Elementary seasonal adjustment of economic data with JDemetra+

Christiane Hofer, Karsten Webel / DG Data & Statistics

CIC Seminar INT202513594, 22–26 September 2025

Disclaimer

The **JDemetra+ teams** listed under

`https://github.com/orgs/jdemetra/teams`

are responsible for the maintenance and improvement of **the JDemetra+** time series **software** for official statistics **that can be downloaded** free of charge **from** one of the following websites:

- `https://github.com/jdemetra`,
- `https://github.com/rjdverse`.

The teams are not responsible for the implementation of methods labelled as “JDemetra+” in other statistical software packages, unless explicitly stated otherwise.

Preface

This presentation has been prepared using the following R packages:

- ☞ {`dplyr`}, {`ggplot2`}, {`ggthemes`}, {`tidyr`}
- ☞ {`kableExtra`}, {`knitr`}
- ☞ {`RJDemetra`}

Additional course material will be made available on the following website:

`https://github.com/bbkrd/CIC-seminar`

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JDemetra+

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Aims

Theory

- Basic understanding → Ideas, concepts
- Pretreatment → RegARIMA models
- Seasonal adjustment → X-11 & ARIMA model-based (AMB) approaches

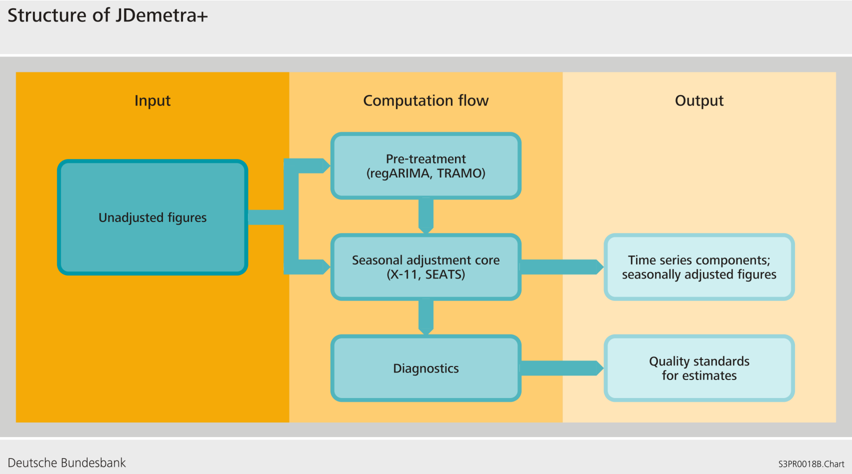
Application

- Software → JDemetra+ (JD+)
- Specification → Key options
- Results → Interpretation, quality assessment

Discussion

- Your questions → Best practices, implementation (business processes), theory

Road map


Structure of JDemetra+
Input
Computation flow
Output
Unadjusted figures
Pre-treatment
(regARIMA, TRAMO)
Seasonal adjustment core
(X-11, SEATS)
Diagnostics
Time series components;
seasonally adjusted figures
Quality standards
for estimates
Deutsche Bundesbank
S3PR0018B.Chart

Agenda

Motivation

Introduction to JD+

Seasonality diagnostics

RegARIMA pretreatment

– Regression equation
– TransReg plug-in
– ARIMA equation

X-11 approach

– Basic principle
– Further issues

ARIMA model-based approach

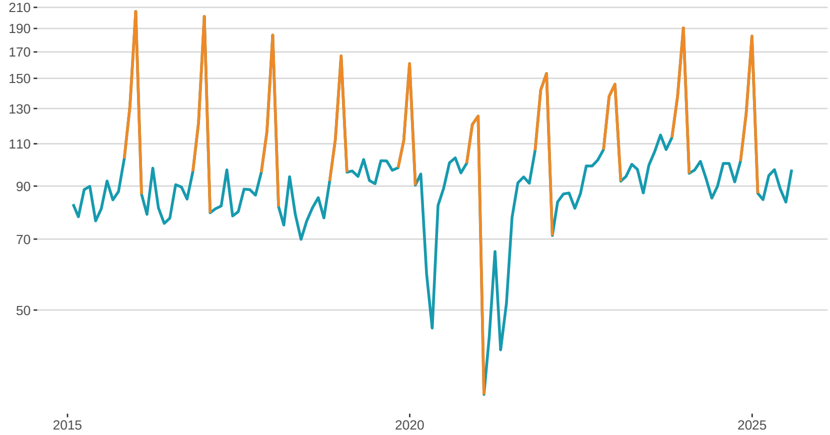
Quality assessment

Revision policies

Composite time series

Summary

What is seasonality? (I/II)



Retail trade turnover: games & toys (value, 2015 = 100) & **usual seasonal fluctuations**

What is seasonality? (II/II)

Deutsche Bundesbank, Statistical Series
“Seasonally adjusted business statistics”

“**Usual seasonal fluctuations**” means those movements which

- ☞ **recur with similar intensity**
- ☞ **in the same season each year**

and which, on the basis of past movements of the time series in question,

- ☞ **can**, under normal circumstances, **be expected to recur**.

Other verbal definitions

Hylleberg (1992)

Seasonality is the systematic, although not necessarily regular, **intra-year movement caused by** the **changes of** the **weather,** the **calendar, and timing of decisions**, directly or indirectly through the production and consumption decisions made by the agents of the economy. These decisions are **influenced by endowments,** the **expectations and preferences of** the **agents, and** the **production techniques** available in the economy.

Nerlove (1964)

In the more general case, then, we may **define seasonality as that characteristic** of a time series **that gives rise to spectral peaks at seasonal frequencies**.

Why does seasonality show up?

Repetitive events

- Natural → Climatic seasons
- Secular → Public holidays
- Clerical → Religious festivals

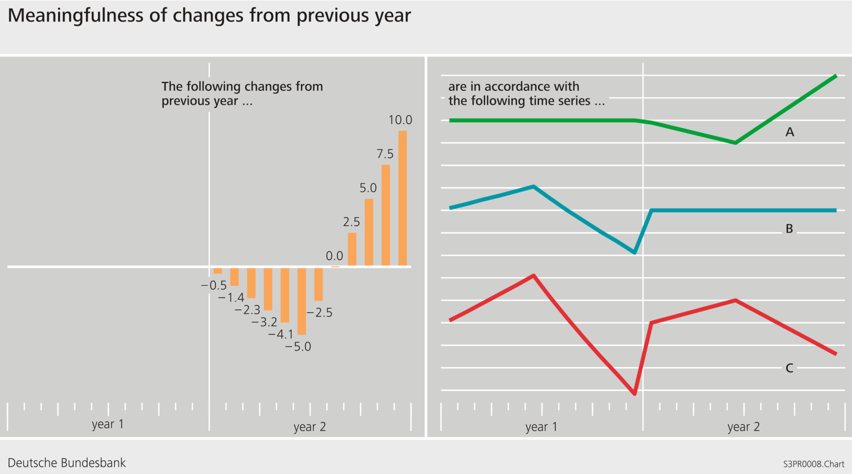
Seasonal economic models

- Agents → Optimising behaviour, rational expectations
  - Endowments, preferences, profit, utility
- Institutions → Conventional business practices, technological changes

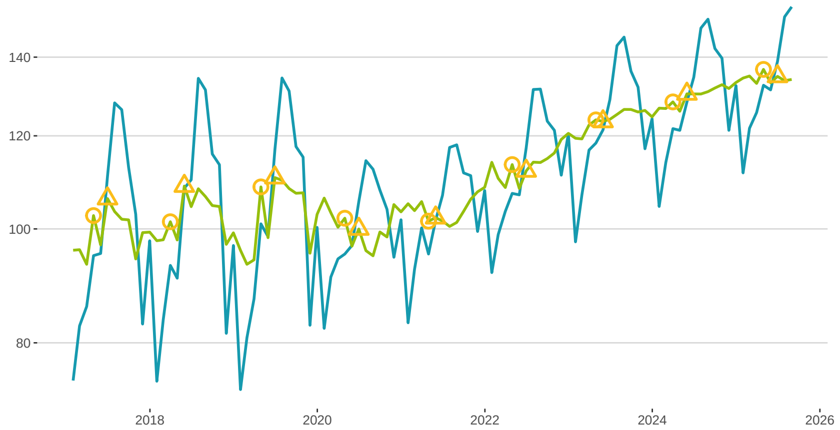
Data compilation

- Sub-components (potentially non-seasonal) → Cross-sectional aggregation

Why not year-on-year changes? (I/III)



Why not year-on-year changes? (II/III)



CPI: package holidays (2020 = 100) & only seasonally adjusted figures
with Easter (*circles*) & Whitsun (*triangles*) identifiers

Why not year-on-year changes? (III/III)

Economic developments

- Current direction → Unclear
- Turning points → Delayed detection

Leakage

- Moving seasonality
- Calendar variation → Moving holidays (e.g. Easter), number of working days

Irregular movements

- Inadequate reflection → Possible

Why seasonal adjustment? (I/II)

Deutsche Bundesbank, Statistical Series
"Seasonally adjusted business statistics"

The […] **purpose** in seasonally adjusting time series **is to filter out the usual seasonal fluctuations** within the movements of the time series under review.

Lovell (1963)

Seasonal adjustment is […] **filtering out the seasonal factor without seriously distorting the other elements** generating the observed data.

Bell & Hillmer (1984)

Seasonal adjustment is done to simplify data so that they may be more easily interpreted by statistically unsophisticated users **without a significant loss of information**.

Why seasonal adjustment? (II/II)

Deutsche Bundesbank, Statistical Series
"Seasonally adjusted business statistics"

Seasonal adjustment also **includes** the

☞ **elimination of working-day variations**

insofar **as influences deriving from**

☞ **differences in** the **number of working days or** the **dates of particular days**

(e.g. public holidays, weekday on the last day of the month in the case of stock series) **can be demonstrated and quantified**.

Visibility in seasonally adjusted data

Deutsche Bundesbank, Statistical Series
“Seasonally adjusted business statistics”

Thus, **fluctuations due to**

- ☞ **exceptionally** strong or weak **seasonal influences**

[...] **will** continue to **be visible in the seasonally adjusted series** to the extent that they

- ☞ exceed, or fall short of, the normal seasonal average.

In general, **other**

- ☞ **random disruptions and unusual movements** that are readily understandable in economic terms

[...] **are** also **not eliminated**.

Quiz: seasonal or not?

Seasonal or non-seasonal movement?		
Time	Observation	Movement
Every winter	Lower output in quarterly construction	
Every February	Lower monthly turnover compared to Jan & Mar	
Leap year February	Higher monthly retail trade turnover	
Easter	Peak in monthly chocolate sales	
August	Higher orders received due to a biennial exhibition	
Christmas	Sales peak	

Tiebreakers: fill in the blank!

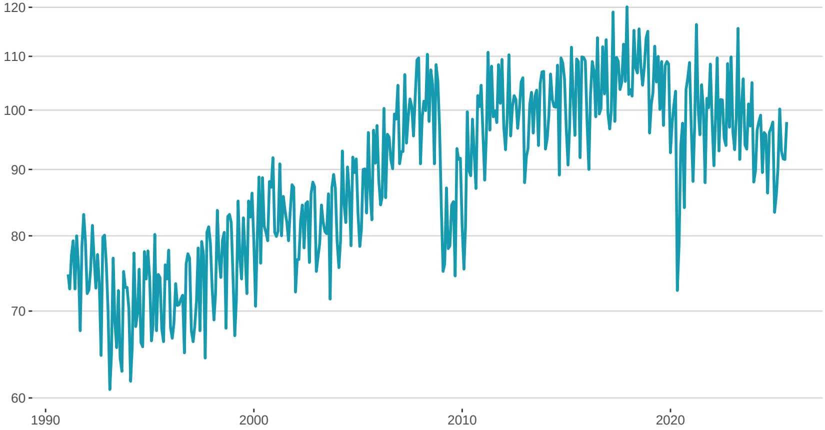
Shiskin & Eisenpress (1957), Seasonal Adjustments by Electronic Computer Methods, JASA 52 (280), 415–449.

The computations of Method II take about […] ____________ for a ten-year monthly series […].

OECD (1960), Seasonal Adjustment on Electronic Computers.

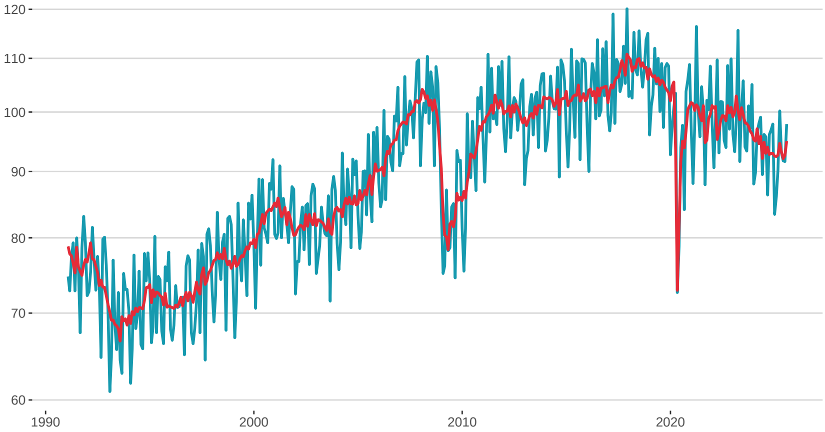
A Method II adjustment done by the Bureau of the Census on its computer costs from **$** ____ to **$** __ for a ten year series, […].

Input



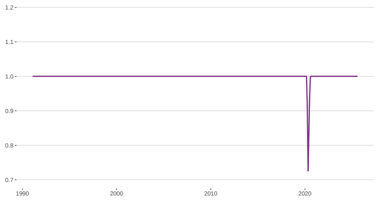
Industrial production (volume, 2021 = 100, Jan 1991 – Jul 2025)

Output

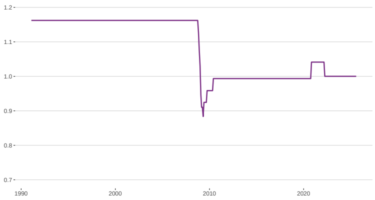


Industrial production (volume, 2021 = 100, Jan 1991 – Jul 2025) &
seasonally adjusted figures

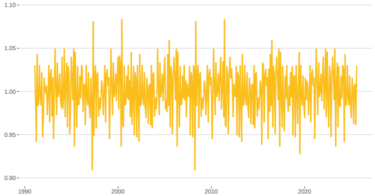
Obstacles



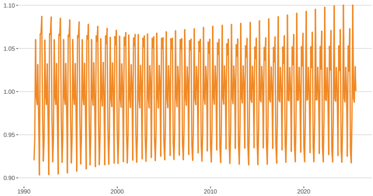
Temporary outliers



Persistent outliers



Calendar variation



Seasonal variation

Seasonal adjustment approaches

Empirical-based (non-parametic)

- Seasonal dynamics → Ad hoc extraction methods
- Weighted moving averages → STL, X-11

Model-based (parametric)

- Seasonal & non-seasonal dynamics → Joint framework
- Top-down strategy → ARIMA model decomposition
- Bottom-up strategy → Structural time series models

Syntheses

- Model-based pretreatment & ad hoc filters → X-12-ARIMA
- Model-based & HP filters → Atomic seasonal models

Key messages

Seasonality

- ☞ Usual seasonal fluctuations → Persistent intra-year swings (same period, similar amplitude)

Year-on-year changes

- ☞ Removal → Deterministic seasonality (constant amplitude)
- ☞ Leakage → Moving seasonality, calendar variation

Seasonal adjustment

- ☞ Unadjusted data → Extraction of unpredictable patterns (“news”)
- ☞ Removal → Deterministic & moving seasonality, calendar variation

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Seasonal time series

- E Ghysels & D R Osborn (2001), The Econometric Analysis of Seasonal Time Series, Cambridge University Press, Cambridge.
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- T Proietti (2000), Comparing Seasonal Components for Structural Time Series Models, International Journal of Forecasting 16 (2), 247–260.

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- H Mendershausen (1937), Methods of Computing and Eliminating Changing Seasonal Fluctuations, Econometrica 5 (3), 234–262.
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Related topics

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- Motivation
- Introduction to JD+
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  - Regression equation
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JDemetra+
History (I/II)

Since the early 19th century seasonal adjustment is a component of the statistics business process. This business has developed over the years and different methods have been implemented in different applications.

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Widely-used methods for seasonal adjustment

- X-11 (U.S. Census Bureau, iterative application of linear filters)
- TRAMO/SEATS (Maravall and Gómez, Signal Extraction in ARIMA Time Series to obtain simultaneously the different components of a time series)
- Structural Models (e.g. Harvey)

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Applications using these methods

- Win X-13 (X-13ARIMA-Seats)
- TSW+ (Tramo seats for Windows)
- Statistical Programs (R, Python, EViews, Matlab, ...)

