Machine Learning Homework 6

R04922034

1.

$$
\begin{aligned}
&\min_{A,B} F(A,B) = \frac{1}{N}\sum_{n=1}^{N} ln(1+exp(-y_n(A_zn+B))) \\
&\frac{\partial F}{\partial A} = \frac{1}{N}\sum_{n=1}^{N} \frac{1}{1+exp(-y_n(Az_n+B))} exp(-y_n(Az_n+B))(-y_n(z_n)) \\
&= \frac{1}{N}\sum_{n=1}^{N} p_n(-y_n z_n) \\
&\frac{\partial F}{\partial B} = \frac{1}{N}\sum_{n=1}^{N} \frac{1}{1+exp(-y_n(Az_n+B))} exp(-y_n(Az_n+B))(-y_n) \\
&= \frac{1}{N}\sum_{n=1}^{N} p_n(-y_n) \\
&\nabla F = [\frac{1}{N}\sum_{n=1}^{N} p_n(-y_n z_n) \quad \frac{1}{N}\sum_{n=1}^{N} p_n(-y_n)]^T
\end{aligned}
$$

2.

$$
\begin{aligned}
\frac{\partial^2 F}{\partial A^2} &= \frac{\partial \frac{1}{N}\sum_{n=1}^{N} p_n(-y_n z_n)}{\partial A} \\
&= -\frac{1}{N}\sum_{n=1}^{N} y_n z_n \frac{\partial p_n}{\partial A} \\
&= -\frac{1}{N}\sum_{n=1}^{N} y_n z_n \frac{\partial}{\partial A}(1 - \frac{1}{1+exp(-y_n(Az_n+B))}) \\
&= -\frac{1}{N}\sum_{n=1}^{N} y_n z_n \frac{\partial}{\partial A}\frac{-1}{1+exp(-y_n(Az_n+B))} \\
&= -\frac{1}{N}\sum_{n=1}^{N} y_n z_n (\frac{1}{1+exp(-y_n(Az_n+B))})^2 exp(-y_n(Az_n+B))(-y_n z_n) \\
&= \frac{1}{N}\sum_{n=1}^{N} z_n^2 (\frac{1}{1+exp(-y_n(Az_n+B))}) p_n \\
&= \frac{1}{N}\sum_{n=1}^{N} z_n^2 (1-p_n) p_n
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 F}{\partial A \partial B} &= \frac{\partial \frac{1}{N}\sum_{n=1}^{N} p_n(-y_n)}{\partial A} \\
&= \frac{1}{N}\sum_{n=1}^{N} (-y_n) \frac{\partial p_n}{\partial A} \\
&= \frac{1}{N}\sum_{n=1}^{N} (-y_n) p_n (1-p_n)(-y_n z_n) \\
&= \frac{1}{N}\sum_{n=1}^{N} z_n p_n (1-p_n)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 F}{\partial B^2} &= \frac{\partial \frac{1}{N}\sum_{n=1}^{N} p_n(-y_n)}{\partial B} \\
&= \frac{1}{N}\sum_{n=1}^{N} (-y_n) \frac{\partial p_n}{\partial B} \\
&= \frac{1}{N}\sum_{n=1}^{N} (-y_n) p_n (1-p_n)(-y_n) \\
&= \frac{1}{N}\sum_{n=1}^{N} p_n (1-p_n)
\end{aligned}
$$

$$
H = \begin{pmatrix} \frac{1}{N}\sum_{n=1}^{N} z_n^2 (1-p_n) p_n & \frac{1}{N}\sum_{n=1}^{N} z_n p_n (1-p_n) \\ \frac{1}{N}\sum_{n=1}^{N} z_n p_n (1-p_n) & \frac{1}{N}\sum_{n=1}^{N} p_n (1-p_n) \end{pmatrix}
$$

3.
$$\beta = (\lambda I + K)^{-1} y w = \sum_{n=1}^{N} \beta_n z_n$$

the dimension of the inverted matrix is N

4.
$$\min_{b,w,\xi_n^{\wedge},\xi_n^{\vee}} \frac{1}{2} w^T w + C \sum_{n=1}^{N} ((\xi_n^{\vee})^2 + (\xi_n^{\wedge})^2) \mid ||y_n - w^T \phi(x_n) - b| - \epsilon| \leq \xi_n$$
$$\implies \min_{b,w,\xi_n^{\wedge},\xi_n^{\vee}} \frac{1}{2} w^T w + C \sum_{n=1}^{N} \max\left(0, |y_n - w^T \phi(x_n) - b| - \epsilon\right)^2$$

