Lab sheet 8

Revisit Lab sheet 4

```
library(prob)
library(distr)
X <- Binom(size = 3, prob = 1/2)
X
```

```
## Distribution Object of Class: Binom
##  size: 3
##  prob: 0.5
```

```
d(X)(1)   # pmf of X evaluated at x = 1
```

```
## [1] 0.375
```

```
p(X)(2)   # cdf of X evaluated at x = 2
```

```
## [1] 0.875
```

Mean, VAR, SD

```
library(distrEx)
E(X)
```

```
## [1] 1.5
```

```
var(X)
```

```
## [1] 0.75
```

```
sd(X)
```

```
## [1] 0.8660254
```

```
E(5*X+3)
```

```
## [1] 10.5
```

```
var(5*X+3)
```

```
## [1] 18.75
```

Functions of Random Variables

```
X <- Binom(size = 3, prob = 0.6)
Y <- 3*X+2
Y
```

```
## Distribution Object of Class: AffLinLatticeDistribution
```

```r
X <- Norm(mean = 0, sd = 1)
Y <- 3*X+2
Y
```

```
## Distribution Object of Class: Norm
##  mean: 2
##  sd: 3
```

```r
library(pracma)
detach("package:pracma", unload = TRUE)
X <- Norm(mean = 0, sd = 1)
Y <- X^2
Y
```

```
## Distribution Object of Class: AbscontDistribution
```

Continued

```r
X <- Norm(mean = 0, sd = 1)
Y <- X^2
Y
```

```
## Distribution Object of Class: AbscontDistribution
```

```r
p(Y)(0.5)
```

```
## [1] 0.5204999
```

```r
Z <- Chisq(df = 1)
p(Z)(0.5)
```

```
## [1] 0.5204999
```

Definition: The quantile function of a random variable X is the inverse of its cumulative distribution function:

$$Q_X(p) = \min\{x : F_X(x) \geq p\},\ 0 < p < 1.$$

Definition: The empirical cumulative distribution function F_n (written ECDF) is the probability distribution that places probability mass $\frac{1}{n}$ on each of the values $x_1, x_2, \ldots, x_n$. The empirical PMF takes the form

$$f_X(x) = \frac{1}{n},\ x \in (x_1, x_2, \ldots, x_n).$$

If the value x_i is repeated k times, the mass at x_i is accumulated to $\frac{k}{n}$.

