Assignment 1 (100 points)

Linear Algebra, Probability and Image Processing

Due 3/29 (Thu) 9:30AM

1. **(Eigen-decomposition: 20 points)** Given a matrix

$$\boldsymbol{A} = \begin{pmatrix} 6 & 2 & 4 \\ -3 & 7 & 2 \\ 3 & -3 & 2 \end{pmatrix}$$

a) Compute the eigenvectors and the corresponding eigenvalues of the matrix with pencil and paper.

b) What are the eigenvectors and the corresponding eigenvalues of $\boldsymbol{A} - 5\boldsymbol{I}$? Can you compute it from the result in a) without another eigen-decomposition? How?

2. **(Bayes Theorem: 25 points)** Suppose that z_k is dependent only on x_k and that x_k is dependent only on x_{k-1} ($k = 1, \dots, n$). Prove the following two equations.

a) $p(x_1, x_2 | z_1, z_2) = \frac{p(z_2|x_2)p(z_1|x_1)p(x_2|x_1)p(x_1)}{p(z_1,z_2)}$

b) $p(x_{1:n}|z_{1:n}) = \frac{p(x_1)\prod_{i=2}^{n} p(x_i|x_{i-1})\prod_{i=1}^{n} p(z_i|x_i)}{p(z_{1:n})}$, where $p(x_{1:n}|z_{1:n}) \equiv p(x_1, \dots, x_n | z_1, \dots, z_n)$.

3. **(Fourier transform: 20 points)** Prove the multiplication property for Fourier transform:

$$F(\omega) * G(\omega) \Leftrightarrow f(x)g(x)$$

4. **(Programming: 35 points)** There are two images, $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$. Suppose that we don't know what the two images look like exactly. However, we know the average image, $(\boldsymbol{I}_1 + \boldsymbol{I}_2)/2$, and two filtered images, $\boldsymbol{h} * \boldsymbol{I}_1$ and $\boldsymbol{h} * \boldsymbol{I}_2$. Given these conditions, answer the following two problems.

a) Compute the 3x3 filter $\boldsymbol{h}$ to minimize the least square error.

b) Reconstruct the two original images, $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ using the solution in a).

Note that you need the mathematical derivation as well as the source code. (The images can be found in the class webpage.)