Problem set 5

Problem 1. Take a coin and flip it 7 times. Print the sample space. How many sequences of Heads and Tails are possible? Ans: 128

Problem 2. Place 3 six-sided dice into a cup. Next, shake the cup well and pour out the dice. How many distinct rolls are possible? Ans: 56

Problem 3. You randomly draw and then replace a card. What's the probability it's an ace? What's the probability it's the 4 of spades?

Problem 4. A coin is tossed three times.

(a) What is the probability of three heads?

(b) If given that an event that shows the first toss was heads, then what is the probability of three heads.

Problem 5. Let X be a random variable that follows the normal distribution with mean 2 and variance 4. If $Y = \sin(X^2)$, find $P[Y > 0.5]$.

Problem 6. There are 11 artists who each submit a portfolio containing 7 paintings for competition in an art exhibition. Unfortunately, the gallery director only has space in the winners' section to accommodate 12 paintings in a row equally spread over three consecutive walls. The director decides to give the first, second, and third place winners each a wall to display the work of their choice. The walls boast 31 separate lighting options apiece. How many displays are possible?

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## [1] 24774195600
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Problem 7. Let X be the maximum and let Y be the minimum of the number of heads obtained when Carlos and Michael each flip a fair coin twice.

(a) Find the probabilities for all values of (X, Y).

(b) Find $P(X = Y)$.

(c) Repeat parts a and b if Carlos uses a biased coin with $P(heads) = \frac{3}{4}$.

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## Answer of (b):  0.375
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## Answer of (c):  0.34375
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Problem 8. Let X have pdf $f(x) = 3x^2,\ 0 < x < 1$. Find $P(0.14 \leq X \leq 0.71)$ using integration in R.

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## 0.355167 with absolute error < 3.9e-15
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Problem 9. Let X have pdf $f(x) = \frac{3}{x^4},\ x > 1$. Find the mean of X using integration in R.

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## 1.5 with absolute error < 1.7e-14
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Problem 10. The amplitudes of two signals X and Y have joint pdf:

$$f_{XY}(x, y) = e^{-x/2} y e^{-y^2} \text{ for } x > 0,\ y > 0.$$

Find $P[X^{1/2} > Y, X \leq 4]$.

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## [1] 0.5321576
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Problem 11. The joint probability density function of two random variables X and Y is $K(1-x-y)$ inside the triangle formed by the axes and the line $x+y=1$ and zero elsewhere. Find the value of K and calculate $P(X<\frac{1}{2}, Y>\frac{1}{4})$.

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## K = 6
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## Value of the probability is 0.40625
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Problem 12. Assume a random variable X follow the normal distribution with mean 0 and standard deviation 3. What is the probability that X takes on a value between -0.5 and 0.5?

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## [1] 0.1323677
```

Problem 13. Suppose the total time taken to complete an assignment is a random variable having a normal distribution of mean (μ) 36 hours and standard deviation (σ) of 2 hours. What is the probability of completing the assignment in the time period of

(a) less than 24 hours

(b) between 34 and 38 hours

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## (a):  9.865876e-10
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## (b):  0.6826895
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Problem 14. Suppose a biased coin is tossed 100 times. What is the prbability of obtaining more than 55 heads if the probabilty of head in a sigle flip is 0.6?

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## [1] 0.8210984
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Problem 15. A fair die is rolled 10 times; find the probability of getting six exactly 3 times.

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## [1] 0.1550454
```

Problem 16. A customer help center receives on average 4 calls every hour.

(a) What is the probability that it will receive at most 5 calls every hour?

(b) What is the probability that it will receive at least 3 calls every hour?

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## (a):  0.7851304
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## (b):  0.5665299
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Problem 17. On the average, a certain computer part lasts ten years. The length of time the computer part lasts is exponentially distributed.

(a) What is the probability that a computer part lasts more than 7 years?

(b) What is the probability that a computer part lasts between nine and 11 years?

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## (a):  0.4965853
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## (b):  0.07369858
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