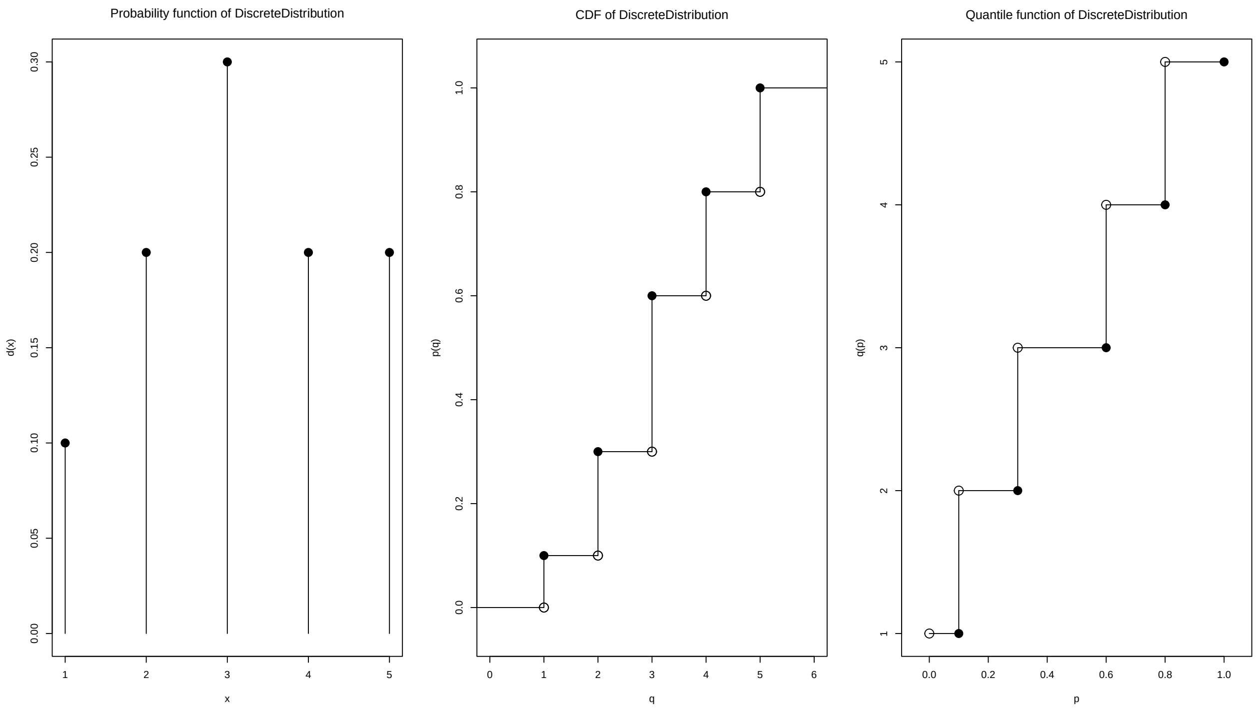
More using distr and distrEx

Defining continuous distribution manually

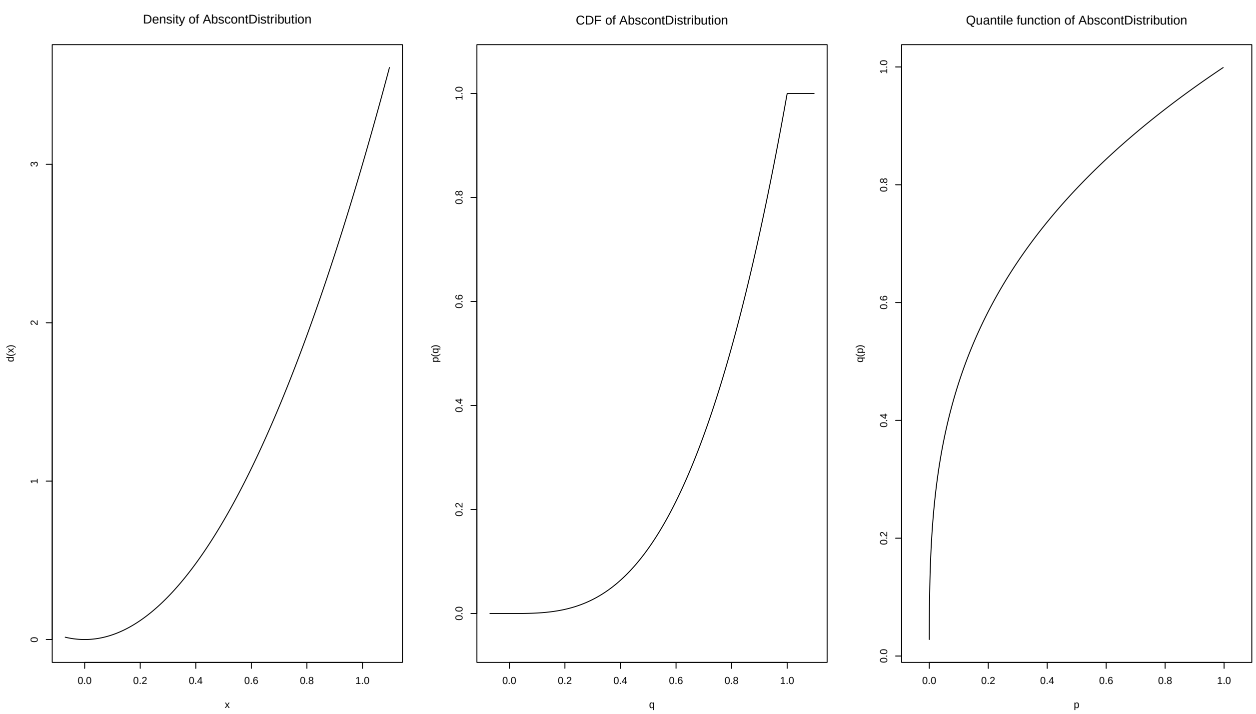
```r
library(distr)
library(distrEx)

# simple discrete distribution
D2 <- DiscreteDistribution(supp = c(1:5), prob = c(0.1, 0.2, 0.3, 0.2, 0.2))
D2
```

```
## Distribution Object of Class: DiscreteDistribution
```

```
plot(D2)
```




```
f <- function(x) 3 * x^2
X <- AbscontDistribution(d = f, low1 = 0, up1 = 1)
plot(X)
```




```
p(X)(Inf)
```

```
## [1] 1
```

```
p(X)(0)
```

