MTP290 Tutorial Sheet - 3

1. Write a MATLAB script to solve the linear system $Ax = b$, where A is an invertible diagonal matrix. Taking $A = diag([1, 2, 3])$ and $b = [1; 1; 1]$, solve for x.

2. Write MATLAB **function** to implement the forward substitution method to solve the linear system $Ax = b$, where A is a non-singular lower triangular matrix. Use it to solve for x if A and b are given as follows:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0.5 & 1 \end{bmatrix}, \; b = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.$$

3. Write a MATLAB **function** for implementing the backward substitution method to solve the system $Ax = b$, where A is a non-singular upper triangular matrix. Use this code to solve for x if A and b are as follows:

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 2 & -3 \\ 0 & 0 & -6.5 \end{bmatrix}, \; b = \begin{bmatrix} 1 \\ 7 \\ 6.5 \end{bmatrix}.$$

4. Write a MATLAB **function** to implement Gauss elimination method for solving a system of linear equations $Ax = b$, where A is a non-singular matrix. Next use this function to write a MATLAB script which solve the following system:

$$\begin{aligned} 4x_1 + x_2 - x_3 &= -2 \\ 5x_1 + x_2 + 2x_3 &= 4 \\ 6x_1 + x_2 + x_3 &= 6. \end{aligned}$$

5. Implement the Gauss elimination method with partial pivoting to solve a system of linear equations $Ax = b$, where A is a non-singular matrix. Use the program to find solution of the linear system $Ax = b$ where,

$$A = \begin{bmatrix} 9 & 3 & 2 & 0 & 7 \\ 7 & 6 & 9 & 6 & 4 \\ 2 & 7 & 7 & 8 & 2 \\ 0 & 9 & 7 & 2 & 2 \\ 7 & 3 & 6 & 4 & 3 \end{bmatrix}, \; b = \begin{bmatrix} 35 \\ 58 \\ 53 \\ 37 \\ 39 \end{bmatrix}.$$

   The correct answer is $[0, 1, 2, 3, 4]^T$.

6. Write a MATLAB function for implementing the LU decomposition (Doolittle's factorization) for a 3×3 matrix. Call the function from a script with the following input matrix

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & -2 \\ -2 & 1 & 1 \end{bmatrix}.$$

   Further, for $b = [1, 1, 1]^T$, solve the system $Ax = b$.

7. Use MATLAB's rand function to generate A, a random 10×10 matrix, and a random vector $b \in \mathbb{R}^{10}$; solve the system $Ax = b$

   (a) using your own code.

   (b) using MATLAB's backslash command: $x = A \setminus b$.

   Obviously, you should get the same results both times.

8. Calculate the condition numbers $cond_p(A)$, $(p = 1, 2, \infty)$ for the following matrix

$$A = \begin{bmatrix} 4.1 & 2.8 \\ 9.7 & 6.6 \end{bmatrix}.$$