Machine Learning Based Position Determination and Prediction Algorithm in Aerial Base Station System

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***Abstract*—A novel framework for the movement of unmanned aerial vehicles (UAVs) based on the prediction of users' position information is proposed. UAV is deployed as aerial base station to serve a group of ground users and obtains a movement as ground users are roaming. The problem of joint deployment of UAV and prediction of users' position information for maintaining high quality of service is formulated. A three-step approach is proposed to obtain the position information of ground users and dynamic movement of UAV. Firstly, an iterative K-Means algorithm is proposed to deploy UAV at the initial time slot. Secondly, an echo state network (ESN) based prediction algorithm is proposed to predict the future positions of users based on the real data set collected from Twitter. Thirdly, the iterative K-Means algorithm is invoked to obtain the optimal position of UAV at each time slot based on the movement of ground users. Numerical results reveal that increasing the reservoir pool size of ESN algorithm leads to a less error between real tracks and predication tracks. Additionally, re-deploying UAV based on the movement of ground users is an effective method to ease the downward trend compared with keeping static.**

I. Introduction

In the modern society, people cannot remove mobile phones from their lives, and most of telecommunication services are provided by ground base station (BS) nowadays. However, preceding surveys [1] shows that the services of telecommunication are usually poor, especially in some remote and busy areas [2]. And [1] also reveals that the poor service quality negatively affects customer satisfaction. To assistant the terrestrial BS in different poor-service scenarios, aerial base station system (ABSS) is proposed and has attracted wide interests from researchers. It has been proved that rapid application of ABSS meets the sudden wireless services demands. We believe that such applications are supposed to provide a realizable solution for current poor telecommunication services and are going to became a mainstream, especially in those areas where the extension plans of ground BS is not available. And the proposed machine learning based algorithm in this paper is supposed to promote the performance of ABSS.

Recently, the applications of ABSS are under active discussions and have achieved several significant achievements. In this paper, several applications of ABSS are investigated. In [3], a method for 3-D placement of unmanned aerial vehicle base station (UAV-BS) was proposed, and resulted in maximizing the number of covered users with the minimum transmit power. The work in [4] and [5] researched a scheme on feasibility of ABSS and after-earthquake telecommunication recovery. The work in [6] developed a model to maximize the revenue of the ABSS network. An efficient method was proposed in [7], researchers not only decreased the number of needed UAVs but also maintained the user coverage. The heuristic algorithm was used to find the positions of drone-BSs in areas with different user densities in [8].

However, all above papers only took a static scenario into consideration. According to reality, the vehicle mounted BSs in ABSS are expected to adjust their positions while the served crowd moves. The absence of consideration about mobility implies relatively lower performance within ABSS system. Besides, [4] uses helium balloon as the platform for ABSS, which is cumbersome and not practical for busy downtowns. As a contrast, UAV is more appropriate for implementing between skyscrapers. For the evaluation part, in [3], [6] and [7], the performance of ABSS was only measured by the user coverage. But the higher coverage cannot directly induce the higher service quality. In this paper, the transmission rate, rather than the user coverage, is used as criteria for evaluation.

And except in [7], the datasets of other researches were generated only by simulator, and some of them were even created randomly. By this mean, they assumed the users are distributed dispersedly and there were no buildings or other obstacles around users. Since the proposed algorithm is data-driven, whose precondition to guarantee the accuracy is sufficient data. And to make a facsimile of the real scenarios, the data for experiment is supposed to cover daily activity trajectories of a huge group of users. To collect such data, in [9], the GPS information was collected automatically, but the purpose of this study was focused on making up a road map, not suitable for the application in this paper. In another related research [10], the researchers used an android application to collect the position information of their volunteers. Such method is an impractical solution either, because the time interval of the experiments in [10] is several weeks, but the peak time of demanding communication service is hours. After comparing above methods, it is decided to collect real-time information from tweets by Twitter API, the data consists of GPS coordinates and recorded timestamps.

With the pros and cons of relevant works, in this paper, UAV manifests its advantages for adapting to the mobility of users in reality. The data in this paper is gathered from real-time tweets to make the simulation results close to the reality. Recently, based on neural network, the accuracy of regression and prediction tasks have been promoted a lot. In order to keep the quality-of-service in network system, the machine learning based algorithms are exploited for UAV placement and positions prediction.