Since we only penalize margin violation ξ_n that is greater than ϵ

5.
$$\min_{b,w,\xi_n^{\wedge},\xi_n^{\vee}} \frac{1}{2} w^T w + C \sum_{n=1}^{N} \max\left(0, |y_n - w^T \phi(x_n) - b| - \epsilon)\right)$$
$$\min_{\beta,w} F(b,\beta) = \frac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} \beta_n \beta_m K(x_n, x_m) + C \sum_{n=1}^{N} (\max(0, |y_n - s_n| - \epsilon))^2$$
$$\frac{\partial F}{\partial \beta_m} = \frac{1}{2} 2 \sum_{n=1}^{N} \beta_n K(x_n, x_m) + 2C \sum_{n=1}^{N} [[|y_n - s_n| > \epsilon]](|y_n - s_n| - \epsilon) sign(|y_n - s_n|) \frac{\partial {-s_n}}{\partial \beta_m}$$
$$= \frac{1}{2} 2 \sum_{n=1}^{N} \beta_n K(x_n, x_m) - 2C \sum_{n=1}^{N} [[|y_n - s_n| > \epsilon]](|y_n - s_n| - \epsilon) sign(|y_n - s_n|)$$
$$\frac{\partial \sum \beta_{m=1}^{N} \beta_m K(x_n, x_m) + b}{\partial \beta_m}$$
$$= \sum_{n=1}^{N} \beta_n K(x_n, x_m) - 2C \sum_{n=1}^{N} [[|y_n - s_n| > \epsilon]](|y_n - s_n| - \epsilon) sign(|y_n - s_n|) K(x_n, x_m)$$

6.
$$Me_t = \sum_{m=1}^{M} g_t(\widetilde{x}_m)^2 - 2 g_t(\widetilde{x}_m) \widetilde{y}_m + \widetilde{y}_m^2$$
$$Me_t = Ms_t - 2 \sum_{m=1}^{M} g_t(\widetilde{x}_m) \widetilde{y}_m + Me_0$$
$$\sum_{m=1}^{M} g_t(\widetilde{x}_m) \widetilde{y}_m = \frac{M(s_t + e_0 - e_t)}{2}$$

7. let a,b denote the two points. With
$$a \neq b$$

we have

$$\begin{aligned} y &= m(x-a)+a^2 \\ m(b-a)+a^2 &= b^2 \\ \implies m &= b+a \iff b \neq a \\ y &= (b+a)(x-a)+a^2 \\ &= (b+a)x - ba - a^2 + a^2 \\ &= (b+a)x - ba \end{aligned}$$

The expected value of (b+a) = the expected value of a + expected value of b = 1

let A denote a matrix of size N, where

$$a_{ij} = \frac{i}{N}\frac{j}{N}$$

Then the expected value of

$$ba \mid b \neq a$$

is equal to

$$\begin{aligned} &\lim_{n\to\infty} \frac{1}{N^2}\left(\sum_{i=1}^{N} \frac{i}{N} \sum_{j=1}^{N} \frac{j}{N} - \sum_{i=1}^{N} \frac{i^2}{N^2}\right) \\ &= \lim_{n\to\infty} \frac{1}{N^4}\left(\sum_{i=1}^{N} i \sum_{j=1}^{N} j - \sum_{i=1}^{N} i^2\right) \\ &= \lim_{n\to\infty} \frac{1}{N^4}\left(\frac{N(N+1)}{2}\frac{N(N+1)}{2} - \frac{N(N+1)(2N+1)}{6}\right) \\ &= \lim_{n\to\infty} \frac{N(N+1)}{2N^4}\left(\frac{N(N+1)}{2} - \frac{(2N+1)}{3}\right) \\ &= \lim_{n\to\infty} \frac{N(N+1)}{2N^4}\left(\frac{3N^2+3N-4N-2}{6}\right) \\ &= \frac{1}{4} \end{aligned}$$

$$g = x - \frac{1}{4}$$

8.

$$\begin{aligned} \min_{w} E_{in}^{u}(w) &= \frac{1}{N}\sum_{n=1}^{N} u_n(y_n - w^T x_n)^2 \\ &= \frac{1}{N}\sum_{n=1}^{N} (\sqrt{u_n}(y_n - w^T x_n))^2 \\ &= \frac{1}{N}\sum_{n=1}^{N} ((\sqrt{u_n}y_n) - w^T(\sqrt{u_n}x_n))^2 \\ &\implies \widetilde{x_n} = \sqrt{u_n}x_n \quad \widetilde{y_n} = \sqrt{u_n}y_n \end{aligned}$$

9. After the first iteration, u+ will be timed by num_error / (num_error + num_correct) = 1/100, while u- will be timed by num_correct / (num_error+num_correct) = 99/100. With

$$u_+^{(1)} = u_-^{(1)} = 1 u_+^{(2)} / u_-^{(2)} = \frac{\frac{1}{100}}{\frac{99}{100}} = \frac{1}{99}$$

10.