JDemetra+
History (II/II)

Demetra

- Program to compare X12 and Tramo Seats (1997/98)
- Integration of the original software in an user-friendly application

Demetra+

- Adoption to the Guidelines on Seasonal Adjustment of the European Statistical System (ESS)
- Cooperation between Eurostat and the National Bank of Belgium
- Release 2012

JDemetra+

- Rewrite of the original software in Java
- Independent of the platform e.g. Unix, Windows,…
- Extendible
- Endorsed 2.0.0 in February 2015
- Release 2.1.0 in April 2016 …
- Release 2.2.6 in February 2025 Release 3.5.1 in June 2025

JDemetra+
Today

- One extendible main application
- Open source application (Java, Maven, NetBeans Framework)
- Officially adopted for the ESS since February 2015
- Recommended software in the ESS Guidelines
- Development is done by the National Bank of Belgium, supported by the Deutsche Bundesbank for the X-11 part

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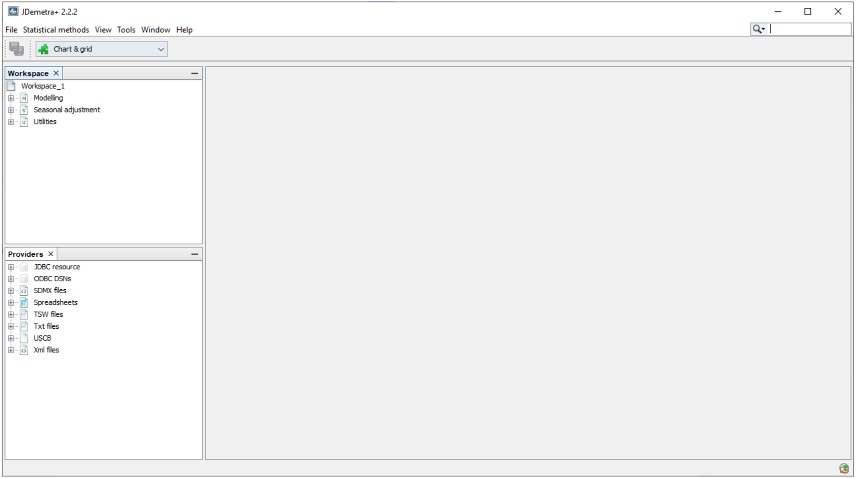
- The release version 2.2.6 (jdemetra-2.2.6-bin-windows.zip incl. Java 21) is available for download on GitHub

  https://github.com/jdemetra/jdemetra-app/releases
- Installation guide

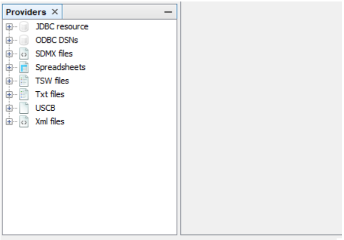
  https://github.com/jdemetra/jdemetra-app/wiki/Quick-install-guide

Basic functionalities
Starting the application for the first time



Basic functionalities
Data import (I/II)

Providers Node structure of supported data sources

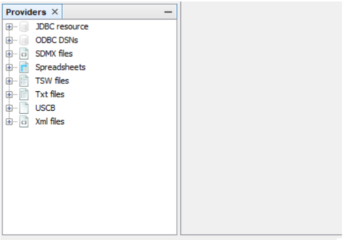


Basic functionalities
Data import (I/II)

Providers Node structure of supported data sources

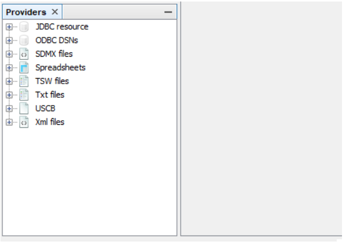
Data can be imported among other providers via

- "Spreadsheets" from Excel files (*.xls, *.xlsx)
- "Txt files" from Text files (*.txt, *.csv, *.tsv)
- "SDMX" from SDMX files (*.xml)
- "ODBC DSNs", "JDBC resource" from standard databases (Oracle, SQLServer, DB2, etc.)
- plug-ins from further providers



Basic functionalities
Data import (II/II)

How to import data:



Basic functionalities
Data import (II/II)

How to import data:

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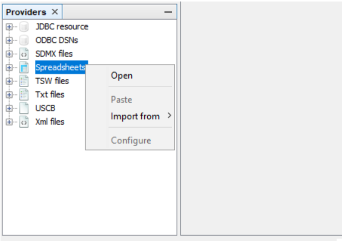
- Select Open in the context menu



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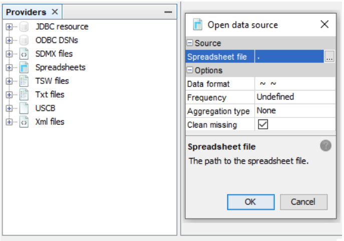

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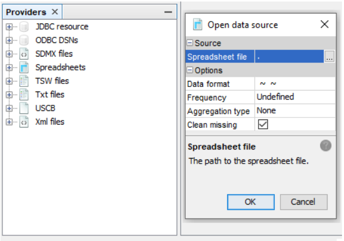

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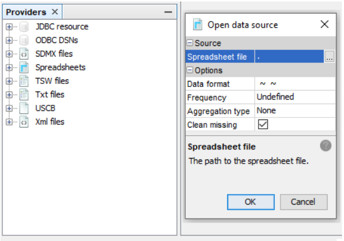
- If the original time series have a higher frequency, you decide how these will be aggregated to the imported ones'



Basic functionalities
Some nice features

- Add Star: Load time series automatically
- Drag and drop for time series
- Customize the window to your own needs
- Key R shows data points in a chart
- Zoom in charts: move mouse right down and other direction to reset

Basic functionalities
Seasonal Adjustment (I/II)

JDemetra+ offers two possibilities to conduct a seasonal adjustment:

Single Analysis:

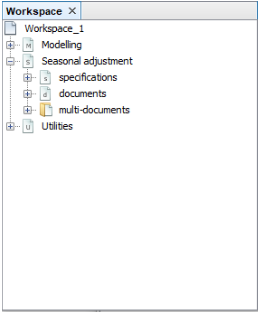
- One time series
- One specification
- Single document

Multi-processing:

- Several time series with individual specifications
- Multi document

Basic functionalities
Seasonal Adjustment (II/II)

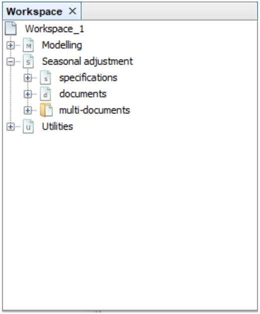
Workspace Node structure containing settings for seasonal adjustment



Basic functionalities
Seasonal Adjustment (II/II)

Workspace Node structure containing settings for seasonal adjustment

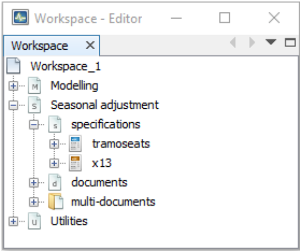
- Modelling find an ARIMA model for a time series
- Seasonal adjustment: specifications both pre-defined and individually customized
- Seasonal adjustment: multi-documents run seasonal adjustment for several time series with individual specifications
- Utilities calendars, user-defined variables



Basic functionalities
SA specifications

tramoseats
- methods of TS

x13
- methods of X-13



Basic functionalities
Saving the current work (I/II)

Workspace Concept for gathering the current workload and making it reusable at a later time and for others.

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Workspace Concept for gathering the current workload and making it reusable at a later time and for others.

- Only a single workspace can be opened.
- A JD+ workspace contains all specifications, multi-documents, variables and time series.

Basic functionalities
Saving the current work (I/II)

Workspace Concept for gathering the current workload and making it reusable at a later time and for others.

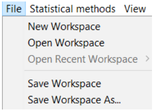
- Only a single workspace can be opened.
- A JD+ workspace contains all specifications, multi-documents, variables and time series.
- It consists of a superior xml file and a directory of the same name with subdirectories containing xml files.

Basic functionalities
Saving the current work (I/II)

Workspace Concept for gathering the current workload and making it reusable at a later time and for others.

- Only a single workspace can be opened.
- A JD+ workspace contains all specifications, multi-documents, variables and time series.
- It consists of a superior xml file and a directory of the same name with subdirectories containing xml files.
- A JD+ workspace is exchangeable between different users.

Basic functionalities
Saving the current work (II/II)



New Workspace	Opens a new empty workspace
Open Workspace	Opens an existing workspace by selecting its superior xml file
Open Recent Workspace	Displays a list of previously opened workspaces to be opened
Save Workspace	Saves the current workspace (create/overwrite)
Save Workspace As...	Saves the current workspace under a selectable name in a selectable location

Plug-ins

Endless possibilities of extensions with plug-ins

It is possible to extend the application with the installation of a plug-in (an add-in which integrates new features in the existing application)

Possible examples:

- New seasonal adjustment
- Additional data provider
- Customize the application to your own needs and style
- New features what you can do with a time series

Enhanced functionalities via plug-ins
How to install a plug-in?

How to install the plug-in `TransReg-1.2.9.nbm`

1. Download the plug-in from `https://github.com/bbkrd/TransReg/releases`

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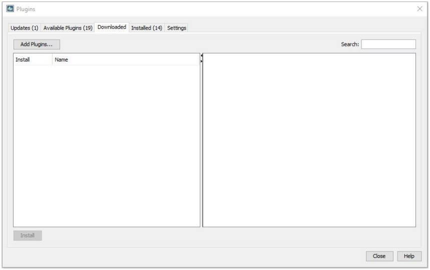

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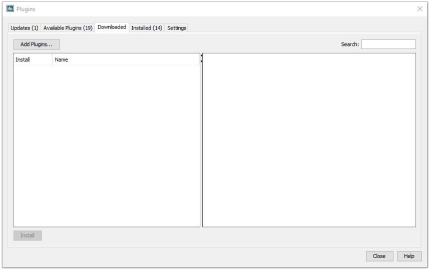

Enhanced functionalities via plug-ins

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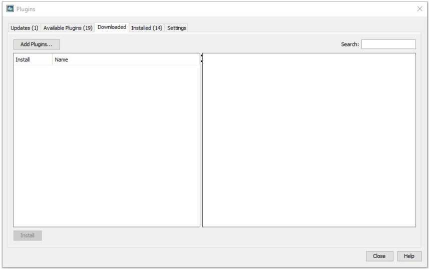
4. Click on the button "Install" and follow the instructions (license agreement, restart)



Useful links (I/II)

Release 2.2.6

- `https://github.com/jdemetra/jdemetra-app/releases/tag/v2.2.6` → Application
- `https://github.com/jdemetra/jwsacruncher/releases` → Cruncher

Documentation & blog

- `https://jdemetra-new-documentation.netlify.app/`
- `https://jdemetra-universe-blog.netlify.app/`

Helpdesk (Eurostat CROS-Portal)

- `https://…/help-jdemetra-and-time-series-analysis`

Useful links (II/II)
Deutsche Bundesbank

Plug-ins (Development)

- `https://github.com/bbkrd/TransReg`
- `https://github.com/bbkrd/CompRes`
- `https://github.com/bbkrd/KIX-UI`, `https://github.com/bbkrd/KIX2.0`

Documentation

- `https://bbkrd.github.io/`

Source Code (Open Source)

- `https://github.com/jdemetra/jdemetra-app`
- `https://github.com/jdemetra/jdemetra-core`

Overview

Key question

- Data → Need for seasonal adjustment?

Answers

- Data → Visualisation
- Descriptive statistics
  - Time domain → Serial dependence (e.g. autocorrelations)
  - Frequency domain → Spectral diagnostics (e.g. periodogram)
- Inferential statistics → Seasonality tests

Seasonality facets

Stable seasonality

- Deterministic portion → Strictly periodic intra-year effects

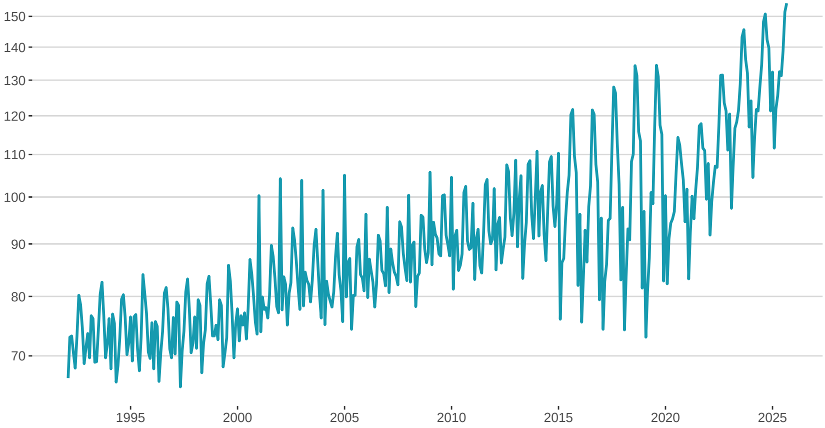
Dynamic seasonality

- Stochastic stationary portion → Time-varying recurrent intra-year effects

Unit root seasonality

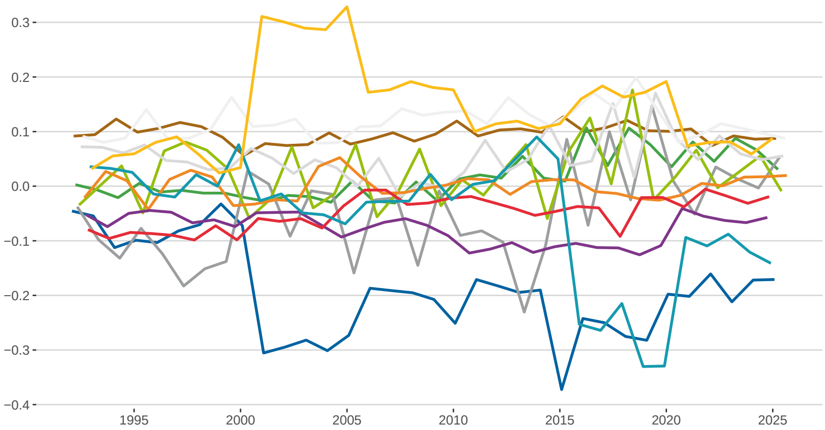
- Stochastic non-stationary portion → Time-varying recurrent intra-year effects
- Possible coverage → Stable seasonality

Data visualisation (I/III)



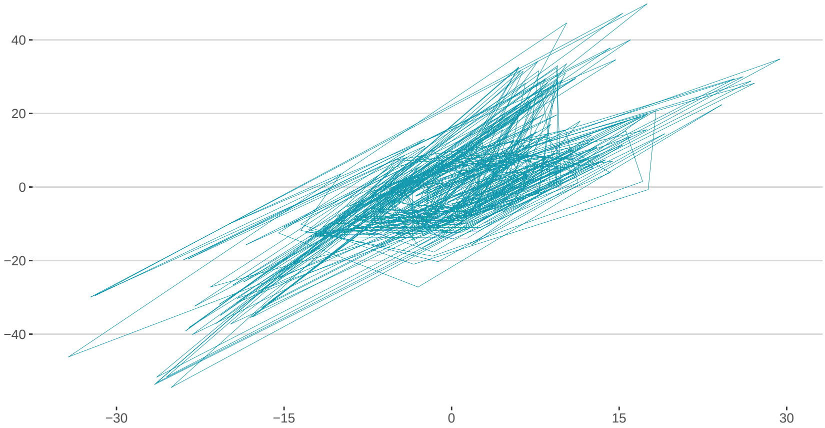
Time series plot for **CPI package holidays** (2020 = 100)

Data visualisation (II/III)



Parallel plot for differenced logged **CPI package holidays** (2020 = 100) by month (Franses, 1994)

Data visualisation (III/III)



Phase-plane plot for first-order versus second-order differenced **CPI package holidays** (2020 = 100)

Measuring serial dependence (I/II)
Autocorrelation function (ACF)

Data_{t-1} Data_{t} Data_{t+1} $\ldots$ Data_{t+h-1} Data_{t+h} Data_{t+h+1}

$\searrow \quad \searrow \quad \searrow \qquad \swarrow \quad \swarrow \quad \swarrow$

Normalised time-constant linear dependence

$\downarrow$

$\text{ACF}(h) \in [-1, 1]$

Assumption

- Data $\rightarrow$ Weakly stationary

Measuring serial dependence (II/II)
Partial autocorrelation function (PACF)

Data$_t$ Data$_{t+1}$ $\ldots$ Data$_{t+h-1}$ Data$_{t+h}$

$\downarrow \quad \searrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \swarrow \quad \downarrow$