The rest of the paper is organized as follows. In Section II, network model consists of the problem formulation for the 3D deployment and movement of UAVs based on the real position information of ground users is presented. In Section III, iterative K-means algorithm is invoked to obtain dynamic movement of UAV. In Section IV, the prediction of ground users' movement based on the real data set from Twitter is proposed, the method to solve this problem is ESN algorithm. Numerical results are presented in Section V, which is followed by conclusions in Section VI.

II. System Goal

In this part, the transmission rate is introduced and calculated. The parameters that affects the rate are investigated. Assuming that all of the coordinates of users are already known, and fixed at $w_n = [x_n, y_n]^T \in \mathbb{R}^2$. At time t, the coordinate of the UAV is denoted as $q(t) = [x(t), y(t)]^T \in \mathbb{R}^2$. With these two equations, the distance between UAV and user k_n at time t is presented as Equation (1):

$$d_n(t) = \sqrt{h^2 + (x(t) - x_n(t))^2 + (y(t) - y_n(t))^2}, \quad (1)$$

in which h means the height of UAV, and the overall distance between UAV and all users at time t is:

$$D_{\text{sum}} = \sum_{n=1}^{N_u} d_n(t). \quad (2)$$

The UAV is enabled with frequency division multiple access (FDMA) system, to serve a group of dispersive N_{u} users. And the communication link between UAV and users is typically an air-to-ground communication. Such communication occurs with two main propagation groups: the first one is that receivers favoring a Line-of-Sight (LoS) condition; and the second corresponds to receivers with no LAP Line-of-Sight (NLoS) but via reflections and diffractions, this group of users still receives coverage. As shown in Equation (3), the probability of resulting LoS is written as:

$$P_{\text{LoS}}(\theta_{k_n}) = b_1 \Big(\frac{180}{\pi} \theta_{k_n} - \zeta \Big)^{b_2}, \quad (3)$$

where $\theta_{k_n} = \sin^{-1}(\frac{h_n(t)}{d_{k_n}(t)})$ is the elevation angle between UAV and user k_n, b_1 and b_2 are constant values of environment impact. And ζ is another constant [11].

For the purpose for simplifying the model, the channel power gain from the UAV to user at time t is defined as:

$$g_n(t) = K_0^{-1} d_n^{-\alpha}(t) [P_{\text{LoS}} \mu_{\text{LoS}} + P_{\text{NLoS}} \cdot \mu_{\text{NLoS}}]^{-1}, \quad (4)$$

where $K_0 = (\frac{4\pi f_c}{c})^2$, μ_{LoS} and μ_{NLoS} are attenuation factors for LoS and NLoS links respectively, and f_c is the carrier frequency, c is the light speed.

Generally, the received signal to interference plus noise ratio (SINR) of user k_n at time t is described by Equation (5):

$$\Gamma_n(t) = \frac{p_n(t) g_n(t)}{\sigma^2}, \quad (5)$$

in the above equations, $\sigma^2 = a_n(t) B N_0$, N_0 means the power spectral density of the additive white Gaussian noise (AWGN) at the receivers, $a_n(t)B$ is the bandwidth allocated to user k_n at time t and p_n is the transmit power.

And then, the proposed algorithm gets the instantaneous transmit rate of user k_n at time t, is expressed as Equation (6):

$$r_n(t) = a_n(t) B log_2 \big(1 + \frac{p_n(t) g_n(t)}{\sigma^2}\big). \quad (6)$$

Suppose that the system requires that each user scheduled satisfy a minimum rate requirement r_0. This means that all users must have capacity greater than a rate r_0. The optimization problem is formulated as:

$$\underset{\text{Q}}{max} \quad R_{\text{sum}} = \sum_{n=1}^{N} r_n(t) \quad (7)$$

$$s.t. \quad r_n(t) \geq r_0, \forall n, t. \quad (8)$$

We aim at trajectory design (obtain the optimal position of UAV at each time slot) to maximize the total transmit rate while satisfy the minimize rate requirement of each ground user.

III. Iterative K-Means Algorithm for the Movement of UAV

The three-step approach for getting the position information of ground users and dynamic movement of UAV is deeply discussed in this section. In this paper, it is assumed that the geographical range of served area is small that one UAV is enough to cover all the demands from ground users. As for the scheme that need two or more UAVs will be carried onto further research in the future. Though the goal of the relevant papers is to maximize user coverage, the ultimate aim for promoting the quality of service in ABSS is to maximize the R_{sum}, since the transmit rate is the direct response of quality of communication service.