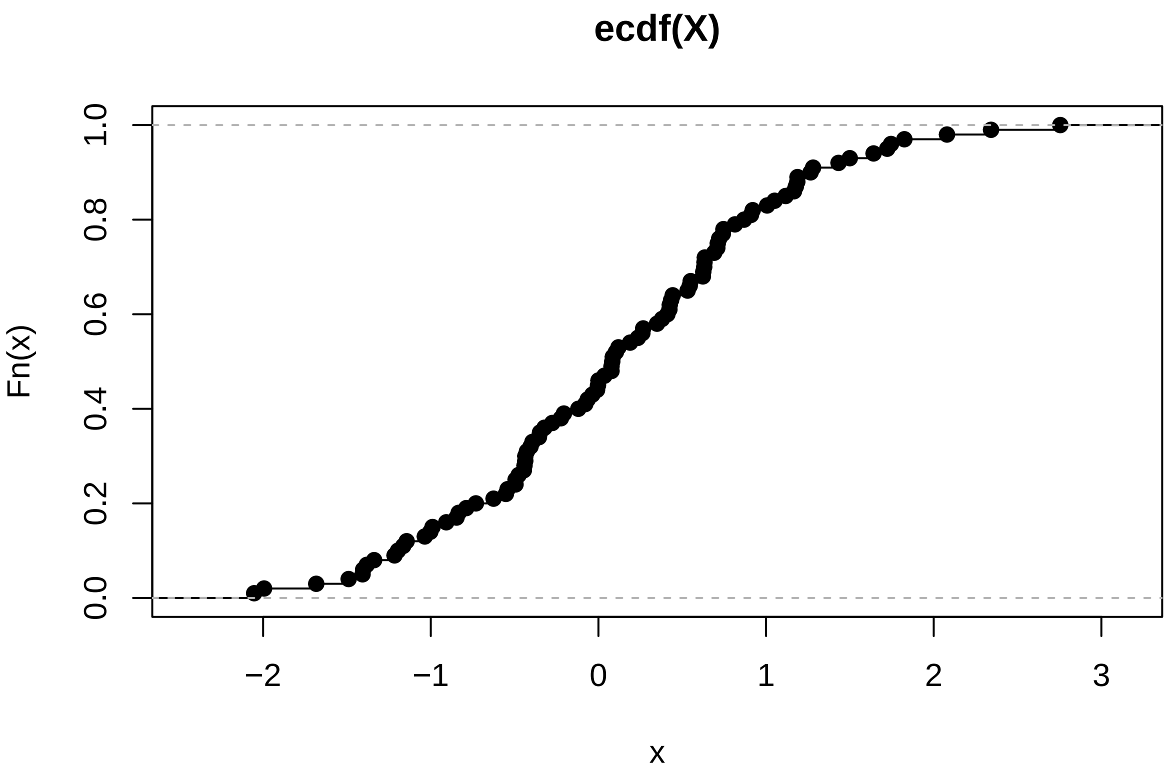
```
## [1] 0
```

```
E(X)
```

```
## [1] 0.7496337
```

Emperical CDF

```
X <- rnorm(100) # X is a sample of 100 normally distributed random variables
ECDF <- ecdf(X) # ECDF is a function giving the empirical CDF of X
plot(ECDF)
```