2d (R-L)+2 = 2 * 2 (6 - 1) +2= 22 where R-L is the number of internal sections and the plus 2 is for the all positive / negative case.

11. K(x,x') = inner product of decisions on x and x' = size of decision stump - 2(for direction)2(for compensate wrong agreement)=

$$2d(R-L) + 2 - 4 \mid x - x^{'} \mid_1$$

12.

Eins(1) = 0.2400 alphas(1) = 0.5763

13. Ein isn't increasing or decreasing through iterations, mainly because we penalized the right-predicted features throughout the iteration process, which should cause Ein to increase. After certain amount of iterations, the first assigned penalty will be balanced by penalties given on later iterations, and result in a reduction on Ein.

14. Ein(300) =0

15. u_s(2)= 0.8542 u_s(300)=0.0055

16. min(eps_s) = 0.1787

17. Eout(1) = 0.2900

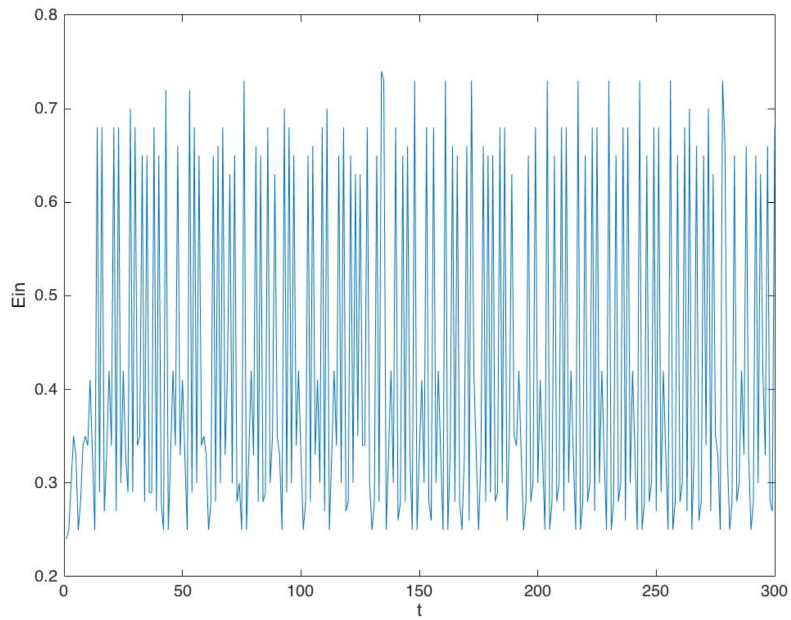
18. Eout(300) = 0.1320

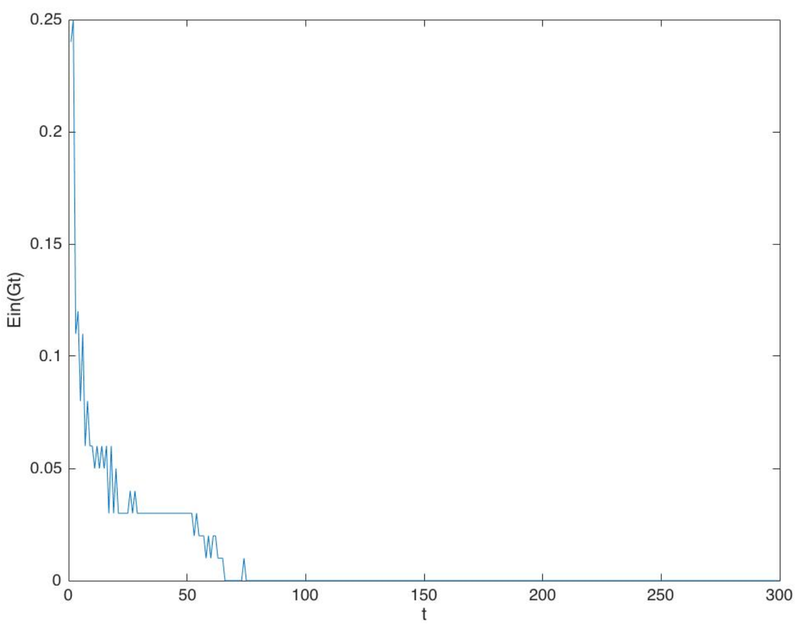
19.

