n$ such that,

$$\alpha = \alpha_1 + \alpha_2 + \alpha_3 + \ldots \alpha_m.$$

Then, for any ~~a~~ assignment

$$x_1 = b_1, x_2 = b_2, \ldots x_n = b_n$$

of values $b_1, b_2, \ldots, b_n \in \{0,1\}$

$\alpha(b_1, b_2, \ldots b_n) = 1$ iff

$\alpha_i(b_1, b_2 \ldots b_n) = 1$ for some i, where $1 \leq i \leq n$.

Q Let $f: B^3 \to B$ be a function. We know that f is complete know if we know the ~~in~~ image of every triple $(b_1, b_2, b_3) \in B^3$ under f.

Suppose that $f(x_1, x_2, x_3)$ is given by the following table.

Row	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
	1	1	1	1
2	1	1	0	0
2	1	0	1	0
3	1	0	0	1
4	0	1	1	1
5	0	1	0	0
6	0	0	1	1
7	0	0	0	0
8				

show that function f is a boolean function clearly from the table $f(x_1 . x_2 . x_3)$ takes the value 1 for assignment 1 or 0 to x_1, x_2, x_3. in the 1^{st}, 4^{th}, 5^{th}, & 7^{th} rows of the table.

for each of these rows, we construct the mean term $\tilde{x}_1, \tilde{x}_2, \tilde{x}$ satisfying the following condition,

$$x_i = \alpha \begin{cases} x_i & if\ x_i = 1 \\ x_i' & if\ x_i = 0 \end{cases}$$

for the 1^{st} row, the mean term is

$$\alpha_1 = x_1 x_2 x_3.$$

for the 4^{th} row, the mean term is

$$\alpha_4 = x_1 x_2' x_3'$$

for the 5^{th} row, the mean term is

$$\alpha_5 = x_1' x_2' x_3$$

for the 7^{th} row, the mean term is

$$\alpha_7 = x_1' x_2' x_3$$

Let us consider $\alpha = \alpha_1 + \alpha_4 + \alpha_5 + \alpha_7$

$$b_1 = 1, b_2 = 1, b_3 = 1$$

$$\alpha_1 (1, 1, 1) = 1.$$

in other words, $\alpha_1 (b_1, b_2, b_3) = 0$.

and $\alpha_1 (b_1, b_2, b_3)$ if $(b_1, b_2, b_3) \neq (1, 1, 1)$

similarly, $\alpha_4 (b_1, b_2, b_3) = 1$

$$b_1 = 1, b_2 = 1, b_3 = 0.$$

iff $(b_1, b_2, b_3) = 1$.

Again,

$\alpha_5(b_1, b_2, b_3) = 1$

iff $\alpha_5(b_1, b_2, b_3) = (0, 1, 1)$

; $\alpha_7(b_1, b_2, b_3) = 1$

iff $\alpha_7(b_1, b_2, b_3) = (0, 0, 1)$

Now, we define $f : B^3 \to B$

$$f\alpha(b_1, b_2, b_3) = \alpha(b_1, b_2, b_3)$$
$$= \alpha_1(b_1, b_2, b_3) + \alpha_4(b_1, b_2, b_3) + \alpha_5(b_1, b_2, b_3) + \alpha_7(b_1, b_2, b_3)$$

$$f\alpha(1, 1, 1) = \alpha(1, 1, 1)$$
$$= \alpha_1(1, 1, 1) + \alpha_4(1, 1, 1) + \alpha_5(1, 1, 1) + \alpha_7(1, 1, 1)$$
$$= 1 + 0 + 0 + 0$$
$$= 1.$$

$\because \alpha_1(1, 1, 1) = 1$, $\alpha_4(1, 1, 1) = 0$

$\alpha_5(1, 1, 1) = 0$, $\alpha_7(1, 1, 1) = 0$.

Similarly,

$$f\alpha(1, 1, 0) = \alpha(1, 1, 0)$$
$$= 0 + 0 + 0 + 0$$
$$= 0$$

$$f\alpha(1, 0, 1) = \alpha(1, 0, 1)$$
$$= 0$$

$$f\alpha(1, 0, 0) = \alpha(1, 0, 0)$$
$$= 1.$$

$$f_\alpha(0,1,1) = \alpha(0,1,1)$$
$$= \underline{1}$$
$$f_\alpha(0,1,0) = \alpha(0,1,0)$$
$$= 0.$$
$$f_\alpha(0,0,1) = \alpha(0,0,1)$$
$$= \underline{1}$$
$$f_\alpha(0,0,0) = \alpha(0,0,0)$$
$$= 0$$

Hence, $f = f_\alpha$

Complete Disjunctive Normal Form

A DNF in n variables, which contains 2^n terms is called the complete DNF in n variable.

