Assignment - 1

1. Approximate $x = \dfrac{1}{a}$ for any given $a \neq 0$ **without using division**. Hint: Use Newton's method with proper justification.

2. Implement the following iterative root finding method to find real roots of the non-linear equation $f(x) = 0$.

$$x_{n+1} = x_n - \frac{f(x_n)}{D(x_n)},\ D(x_n) = \frac{f(x_n + f(x_n)) - f(x_n)}{f(x_n)},\ n = 0, 1, 2 \ldots$$

   Additionally, calculate the order of convergence numerically and print/display it after each iteration. Use this program to find a root of $f(x) = x^4 - 16x^3 + 89x^2 - 194x + 120$ near 0.5.

3. Use the following method to find two sequences u_k and v_k such that all the values $f(u_k)$, $k = 0, 1, 2, \ldots$ have one sign and all the values $f(v_k)$ have the opposite sign.
   Method: We define $u_{k+1} = w_k$, $v_{k+1} = v_k$ if $f(w_k)$ has the same sign as $f(u_k)$, and otherwise $u_{k+1} = u_k$, $v_{k+1} = w_k$ where

$$w_k = \frac{u_k f(v_k) - v_k f(u_k)}{f(v_k) - f(u_k)},\ k = 0, 1, 2, \ldots.$$

4. Write a function that combines the bisection and Newton's method in the following way:

   Start with the bisection method with an initial interval $[a, b]$ and switch to Newton's method when the length of the current interval in the bisection method becomes less than $s(b - a)$. Try the new method on $\tanh(x) = 0$ with an initial interval $[-10, 15]$ and $s = 0.1$.

Instructions:

- Each problem carries 5 marks.
- Any descriptive answer should be written at the top of the code. Use '%' to comment inside the code.
- Make Matlab script for each of the above problems and submit only the '.m' file in gradescope.
- The final code should run without any error.
- Code will be checked manually. Checker will only hit run, and he/she will not provide any input during checking. Everything should be specified in each code.
- **Please do not take the risk of copying or sharing your code with classmates. Code similarity will be checked.**
- Use $tol = 1e - 10$ for the stopping criterion of the iterations.
- Any other input required for the code can be specified by yourself.