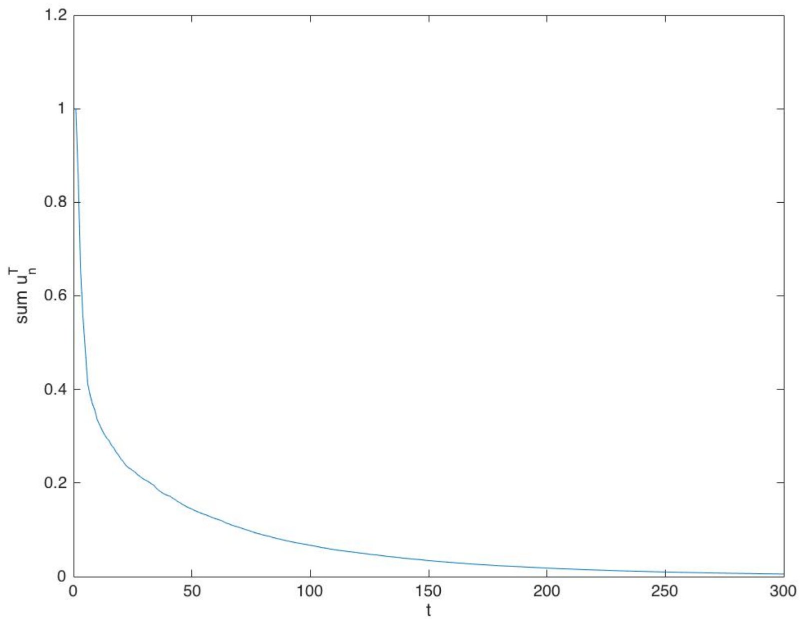
rs = 32 lambda_index = 1.000000e-03 Ein = 0
rs = 32 lambda_index = 1 Ein = 0
rs = 32 lambda_index = 1000 Ein = 0
rs = 2 lambda_index = 1.000000e-03 Ein = 0
rs = 2 lambda_index = 1 Ein = 0
rs = 2 lambda_index = 1000 Ein = 0
rs = 1.250000e-01 lambda_index = 1.000000e-03 Ein = 0
rs = 1.250000e-01 lambda_index = 1 Ein = 3.000000e-02
rs = 1.250000e-01 lambda_index = 1000 Ein = 2.425000e-01

20.

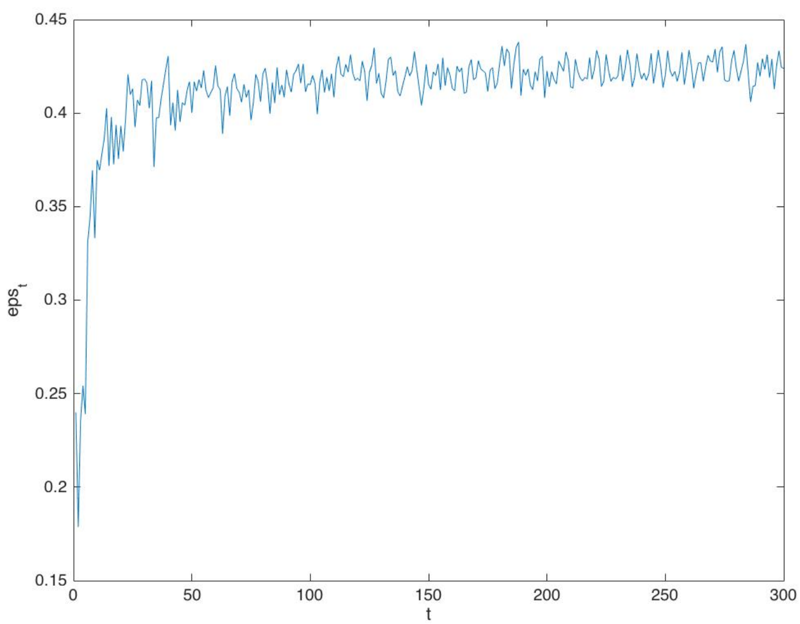
rs = 32 lambda_index = 1.000000e-03 Eout = 4.500000e-01
rs = 32 lambda_index = 1 Eout = 4.500000e-01
rs = 32 lambda_index = 1000 Eout = 4.500000e-01
rs = 2 lambda_index = 1.000000e-03 Eout = 4.400000e-01
rs = 2 lambda_index = 1 Eout = 4.400000e-01
rs = 2 lambda_index = 1000 Eout = 4.400000e-01
rs = 1.250000e-01 lambda_index = 1.000000e-03 Eout = 4.600000e-01
rs = 1.250000e-01 lambda_index = 1 Eout = 4.500000e-01
rs = 1.250000e-01 lambda_index = 1000 Eout = 3.900000e-01