Latent dependencies

$\downarrow \quad \downarrow \quad \downarrow$

Normalised time-constant linear dependence
corrected for latent factors

$\downarrow$

PACF$(h) \in [-1, 1]$

Spectral density (I/II)

Basic idea

Data

- Periodic base functions (e.g. cosine, sine) → Weighted sum
- Detection → Influential frequencies

Interpretation

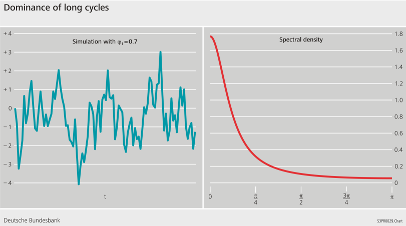
- ACF → Frequency-domain analogue
- Frequency $\in [0, \pi]$ → Importance
- Estimators → AR(30) (model-based), periodogram (non-parametric)

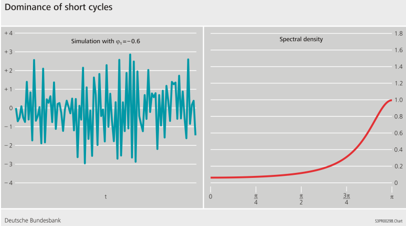
Squared gain (of linear filter)

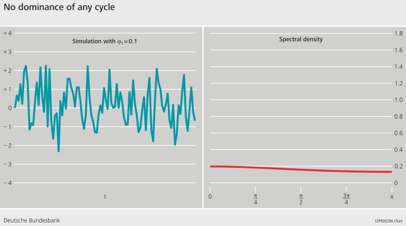
- Data & filtered data → Link between spectral densities
- Frequency $\in [0, \pi]$ → Amplification, suppression
- Examples → X-11 trend & seasonal filters

Spectral density (II/II)

Periodogram: AR (1) process, unit WN variance







Cycles per year	1	2	3	4	5	6
Duration in months	12	6	4	3	2.4	2
Seasonal frequency	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	π
Duration in quarters	4	2				
Seasonal frequency	$\frac{\pi}{2}$	π				

Initial question: answers

First things first

- Data → Visualisation

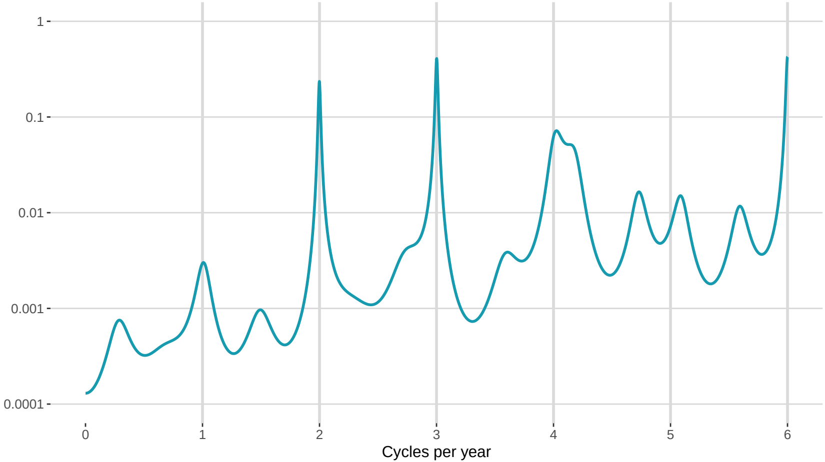
Descriptive statistics

- ACF, PACF → Peaks at "intra-year cycle" lags (especially 2, 3, 4, 6, 12)
- Estimated spectral density → Peaks at seasonal frequencies
  - Width → Stability indicator

Need for SA?

- Unadjusted data
- Transformed (modified) data
- Linearised data

Data visualisation: a look through the spectral lens



AR(30) spectral density estimate for **industrial production** (volume, 2021 = 100, Jan 1991 – Jul 2025)

Seasonality tests in JD+ (I/X)
Overview

Name	Variable checked for significance	Short
Modified QS test	Autocorrelations at seasonal lags	QS
Friedman test	ANOVA with repeated measures on intra-year ranks	FT
Kruskal-Wallis test	ANOVA without repeated measures on overall ranks	KW
Test for spectral peaks	Tukey and AR(30) spectra at seasonal frequencies	SP
Periodogram test	Weighted sum of periodogram at seasonal frequencies	PD
F-test on seasonal dummies	Effects of seasonal dummies in the "$(pdq)(000)$ $+$ mean $+$ seasonal dummies" model*	SD

* Variant 1: $(pdq) = (011)$, variant 2: (pdq) according to automatic model identification.

Seasonality tests in JD+ (II/X)

Ljung-Box test

$$LB_H = \sum_{h=1}^{H} w(h)\,\mathrm{ACF}^2(h), \quad \text{with } w(h) = \frac{T(T+2)}{T-h}$$

Hypothesis

- Data → Zero autocorrelations (i.e. independence)

Decision

- Rejection if LB_H too large

Seasonality tests in JD+ (III/X)

Modified QS test

$$QS = \begin{cases} w(12)\,\text{ACF}^2(12) + w(24)\left[\max\{0, \text{ACF}(24)\}\right]^2, & \text{ACF}(12) > 0 \\ 0, & \text{ACF}(12) \leqslant 0 \end{cases}$$

Hypothesis

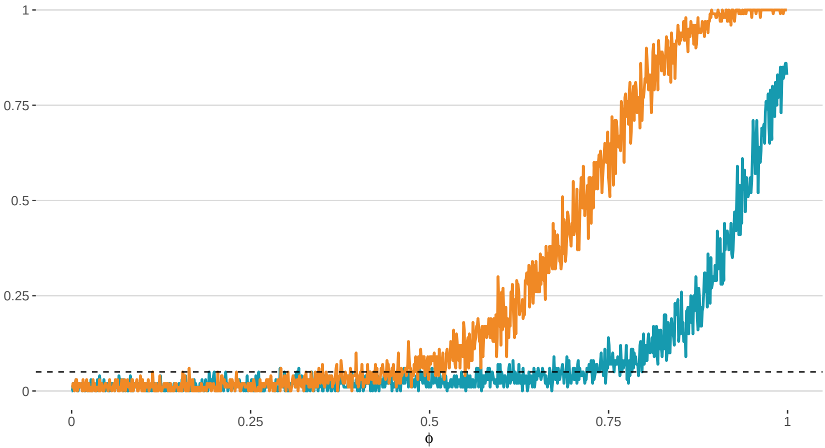
- Data → No positive autocorrelation at seasonal lags (i.e. no dynamic seasonality)

Decision

- Rejection if QS too large

Seasonality tests in JD+ (IV/X)

Modified QS test: digression



Empirical rejection rates for simulated **non-seasonal** AR(1) **processes** with parameter $\phi \in [0, 1)$ for $\boldsymbol{\tau = 4}$ & $\boldsymbol{\tau = 12}$ $(T = 120, \alpha = 0.05)$

Seasonality tests in JD+ (V/X)

Friedman test

$$FT = \frac{\text{Variance of month-specific mean ranks}}{\text{Variance of ranks within months}}$$

Rank assignment

- Within each year → $1 \leqslant \text{rank}_{ij} \leqslant \tau$

Hypothesis

- Data → Same month-specific mean ranks (i.e. no stable seasonality)

Decision

- Rejection if FT too large

Seasonality tests in JD+ (VI/X)

Kruskal-Wallis test

$$KW = \frac{\text{Variance of month-specific mean ranks}}{\text{Variance of ranks within months}}$$

Rank assignment

- Within observation period $\rightarrow 1 \leqslant \text{rank}_{ij} \leqslant T$

Hypothesis

- Data $\rightarrow$ Same month-specific mean ranks (i.e. no stable seasonality)

Decision

- Rejection if KW too large

Seasonality tests in JD+ (VII/X)
Test for spectral peaks

Idea

- Spectral peaks → Visual significance
- Tukey & AR(30) spectra → Combination

Hypothesis

- Data → No visually significant seasonal peaks (i.e. no dynamic seasonality)

Decision

- Rejection if too many visually significant seasonal peaks
- Consideration → Location of seasonal peaks

Seasonality tests in JD+ (VIII/X)
Periodogram test

$$PD = \text{Rescaled}\left[2\sum_{j=1}^{5} I(\mathsf{SF}_j) + I(\mathsf{SF}_6) \times \begin{Bmatrix} 1, & T \text{ even} \\ 0, & T \text{ odd} \end{Bmatrix}\right]$$

Hypothesis

- Data → No seasonal peaks (i.e. no dynamic seasonality)

Decision

- Rejection if PD too large

Seasonality tests in JD+ (IX/X)
F-test on seasonal dummies

$$SD = \text{Rescaled}\left(\hat{\boldsymbol{\beta}}^\top \hat{\boldsymbol{\Sigma}}_{\hat{\boldsymbol{\beta}}}^{-1} \hat{\boldsymbol{\beta}}\right) \text{ from model}$$
$$\phi_p(B)(1-B)^d\left(\text{Data}_t - \hat{\boldsymbol{\beta}}^\top \textbf{Dummies}_t\right) = \mu + \theta_q(B)\,\varepsilon_t$$

Hypothesis

- Data → Zero effects for seasonal dummies (i.e. no stable seasonality)

Decision

- Rejection if SD too large

Seasonality tests in JD+ (X/X)
Examples

Industrial production
Volume, 2021 = 100

Test	Linearised data Statistic	Linearised data p-value	SA data Statistic	SA data p-value
Modified QS	724.365	0	0.190	0.910
Friedman	350.593	0	4.353	0.958
Kruskal-Wallis	35.771	0	0.069	1.000
Periodogram	614.541	0	0.672	0.766
Seasonal dummies	561.823	0	0.642	0.793

Warning

Test statistics and p-values have been obtained through the `{RJDemetra}` package and may differ from the figures displayed in the graphical user interface of JD+.

Key messages

Main questions

☞ Data → Seasonality? Need for seasonal adjustment?

Tools

☞ Data → Visualisation

☞ Descriptive statistics

- ACF, PACF → Peaks at seasonal lags
- Estimated spectral density → Peaks at seasonal frequencies

☞ Inferential statistics → Seasonality tests

- Seasonality → Different manifestations

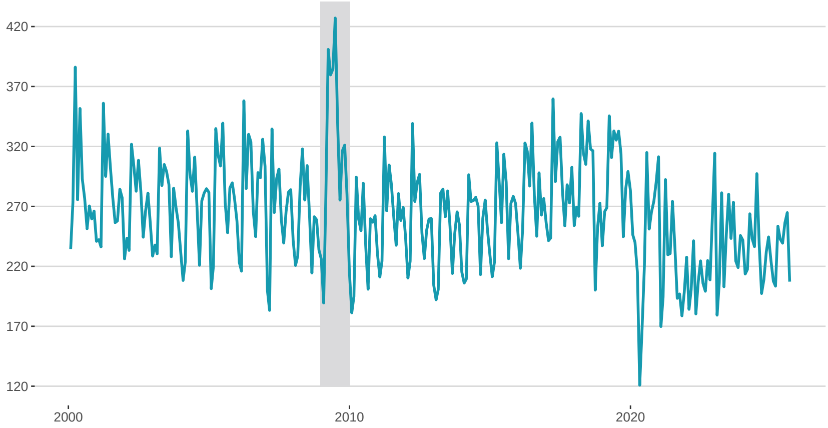
References

- W R Bell, K M McDonald-Johnson, T S McElroy, O Pang, B C Monsell & B Chen (2022), Identifying Seasonality, Interagency Seasonal Adjustment Team, Subgroup A.
- P H Franses (1994), A Multivariate Approach to Modeling Univariate Seasonal Time Series, Journal of Econometrics 63 (1), 133–151.
- D Ollech & K Webel (2023), A Random Forest-based Approach to Combining and Ranking Seasonality Tests, Journal of Econometric Methods 12 (1), 117–130.
- T Proietti (2025), Another Look at Dependence: The Most Predictable Aspects of Time Series, Journal of Business & Economic Statistics 43 (3), 723–736.
- K Webel & D Ollech (2025), A Novel Tree-Based Combined Test for Seasonality, Data Science in Science 4 (1), Article 2517006.

Quiz: guess the year!

In almost every true series of observations, some are found, which differ so much from the others as to indicate some abnormal source of error not contemplated in the theoretical discussions, and the introduction of which into the investigations can only serve, in the present state of science, to perplex and mislead the inquirer.

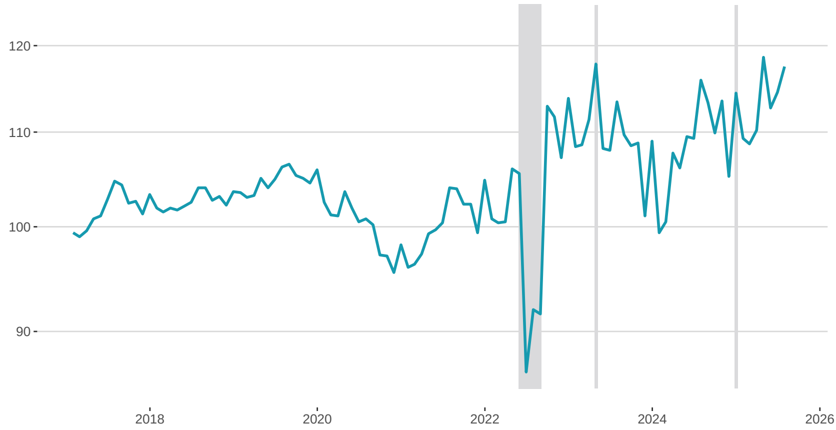
Messy data (I/V)
Temporary outliers: monetary incentive



Registered passengar cars in Germany (thousands) with **governmental fund identifier** (*shaded area*)

Messy data (II/V)

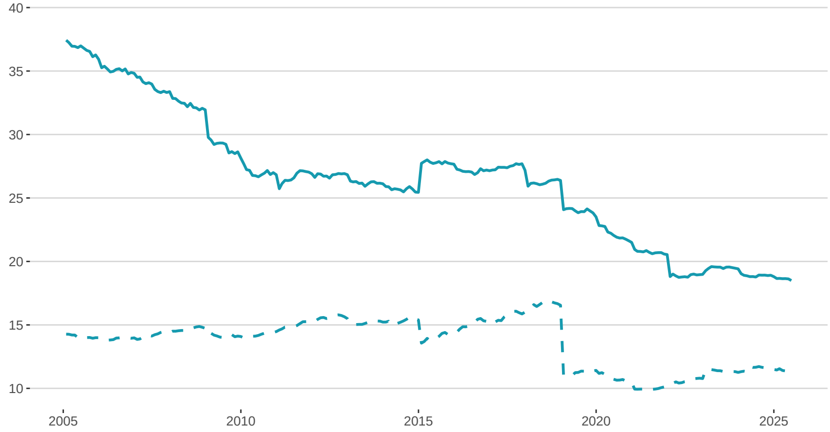
Temporary outliers: monetary incentive



CPI: transport services (2020 = 100) with **€9 ticket** (*shaded area*) & **€49 & €58 ticket identifiers** (*verticals*)

Messy data (III/V)

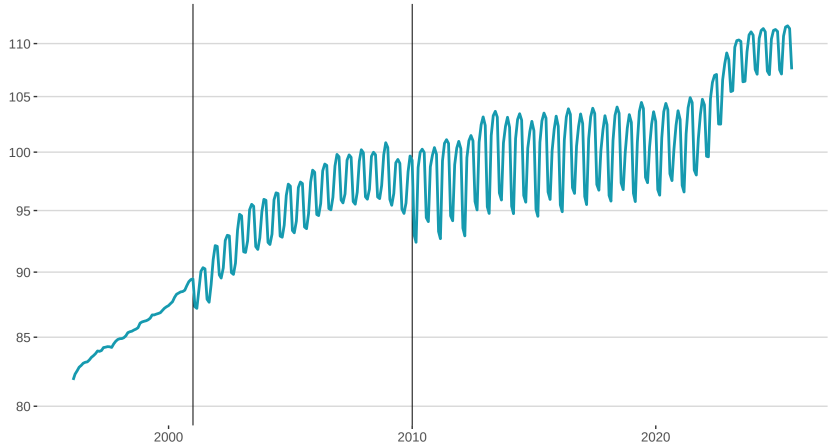
Persistent outliers: change in classification



Employed persons (thousands) **in the manufacture of wearing apparel** (*solid*) & **of leather & related products** (*dashed*)

Messy data (IV/V)

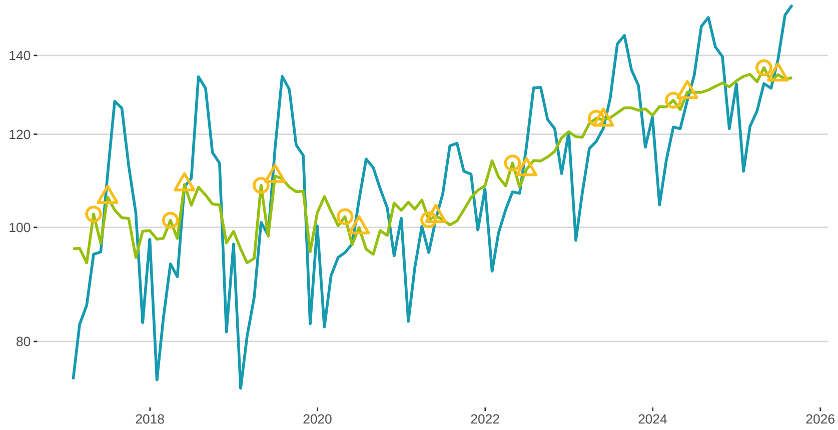
Structural breaks: change in data compilation



HICP for Spain: industrial goods excluding energy (2015 = 100) with **implementation date identifiers** (*verticals*)

Messy data (V/V)

Calendar variation: moving holidays



CPI: package holidays (2020 = 100) & **only seasonally adjusted figures** with **Easter** (*circles*) **& Whitsun** (*triangles*) **identifiers**

Reasons (I/II)
Outliers

Temporary unusual circumstances

- Atypical holiday constellations
- Extreme weather conditions
- Large-scale orders
- Major sport events
- Pandemics
- Strikes

Persistent changes

- Classification
- Policiy → Economic, political
- Legislation, regulation

Reasons (II/II)
Calendar variation

Stock data

- Reporting → Day of the week
- Currency in circulation, overnight deposits, etc.