Once the service stress of terrestrial BS is overloading, the initial position is needed to deploy the UAV. The algorithm finds a point to minimize the D_{sum}. And then based on the found point to obtain the local optimal solution point. For the first step, this paper clusters users in real-time data collections with Iterative K-Means algorithm. Iterative K-Means is an improved version of K-Means algorithm [12], the algorithm calculates a new weight for each variable based on the former variance with cluster distances. And the new weights are used to calculate the new cluster members from datasets in the next iteration. With iterative K-Means algorithm, position C, the center of the current user cluster, is supposed to be found easily. And naturally, position C is the point for UAV that make D_{sum} at the minimum. After getting the result position from iterative K-Means algorithm, the algorithm then do traversal search for the final position P to locally maximize R_{sum} within a circle with C as center, $10m$ as radium and $0.2m$ as step. In the numerical simulation results, the local optimal point P brings an enhancement on overall transmit rate.

After the UAV gets to the service position in the initial time slot, the proposed algorithm starts predicting the move trend of ground users in the second step. Since the flight to the next service point would take some time, to promote the overall rate as much as possible and to keep the stability of service, we need the UAV to know the next positions of users in advance, then it is supposed to fly to next local optimal point P, before the crowd moves. For this reason, this paper does not simply use immediately positions of the crowd. The details of prediction algorithm are discussed in the next section, and the superiority of the prediction is certified by numerical results.

In the third step of the proposed approach, the iterative K-Means is implemented again for the optimal solution in next time slot. The difference with the first step is that the datasets used are prediction on real-time values, rather than use the real-time data directly.

The algorithm of above three-step for determination positions of UAV is given in Algorithm 1.

Algorithm 1 Iterative algorithm for UAV positions determination

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Input: The number of users: N_u, and the position information of each user: $w_n = [x_n, y_n]^T$

1: Use iterative K-Means algorithm to find the center of user cluster C to minimize D_{sum} with input.
2: Find the local optimal position P to maximize the overall transmit rate R_{sum} by traversal research around C.
3: Make the UAV get to position P and begin providing service in the initial time slot.
4: Use prediction algoriyhm and take the prediction information as new inputs periodically.
5: Update the position of UAV for next time slot by repeating from Step 1) to Step 4) until terrestrial BS is competent for the demand of telecommunication services.

Output: The optimal position to maximize R_{sum}: P

IV. Prediction of Ground Users' Movement

In this part, the algorithm for prediction is introduced, the prediction information is used for iterative K-Means except in the first time-slot.

A. Data Collection

As mentioned, the datasets for this paper is gathered from Twitter, when people post tweets, their GPS coordinates are recorded and legally visible. The chosen zone for data collecting was Oxford Street, where is known as the busiest area in the London city, and the duration of collection was set to be in the afternoon of 14th, March 2018 and in the early morning of 15th, March 2018.

In the experiments, more than 10,000 pieces of information were gathered from Twitter, during the afternoon and the early morning. After cleaning and pre-operating the data, the dimension of the data is four: user number, latitude distance from the Oxford Street, longitude distance from the Oxford Street and time information respectively. The notation of the dataset is $u(n) = [u_1(n), u_2(n), \ldots, u_{N_u}(n)]^T$.

Fig. 1 is drawn to show the initial positions of users with Google Map, their positions were distributed around the pentastar *i.e.* Oxford Street. Most of users were concentrated in the streets and malls, this proves the collected data has reached the expectations.

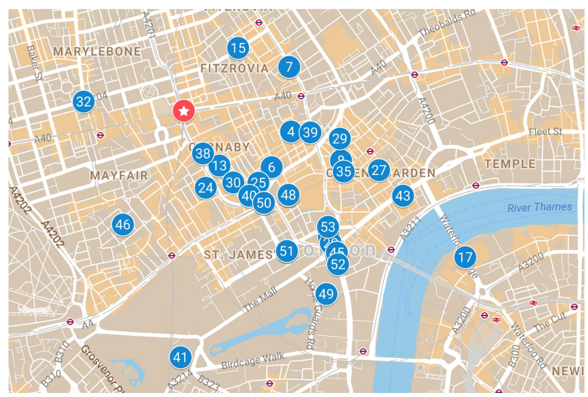

Fig. 1. The initial positions of users in daytime. The red pentastar is the Oxford Street.