Multivariate Distributions

```
S <- rolldie(2, makespace = TRUE)
S <- addrv(S, FUN = sum, invars = c("X1", "X2"),
           name = "U")
S <- addrv(S, FUN = min, invars = c("X1", "X2"),
           name = "V")
head(S)
```

```
##   X1 X2 U V      probs
## 1  1  1 2 1 0.02777778
## 2  2  1 3 1 0.02777778
## 3  3  1 4 1 0.02777778
## 4  4  1 5 1 0.02777778
## 5  5  1 6 1 0.02777778
## 6  6  1 7 1 0.02777778
```

```
JointD <- marginal(S, vars = c("U", "V"))
head(JointD)
```

```
##   U V      probs
## 1 2 1 0.02777778
## 2 3 1 0.05555556
## 3 4 1 0.05555556
## 4 5 1 0.05555556
## 5 6 1 0.05555556
## 6 7 1 0.05555556
```

```
xtabs(round(probs, 3) ~ U + V, data = JointD)
```

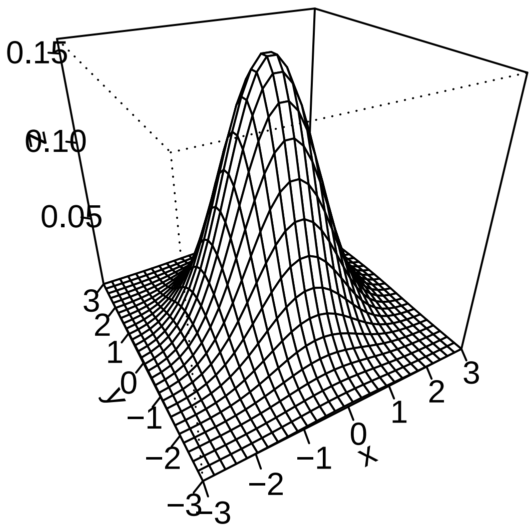
```
##     V
## U        1     2     3     4     5     6
##   2  0.028 0.000 0.000 0.000 0.000 0.000
##   3  0.056 0.000 0.000 0.000 0.000 0.000
##   4  0.056 0.028 0.000 0.000 0.000 0.000
##   5  0.056 0.056 0.000 0.000 0.000 0.000
##   6  0.056 0.056 0.028 0.000 0.000 0.000
##   7  0.056 0.056 0.056 0.000 0.000 0.000
##   8  0.000 0.056 0.056 0.028 0.000 0.000
##   9  0.000 0.000 0.056 0.056 0.000 0.000
##   10 0.000 0.000 0.000 0.056 0.028 0.000
##   11 0.000 0.000 0.000 0.000 0.056 0.000
##   12 0.000 0.000 0.000 0.000 0.000 0.028
```

```
marginal(JointD, vars = "U")
```

```
##     U      probs
## 1   2 0.02777778
## 2   3 0.05555556
## 3   4 0.08333333
## 4   5 0.11111111
## 5   6 0.13888889
## 6   7 0.16666667
## 7   8 0.13888889
## 8   9 0.11111111
## 9  10 0.08333333
## 10 11 0.05555556
## 11 12 0.02777778
```

Bivariate normal distribution

```
library(mvtnorm)
x <- y <- seq(from = -3, to = 3, length.out = 30)
f <- function(x, y) dmvnorm(cbind(x, y), mean = c(0, 0),
                            sigma = diag(2))
z <- outer(x, y, FUN = f)
persp(x, y, z, theta = -30, phi = 30, ticktype = "detailed")
```



Differentation

```
f<-expression(x^3+2*x+3)
d<-D(f,'x')
d
```

```
## 3 * x^2 + 2
```

```
d1<-D(D(f,'x'),'x')
d1
```

```
## 3 * (2 * x)
```

Differentiation in two variables

```
f<-expression(x^2+y^2+2*x*y-3*x+4*y+4)
D1=D(f,'x')
D1
```

```
## 2 * x + 2 * y - 3
```

```
f<-expression(x^2+y^2+2*x*y-3*x+4*y+4)
D2=D(f,'y')
D2
```

```
## 2 * y + 2 * x + 4
```