Flow data

- Each weekday → Varying number over months
- Orders received, output, turnover, etc.

Moving holidays

- Easter, Whitsun, etc.

Pretreatment regression model

Goals

- Outliers → Temporary removal
- Calendar effects → Permanent removal
- Data → Log transformation, “optimal” forecasts

Regression equation

- Linear regression model → Exogenous effects
- Independent variables → Outlier & calendar regressors
- Disturbances → ARIMA process

ARIMA equation

- Seasonal ARIMA model → Dependence structure

Reminder: outlier concept is a relative one

Unobserved component (UC) model

$$
\begin{array}{ccccccccc}
\text{Data} & = & \text{Trend-cycle} & \circ & \text{Seasonal} & \circ & \text{Calendar} & \circ & \text{Irregular} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
y_t & = & t_t & \circ & s_t & \circ & c_t & \circ & i_t \\
 & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
 & & \text{Long-term growth path} & & \text{Recurring intra-year movements} & & \text{Calendar-related movements} & & \text{Remaining movements} \\
 & & \searrow & & & & & & \swarrow \\
 & & & & \text{Seasonally adjusted (SA) data} & & & &
\end{array}
$$

Assumptions

- Data → Monthly
- Decomposition → Additive ($\circ = +$), multiplicative ($\circ = \times$)

Non-technical setup

Data = Outliers + Calendar variation + Disturbances

Data	Outliers	TD	Holiday	User	Disturbances
↓	↓	↓	↓	↓	↓
↓	↓	↓	↓	↓	↓
	Calculation of scaling factors from estimated effects				
↓	↓	↓	↓	↓	↓
Table A 1	Table A 8	Table A 6	Table A 7	Table A 9	Table B 1

Technical setup

$$f(y_t) = \sum_k \beta_k \times k\text{-th regressor}_t + z_t$$

Notations

- f → Transformation
  - Additive decomposition → Identity
  - Multiplicative decomposition → Log
  - Automatic selection → Log-level test
- β_k → Unknown outlier & calendar effects
- z_t → ARIMA disturbances

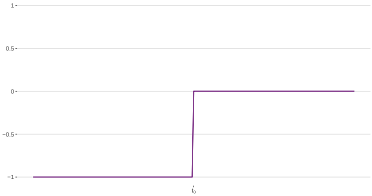
Outliers (I/VI)
Types

Effect	Type	UC	Correction	Visibility in SA figures	Frequency of use (BBk)
Tempo-rary	Additive outlier	Irregular	Present	Yes	Common
	Transitory change	Irregular	Present	Yes	Occasional
	Transitory level shift [1)]	Trend-cycle	Present	Yes	N/A
Persis-tent	Level shift	Trend-cycle	Past	Yes	Common
	Linear ramp	Trend-cycle	Past	Yes	Rare
	Quadratic ramp [1)]	Trend-cycle	Past	Yes	N/A
	Seasonal outlier	Seasonal	Past	No	Rare

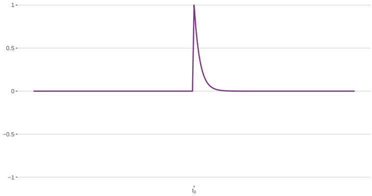
1 This type is currently not implemented in JD+.

Outliers (II/VI)
Regression variables

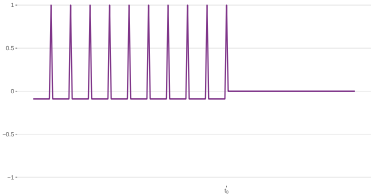
Additive outlier (AO)



Level shift (LS)

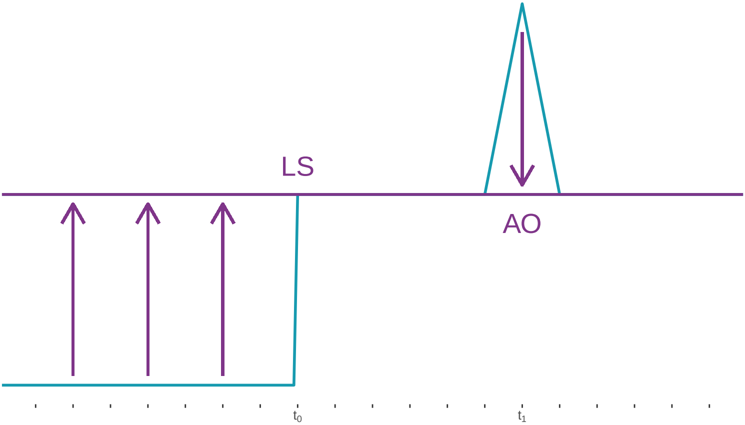


Transitory change (TC)



Seasonal outlier (SO)

Outliers (III/VI)
Correction principle



Unadjusted figures & outlier adjusted figures

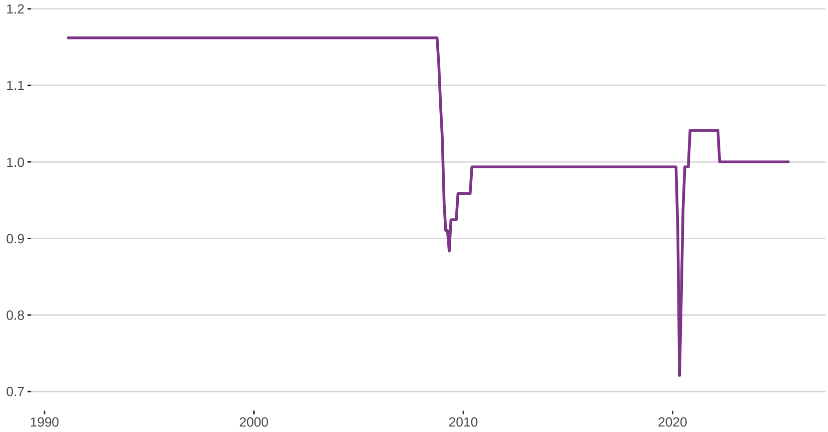
Outliers (IV/VI)
Estimated effects

Industrial production
Volume, 2021 = 100

LS	Estimate	Std. err.	t-value	AO	Estimate	Std. err.	t-value
10-2008	-0.0308	0.01522	-2.02	03-2020	-0.0875	0.01398	-6.26
11-2008	-0.0471	0.01612	-2.92	04-2020	-0.3206	0.01509	-21.25
12-2008	-0.0426	0.01611	-2.65	05-2020	-0.1864	0.01509	-12.35
01-2009	-0.0846	0.01610	-5.25	06-2020	-0.0581	0.01394	-4.17
02-2009	-0.0389	0.01530	-2.54				
04-2009	-0.0300	0.01530	-1.96				
05-2009	0.0453	0.01536	2.95				
09-2009	0.0362	0.01430	2.53				
05-2010	0.0358	0.01444	2.48				
10-2020	0.0469	0.01434	3.27				
03-2022	-0.0404	0.01447	-2.79				

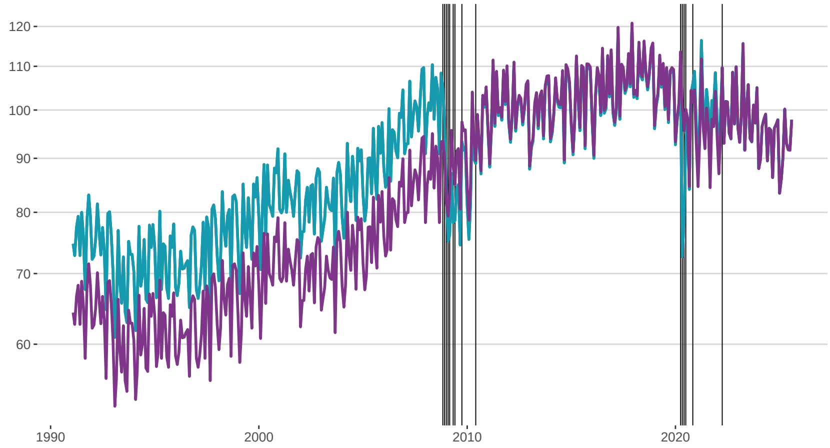
Outliers (V/VI)

Estimated component



Combined outlier component (A 8) for industrial production

Outliers (VI/VI)
Temporary removal



Industrial production (volume, 2021 = 100) & outlier adjusted figures with **AO & LS identifiers** *(verticals)*

Calendar variation (I/X)

Types

Calendar variation	=	Seasonal calendar variation	+	Structural calendar variation
		↓		↓
		Properties of the calendar that		
		recur every year		vary from year to year
		↓		↓
		Seasonal component		Calendar component

Calendar variation (II/X)

Example: length-of-month (LOM) effect in flows (Bell, 1984)

LOM effect	=	Average LOM effect	+	Deviation from average LOM (excluding LY) effect	+	Leap-year (LY) effect
		↓		↓		↓
		Constant level		Seasonal variation		Structural calendar variation
		↓		↓		↓
		Trend-cycle		Seasonal component		Calendar component

Calendar variation (III/X)

Predefined regression variables

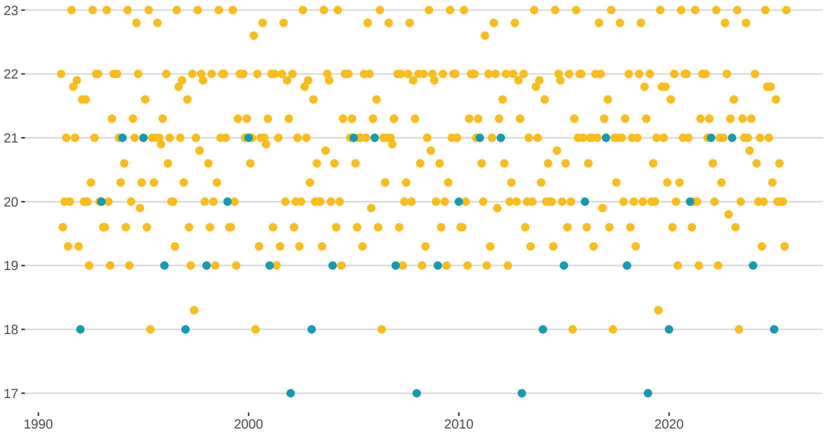
Selected variables

- Trading-day contrasts → Individual day-of-the-week effects
- Working-day contrast → Common MON–FRI & SAT–SUN effects
- Leap year → February dummy
- Easter → February/March/April share of pre-Easter period of interest (in days)

Pros & cons

- Advantage → Easy use, instant results, indication
- Disadvantage → No customisation, possible mismatch with national calendar

Calendar variation (IV/X)
User-defined regression variables



Working days in Germany (including partial working days, non-centred) for January through November & December

Calendar variation (V/X)
Estimated working-day effects

Industrial production Volume, 2021 = 100			
Working days in ...	Estimate	Std. err.	t-value
January, ..., November	0.0358	0.00064	56.18
December	0.0232	0.00151	15.41

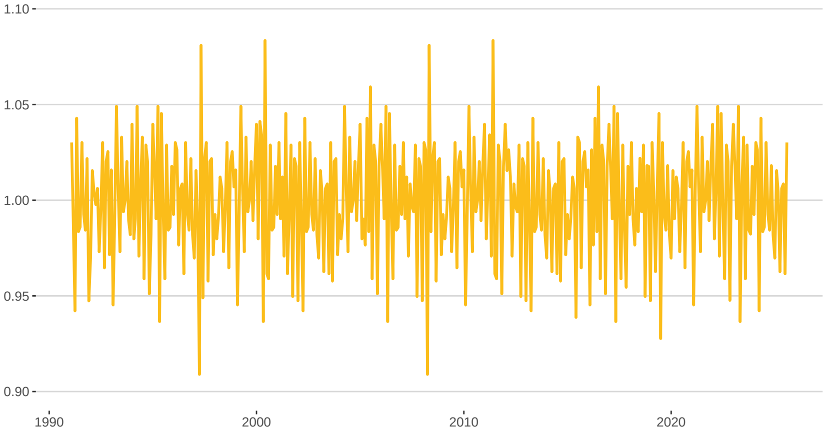
Working-day (WD) regression variables

- Centred → Month-specific means

Interpretation

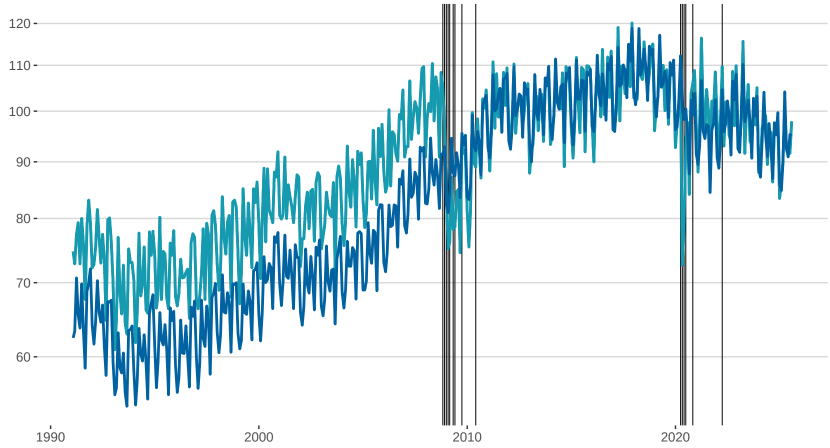
- One additional WD → Average rise in production
    - January, ..., November → 3.6 %
    - December → 2.3 %

Calendar variation (VI/X)
Estimated working-day component



User-defined TD component (A 6) for industrial production

Calendar variation (VII/X)
Permanent removal



Industrial production (volume, 2021 = 100) & **outlier & calendar adjusted** (linearised) **figures** with **AO & LS identifiers** (*verticals*)

Calendar variation (VIII/X)
Estimated moving-holiday effects

CPI: package holidays 2020 = 100			
Location of …	Estimate	Std. err.	t-value
Easter Sunday in March	0.0402	0.01232	3.26
Easter Sunday in April	0.0621	0.01299	4.78
Whit Sunday in May/June	0.0298	0.00685	4.35

Moving-holiday (MH) regression variables

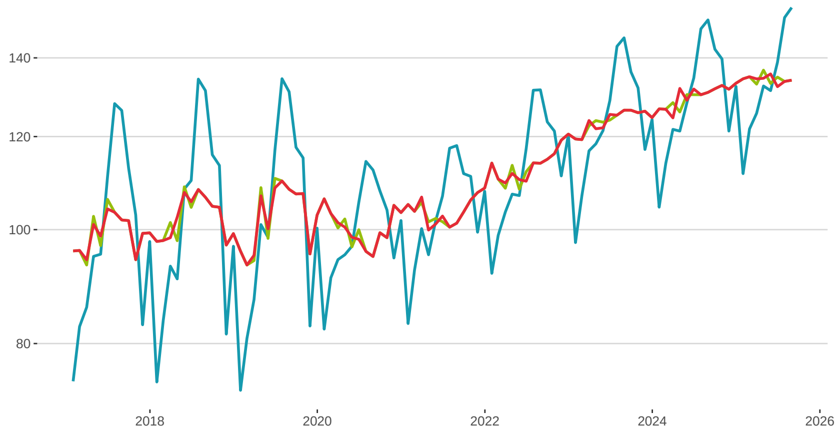
- Easter → March & April dummies
- Whitsun → Joint May/June dummy

Interpretation

- Location of MH → Average rise in consumer prices
  - Easter Sunday → 4 % (March) & 6.2 % (April)
  - Whit Sunday → 3 % (May & June)

Calendar variation (IX/X)

Permanent removal



CPI: package holidays (2020 = 100), **only seasonally adjusted figures** & **seasonally adjusted figures**

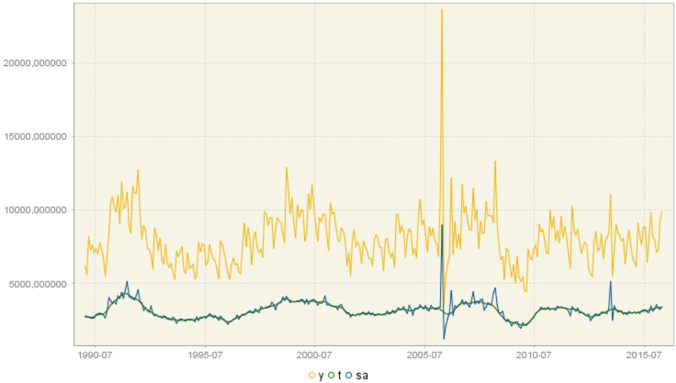
Calendar variation (X/X)
Further aspects

Calendar adjustment in official statistics

Criterion [1]	Average impact of: Working days	Bridge days	School holidays	Weather
Estimated effect ...				
significant	Yes	Yes	Yes	Yes
plausible	Yes	Yes	No	Yes
Majority of adjusted figures plausible	Yes	Yes	Yes	No
Systematic over-/under-adjustment irrelevant	Yes	No	No	No
Catch-up effects quantifiable	Yes	No	No	No

1 As described in Section 4.5 of the ESS Guidelines on Seasonal Adjustment (2024 edition).

Question?