B. Positions Prediction of UAV

*1) **Regression Method**:* The learning process of prediction is a process of regression, this paper uses an usual and light regression method in co-linear data analysis, called ridge regression. By abandoning unbiased property of least square method (LSM), ridge regression gets a more plausible and reliable outputs [13]. The target function of ridge regression is shown in Equation (9):

$$\Lambda^{ridge} = \sum \Big(y - \big(\beta_0 + \sum_{i=1}^{N_y} \beta_i x_i\big) \Big)^2 + \alpha \sum_{i=1}^{N_y} \beta_i^2, \quad (9)$$

in this equation, β is the constant matrix, and α is a constant.

*2) **Model Description and Evaluation Function**:* Machine learning prediction algorithms are currently proposed for automatic prediction tasks. Unlike the traditional methods, the neural machine prediction aims at building a neural network [14] rather than measuring tons of numbers.

The recurrent neural network (RNN) is a popular method for prediction task [15]. However, within most of RNN algorithm, to increase the performance of the network, backpropagation (BP) is one of the most indispensable methods in neural network training. Nevertheless, such method consumes plenty of computational resources. As consequence, to save on energy, this paper selects echo state network (ESN) model [16] for prediction of users' traces. The structure of ESN model is drawn in Fig. 2.

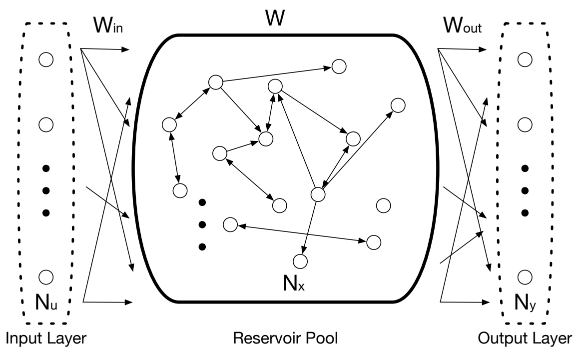

Fig. 2. The structure of Echo State Network.

In the classic ESN approach, the core part of a network is reservoir pool, who makes the whole network has a memory on previous. ESN provides an architecture and supervised learning principle like RNN algorithm, thus ESN model is also capable of short-term prediction tasks. Since only the readout from the pool is trained [17], the BP method is not helpful therefore the algorithm is energy-efficient. Though ESN algorithm costs few resources but is surprisingly found to yield excellent performance of the demanding tasks in [18], [19] and [20].

The ESN model is approximately understood as a function, whose input is the vector $u(n) = [u_1(n), u_2(n), \dots, u_{N_u}(n)]^T$ and output is $y(n) = [y_1(n), y_2(n), \dots, y_{N_y}(n)]^T$. As shown in Fig. 2, basic ESN model mainly consists of three layers: input layer, reservoir pool and output layer. The W_{in} and W_{out} are the connections between three layers, presented as matrices. The W is another matrix that presents the connections between the neurons in reservoir pool. Every segment is fixed once the whole network is established, except W_{out}, the only trainable part in the network.

As to evaluation function, in the field of prediction, mean square error (MSE) is a widely used error measurement [21], shown as Equation (10):

$$MSE = \frac{1}{N_y} \sum_{n=1}^{N_y} \sqrt{\frac{1}{T} \sum_{i=1}^{T} (y_i(n) - y_i^{target}(n))^2}, \qquad (10)$$

where the $y_i(n)$ is the prediction and $y_i^{target}(n)$ is the real value.

The algorithm of ESN is applied to supervised machine learning tasks where for a given training input signal $u(n) \in \mathbb{R}^{N_u}$ and a desired target output signal $y^{target}(n) \in \mathbb{R}^{N_y}$ is known. According to Equation (10), the task of the prediction algorithm is clear: to learn a model with output $y(n) \in \mathbb{R}^{N_y}$, minimizing the error measure and making $y(n)$ matches to $y^{target}(n)$ as well as possible.

