The Solution of $u_t + (f(u))_x = 0$ with the Riemann data $u(x, 0) = \begin{cases} uL & x < 0 \\ uR & x \geq 0 \end{cases}$

The exact solution is calculated using the article by Osher which is available here: http://www.ams.org/journals/proc/1983-089-04/S0002-9939-1983-0718989-X/S0002-9939-1983-0718989-X.pdf

Also refer "Finite-Volume Methods For Hyperbolic Problems" By LeVeque.

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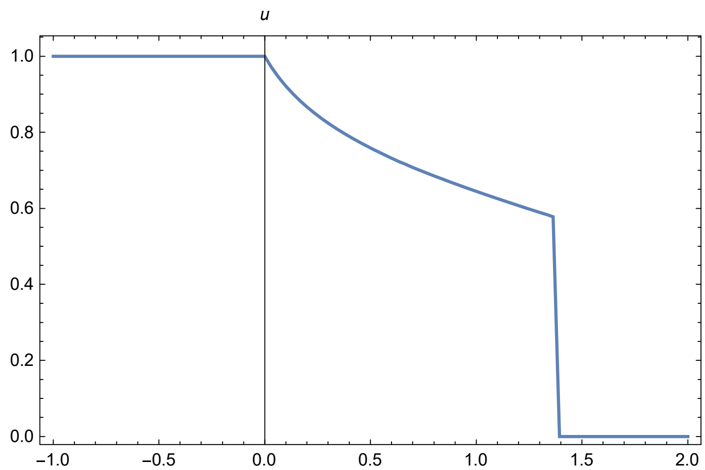
By using this code one can get the analytic solution of known scalar Riemann problems for conservation laws.

```
xL = -1; xR = 2; (* The domain *)
(*uL=π/4;uR=15 π/4;*) (* Note that uL < uR *)
(*f[v_]:=Sin[v];*) (* Define flux here *)
uL = 1; uR = 0; (* Note that uL > uR *)
f[v_] := v^2/(v^2 + 1/2 (1 - v)^2); (* Define flux here *)
Ngrid = 100;(* Define grid number here *)
dx = (xR - xL)/(Ngrid - 1); (* Mesh size *)
t = 1; (* The time *)
For[i = 1, i ≤ Ngrid, i++, xgrid[i] = xL + (i - 1) dx ];
(*uexact=Table[ ArgMin[{f[v]-xgrid[i]/t v, uL≤v≤uR},v],{i,1,Ngrid}];*)
(* This is the case when uL ≤ uR *)
uexact = Table[xi = xL + (i - 1) dx ; ArgMax[{f[v] - xi/t v, uR ≤ v ≤ uL}, v], {i, 1, Ngrid}];
(* This is the case when uL ≥ uR *)
x1 = Array[xgrid, Ngrid];
```

```
ListLinePlot[Thread[{x1, uexact}],
 AxesLabel → {HoldForm[x], HoldForm[u]}, Frame → True] (* To plot *)
```




```
(*Export["uexact.mat",Thread[{x,uexact}],"MAT"] ;*)(* To export in MAT file *)
```