Adjusting German car registrations using a non-centred working-day regressor

Problem

JD+

- No automatic centring of user-defined variables
- Non-centred regressors lead to a level mismatch
- No built-in centring option
- Centring outside JD+ and then loading centred regressors necessary

Solution (I/II)

Idea

- Centring user-defined regressors
- Deviation from long-term average
- centeruser = mean
- centeruser = seasonal

Implementation

- Transformation of regressors
- A plug-in to create, generate, transform and work with regressors

Solution (II/II)

Install

TransReg-1.2.9.nbm

Create a TransReg document

Workspace → TransReg → documents → New

Create a new variable

Utilities → Variables → New

Use the transformed variable

Open with Chart & Grid → Drag & Drop to variable

Useful links

Plug-ins

https://github.com/bbkrd/TransReg

Documentation

https://bbkrd.github.io/pages/transreg/

Reminder: regression equation

Goals

- Data → Log transformation, “optimal” forecasts
- Outliers → Temporary removal
- Calendar variation → Permanent removal

Disturbances

- Trend & seasonal dynamics → Non-zero autocorrelations at multiple lags
- White-noise assumption → Invalid
- Valid alternative → ARIMA models

Why ARIMA models?

STATCAN requirements (Dagum, 1978)

- Models → Parsimony (number of parameters), stability (across time), simplicity
- Forecasts → Mean squared error (MSE) optimality, reflectivity (infra-year dynamics), short-term effectiveness

Key concept

- Data (general structure) → Past observations & lagged random shocks

Ingredients

Non-seasonal/seasonal operators

- Stationary autoregressive → AR/SAR
- Differencing → D/SD
- Invertible moving average → MA/SMA

White noise (WN)

- Mean → Zero
- Variance → Finite, constant
- Dependence structure → Zero (all lags)

Non-technical setup

Disturbances of regression equation

$$\downarrow$$

$$\begin{array}{ccccccccccccccc}
\mathrm{AR} & \circ & \mathrm{SAR} & \circ & \mathrm{D} & \circ & \mathrm{SD} & \circ & \mathrm{B}\,1 & = & \mathrm{MA} & \circ & \mathrm{SMA} & \circ & \mathrm{WN} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & & & \downarrow & & \downarrow & & \\
p & & P & & d & & D & & & & q & & Q & & \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & & & \downarrow & & \downarrow & &
\end{array}$$

Order identification & parameter estimation

Technical setup

AR	SAR	D	SD	B 1	MA	SMA	WN
↓	↓	↓	↓	↓	↓	↓	↓

$$\phi_p(B)\,\Phi_P\left(B^{12}\right)(1-B)^d\left(1-B^{12}\right)^D z_t = \theta_q(B)\,\Theta_Q\left(B^{12}\right)\varepsilon_t$$

Notations

- B → Backshift operator, i.e. $B^k z_t = z_{t-k}$
- Short → $f(pdq)(PDQ)$

Details (I/VI)
Non-seasonal AR operator

$$\phi_p(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$$

Example $(p = 2)$

$$\begin{aligned} \phi_2(B) z_t &= \left(1 - \phi_1 B - \phi_2 B^2\right) z_t \\ &= z_t - \phi_1 z_{t-1} - \phi_2 z_{t-2} \end{aligned}$$

Details (II/VI)
Seasonal AR operator

$$\Phi_P\left(B^{12}\right) = 1 - \Phi_1 B^{12} - \Phi_2 B^{24} - \cdots - \Phi_P B^{12P}$$

Example $(P = 2)$

$$\begin{aligned}\Phi_2\left(B^{12}\right) z_t &= \left(1 - \Phi_1 B^{12} - \Phi_2 B^{24}\right) z_t \\ &= z_t - \Phi_1 z_{t-12} - \Phi_2 z_{t-24}\end{aligned}$$

Details (III/VI)

Non-seasonal differencing operator

$$(1 - B)^d$$

Example $(d = 1)$

$$(1 - B)z_t \quad = \quad z_t - z_{t-1}$$

Details (IV/VI)
Seasonal differencing operator

$$\left(1 - B^{12}\right)^{D}$$

Example $(D = 1)$

$$\left(1 - B^{12}\right) z_t \quad = \quad z_t - z_{t-12}$$

Details (V/VI)
Non-seasonal MA operator

$$\theta_q(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q$$

Example ($q = 2$)

$$\begin{aligned} \theta_2(B)\varepsilon_t &= \left(1 - \theta_1 B - \theta_2 B^2\right)\varepsilon_t \\ &= \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} \end{aligned}$$

Details (VI/VI)
Seasonal MA operator

$$\Theta_Q\left(B^{12}\right) = 1 - \Theta_1 B^{12} - \Theta_2 B^{24} - \cdots - \Theta_Q B^{12Q}$$

Example ($Q = 1$)

$$\begin{aligned}\Theta_1\left(B^{12}\right)\varepsilon_t &= \left(1 - \Theta_1 B^{12}\right)\varepsilon_t \\ &= \varepsilon_t - \Theta_1 \varepsilon_{t-12}\end{aligned}$$

Airline model (I/III)
Definition

$$(1-B)\left(1-B^{12}\right)z_t = (1-\theta_1 B)\left(1-\Theta_1 B^{12}\right)\varepsilon_t$$

Notations

- $-1 \leqslant \theta_1, \Theta_1 \leqslant 1$
- Short $\rightarrow (pdq)(PDQ) = (011)(011)$

Interpretation

- Θ_1 close to one/zero $\rightarrow$ Stable/unstable seasonality
- θ_1 close to one/zero (given Θ_1) $\rightarrow$ Stable/unstable trend

Airline model (II/III)
Explanation: model for input series

$$z_t = a + bt + s_t + \text{residual}_t$$

Deterministic components

- Trend
- Seasonal factors

Consequence

$$(1 - B)\left(1 - B^{12}\right)\text{residual}_t = (1 - \theta_1 B)\left(1 - \Theta_1 B^{12}\right)\varepsilon_t$$

Airline model (III/III)
Application

Choice of Henderson & seasonal filters in X-11

θ_1	Θ_1 0.95		0.80		0.70		0.60	
0.95	s3x15	H23	s3x15	H23	s3x9	H23	s3x5	H23
0.80	s3x15	H23	s3x15	H23	s3x9	H23	s3x5	H23
0.70	s3x15	H23	s3x15	H23	s3x9	H23	s3x5	H23
0.60	s3x15	H17	s3x15	H17	s3x9	H17	s3x5	H17
0.50	s3x15	H13	s3x15	H13	s3x9	H13	s3x5	H13
0.40	s3x15	H13	s3x15	H13	s3x9	H13	s3x5	H13
0.20	s3x15	H9	s3x15	H9	s3x9	H9	s3x5	H9
0.00	s3x15	H9	s3x15	H9	s3x9	H9	s3x5	H9
θ_1	Θ_1 0.50		0.40		0.20		0.00	
0.95	s3x3	H23	s3x3	H23	s3x3	H23	s3x3	H23
0.80	s3x3	H23	s3x3	H23	s3x3	H23	s3x3	H23
0.70	s3x3	H23	s3x3	H23	s3x3	H23	s3x3	H23
0.60	s3x5	H23	s3x3	H23	s3x3	H23	s3x3	H23
0.50	s3x5	H13	s3x3	H17	s3x3	H23	s3x3	H23
0.40	s3x5	H13	s3x3	H13	s3x3	H17	s3x3	H23
0.20	s3x3	H9	s3x3	H9	s3x3	H13	s3x3	H23
0.00	s3x3	H9	s3x3	H9	s3x3	H9	s3x3	H23

Source: R Depoutot & C Planas (1998), Comparing Seasonal Adjustment and Trend Extraction Filters with Application to a Model-Based Selection of X11 Linear Filters, Discussion Paper No 9/1998/A/9, Eurostat.

Keys messages

Pretreatment

- ☞ Temporary & persistent outliers → Temporary removal
- ☞ Structural calendar variation → Permanent removal

RegARIMA model

- ☞ Regression equation → Exogenous effects
  - Outliers → Predefined JD+ variables
  - Calendar variation → Predefined JD+ variables & customisation
- ☞ ARIMA equation → Disturbances
  - Dependence structure → Seasonal ARIMA model

User-defined calendar regression variables

- ☞ JD+ assumption → Centred form

References (I/IV)
Transformations & outliers

- G E P Box & D R Cox (1964), An Analysis of Transformations, Journal of the Royal Statistical Society B 26 (2), 211–252.
- G E P Box & G C Tiao (1975), Intervention Analysis with Applications to Economic and Environmental Problems, Journal of the American Statistical Association 70 (349), 70–79.
- A J Fox (1972), Outliers in Time Series, Journal of the Royal Statistical Society B 34 (3), 350–363.
- G M Ljung (1993), On Outlier Detection in Time Series, Journal of the Royal Statistical Society B 55 (2), 559–567.

References (II/IV)
Calendar variation

- W R Bell (1984), Seasonal Decomposition of Deterministic Effects, Research Report 84/01, U.S. Bureau of the Census.
- W R Bell & S C Hillmer (1983), Modeling Time Series With Calendar Variation, Journal of the American Statistical Association 78 (383), 526–534.
- W S Cleveland & S J Devlin (1980), Calendar Effects in Monthly Time Series: Detection by Spectrum Analysis and Graphical Methods, Journal of the American Statistical Association 75 (371), 487–496.
- W S Cleveland & S J Devlin (1982), Calendar Effects in Monthly Time Series: Modeling and Adjustment, Journal of the American Statistical Association 77 (379), 520–528.

References (III/IV)
Calendar variation

- Deutsche Bundesbank (2012), Calendar effects on economic activity, Monthly Report 64 (12), 51–60.
- O C Pang, W R Bell & B C Monsell (2022), Accommodating Weather Effects in Seasonal Adjustment: A Look into Adding Weather Regressors for Regional Construction Series, Research Report No. 2022-01, Center for Statistical Research & Methodology, US Census Bureau, Washington, DC.
- M F Tüzen, G M Gökçin & Ö Yiğit (2023), The Calculation of Composite Calendar Regressors Including Moving Holidays in Türkiye, Journal of Statistical Research 13 (2), 13–27.

References (IV/IV)
ARIMA models

- O D Anderson (1977), The Interpretation of Box-Jenkins Time Series Models, Journal of the Royal Statistical Society D 26 (2), 127–145.
- K Chiu, J Higginson & G Huot (1985), Performance of ARIMA Models in Time Series, Survey Methodology 11 (1), 51–64.
- E B Dagum (1978), Modelling, Forecasting and Seasonally Adjusting Economic Time Series with the X-11-ARIMA Method, Journal of the Royal Statistical Society D 27 (3–4), 203–216.
- P Newbold & C W J Granger (1974), Experience with Forecasting Univariate Time Series and the Combination of Forecasts, Journal of the Royal Statistical Society A 137 (2), 131–146.

UC model after linearisation

Pretreated data	$=$	Trend-cycle	$\circ$	Seasonal	$\circ$	Irregular
$\downarrow$		$\downarrow$		$\downarrow$		$\downarrow$
z_t	$=$	t_t	$\circ$	s_t	$\circ$	i_t
		$\downarrow$		$\downarrow$		$\downarrow$
		Long-term growth path		Recurring intra-year movements		Remaining movements

$\searrow$ Seasonally adjusted (SA) data $\swarrow$

Decomposition ($\circ$)

- Multiplicative $\rightarrow$ Seasonal proportional to trend-cycle
- Additive $\rightarrow$ Seasonal independent of level of trend-cycle

Reminder & outlook

Pretreated data

- Outliers → Temporary removal
- Calendar variation → Permanent removal
- Data → Log transformation (multiplicative decomposition), “optimal” forecasts

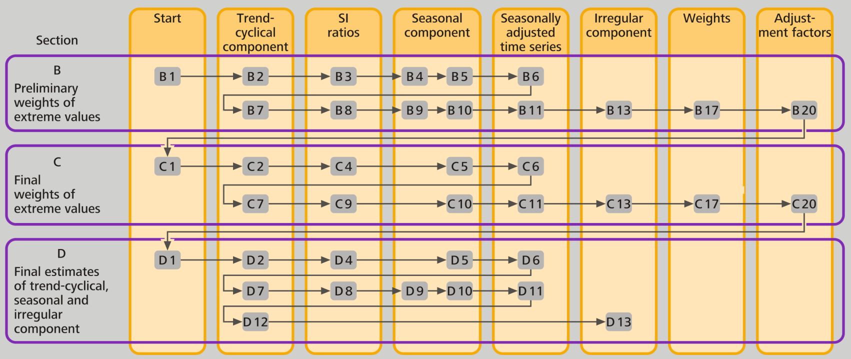
X-11 approach

- Assumptions
  - Data → Monthly
  - Decomposition → Multiplicative ($\circ = \times$)
- Illustration → Section D (see slides on X-11 strategy)

Strategy (I/II)

Basic principle of the X-11 seasonal adjustment algorithm in JDemetra+*

Workflow diagram



* In X-13 terminology, Section A is solely devoted to the treatment of outliers and calendar effects within a regARIMA modelling framework which is done prior to the application of the X-11 core.

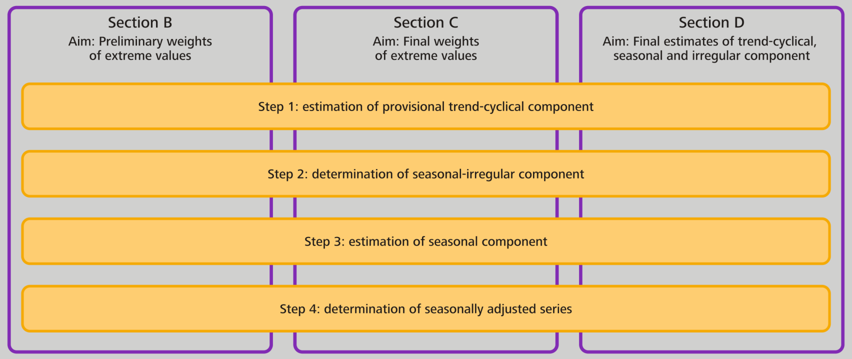
Deutsche Bundesbank

S3PR0037D.Chart

Strategy (II/II)

Basic principle of the X-11 seasonal adjustment algorithm*

Workflow diagram, simplified version



* In X-13 terminology, Section A is solely devoted to the treatment of outliers and calendar effects within a regARIMA modelling framework which is done prior to the application of the X-11 core.