For example, the expression, $xy + x'y + xy' + x'y'$ is called the complete DNF in two variables x & y.

Complement of a Boolean expression in DNF

It is the boolean expression which is formed by the sum of exactly those terms of the complete ~~&~~ DNF missing from given DNF

Example — The complement of the boolean expression in DNF $xy' + xy$ is $x'y + x'y'$

Q. Find the complement of the boolean expression

$\cancel{xyz + x'y'z'}$.

$xyz' + x'yz + x'y'z' + x'y'z + xy'z'$

Solⁿ: The complete DNF in 3 variable is

$xyz + x'yz + x'y'z' + x'y'z + xy'z' +$
$x'y'z' +$

$xyz + x'yz + xy'z + xyz' + x'y'z$

$+ x'yz' + xy'z' + x'y'z'$

$\therefore$ Complement is $- xy'z + xyz' + x'yz'$

Q. Consider the Boolean function $f(x, y, z)$ given by the following table.

x	y	z	$f(x,y,z)$
0	0	0	0
0	0	1	0
0	1	0	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Q Find the Boolean exp. α in DNF such that $f_\alpha = f$

Combinatorics

<u>Pigeonhole principle</u> — In mathematics the ~~pect~~ pigeonhole principle states that, if 'n' items are put into 'm' containers, with $n > m$ then at least one container must contain more than one item.

Eg — (i) Birthday program —

If there are 367 people in a room then by pigeon hole principle there is at least ~~one~~ one pair of people who share the same ~~room~~ b'day with 100% probability, as there are 366 possible b'days to choose from (including 29 feb if present).

(ii) Irreversible gloves —

If a person has 3 gloves (none is reversible) then there must be at least 2 right handed gloves or 2 left handed gloves.

Logic

Connectives :

The operational symbol $\sim, \wedge, \vee, \rightarrow, \leftrightarrow$ are called connectives.

Statement formula :

A statement formula is defined as followed :-

(i) Any statement letter is a statement formula

(ii) If A & B are statement formulae, then the expressions $\sim A$, $A \wedge B$, $A \vee B$, $A \rightarrow B$, and $A \leftrightarrow B$ are also statement formulae.

(iii) Only those expressions are statement formulae which are constructed by using one & two.

Use of ~~Brackets in~~ Brackets in statement formula

(i) The expression ~~$p \wedge q$~~ $p \wedge q \rightarrow r$ without any bracket might mean the same statement formula $(p \wedge q) \rightarrow r$ or $p \wedge (q \rightarrow r)$. But these two formulas are not equivalent. So, we need to bracket properly to write a statement formula.

(ii) $(\sim p)$ can be written as $\sim p$.

(iii) $p \to q \to r \to p \to s$

stands for ~~$p \to q$~~

$$((((p \to q) \to r) \to p) \to s)$$

(iv) $((p \wedge q) \wedge p)$ is equivalent to $p \wedge q \wedge r$

(v) The connectives are ordered according to their scopes (in increase order) as follows —

$\sim, \wedge, \vee, \to, \leftrightarrow$

i.e. '$\sim$' has the least scope and $\leftrightarrow$ has the maximum scope.

(vi) The component of a statement formula is, by ~~is~~ itself a statement formula.

Eg — $(p \wedge \sim r)$ is a component of the statement formula $(p \wedge \sim r) \to q$

Removal of brackets

Brackets are eliminated from the formula

$$(((p \vee (\sim q)) \rightarrow r) \Leftrightarrow p)$$

(i) $((p \vee (\sim q)) \rightarrow r) \Leftrightarrow p$

Outer brackets are removed.

(ii) $((p \vee \sim q) \rightarrow r) \Leftrightarrow p$

brackets enclosing $\sim q$ are removed.

(iii) $(p \vee \sim q \rightarrow r) \Leftrightarrow p$

brackets surrounding $p \vee \sim q$ are removed

(iv) $p \vee \sim q \rightarrow r \Leftrightarrow p$

brackets enclosing $p \vee \sim q \rightarrow r$ are removed

(following the scope)

So, the given formula can be written as

$$p \vee \sim q \rightarrow r \Leftrightarrow p$$

Similarly, we can restore bracket in a statement formula.

Q Write a truth ~~tap~~ table for the statement formula

$$\sim(\sim p \wedge q)$$

p	q	$\sim p$	$\sim p \wedge q$	$\sim(\sim p \wedge q)$
T	T	F	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

<u>Tautology</u> — A statement formula is called a tautology if ~~A~~ in its truth table, the column under the statement formula contains only T's (Truth) i.e. if its truth function always takes the value T. Eg — $P \vee \sim P$ ~~are tautology~~

$((P \to q) \wedge (q \to r)) \to (P \to r)$ are tautology

<u>Contradiction</u> — A statement formula is called a contradiction if in its truth table, the column under the formula contain only F's i.e. if its truth function takes always the value F.