Other differentiation

```
exp<- expression(sin(cos(x + y^2)))
dx<-D(exp,'x');dx
```

```
## -(cos(cos(x + y^2)) * sin(x + y^2))
```

```
dy<-D(exp,'y'); dy
```

```
## -(cos(cos(x + y^2)) * (sin(x + y^2) * (2 * y)))
```

Integration

```
func<-function(x)(x^3+2*x^2+3*x+9)
I<-integrate(func,lower=0,upper=3)
I
```

```
## 78.75 with absolute error < 8.7e-13
```

Other Integration

```
integrate(dnorm, -1.96, 1.96)
```

```
## 0.9500042 with absolute error < 1e-11
```

```
integrate(dnorm, -Inf, Inf)
```

```
## 1 with absolute error < 9.4e-05
```

How to evaluate tripple integral?

Evaluate

$$\int_0^{0.5}\int_0^{0.5}\int_0^{0.5}\frac{2}{3}\left(x^2+y^2+z^2\right)\,dx\,dy\,dz.$$

```
library(cubature)

f <- function(x) { 2/3 * (x[1] + x[2] + x[3]) }
# "x" is vector x[1], x[2], x[3] are referring to x1, x2 and x3 respectively.
adaptIntegrate(f, lowerLimit = c(0, 0, 0), upperLimit = c(0.5, 0.5, 0.5))
```

```
## $integral
## [1] 0.0625
##
## $error
## [1] 1.387779e-17
##
## $functionEvaluations
## [1] 33
##
## $returnCode
## [1] 0
```

```
## a slowly-convergent integral

integrand <- function(x) {1/((x+1)*sqrt(x))}
integrate(integrand, lower = 0, upper = Inf)
```

```
## 3.141593 with absolute error < 2.7e-05
```

Another package to integrate

Evaluate

$$\int_0^1\int_0^1\cos x\cos y\,dx\,dy.$$

```r
library('pracma')
fun <- function(x, y) cos(x) * cos(y)
integral2(fun, 0, 1, 0, 1, reltol = 1e-10)
```

```
## $Q
## [1] 0.7080734
##
## $error
## [1] 0
```

Compute the volume of a sphere

Evaluate

$$\int_{x=0}^{1} \int_{y=0}^{\sqrt{1-x^2}} \sqrt{1 - x^2 - y^2}\ dy\,dx.$$

```r
f <- function(x, y) sqrt(1 -x^2 - y^2)
xmin <- 0; xmax <- 1
ymin <- 0; ymax <- function(x) sqrt(1 - x^2)
I <- integral2(f, xmin, xmax, ymin, ymax)
I$Q
```

```
## [1] 0.5236076
```

Compute the volume over a sector

```r
I <- integral2(f, 0,pi/2, 0,1, sector = TRUE)
I$Q
```

```
## [1] 0.5236226
```

Integrate $\dfrac{1}{\sqrt{x+y}(1+x+y)^2}$ over the triangle $0 \le x \le 1,\ 0 \le y \le 1-x$. Note that the integrand is infinite at $(0,0)$.

```r
f <- function(x,y) 1/( sqrt(x + y) * (1 + x + y)^2 )
ymax <- function(x) 1 - x
I <- integral2(f, 0,1, 0,ymax)
I$Q + 1/2 - pi/4
```

```
## [1] -3.247091e-08
```

```r
##  Compute this integral as a sector
rmax <- function(theta) 1/(sin(theta) + cos(theta))
I <- integral2(f, 0,pi/2, 0,rmax, sector = TRUE, singular = TRUE)
I$Q + 1/2 - pi/4
```

```
## [1] -4.998646e-11
```

Evaluate

$$\int_{x=2}^{4} \int_{x-1}^{x+6} \int_{-2}^{4+y^2} (x^2 + y^2 + z)\ dz\,dy\,dx.$$

```r
integrand3 <- function(x, y, z) x^2 + y^2 + z
xmin <- 2; xmax <- 4
ymin <- function(x) x - 1
ymax <- function(x) x + 6
```

```r
zmin <- -2
zmax <- function(x, y) 4 + y^2
integral3(integrand3, xmin, xmax, ymin, ymax, zmin, zmax)
```

```
## [1] 47416.76
```