Deutsche Bundesbank

S3PR0023.Chart

Step 1: provisional trend-cycle (I/II)

Preliminary trend-cycle

$$\hat{t}_t = \frac{1}{24} z_{t-6} + \frac{1}{12} z_{t-5} + \cdots + \frac{1}{12} z_t + \cdots + \frac{1}{12} z_{t+5} + \frac{1}{24} z_{t+6}$$

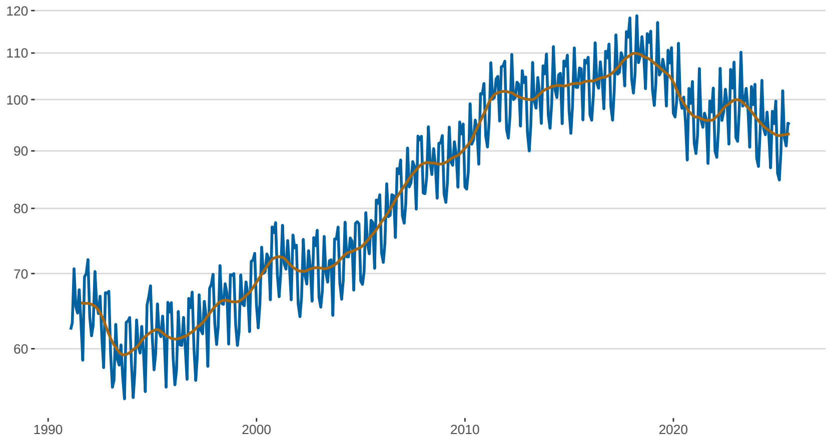
Interpretation

- Linearised data → Smoothed by centred moving average (MA) over 13 months

Boundary issue

- MA application → 6 missing values (at either end)

Step 1: provisional trend-cycle (II/II)



Modified industrial production (D 1) & **preliminary trend-cycle** (D 2)

Step 2: seasonal-irregular component

Preliminary seasonal-irregular (SI)

$$(\widehat{si})_t = \frac{z_t}{\hat{t}_t}$$

Interpretation

– Linearised data → Removal of preliminary trend-cycle

Step 3: seasonal component (I/III)

Preliminary seasonal

$$\hat{s}_t = \frac{1}{9}\,(\widehat{si})_{t-24} + \frac{2}{9}\,(\widehat{si})_{t-12} + \frac{3}{9}\,(\widehat{si})_t + \frac{2}{9}\,(\widehat{si})_{t+12} + \frac{1}{9}\,(\widehat{si})_{t+24}$$

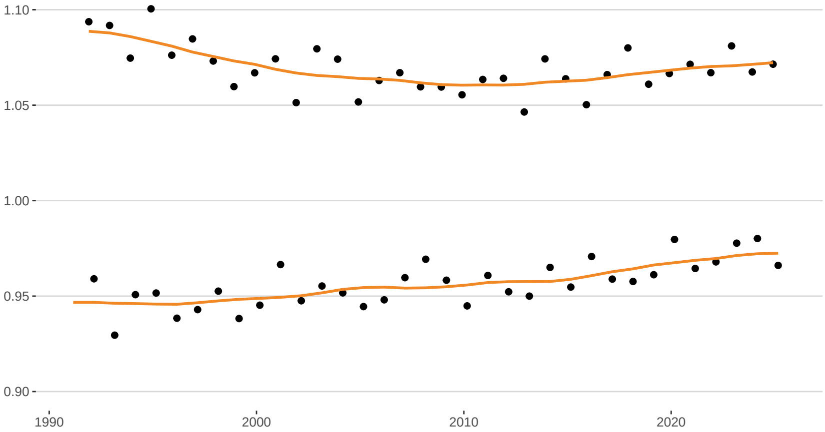
Interpretation

- Preliminary SI ratios → Smoothed by 3×3 seasonal filter within each month
- Normalisation → Centred MA over 13 months

Boundary issue

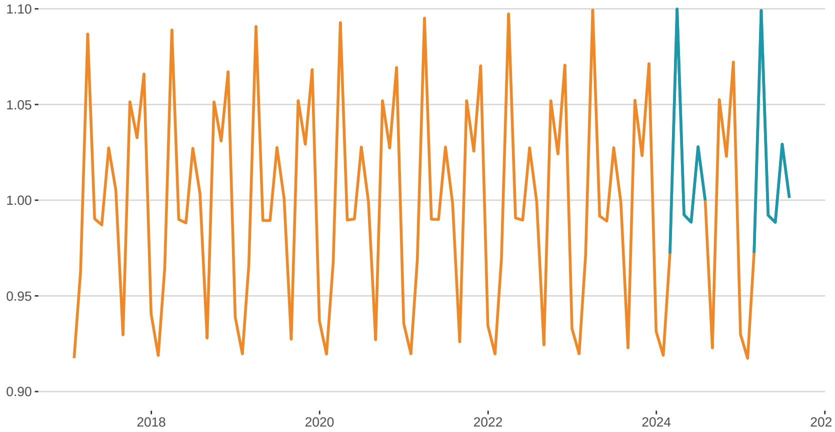
- Missing values → Imputation (copy-paste in case of no regARIMA forecasts)

Step 3: seasonal component (II/III)



Preliminary SI ratios (D 4) & normalised **preliminary seasonal** (D 5) for industrial production in **February** *(bottom)* & **November** *(top)*

Step 3: seasonal component (III/III)



Normalised **preliminary seasonal** (D 5) for industrial production with **imputed values** (*final six values*)

Step 4: seasonally adjusted series (I/II)

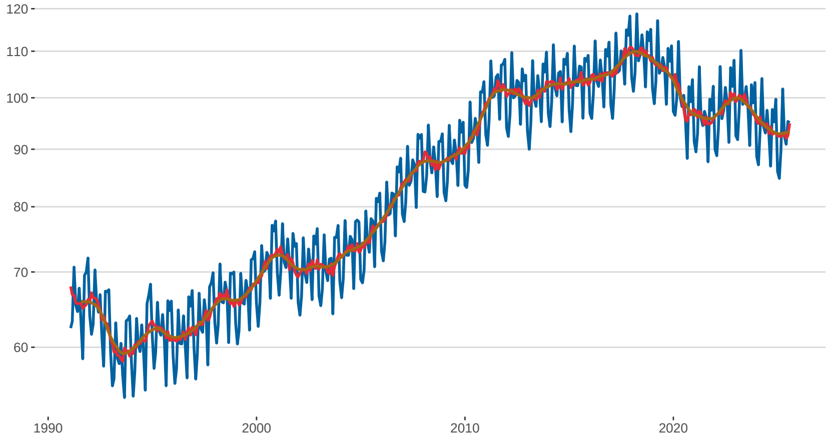
Preliminary seasonally adjusted data

$$\hat{z}_t^{(sa)} = \frac{z_t}{\hat{s}_t}$$

Interpretation

- Linearised data → Removal of normalised preliminary seasonal

Step 4: seasonally adjusted series (II/II)



Modified industrial production (D 1), **preliminary trend-cycle** (D 2) &
preliminary seasonally adjusted figures (D 6)

Next loop

Final UC estimates

- Preliminary estimates → Refinement

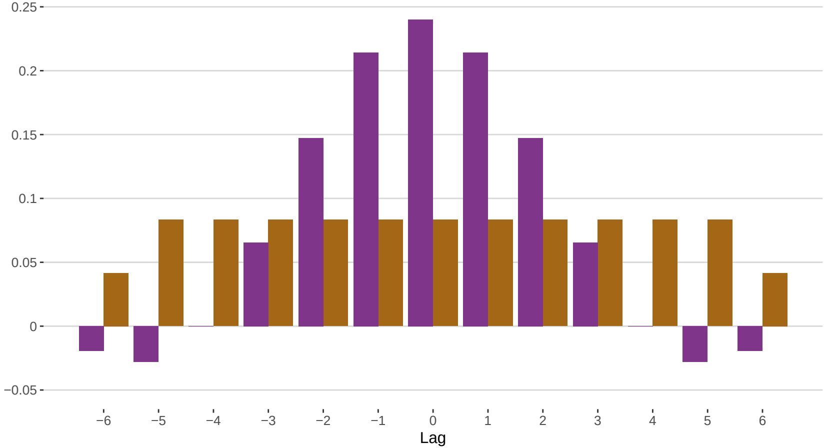
User customisation

- Options → Extended
- Trend-cycle filters → Henderson types
- Seasonal filters → Period-specific
- Extreme SI ratios → Down-weighting, replacement

Boundary issue

- Symmetric Henderson & seasonal filters → Asymmetric non-centred variants

Henderson filters (I/III)



Weights of X-11 trend-cycle filters (*symmetric 13-term filters*): **Henderson** & **centred MA**

Henderson filters (II/III)

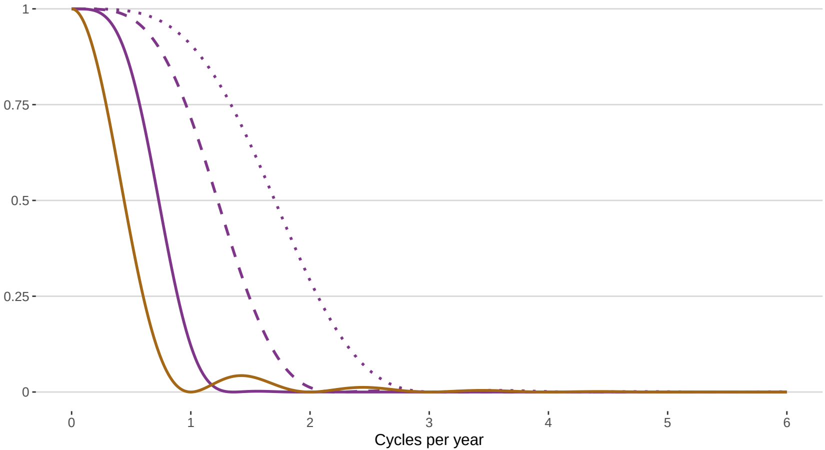
	Automatic selection rule		
"Noise level"	Low	Medium	High
I/C*	$[0, 1)$	$[1, 3.5)$	$[3.5, \infty)$
Monthly data	9-term	13-term	23-term
Quarterly data	5-term	5-term	7-term

* I/C is the ratio between the average absolute period-on-period changes of a temporary irregular and a temporary trend-cyclical component.

Interpretation

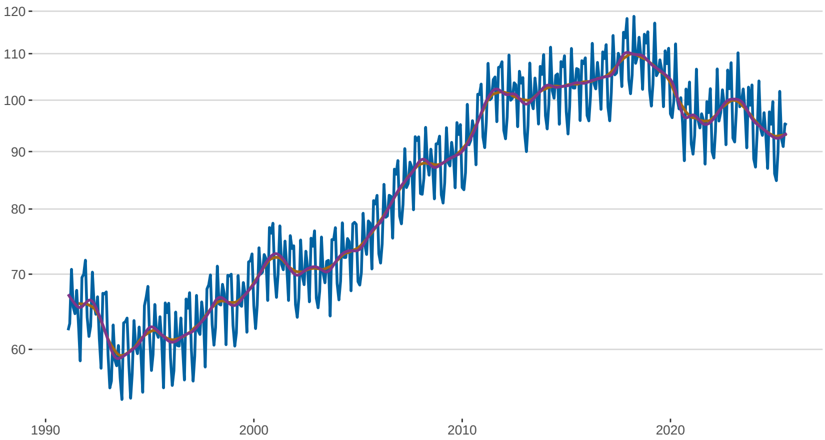
- I/C large → Dominance of irregular (i.e. long filter)
- I/C small → Dominance of trend-cycle (i.e. short filter)

Henderson filters (III/III)



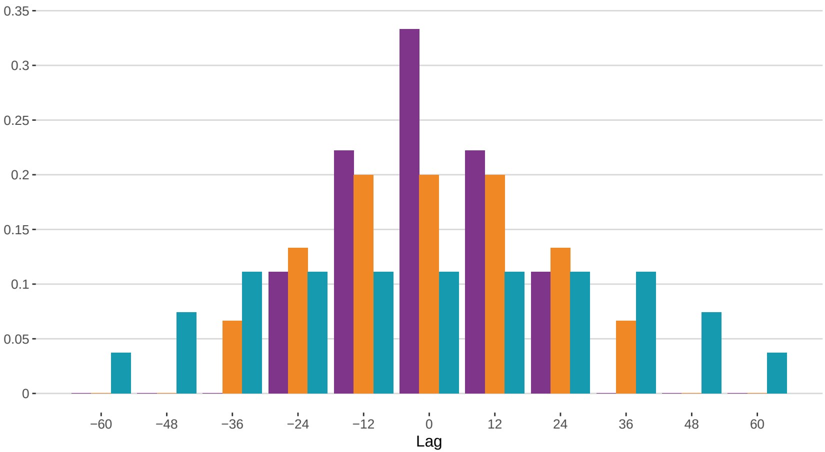
Squared gains of symmetric X-11 trend-cycle filters: **9-term** *(dotted)*, **13-term** *(dashed)* & **23-term** *(solid)* **Henderson** & **centred 12-term MA**

Refined preliminary trend-cycle



Modified industrial production (D 1), preliminary trend-cycle (D 2) & refined preliminary trend-cycle (D 7)

Seasonal filters (I/VI)



Weights of symmetric X-11 seasonal filters: $\mathbf{3} \times \mathbf{3}$, $\mathbf{3} \times \mathbf{5}$ & $\mathbf{3} \times \mathbf{9}$

Seasonal filters (II/VI)

Seasonal estimate ($3 \times k$ filter)

$$\hat{s}_t = \frac{1}{3}\left[(\widetilde{si})^{(n)}_{t-12} + (\widetilde{si})^{(n)}_{t} + (\widetilde{si})^{(n)}_{t+12}\right], \quad n = (k-1)/2$$

Tentative averages (SI ratios)

$$(\widetilde{si})^{(n)}_t = \frac{1}{2n+1}\sum_{j=-n}^{n}(\widehat{si})_{t+12j}$$

Seasonal filters (III/VI)

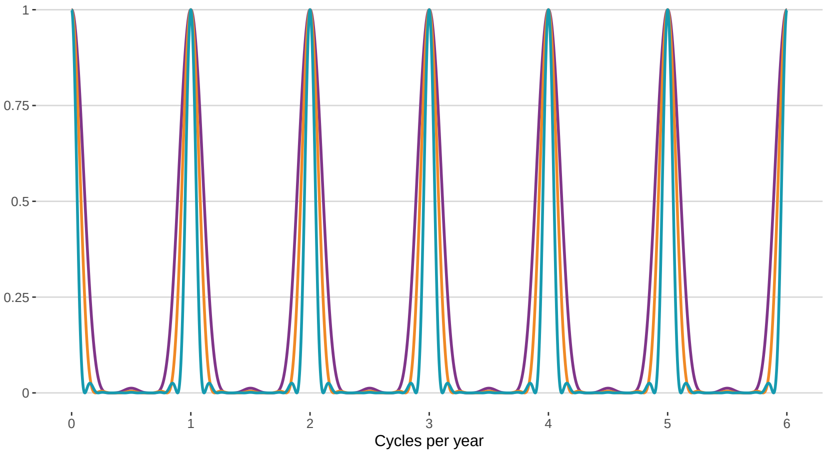
	Automatic selection rule				
"Noise level"	Low		Medium		High
I/S*	$[0, 2.5)$	$[2.5, 3.5]$	$(3.5, 5.5)$	$[5.5, 6.5]$	$(6.5, \infty)$
Monthly data	3×3	↻	3×5	↻	3×9
Quarterly data	3×3	↻	3×5	↻	3×9

* I/S is the ratio between the average absolute year-on-year changes of a temporary irregular and a temporary seasonal component. ↻ Omit last year from D 9 and recalculate I/S; if necessary, repeat as long as D 9 is at least 5 years long; apply 3×5 if still no decision has been found.

Interpretation

- I/S large → Dominance of irregular (i.e. long filter)
- I/S small → Dominance of seasonal (i.e. short filter)

Seasonal filters (IV/VI)



Squared gains of symmetric X-11 seasonal filters: $\mathbf{3 \times 3}$, $\mathbf{3 \times 5}$ & $\mathbf{3 \times 9}$

Seasonal filters (V/VI)

Seasonal factors (D 10) vs. SI ratios (D 8)

- Long-term evolution → Reflection
- Current end → Typical behaviour

Airline model

- Parameter estimates → Guidance
- Details → ARIMA equation

Additional information

- Data → Length
- Seasonality → Weak degree, changes (e.g. structural breaks)

Seasonal filters (VI/VI)

	Seasonal filter for X-11 table					
Option	B 5	B 10	C 5	C 10	D 5	D 10
X11Default	3×3	3×5	3×3	3×5	3×3	3×5
MSR	3×3	3×5	3×3	3×5	3×3	I/S
Stable*	$\overline{(si)}_j$	$\overline{(si)}_j$	$\overline{(si)}_j$	$\overline{(si)}_j$	$\overline{(si)}_j$	$\overline{(si)}_j$
S3xk	$3 \times k$	$3 \times k$	$3 \times k$	$3 \times k$	$3 \times k$	$3 \times k$

* $\overline{(si)}_j$ means that the seasonal component is estimated by the period-specific averages of the (extreme-value corrected) SI ratios stored in the preceding X-11 table.