*3) **Parameters in the Reservoir**:* Reservoir is a sparse network, consists of sparsely connected neurons, which has short-term memory on previous states and is the core part of ESN [17]. In reservoir pool, the typical update equations are

$$\tilde{x}(n) = tanh\Big(W_{in}[0 : u(n)] + W \cdot x(n-1)\Big), \qquad (11)$$

$$x(n) = (1-\alpha)x(n-1) + \alpha\tilde{x}(n), \qquad (12)$$

where $x(n) \in \mathbb{R}^{N_x}$ is the updated variable of $\tilde{x}(n)$, $tanh(\cdot)$ is the activation function of reservoir neurons, $W_{in} \in \mathbb{R}^{N_x \cdot (1+N_u)}$ and $W \in \mathbb{R}^{N_x \cdot N_x}$ are the input and recurrent weight matrices respectively.

After data echoes in the pool, it flows to output layer, which is defined as

$$y(n) = W_{out}[0; x(n)], \qquad (13)$$

where $y(n) \in \mathbb{R}^{N_y}$ stands for network outputs, and $W^{out} \in \mathbb{R}^{N_y \cdot (1+N_u+N_x)}$ is described as the weight matrix of outputs.

Here are four parameters involved to define a reservoir pool. Now they are proceeded in this order to give more details on each parameter.

- **Size of Pool**: it is the number of neurons in reservoir pool. The larger, the more precise. This is the most crucial parameter of the pool and in Equations (11) and (12) it is N_x.
  In the experiments, several different sizes of the pool have been tried, to find smallest size and meet the requirements.
- **Sparsity**: it reveals the degree of the connections between neurons in the reservoir, every neuron does not connect with all others.
  More sparse connections only give a bit better performance on position prediction task in our experiments. And to lower complexity of ESN model, a small sparsity is more ideal.
- **Distribution of Nonzero Elements**: the matrix W, which one is typically generated as a sparse network, with normal distribution and centered around zero.
  To meet the requirements in experiments, this paper uses uniform distribution for its continuity of values and boundedness, which gives almost the best performance comparing to other kinds of distributions.
- **Spectral Radius of *W***: it is one of the characteristic values of the matrix W, whose absolute value is the

largest among all. It scales the matrix W, or in another word, scales the width of the distribution of its nonzero elements.

With ESN model, the proposed algorithm easily obtains predictions of ground users' positions with low computational costs. And use iterative K-Means again to cluster the predicted positions, the next position of UAV is obtained. The accuracy of predictions is illustrated in the next section.

V. Numerical Results

In this section, numerical results are presented to facilitate the performance of the proposed machine learning based algorithm. Considering the features and amounts of the datasets, the ESN model was customized in experiments that the sparsity of reservoir pool is assumed to be 5%; the leaking rate is set as $\alpha = 0.2$.

The ensemble average MSE between predictions and real data gathered in daytime and nighttime are 1.19×10^5 and 1.25×10^5 respectively. Comparing with the MSE values of the results from random walk model [22], 5.60×10^5 in daytime and 6.42×10^5 in nighttime. And such performance was made without heavy backpropagation. Another interesting thing is that the disturbance of numerical result in daytime is less than its counterpart in the night, there are two explanations: one is that people are relatively less active in daytime, so their tracks are easier to be predicted; the other one is that the scale of the data collection is not large enough to cover all comprehensive situations.

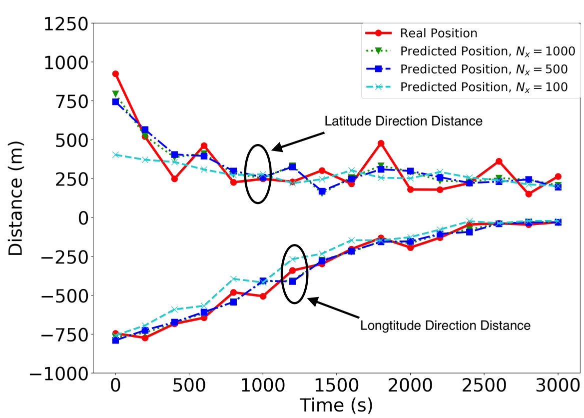

Fig. 3. The difference between real tracks and predictions in daytime.

