MTP290 Tutorial Sheet - 2

1. Write a MATLAB function to approximate the root of $f(x) = 0$ using Newton's method. Then provide $f(x) = x^{1/3}$ and $x_0 = 1$ as input to the MATLAB function and observe the behavior of iteration.

2. Let $f(x) = 27x^4 + 162x^3 - 180x^2 + 62x - 7$.

   (a) Show that $f(x)$ has a zero of multiplicity 3 at $x = \frac{1}{3}$.

   (b) Use Newton's Method to solve this equation with $x_0 = 0$ within 10^{-8}.

   (c) Choose modified Newton's Method to solve this equation with $x_0 = 0$ within 10^{-8}.

3. Apply Newton's method to find the approximation of the root of $x = \tan x$, starting with initial guess $x_0 = 4$ and $x_0 = 4.6$. Compare the results obtained from these two initial guesses. Does the method converge?

4. $f(x) = x - 2 + \log(x)$ has a root near $x = 1.5$. Use the Newton Raphson formula to obtain the better estimate.

5. Obtain an estimation (accurate till 4 decimal point) of the point of intersection of the curves $y = x - 1$ and $y = \cos x$.

6. Apply Newton's method to the function

$$f(x) = \begin{cases} x^{2/3}, & x \geq 0 \\ -x^{2/3}, & x < 0 \end{cases}$$

   with the root $x^* = 0$. What is the behavior of the iterates? Do they converge, if yes, at what order?

7. Use Newton's method and secant method for finding the approximations of the two zeros, one in $[-1, 0]$ and other in $[0, 1]$ to within $1e-3$ accuracy of $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$. Use the end points of the interval as initial guesses for the secant method and the midpoint for Newton's method.

8. Use Newton's method to find solutions accurate within $1e-5$ for the following problems:

   (a) $x^3 - 2x^2 - 5 = 0$ on the interval $[1, 4]$.

   (b) $x\cos(x) = 0$ on the interval $(0, \pi)$.

9. Solve the Problem 8 using MATLAB inbuilt function 'fzero' and compare the results obtained with the bisection and secant method.

10. The sum of two real numbers is 20. If each number is added to its square root, the product of the two sums is 155.55. Determine the two numbers to within 10^{-8} by the Newton's Method.

11. Use Newton's method to approximate, to within 10^{-3}, the value of x that produces the point on the graph of $y = x^2$ that is closest to $(1, 0)$.

12. Let $f(x) = x^3 - 2x + 2$ and consider the initial approximation as $x_0 = 0$. Find the root of $f(x)$ using Newton's method.(Check whether this method converge or not?)