General problems

Symmetric filters

- Finite input → Boundary issue
- Remedy → Asymmetric non-centred variants (Henderson & seasonal filters)

Linear filters

- Extreme input → High sensitivity
- Remedy → Down-weighting, replacement (SI ratios)

Extreme SI ratios (I/V)

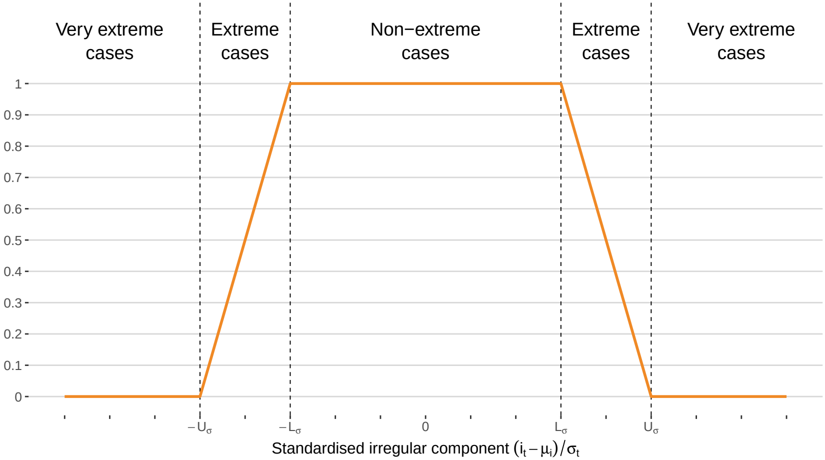
Identification

- Irregular → Large deviation from expectation $\mathbb{E}\left(i_t\right) = \mu_i$
  - Additive decomposition → $\mu_i = 0$
  - Multiplicative decomposition → $\mu_i = 1$
- Benchmark → Moving standard deviation (σ_t)
- Tolerances → Lower & upper σ-limits (L_σ, U_σ)

Replacement value (D 9)

- Weighted average → Extreme SI ratio & four non-extreme close SI ratios (same period)
  - Extreme SI ratio → Weight $\in [0, 1)$ (C 17)
  - Non-extreme SI ratios → Weight $= 1$

Extreme SI ratios (II/V)



Weighting function for X-11 extreme value detection with **σ-limit identifiers** (*dashed verticals*)

Extreme SI ratios (III/V)

Non-extreme cases

$$\hat{i}_t \in [\mu_i - L_\sigma \, \sigma_t, \mu_i + L_\sigma \, \sigma_t]$$

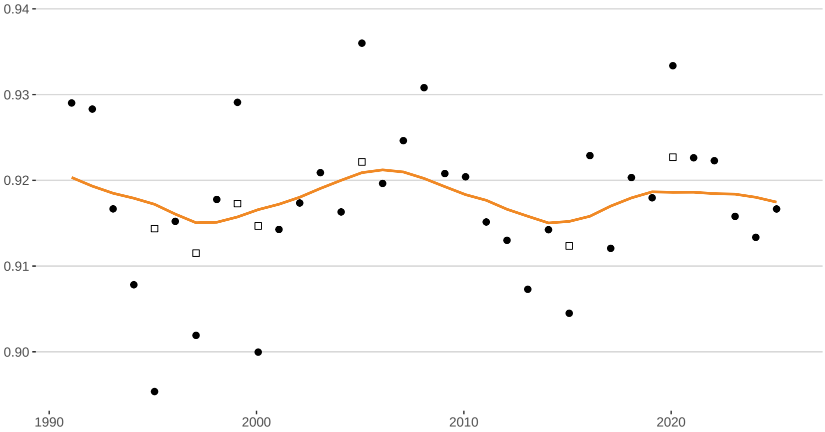
Extreme cases

$$\hat{i}_t \in (\mu_i - U_\sigma \, \sigma_t, \mu_i - L_\sigma \, \sigma_t) \cup (\mu_i + L_\sigma \, \sigma_t, \mu_i + U_\sigma \, \sigma_t)$$

Very extreme cases

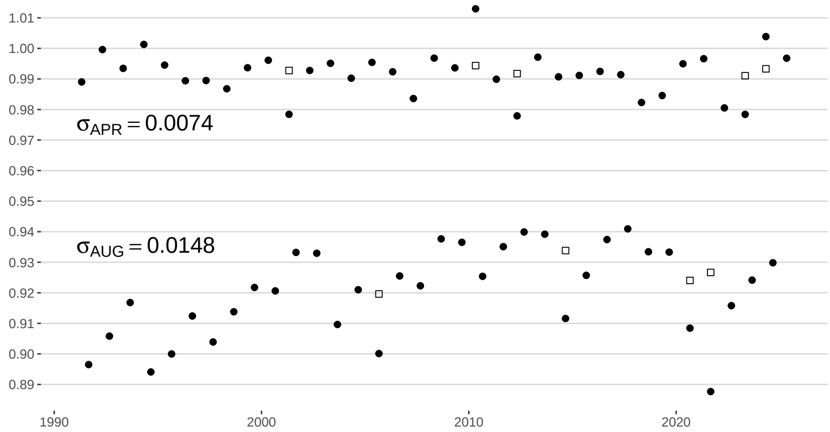
$$\hat{i}_t \notin (\mu_i - U_\sigma \, \sigma_t, \mu_i + U_\sigma \, \sigma_t)$$

Extreme SI ratios (IV/V)



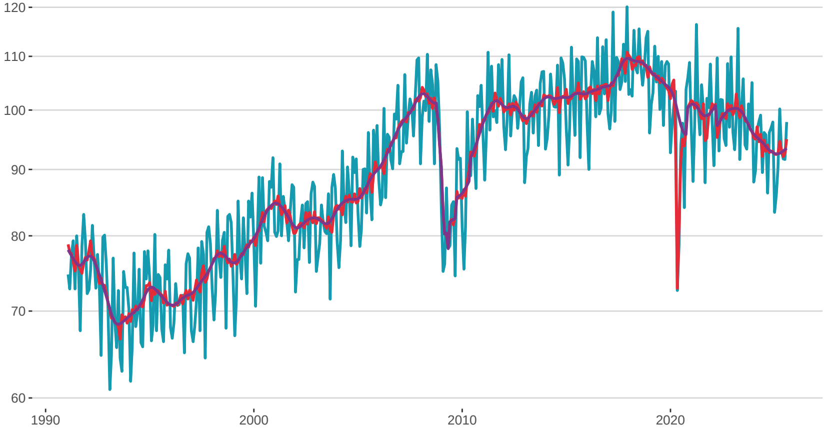
Final SI ratios (D 8, *dots*), **replacement values** (D 9, *squares*) & **final seasonal component** (D 10) for industrial production in **January**

Extreme SI ratios (V/V)



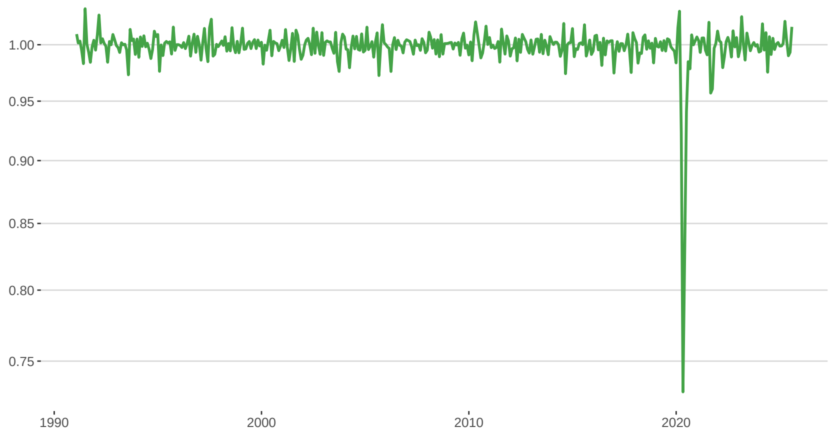
Final SI ratios (D 8, *dots*) & **replacement values** (D 9, *squares*) for industrial production in **April** (*top*) & **August** (*bottom*)

Final estimates (I/II)



Industrial production (volume, 2021 = 100), final seasonally adjusted figures (D 11) & final trend-cycle (D 12)

Final estimates (II/II)



Final irregular component (D 13) for industrial production

Key messages

X-11 approach

☞ Predefined trend-cycle & seasonal filters → Sequential application

☞ Basic 4-step principle → Trend-cycle estimation, detrending, seasonal estimation, seasonal adjustment

Filter selection

☞ General idea → Volatility ratios

- Dominant irregular → Longer filter
- Dominant trend-cycle/seasonal → Shorter filter

☞ Automatic rules → I/C & I/S ratios

X-11 extreme values

☞ Aberrant irregular values → Down-weighting & replacement

☞ Identification → Benchmark volatility & tolerance multipliers

References (I/III)

General aspects & overviews

D F Findley, B C Monsell, W R Bell, M C Otto & B-C Chen (1998), New Capabilities and Methods of the X-12-ARIMA Seasonal-Adjustment Program, Journal of Business & Economic Statistics 16 (2), 127–152.

D Ladiray & B Quenneville (2001), Seasonal Adjustment with the X-11 Method, Lecture Notes in Statistics Vol. 158, Springer, New York.

J Shiskin & H Eisenpress (1957), Seasonal Adjustments by Electronic Computer Methods, Journal of the American Statistical Association 52 (280), 415–449.

J Shiskin, A H Young & J C Musgrave (1967), The X-11 Variant of the Census Method II Seasonal Adjustment Program, Technical Paper No. 15, US Department of Commerce, Bureau of the Census, Washington, DC.

References (II/III)
Properties of X-11 filters & UC estimates

- E B Dagum (1983), Spectral Properties of the Concurrent and Forecasting Seasonal Linear Filters of the X-11-ARIMA Method, Canadian Journal of Statistics 11 (1), 73–90.
- E B Dagum, N Chhab & K Chiu (1996), Derivation and Properties of the X11ARIMA and Census X11 Linear Filters, Journal of Official Statistics 12 (4), 329–347.
- E B Dagum, N Chhab & B Solomon (1991), The Autocorrelation of Residuals from the X11ARIMA Method, Journal of Official Statistics 7 (2), 181–194.
- M Doherty (2001), The Surrogate Henderson Filters in X-11, Australian & New Zealand Journal of Statistics 43 (4), 385–392.

References (III/III)

Deutsche Bundesbank

- Deutsche Bundesbank (1970), Seasonal adjustment by the Census Method, Monthly Report 22 (3), 37–41.
- Deutsche Bundesbank (1991), Data, adjusted for seasonal and working-day variations, on the expenditure components of GNP, Monthly Report 43 (4), 35–40.
- Deutsche Bundesbank (1999), The changeover from the seasonal adjustment method Census X-11 to Census X-12-ARIMA, Monthly Report 51 (9), 39–50.
- Deutsche Bundesbank (2018), Gradual changeover to JDemetra+ software for seasonal adjustment of the official statistics, Monthly Report 70 (5), 50.

Reminder: UC model after linearisation

Pretreated data	=	Trend-cycle	+	Seasonal	+	Irregular
$\downarrow$		$\downarrow$		$\downarrow$		$\downarrow$
z_t	=	t_t	+	s_t	+	i_t
		$\downarrow$		$\downarrow$		$\downarrow$
		Long-term growth path		Recurring intra-year movements		Remaining movements

$\searrow$ Seasonally adjusted (SA) data $\swarrow$

Additional decomposition (optional)

- Irregular → Transitory & white noise
- Transitory → Stationary short-term movements

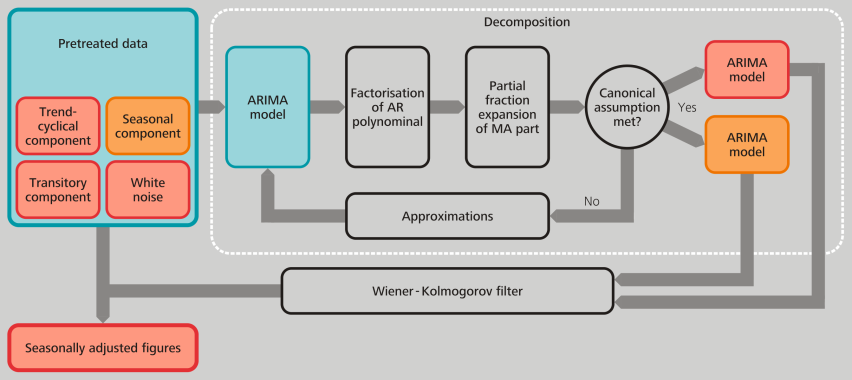
Bottom-up principle

$$
\begin{array}{ccccccc}
\text{Pretreated data} & & & & & & \\
\uparrow & & & & & & \\
\text{ARIMA} & = & \text{ARIMA} & + & \text{ARIMA} & + & \text{ARIMA} \\
 & & \uparrow & & \uparrow & & \uparrow \\
 & & \multicolumn{5}{c}{\text{Stochastic extensions}} \\
 & & \uparrow & & \uparrow & & \uparrow \\
 & & \multicolumn{5}{c}{\text{Main deterministic characteristics}} \\
 & & \uparrow & & \uparrow & & \uparrow \\
 & & \text{Trend-cycle} & & \text{Seasonal} & & \text{Irregular}
\end{array}
$$

Top-down estimation

Basic principle of the SEATS seasonal adjustment algorithm

Workflow diagram



Deutsche Bundesbank S3PR0086A.Chart

Assumptions

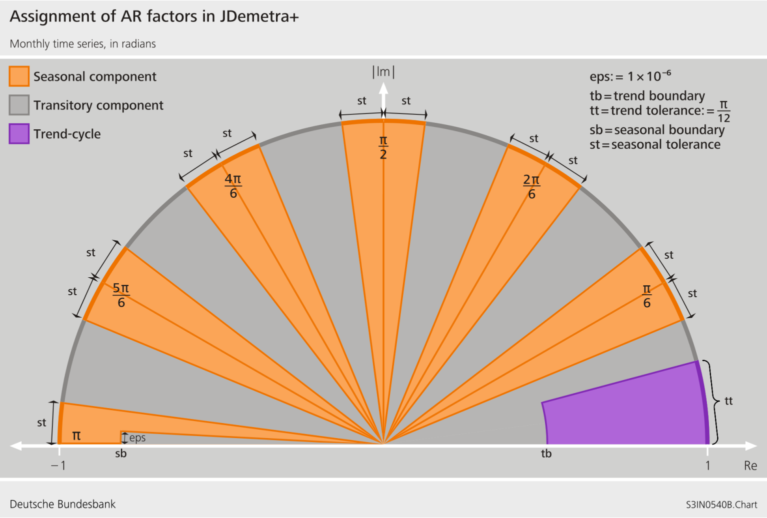
Component models

- Linearity → ARIMA models
- Separability → No common AR roots
- Invertibility (of aggregate model) → No common MA unit roots
- Identifiability → Contemporaneously uncorrelated innovations & ARI orders ⩾ MA orders
  - Burman (1980) → Balanced & "bottom-heavy" models

Admissible decompositions

- Uniqueness → Canonical components

Factorisation of AR polynomial



Partial fraction expansion of MA part

Pretreated data

$\downarrow$

Two representations

(given allocation of AR factors)

$\swarrow$ $\searrow$

Aggregate model | Sum of UC models

$\searrow$ $\swarrow$

Same dependence structure

Identifiable solution

– Autocovariances $\rightarrow$ Linear equation system

Canonical assumption (I/II)

Admissible decompositions

- UC spectra → Non-negative
- Number → Infinite

Core issue

- White noise → Allocation amongst UCs

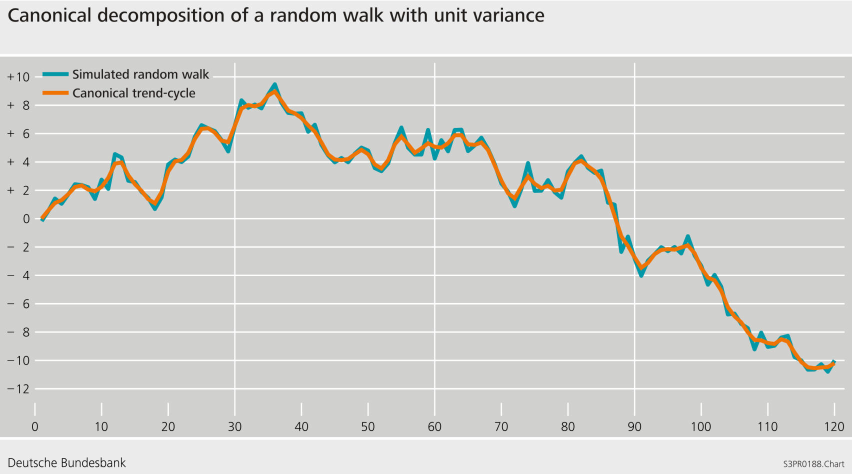
Uniqueness

- Trend-cycle, seasonal, transitory → Maximum smoothness (i.e. absence of extractable additive white noise)

