MTP290 Tutorial Sheet - 4

1. Write a MATLAB function for implementing the Gauss-Jacobi method to solve the system $Ax = b$, where A is a non-singular matrix. Stop iteration when the error in l_∞ norm is less than the tolerance $1e-10$.

   (a) Solve the following linear system by Gauss-Jacobi method. Choose initial guess as $x_1 = x_2 = x_3 = 0.5$.

   $$\begin{aligned} 13x_1 + x_2 + x_3 &= 15 \\ x_1 + 13x_2 + x_3 &= 15 \\ x_1 + x_2 + 13x_3 &= 15. \end{aligned}$$

   (b) Solve the following linear system using Gauss Jacobi method with the initial guess $x_1 = x_2 = x_3 = 0$.

   $$\begin{aligned} 4x_1 + x_2 - x_3 &= 3 \\ 2x_1 + 7x_2 + x_3 &= 19 \\ x_1 - 3x_2 + 12x_3 &= 31. \end{aligned}$$

   (c) Use Gauss-Jacobi method to attempt solving the linear system

   $$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 5 \\ 2x_1 - x_2 + 2x_3 &= 1 \\ 3x_1 + x_2 - 2x_3 &= -1. \end{aligned}$$

2. Write a MATLAB function for implementing the Gauss-Seidel method to solve the system $Ax = b$, where A is a non-singular matrix. Stop iteration when the error in l_∞ norm is less than the tolerance $1e-10$.

   (a) Redo Problem 1a using Gauss-Seidel method.

   (b) Solve the following system using Gauss-Seidel method.

   $$\begin{aligned} 5x + 6y + 7z &= 18 \\ 6x + 3y + 9z &= 18 \\ 7x + 9y + 10z &= 26 \end{aligned}$$

   Choose different initial guesses $[2, 5, 7]$ and $[0.99, 0.995, 0.997]$ to discuss the convergence. Is the coefficient matrix diagonally dominant?

3. The upward velocity of a rocket is given at three different times in the following table

Time (s) (t)	Velocity (m/s) (v)
5	106.8
8	177.2
12	279.2
5	106.8

   The velocity data is approximated by a polynomial as

   $$v(t) = at^2 + bt + c, \ t \in [5, 12].$$

   Find the values of a, b and c using the Gauss-Seidel method. Assume an initial guess of the solution as $[a, b, c] = [1, 2, 5]$.