Eg — $P \wedge \sim P$

$(P \wedge q) \wedge \sim (P \vee q)$

<u>Note</u>: A statement formula which is neither a tautology nor a contradiction is called a contingent.

<u>Logical Consequence</u>:

If $A \rightarrow B$ is a tautology, then, A is said to <u>logically imply</u> B. Or alternatively B is said to be a <u>logical consequence</u> of A

<u>Logical Equivalence</u>:

If $A \Leftrightarrow B$, is a tautology then A & B are said to be logically equivalent and we denote this by $A \equiv B$

Q1 Construct a truth table for the following formulas

$$(p \rightarrow q) \longrightarrow (\sim q \rightarrow \sim p)$$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \rightarrow q$
T	F	T
T	F	F
F	T	T
F	F	T

$p \rightarrow q$

Q p = college
q = Out

---

sol^n 1

p	q	$\sim p$	$\sim q$	$(p \rightarrow q)$	$(\sim q \rightarrow \sim p)$	A
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

∴ Tautology

Q Using truth ~~tables~~ table show the following.

(a) $\sim(p \wedge q) \equiv \sim p \vee \sim q$

(b) $p \longrightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow q)$

(a)

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

∴ $\sim(p \wedge q) \equiv \sim p \vee \sim q$

~~Transformation of a normal form to the other form~~

CNF

A Boolean expression α in the variables $x_1, x_2, \ldots x_n$ is called a ~~maxim~~ maximum term if $\alpha = \tilde{x}_1 + \tilde{x}_2 + \cdots + \tilde{x}_n$ where $\tilde{x}_i$ denotes ~~x_i or~~ $x_i \wedge x_i'$

Eg- The expression $x+y+z'$, $x'+y+z'$, $x'+y'+z'$ are examples of maximum terms in the variables x, y, z.

A boolean expression $\alpha(x_1, x_2, \ldots, x_n)$ is said to be in CNF if there exist distinct max terms $\alpha_1, \alpha_2, \ldots \alpha_m$ in the variables $x_1, x_2, \ldots, x_n$. Such that

$\alpha = \alpha_1 \alpha_2 \cdots \alpha_m$

Eg - The boolean expressions $(x+x')$, ~~$(x+y')$~~, $(x+y')(x'+y)(x+y)(x'+y')$, $(x+y+z)(x'+y+z)(x+y'+z)(x+y+z')$ are examples of CNF in 1, 2, 3 variables respectively.

Note: In a DNF, the expression is a sum of products whereas, in a CNF, the expression is a product of sums.

* Poset — a set equiped with a partial order relation R defined on A

~~Q3 Express the Boolean expression~~

~~A set~~

Soln: Let A is any ~~st set~~ subset of real number then $[A : \leq]$ is a partial order relation is a poset

Q3. Express the Boolean expression in CNF

$$(x+y+z)(xy+x'z)'$$

$$= (x+y+z)\{(xy)' \cdot (x'z)'\} \quad \text{(using De Morgan's Law)}$$

$$= (x+y+z)\{(x'+y')\cdot(x+z')\} \quad \text{(using " ")}$$

$$= (x+y+z)\{(x'+y')+0\}\{(x+z')+0\}$$

$$= (x+y+z)\{(x'+y')+z\cdot z'\}\{(x+z')+y\cdot y'\}$$

$$\cancel{= (x+y+z)\{x'zz'+y'zz'\}\{xyy'+z'y\cdot y'\}}$$

$$\cancel{= (x+y+z)\,x}$$

$$= (x+y+z)\{(x'+y'+z)\cdot(x'+y'+z')\}\{(x+z'+y)(x+z'+y')\}$$

(Distributive Law)

$$= (x+y+z)(x'+y'+z)(x'+y'+z')(x+z'+y)(x+z'+y')$$

Transformation of a normal form to the other form

We can transform a Boolean expression α in DNF to an expression in CNF or a CNF to a DNF simply by the following method based on the ~~basic~~ fact that

$$(\alpha')' = \alpha$$

Let α be a Boolean expression in DNF, we first find α' using the complete DNF method i.e. α' will be the sum of the terms of the complete DNF missing from α. Then we will find $(\alpha')'$ using De Morgan's Law. The resulting exp. is equivalent to α in CNF. Here the order of using the above 2 methods to find the complements is immaterial.

Similarly, we can transform an exp. in CNF & DNF.