Fig. 3 shows the disparity between real tracks and prediction results with daytime datasets. In this figure, the red lines are the real track of a single user, and the lines of other colors are the predicted outputs with different sizes of reservoir. In the procedure of experiments, three different sizes of reservoir pool are tried, they are 100, 500 and 1000. From this figure, the differences between $N_x = 500$ and 1000 are ignorable, while the errors between $N_x = 100$ and others are obvious, especially in the first half part of the prediction. Besides, considering with average time in each program with Intel® CORE 7700HQ, 0.737s for $N_x = 1000$, 0.152s for $N_x = 500$ and 0.014s for $N_x = 100$, $N_x = 500$ is taken as the final solution. In Fig. 3, the real track is very complex but the result of prediction reflects the moving trend of the user with high accuracy without overfitting. Moreover, since the radium of the gathered information collection is $1km$, thus the deviations are too small to have perceptible impacts on the real service quality.

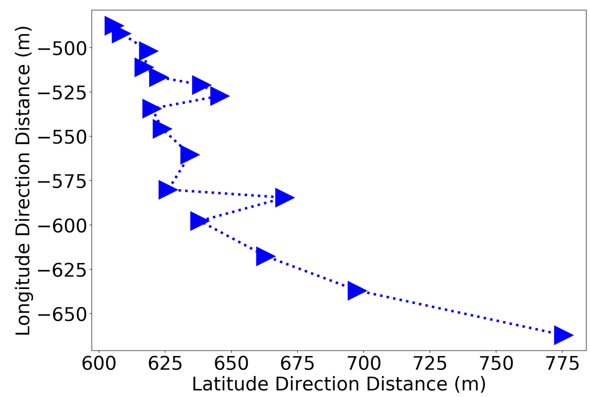

Fig. 4. The simulation track of the UAV.

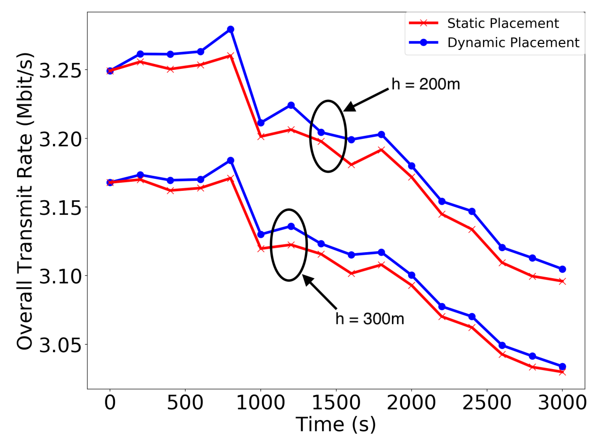

Fig. 5. The improvement of the algorithm on transmit rate.

Fig. 4 is the trajectory of UAV in the duration of 3,000 seconds, generated by iterative K-Means algorithm with the predicted data by ESN model. The direction of triangles indicates the moving trend of UAV, and Oxford Street is the origin point of this coordinate. When the served crowd moves, the overall distance between the UAV and users, D_{sum}, would change in experiments. And the moving trend of the crowd is away from the Oxford Street, which is implied in Fig. 4.

Fig. 5 expresses the improvement of the proposed algorithm on transmit rate, comparing with a UAV provides services and just stays at the initial position. As demonstrated, a significant increment on transmit rate was made by this proposed algorithm. Two situations are simulated, where the only difference was the height of UAV. To avoid skyscrapers in metropolis, simulations were tried with various high altitudes. And the lower UAV flies, the more this algorithm brings. The average distance between each user is going further in the collected datasets, therefore, the transmit rate in the simulations is going lower, as shown in Fig. 5.

VI. Conclusion

The deployment of UAV and prediction of users' position information for maintaining high quality of service were jointly studied. Three steps were provided to tackle the proposed problem. More particularly, iterative K-Means based deployment algorithm was proposed to deploy UAV at the initial time slot. Real data set of ground users' position information was collected from Twitter and ESN based prediction algorithm was proposed to predict the future positions of users. Iterative K-Means algorithm was invoked to obtain the optimal position of UAV at each time slot. It is demonstrated that the proposed algorithm was able to predict the movement of users with a praiseworthy accuracy. Additionally, re-deploying UAV based on the movement of ground users was an effective method to maintain high quality of service from UAV to users. The results are able to provide practical guidelines for the placement and movement of UAV in different scenarios to satisfy ground users' service requirements

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