Ein
t

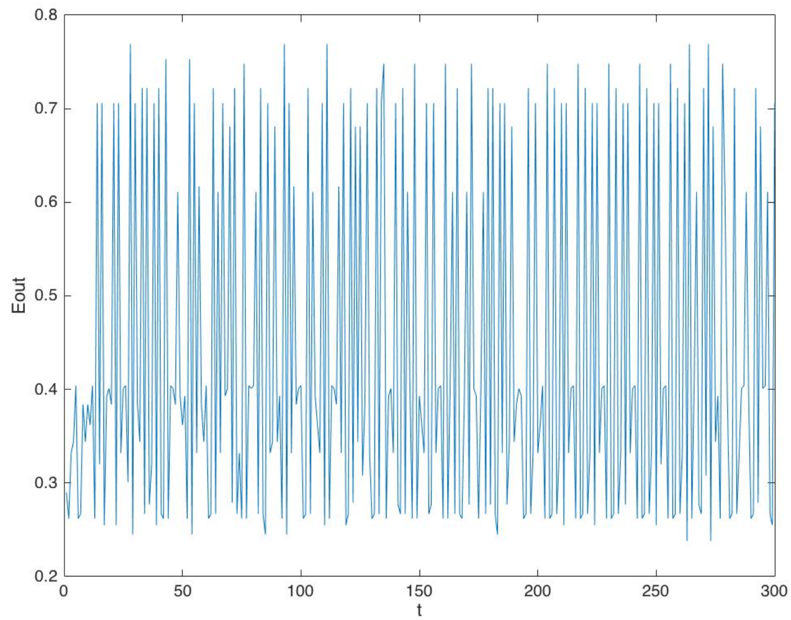

Ein(Gt)
t

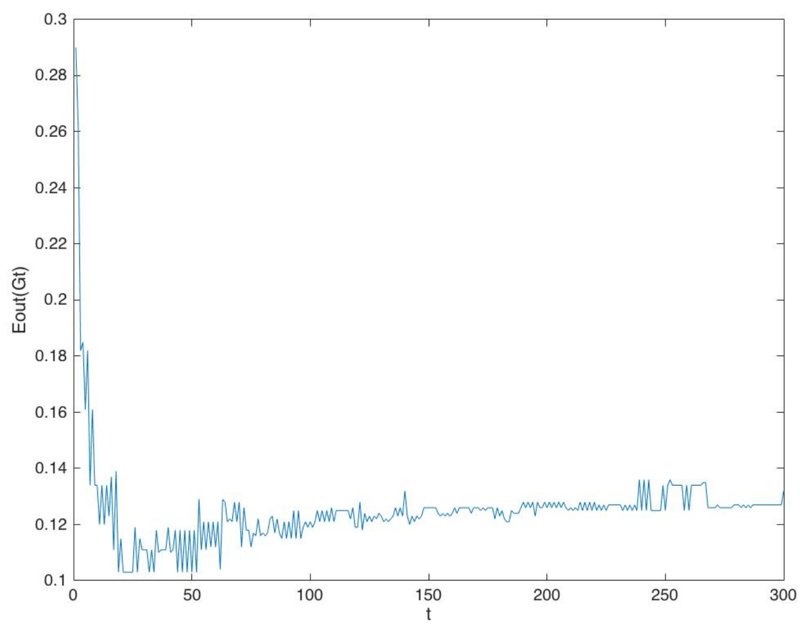


1.2
1
0.8
0.6
0.4
0.2
0
sum u$_n^T$
0
50
100
150
200
250
300
t



0.45
0.4
0.35
0.3
0.25
0.2
0.15
eps$_t$
0
50
100
150
200
250
300
t


0.8
0.7
0.6
0.5
0.4
0.3
0.2
Eout
0
50
100
150
200
250
300
t


0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
Eout(Gt)
0
50
100
150
200
250
300
t