Eg —

$$\alpha = xy + x'y + x'y'$$

$$\alpha' = xy'$$

$$\alpha = (\alpha')'$$

$$= (xy')' = x' + y$$

Alternatively,

$$\alpha = xy + x'y + x'y'$$

$\therefore$ α' using de-margan law can be obtained as

$$\alpha' = (xy + x'y + x'y')'$$
$$= (xy)' \cdot (x'y)' \cdot (x'y')'$$
$$= (x'+y')(x+y')(x+y)$$

$$(\alpha')' = \alpha = x'+y$$

H/W. 1) Find the complement of each of the following boolean expressions in CNF

(a) $(x+y)(x+y')(x'+y')$

(b) $(x+y+z')(x+y'+z')(x'+y+z')(x'+y'+z)$
$(x'+y'+z')$

2) Transform each of the following DNF into CNF.

(a) $xy + x'y$

(b) $xyz + x'yz' + xy'z' + x'y'z + x'y'z'$

<u>Logic</u> — Two important pillars of mathematical logic are propositional logic ~~and~~ (propositional calculus) and predicate logic (predicate Calculus)

Two basic objects in a propositional logic are statements (or proposition) and statement formulas

<u>Statement</u> — By a statement we mean a declarative sentence that can be classified as true or false but not both.

The truth or falsity of a statement is called its truth value denoted by 'T' or 1. and 'F' or 0 respectively.

The truth value of a statement is determined or assigned from our existing knowledge or by some rules. We will denote statements by p, q, r, ---

Eg — (i) p : 3 is a real number.
∴ p is a statement & its truth value is T or 1.

(ii) q : What is your name?

q is not a statement because it is not a declarative sentence, and has no truth value.

(iii) r : I am a liar.

Even though r is a declarative sentence, it is not a statement. For if the statement is true then I am ~~not~~ not lying and hence not a liar, so the sentence will be false. Again if the sentence is false then I am lying and hence a liar. Hence the sentence has both truth values T & F simultaneously and has no definite truth value.

(iv) s : x is a real number.

Even though s is a declarative sentence, it is not a statement, because we cannot assign any truth values to s.

Operations on Statement

(i) Negation (Not) — Negation is an unary operation on statement. If p is a statement then, its negation denoted ~~by~~ by '$\sim p$' is another statement

~~for conjunction~~

(∧) Conjunction (AND) — It is a binary operation on statement, two statements ~~p&~~ p, q can be combined by writting the word 'AND' in between them to form a new statement, called the conjunction of p & q denoted by $p \wedge q$.

(∨) Disjunction (OR) — Two statements p, q can be combined by ~~writting~~ writing the word 'OR' in between them, to form a new statement P OR Q called ~~dis~~ disjunction of P and Q denoted by ~~P∨Q~~

$p \vee q$

Conditional (or Implication): It is a binary operation on statement. Two statements P, q can be combined by writting the word 'if' before p and the word 'than' ~~the~~ before q. ~~B~~ Thus, the conditional is ~~$~~ If p then q denoted by $p \rightarrow q$, where p is called ~~antecedent~~ ~~ante~~ 'antecedent' and q is called 'consequent'

Biconditional (or Biimplication) : It combines two statements p, q to form a new statement, p if and only if q.

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$$\boxed{A \cdot adj(A) = |A| I_n}$$

$$A_{11} = (-1)^{1+1}\left[(a_{22}-\lambda)(a_{33}-\lambda) - a_{23}a_{32}\right]$$

$$A_{12} = (-1)^{1+2}\left[\right.$$

$(n-1)$ or less

$$(A-\lambda E)\left(B_0\lambda^{n-1}+B_1\lambda^{n-2}+\cdots+B_{n-1}\right)$$
$$=(-1)^n\left[\lambda^n+a_1\lambda^{n-1}+a_2\lambda^{n-2}+\cdots+a_n\right]I_n$$

$$\Rightarrow AB_0\lambda^{n-}+AB_1\lambda^{n-2}+\cdots+AB_{n-1}-\lambda B_0\lambda^{n-1}$$
$$-\lambda B_1\lambda^{n-2}+\cdots-\lambda B_{n-1}$$
$$=(-1)^n\left[\lambda^n+a_1\lambda^{n-1}+\cdots+a_n\right]I_n$$

$$\Rightarrow AB_0-B_1=(-1)^n a_1 I_n$$

$\vdots$

$(a+b)(a'+b)(a+b')$

$(xy + x' + y')$

$= xy + x' \cdot 1 + y' \cdot 1$

$= xy + x'(y+y') + y'(x+x')$

$= xy + x'y + x'y' + y'x + y'x'$

(H/W) Write the definations of one-one, onto, bijection and give examples of them.