Problem sheet 7

***Problem* 1.** Suppose that when a signal having value μ is transmitted from location A the value received at location B is normally distributed with mean μ and variance 4. That is, if μ is sent, then the value received is $\mu + N$ where N, representing noise, is normal with mean 0 and variance 4. To reduce error, suppose the same value is sent 9 times. If the successive values received are 5, 8.5, 12, 15, 7, 9, 7.5, 6.5, 10.5, find a 95 percent confidence interval for μ.

```
x<- c(5, 8.5, 12, 15, 7, 9, 7.5, 6.5, 10.5)
mean(x)+c(-1,1)*qnorm(0.05/2,lower.tail = FALSE)*2/sqrt(length(x))
```

```
## [1]  7.693357 10.306643
```

***Problem* 2.** Reapeat Problem 1 assuming the variance is unknown.

```
x<- c(5, 8.5, 12, 15, 7, 9, 7.5, 6.5, 10.5)
mean(x)+c(-1,1)*qt(0.05/2,length(x)-1,lower.tail = FALSE)*sd(x)/sqrt(length(x))
```

```
## [1]  6.630806 11.369194
```

***Problem* 3.** Determine a 95 percent confidence interval for the average resting pulse of the members of a health club if a random selection of 15 members of the club yielded the data 54, 63, 58, 72, 49, 92, 70, 73, 69, 104, 48, 66, 80, 64, 77.

```
x <- c(54, 63, 58, 72, 49, 92, 70, 73, 69, 104, 48, 66, 80, 64, 77)
mean(x)+c(-1,1)*qt(0.05/2,length(x)-1,lower.tail = FALSE)*sd(x)/sqrt(length(x))
```

```
## [1] 60.86694 77.66640
```

***Problem* 4.** A standardized procedure is expected to produce washers with very small deviation in their thicknesses. Suppose that 10 such washers were chosen and measured. If the thicknesses of these washers were, in inches,

$$0.123, 0.133, 0.124, 0.125, 0.126, 0.128, 0.120, 0.124, 0.130, 0.126$$

what is a 90 percent confidence interval for the standard deviation of the thickness of a washer produced by this procedure?

```
x <- c(0.123, 0.133, 0.124, 0.125, 0.126, 0.128, 0.120, 0.124, 0.130, 0.126)
n<- length(x)
sig2in <- (n-1)*var(x)*1/c(qchisq(0.05,n-1,lower.tail = F), qchisq(0.95,n-1,lower.tail = F))
sqrt(sig2in)
```

```
## [1] 0.002695187 0.006079568
```

Problem 5. It is known that if a signal of value μ is sent from location A, then the value received at location B is normally distributed with mean μ and standard deviation 2. That is, the random noise added to the signal is an $N(0, 4)$ random variable. There is reason for the people at location B to suspect that the signal value $\mu = 8$ will be sent today. Test this hypothesis if the same signal value is independently sent five times and the average value received at location B is $\bar{X} = 9.5$.

We will compute the test statistic

$$\sqrt{n}\frac{|\bar{X} - \mu|}{\sigma}.$$

```
T<- sqrt(5)*abs(9.5-8)/2
T
```

```
## [1] 1.677051
```

```
H0 <- T <= qnorm(0.05/2,lower.tail = F)
H0
```

```
## [1] TRUE
```

Problem 6. A public health official claims that the mean home water use is 350 gallons a day. To verify this claim, a study of 20 randomly selected homes was instigated with the result that the average daily water uses of these 20 homes were as follows:

340	344	356	386	332
402	362	322	318	360
362	354	340	372	338
375	364	355	324	370

Do the data contradict the official's claim?

```
x=c(340, 344, 356, 386, 332, 402, 362, 322, 318, 360, 362, 354,
    340, 372, 338, 375, 364, 355, 324, 370)
n=length(x)
T <- sqrt(n)*(mean(x)-350)/sd(x)
H0 <- abs(T)<=qt(0.05,n-1,lower.tail = F)
p.value <- 2*(1-pt(T,n-1))
cat("p-value =",p.value)
```

```
## p-value = 0.4462411
```

```
t.test(x, y = NULL, alternative = "two.sided", mu = 350, conf.level = 0.95)
```

```
##
##  One Sample t-test
##
## data:  x
## t = 0.77784, df = 19, p-value = 0.4462
## alternative hypothesis: true mean is not equal to 350
## 95 percent confidence interval:
##  343.5749 364.0251
## sample estimates:
## mean of x
##     353.8
```