Lab 9: Probability in R

Prog. Workshop

08-11-2021

Sample space

How do you roll a dice or flip a coin in R?

```
sample(6, size = 10, replace = TRUE)
```

```
##  [1] 2 1 3 2 4 4 1 1 4 6
```

```
sample(7:10, size = 10, replace = TRUE)
```

```
##  [1]  8  8  9  8  9 10  8 10  7 10
```

```
sample(c("H","T"), size = 10, replace = TRUE)
```

```
##  [1] "T" "T" "T" "H" "T" "H" "T" "H" "H" "T"
```

Other ways to do the same

```
library(prob)
tosscoin(1)
```

```
##   toss1
## 1     H
## 2     T
```

```
tosscoin(3)
```

```
##   toss1 toss2 toss3
## 1     H     H     H
## 2     T     H     H
## 3     H     T     H
## 4     T     T     H
## 5     H     H     T
## 6     T     H     T
## 7     H     T     T
## 8     T     T     T
```

continued

```
rolldie(1)
```

```
##   X1
## 1  1
## 2  2
## 3  3
## 4  4
## 5  5
## 6  6
```

```
head(rolldie(3, nsides = 3))
```

```
##   X1 X2 X3
## 1  1  1  1
## 2  2  1  1
## 3  3  1  1
## 4  1  2  1
## 5  2  2  1
## 6  3  2  1
```

continued

```
head(cards())
```

```
##   rank suit
## 1    2 Club
## 2    3 Club
## 3    4 Club
## 4    5 Club
## 5    6 Club
## 6    7 Club
```

continued

```
urnsamples(1:3, size = 2, replace = TRUE, ordered = TRUE)
```

```
##    X1 X2
## 1   1  1
## 2   2  1
## 3   3  1
## 4   1  2
## 5   2  2
## 6   3  2
## 7   1  3
## 8   2  3
## 9   3  3
```

Events

```
S <- tosscoin(2, makespace = TRUE)
S[1:3,]
```

```
##   toss1 toss2 probs
## 1     H     H  0.25
## 2     T     H  0.25
## 3     H     T  0.25
```

```
subset(S, toss2 == "T")
```

```
##   toss1 toss2 probs
## 3     H     T  0.25
## 4     T     T  0.25
```

continued

```
S <- cards()
subset(S, suit == "Spade")
```

```
##    rank  suit
## 40    2 Spade
## 41    3 Spade
## 42    4 Spade
## 43    5 Spade
## 44    6 Spade
## 45    7 Spade
## 46    8 Spade
## 47    9 Spade
## 48   10 Spade
## 49    J Spade
## 50    Q Spade
## 51    K Spade
## 52    A Spade
```

continued

```
subset(S, rank %in% c('A','Q'))
```

```
##    rank    suit
## 11    Q    Club
## 13    A    Club
## 24    Q Diamond
## 26    A Diamond
## 37    Q   Heart
## 39    A   Heart
## 50    Q   Spade
## 52    A   Spade
```

continued

```
subset(rolldie(3), X1 + X2 + X3 > 15)
```

```
##     X1 X2 X3
## 144  6  6  4
## 174  6  5  5
## 179  5  6  5
## 180  6  6  5
## 204  6  4  6
## 209  5  5  6
## 210  6  5  6
## 214  4  6  6
## 215  5  6  6
## 216  6  6  6
```

continued

```
subset(rolldie(3), X1 + X2 + X3 > 16)
```

```
##     X1 X2 X3
## 180  6  6  5
## 210  6  5  6
## 215  5  6  6
## 216  6  6  6
```

Union and intersection of events

```
S = cards()
A = subset(S, suit == "Heart")
B = subset(S, rank %in% c('Q','K'))
head(union(A,B))
```

```
##    rank    suit
## 11    Q    Club
## 12    K    Club
## 24    Q Diamond
## 25    K Diamond
## 27    2   Heart
## 28    3   Heart
```

```
intersect(A,B)
```

```
##    rank  suit
## 37    Q Heart
## 38    K Heart
```

Probability space

Equally likely outcomes

```
probspace(1:6)
```

```
##   x     probs
## 1 1 0.1666667
## 2 2 0.1666667
## 3 3 0.1666667
## 4 4 0.1666667
## 5 5 0.1666667
## 6 6 0.1666667
```

```
rolldie(1,makespace = TRUE)
```

```
##   X1     probs
## 1  1 0.1666667
## 2  2 0.1666667
## 3  3 0.1666667
## 4  4 0.1666667
## 5  5 0.1666667
## 6  6 0.1666667
```

Biased outcomes

```
probspace(tosscoin(1), probs = c(0.6, 0.4))
```

```
##   toss1 probs
## 1     H   0.6
## 2     T   0.4
```

Calculating probability

```
S = cards(makespace = TRUE)
A = subset(S, suit == "Heart")
B = subset(S, rank %in% c('Q','K'))

prob(A)
```

```
## [1] 0.25
```

```
prob(B)
```

```
## [1] 0.1538462
```

```
prob(setdiff(A,B)) # P[A\B]
```

```
## [1] 0.2115385
```

Sampling k from n objects with urnsamples

	ordered = FALSE	ordered = TRUE
replace = TRUE	n^k	$\frac{(n-1+k)!}{(n-1)!k!}$
replace = FALSE	$\frac{n!}{(n-k)!}$	$\binom{n}{k}$

