MTP290 Tutorial Sheet - 5

1. Write a MATLAB function for implementing the Euler method to solve the first order ODE,

$$y' = f(x, y), \quad y(x_0) = y_0.$$

Find solution after 10 steps with specified step-size h for the following problems:

(a) $y' + 0.2y = 0$, $y(0) = 5$, $h = 0.2$.

(b) $y' = \frac{1}{2}\pi\sqrt{1 - y^2}$, $y(0) = 0$, $h = 0.1$.

(c) $y' = -20y + 20x^2 + 2x$, $y(0) = 1$, $h = 0.1$. Plot the solution with the exact solution $y = \exp(-20x) + x^2$.

2. Write a MATLAB function for implementing the improved (also called modified) Euler method to solve the first order ODE,

$$y' = f(x, y), \quad y(x_0) = y_0.$$

Find solution after 10 steps with specified step-size h for the following problems:

(a) $y' - xy^2 = 0$, $y(0) = 1$, $h = 0.1$.

(b) $y' = y - y^2$, $y(0) = 0.2$, $h = 0.1$.

(c) Solve Problem 1b using improved Euler method and compare the results with the Euler method.

3. Write a MATLAB function for implementing the classical Runge-Kutta method of fourth order to solve the first order ODE,

$$y' = f(x, y), \quad y(x_0) = y_0.$$

Find solution after 10 steps with specified step-size h for the following problems:

(a) $y' + y \tan x = \sin 2x$, $y(0) = 1$, $h = 0.1$.

(b) Redo Problem 2b using classical Runge-Kutta method of fourth order and compare the results.

4. Use finite difference method to solve the following boundary value problems with $n = 4,\ 8$:

(a) $y'' = 6x$, $y(0) = 0$, $y(2) = 8$.

(b) $y'' = 24x^2$, $y(0) = 0$, $y(2) = 32$.

(c) $y'' + y = 1$, $y(0) = 1$, $y(\pi/2) = 0$.

Also, plot the discrete solution.

5. Write a MATLAB function for implementing the Euler method to solve the system of ODE,

$$y_1' = f_1(t, y_1, y_2), \quad y_1(t_0) = y_{1,0},$$
$$y_2' = f_2(t, y_1, y_2), \quad y_2(t_0) = y_{2,0}.$$

Use the function to find solution of the following problem at $t = 1$ using time step $h = 0.1$:

$$y_1' = y_1 - y_1 y_2 + \sin \pi t, \quad y_1(0) = 2,$$
$$y_2' = y_1 y_2 - y_2, \quad y_2(0) = 1.$$

6. Write a MATLAB function for implementing the classical Runge-Kutta method of second order to solve the system of ODE,

$$
\begin{aligned}
y_1' &= f_1(t, y_1, y_2), \quad y_1(t_0) = y_{1,0}, \\
y_2' &= f_2(t, y_1, y_2), \quad y_2(t_0) = y_{2,0}.
\end{aligned}
$$

Use the function to find solution of the following problem at $t = 1$ using time step $h = 0.1$:

$$
\begin{aligned}
y_1' =& -4y_1 + y_2 + \sin \pi t, \quad y_1(0) = 1, \\
y_2' =& y_1 - 4y_2, \quad y_2(0) = 2.
\end{aligned}
$$