Problem sheet 6

Problem 1. Take a coin and flip it 7 times. Print the sample space. How many sequences of Heads and Tails are possible? Ans: 128

Problem 2. Place 3 six-sided dice into a cup. Next, shake the cup well and pour out the dice. How many distinct rolls are possible? Ans: 56

Problem 3. You randomly draw and then replace a card. What's the probability it's an ace? What's the probability it's the 4 of spades?

Problem 4. A coin is tossed three times.

(a) What is the probability of three heads?

(b) If given that an event that shows the first toss was heads, then what is the probability of three heads.

Problem 5. Let X be a random variable that follows the normal distribution with mean 2 and variance 4. If $Y = \sin(X^2)$, find $P[Y > 0.5]$.

Problem 6. There are 11 artists who each submit a portfolio containing 7 paintings for competition in an art exhibition. Unfortunately, the gallery director only has space in the winners' section to accommodate 12 paintings in a row equally spread over three consecutive walls. The director decides to give the first, second, and third place winners each a wall to display the work of their choice. The walls boast 31 separate lighting options apiece. How many displays are possible?

```
library(prob)
n <- c(11, 7, 31)
k <- c(3, 4, 3)
r <- c(FALSE, FALSE, TRUE)
x <- nsamp(n, k, rep = r, ord = TRUE)
prod(x)
```

```
## [1] 24774195600
```

Problem 7. Let X be the maximum and let Y be the minimum of the number of heads obtained when Carlos and Michael each flip a fair coin twice.

(a) Find the probabilities for all values of (X, Y).

(b) Find $P(X = Y)$.

(c) Repeat parts a and b if Carlos uses a biased coin with $P(heads) = \frac{3}{4}$.

```
coinflip <- iidspace(c(0,1), ntrials = 2, probs = c(0.5, 0.5))
coinflip <- addrv(coinflip, FUN = sum, invars = c("X1", "X2"),
           name = "nhead")
coinflip <- marginal(coinflip, vars = c("nhead"))
S <- iidspace(coinflip$nhead, ntrials = 2, probs = coinflip$probs)
S <- addrv(S, FUN = max, invars = c("X1", "X2"),
           name = "X")
S <- addrv(S, FUN = min, invars = c("X1", "X2"),
           name = "Y")
JointD <- marginal(S, vars = c("X", "Y"))
```

```
# JointD
cat("Answer of (b): ",prob(S, X==Y))
```

```
## Answer of (b):  0.375
```

```
coinflip1 <- iidspace(c(0,1), ntrials = 2, probs = c(0.5, 0.5))
coinflip1 <- addrv(coinflip1, FUN = sum, invars = c("X1", "X2"),
                   name = "nhead")
coinflip1 <- marginal(coinflip1, vars = c("nhead"))

coinflip2 <- iidspace(c(0,1), ntrials = 2, probs = c(0.25, 0.75))
coinflip2 <- addrv(coinflip2, FUN = sum, invars = c("X1", "X2"),
                   name = "nhead")
coinflip2 <- marginal(coinflip2, vars = c("nhead"))

df <- cbind(X1=rep(c(0,1,2),times=3),X2=rep(c(0,1,2),each=3))
S <- probspace(df,probs = c(outer(coinflip1$probs,coinflip2$probs,FUN="*")))

S <- addrv(S, FUN = max, invars = c("X1", "X2"),
           name = "X")
S <- addrv(S, FUN = min, invars = c("X1", "X2"),
           name = "Y")
JointD <- marginal(S, vars = c("X", "Y"))
#JointD
cat("Answer of (c): ",prob(S, X==Y))
```

```
## Answer of (c):  0.34375
```

Problem 8. Let X have pdf $f(x) = 3x^2,\ 0 < x < 1$. Find $P(0.14 \leq X \leq 0.71)$ using integration in R.

```
f <- function(x) 3 * x^2
integrate(f, lower = 0.14, upper = 0.71)
```

```
## 0.355167 with absolute error < 3.9e-15
```

Problem 9. Let X have pdf $f(x) = \dfrac{3}{x^4},\ x > 1$. Find the mean of X using integration in R.

```
f <- function(x) 3/x^3
integrate(f, lower =1, upper = Inf)
```

```
## 1.5 with absolute error < 1.7e-14
```

Problem 10. The amplitudes of two signals X and Y have joint pdf:

$$f_{XY}(x, y) = e^{-x/2} y e^{-y^2} \text{ for } x > 0,\ y > 0.$$

Find $P[X^{1/2} > Y, X \leq 4]$.

```
library(pracma)
fun <- function(x, y) exp(-x/2)*y*exp(-y^2)
xmin <- 0; xmax <- 4
ymin <- 0; ymax <- function(x) sqrt(x)
I <- integral2(fun, xmin, xmax, ymin, ymax)
I$Q
```

```
## [1] 0.5321576
```

Problem 11. The joint probability density function of two random variables X and Y is $K(1 - x - y)$ inside the triangle formed by the axes and the line $x + y = 1$ and zero elsewhere. Find the value of K and calculate

$P(X < \frac{1}{2}, Y > \frac{1}{4})$.

```
f <- function(x, y) 1-x-y
xmin <- 0; xmax <- 1
ymin <- 0; ymax <- function(x) 1 - x
I1 <- integral2(f, xmin, xmax, ymin, ymax)
K<-1/I1$Q
cat("K =", K)
```

```
## K = 6
```

```
I2 <- integral2(f, 0,1/2, 1/4, ymax)
cat("Value of the probability is", K*I2$Q)
```

```
## Value of the probability is 0.40625
```