Example

```
urnsamples(1:3, size = 2, replace = TRUE, ordered = TRUE)
```

```
##    X1 X2
## 1   1  1
## 2   2  1
## 3   3  1
## 4   1  2
## 5   2  2
## 6   3  2
## 7   1  3
## 8   2  3
## 9   3  3
```

```
nsamp(n = 3, k = 2, replace = TRUE, ordered = TRUE)
```

```
## [1] 9
```

Conditional Probability

```
S <- rolldie(2, makespace = TRUE)
head(S)
```

```
##   X1 X2      probs
## 1  1  1 0.02777778
## 2  2  1 0.02777778
## 3  3  1 0.02777778
## 4  4  1 0.02777778
## 5  5  1 0.02777778
## 6  6  1 0.02777778
```

```
A <- subset(S, X1==6)
B <- subset(S, X1+X2>=8)

prob(B, given=A)
```

```
## [1] 0.8333333
```

Independent repeated experiments

```
iidspace(c("H","T"), ntrials = 3, probs = c(0.6, 0.4))
```

```
##   X1 X2 X3 probs
## 1  H  H  H 0.216
## 2  T  H  H 0.144
## 3  H  T  H 0.144
## 4  T  T  H 0.096
## 5  H  H  T 0.144
## 6  T  H  T 0.096
## 7  H  T  T 0.096
## 8  T  T  T 0.064
```

Random variables

```
S <- rolldie(3, makespace = TRUE)
S <- addrv(S, U = X1 + X2 + X3)
head(S)
```

```
##   X1 X2 X3 U      probs
## 1  1  1  1 3 0.00462963
## 2  2  1  1 4 0.00462963
## 3  3  1  1 5 0.00462963
## 4  4  1  1 6 0.00462963
## 5  5  1  1 7 0.00462963
## 6  6  1  1 8 0.00462963
```

Random variables continued

```
S <- rolldie(3, makespace = TRUE)
S <- addrv(S, FUN = sum, invars = c("X1", "X2", "X3"),
           name = "X")
head(S)
```

```
##   X1 X2 X3 X      probs
## 1  1  1  1 3 0.00462963
## 2  2  1  1 4 0.00462963
## 3  3  1  1 5 0.00462963
## 4  4  1  1 6 0.00462963
## 5  5  1  1 7 0.00462963
## 6  6  1  1 8 0.00462963
```

Probability distribution continued

```
library(distr)
X <- Binom(size = 3, prob = 1/2)
X
```

```
## Distribution Object of Class: Binom
##  size: 3
##  prob: 0.5
```

```
d(X)(1)   # pmf of X evaluated at x = 1
```

```
## [1] 0.375
```

```
p(X)(2)   # cdf of X evaluated at x = 2
```

```
## [1] 0.875
```

Mean, VAR, SD

```
library(distrEx)
E(X)
```

```
## [1] 1.5
```

```
var(X)
```

```
## [1] 0.75
```

```
sd(X)
```

```
## [1] 0.8660254
```

```
E(5*X+3)
```

```
## [1] 10.5
```

```
var(5*X+3)
```

```
## [1] 18.75
```

Functions of Random Variables

```
X <- Binom(size = 3, prob = 0.6)
Y <- 3*X+2
Y
```

```
## Distribution Object of Class: AffLinLatticeDistribution
```

```
X <- Norm(mean = 0, sd = 1)
Y <- 3*X+2
Y
```

```
## Distribution Object of Class: Norm
##  mean: 2
##  sd: 3
```

```
X <- Norm(mean = 0, sd = 1)
Y <- X^2
Y
```

```
## Distribution Object of Class: AbscontDistribution
```

Continued

```
X <- Norm(mean = 0, sd = 1)
Y <- X^2
Y
```

```
## Distribution Object of Class: AbscontDistribution
```

```
p(Y)(0.5)
```

```
## [1] 0.5204999
```

```
Z <- Chisq(df = 1)
p(Z)(0.5)
```

```
## [1] 0.5204999
```

Multivariate Distributions

```
S <- rolldie(2, makespace = TRUE)
S <- addrv(S, FUN = sum, invars = c("X1", "X2"),
           name = "U")
S <- addrv(S, FUN = min, invars = c("X1", "X2"),
           name = "V")
head(S)
```

```
##   X1 X2 U V      probs
## 1  1  1 2 1 0.02777778
## 2  2  1 3 1 0.02777778
## 3  3  1 4 1 0.02777778
## 4  4  1 5 1 0.02777778
## 5  5  1 6 1 0.02777778
## 6  6  1 7 1 0.02777778
```

Continued

```
JointD <- marginal(S, vars = c("U", "V"))
head(JointD)
```

```
##   U V      probs
## 1 2 1 0.02777778
## 2 3 1 0.05555556
## 3 4 1 0.05555556
## 4 5 1 0.05555556
## 5 6 1 0.05555556
## 6 7 1 0.05555556
```

```
marginal(JointD, vars = "U")
```

```
##     U      probs
## 1   2 0.02777778
## 2   3 0.05555556
## 3   4 0.08333333
## 4   5 0.11111111
## 5   6 0.13888889
```