Consequences

- UC spectra → Zeros
- UC innovations → Variance minimisation
- White noise → Variance maximisation

Canonical assumption (II/II)



Overcoming inadmissibility

Legacy solutions

- ARMA parameters → Heuristic replacement
  - ARIMA orders → No changes
  - Implementation → 2-parameter models with analytic expression for admissibility region
- ARIMA orders → Heuristic changes

Variance inflation

- ARIMA model → Additional artificial white noise

Optimal filtering

Optimal estimator (seasonal)

$$\hat{s}_{t|T} = \mathbb{E}\left(s_t \,|\, \mathbf{z}_T\right)$$

- $\mathbf{z}_T = (z_1, \ldots, z_T)^\top$ → Finite sequence

Conditional expectation

- Estimator → Linear projection
- Optimality → MSE sense
- Computation → Kalman or Wiener-Kolmogorov filter

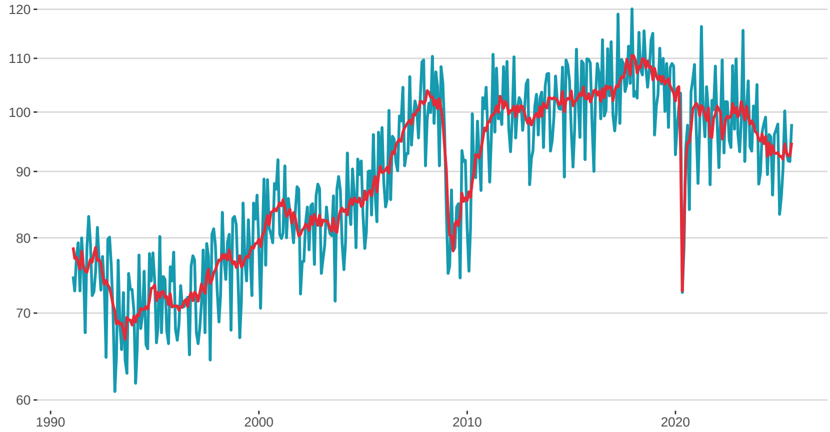
Wiener-Kolmogorov filter

- General solution → Infinite & stationary observations
- AMB extension → Finite & non-stationary observations
  - ARIMA polynomials (UC models) → Appropriate combination

Implementation

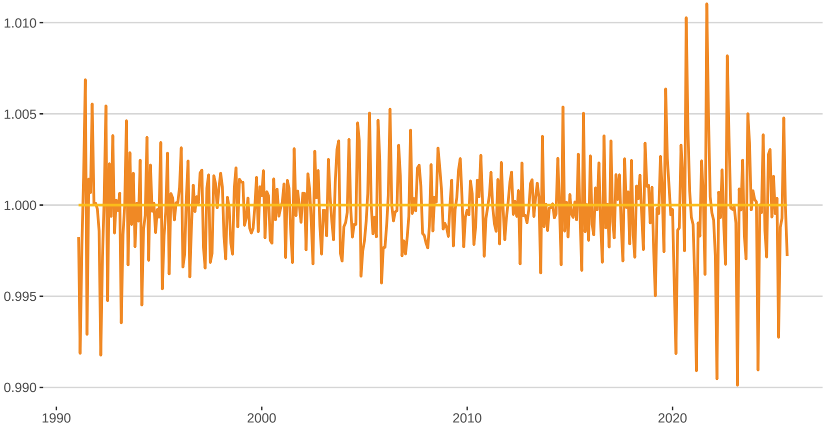
AMB parameters JD+ version 2.2.6		
Name	Parameter	Range (default)
Approximation mode	Modification type for inadmissible models	None, Legacy, Noisy
MA unit root boundary	Modulus threshold for resetting MA “near-unit” roots	$[0.9, 1]$ (0.95)
Trend boundary	Modulus threshold for assigning positive real AR roots	$[0, 1]$ (0.5)
Seasonal tolerance	Degree threshold for assigning complex AR roots	$[0, 10]$ (2)
Seasonal boundary	Modulus threshold for assigning negative real AR roots	$[0, 1]$ (0.8)
Seas. boundary (unique)	Same modulus threshold for unique seasonal AR roots	$[0, 1]$ (0.8)
Method	Estimation algorithm	Burman, Kalman, McElroy

Final estimates



Industrial production (volume, 2021 = 100) & **final seasonally adjusted figures** (*default specification*)

Comparison with X-11



Ratios between X-11 & AMB estimates for **calendar component** & **seasonal component**

Inference

Model adequacy

- UCs & UC estimators → Different stochastic properties
- Meaningful comparison → Auto- & cross-covariances
  - UC estimators → Theoretical benchmark
  - UC estimates → Empirical counterpart

Error analysis

- Total estimation error → Independent stationary ARMA components
  - Final (“historical”) estimator → Estimation error
  - Preliminary estimator → Revision error
- Revision error diagnostics → Phase delay, speed of convergence, variance

Key messages

AMB approach

- ☞ Data model → Canonical decomposition into UC models
- ☞ UC estimators → MSE optimality
- ☞ Model adequacy → UC estimators versus estimates

Advantages

- ☞ Data adaptability → Mass production
- ☞ UC estimates & forecasts → Standard errors
- ☞ Inference → Error analysis, model adequacy

References (I/III)
General aspects & overviews

- D F Findley, D P Lytras & A Maravall (2016), Illuminating ARIMA model-based seasonal adjustment with three fundamental seasonal models, SERIEs 7 (1), 11–52.
- A Maravall & D Pérez (2012), Applying and Interpreting Model-Based Seasonal Adjustment—The Euro-Area Industrial Production Series, in: W R Bell, S H Holan & T S McElroy (eds), Economic Time Series – Modeling and Seasonality, CRC Press, Boca Raton, 281–313.
- A Maravall & D A Pierce (1987), A Prototypical Seasonal Adjustment Model, Journal of Time Series Analysis 8 (2), 177–193.
- G Mélard (2016), On some remarks about SEATS signal extraction, SERIEs 7 (1), 53–98.

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Theoretical foundation

- W R Bell & S C Hillmer (1984), Issues Involved With the Seasonal Adjustment of Economic Time Series, Journal of Business & Economic Statistics 2 (4), 291–320.
- J P Burman (1980), Seasonal Adjustment by Signal Extraction, Journal of the Royal Statistical Society A 143 (3), 321–337.
- D M Grether & M Nerlove (1970), Some Properties of “Optimal” Seasonal Adjustment, Econometrica 38 (5), 682–703.
- A Maravall (1988), A Note on Minimum Mean Squared Error Estimation of Signals with Unit Roots, Journal of Economic Dynamics & Control 12 (2–3), 589–593.

References (III/III)

Canonical components & models

- S C Hillmer & G C Tiao (1982), An ARIMA Model-Based Approach to Seasonal Adjustment, Journal of the American Statistical Association 77 (377), 63–70.
- A Maravall (1987), Minimum Mean Squared Error Estimation of the Noise in Unobserved Component Models, Journal of Business & Economic Statistics 5 (1), 115–120.
- A Maravall (1989), On the Dynamic Structure of a Seasonal Component, Journal of Economic Dynamics & Control 13 (1), 81–91.
- A Maravall (1993), Stochastic Linear Trends – Models and Estimators, Journal of Econometrics 56 (1–2), 5–37.

Overview

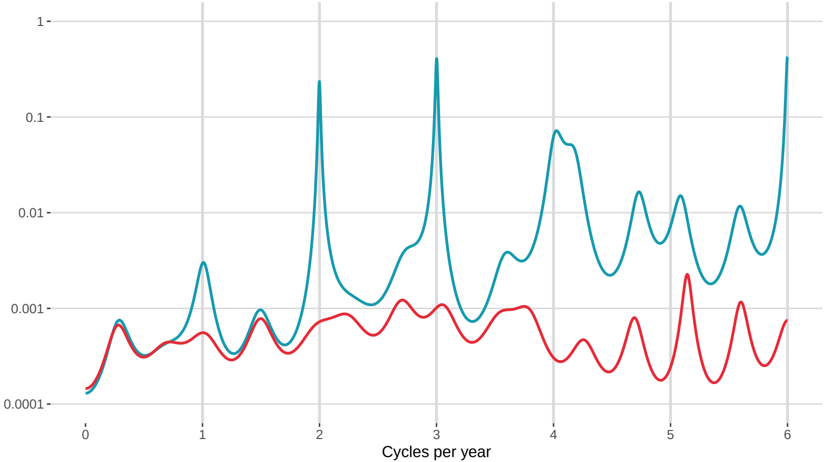
Key questions

- SA data → No sesonality?
- SA adequacy → Over- or under-adjustment?

Answers

- Seasonality diagnostics → Mostly as before
  - Time domain → Serial dependence
  - Frequency domain → Spectral density
  - Seasonality tests → Dynamic seasonality
- Approach-specific diagnostics
  - X-11 heuristics → M- & Q-statistics
  - AMB diagnostics → UC estimators versus estimates

Data visualisation: a look through the spectral lens



AR(30) spectral density estimate for **industrial production** (volume, 2021 = 100, Jan 1991 – Jul 2025) & **X-11 seasonally adjusted figures**

Additional X-11 diagnostics (I/VI)

F-test: stable seasonality

$$F_S = \frac{\text{Variance of month-specific mean SI ratios}}{\text{Variance of SI ratios within months}}$$

Hypothesis

- Seasonal factors $\rightarrow$ Same month-specific averages (i.e. no stable seasonality)

Decision

- Rejection if F_S too large

Additional X-11 diagnostics (II/VI)

F-test: moving seasonality

$$\left|(\widehat{si})_{ij} - 100\right| = \text{Year}_i + \text{Month}_j + \text{Residual}_{ij}$$

Hypothesis

- SI ratios $\rightarrow$ Absence of annual effects (i.e. no moving seasonality, $\text{Year}_1 = \text{Year}_2 = \cdots = \text{Year}_T$)

Decision

- Rejection if F_M-statistic too large

Additional X-11 diagnostics (III/VI)

M-statistics: basic idea

Indication

- Seasonal adjustment → Quality
- Development → Statistics Canada

Construction

- Domain → $[0, 3]$
- Acceptance region → $[0, 1]$

Warning

- Relevance → Limited

Additional X-11 diagnostics (IV/VI)

M-statistics: $M7$

$$M7 = \sqrt{\frac{1}{2}(T_1 + T_2)}, \quad \text{with } T_1 = \min\left\{\frac{7}{F_S}, 9\right\}, T_2 = \min\left\{\frac{3F_M}{F_S}, 9\right\}$$

Interpretation

- "Ratio" → Stable (F_S) & moving (evolutive) seasonality (F_M)

Additional X-11 diagnostics (V/VI)
Q-statistic

M-statistic [1)]	Weight as a percentage	M-statistic [1)]	Weight as a percentage
$M1$	13	$M7$	16
$M2$	13	$M8$	7
$M3$	10	$M9$	7
$M4$	5	$M10$	4
$M5$	11	$M11$	4
$M6$	10		

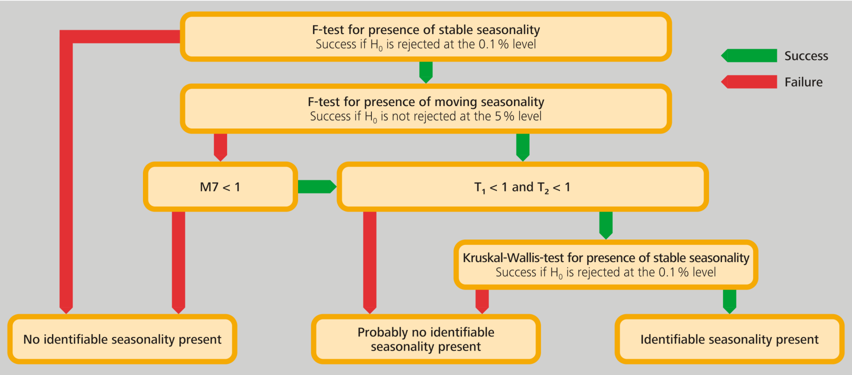
1 $M8$ to $M11$ are not calculated for series shorter than six years and, hence, other weights apply.

Additional X-11 diagnostics (VI/VI)

Overall test: identifiable seasonality

Combined test for presence of identifiable seasonality

Each single null hypothesis (H_0) states “no seasonality”



Source: Morry and Lothian (1978).
Deutsche Bundesbank
S3PR0139.Chart

Additional AMB diagnostics
UC estimators versus estimates: ACF

Trend

Lag	Component	Estimator	Estimate	P-Value
1	0,0004	0,3874	0,3327	0,0774
2	-0,4996	-0,3638	-0,3925	0,5474
3	0,0000	-0,3378	-0,2512	0,0889
4	0,0000	-0,1164	-0,0819	0,6191
5	0,0000	-0,0395	-0,0782	0,5949
6	0,0000	-0,0117	0,0591	0,2499
7	0,0000	0,0013	0,1261	0,0848
8	0,0000	0,0159	0,0253	0,8946
9	0,0000	0,0501	-0,0044	0,4222
10	0,0000	0,0543	-0,0297	0,2287
11	0,0000	-0,0579	-0,1641	0,1331
12	0,0000	-0,1494	-0,1212	0,6465

Seasonally adjusted

Lag	Component	Estimator	Estimate	P-Value
1	-0,6187	-0,6196	-0,6298	0,7411
2	0,1188	0,1197	0,0966	0,7192
3	0,0000	-0,0000	0,0668	0,3326
4	0,0000	0,0000	-0,0260	0,7056
5	0,0000	0,0000	-0,0093	0,8880
6	0,0000	0,0000	-0,0516	0,3974
7	0,0000	0,0000	0,1412	0,0331
8	0,0000	0,0000	-0,1293	0,0606
9	0,0000	0,0000	0,0434	0,5287
10	0,0000	-0,0179	0,0212	0,5701
11	0,0000	0,0926	0,0164	0,2462
12	0,0000	-0,1495	-0,1368	0,8351

Seasonal

Lag	Component	Estimator	Estimate	P-Value
1	0,9263	0,7689	0,7643	0,8844
2	0,7868	0,3994	0,3815	0,6847
3	0,6164	-0,0037	-0,0213	0,6345
4	0,4414	-0,3593	-0,3771	0,6759
5	0,2810	-0,6095	-0,5975	0,8677
6	0,1478	-0,7197	-0,6621	0,5092
7	0,0485	-0,6772	-0,6419	0,5983
8	-0,0150	-0,4914	-0,4788	0,7921
9	-0,0449	-0,1928	-0,1694	0,7461
10	-0,0468	0,1672	0,1921	0,7950
11	-0,0286	0,5168	0,5493	0,7247
12	0,0000	0,7642	0,7883	0,7024

Irregular

Lag	Component	Estimator	Estimate	P-Value
1	-0,0000	-0,3273	-0,3858	0,1643
2	-0,0000	-0,1130	-0,1153	0,9687
3	-0,0000	-0,0390	0,0501	0,1205
4	-0,0000	-0,0135	-0,0257	0,8304
5	-0,0000	-0,0046	-0,0380	0,5622
6	-0,0000	-0,0014	-0,0153	0,7829
7	-0,0000	0,0001	0,1346	0,0197
8	-0,0000	0,0018	-0,1012	0,0714
9	-0,0000	0,0058	0,0225	0,7695
10	-0,0000	0,0169	0,0385	0,7056
11	-0,0000	0,0489	-0,0417	0,1124
12	-0,0000	-0,1495	-0,1310	0,7152

Residual seasonality

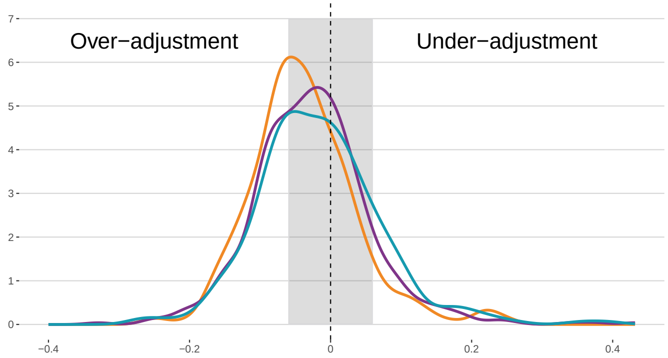
Under-adjustment error

- Options, parameters → Inadequate choices
  - Seasonal filters → Overly stable
- SA data, irregular → Detectable seasonality
  - Mild form → Evolutive pattern, reduced amplitude, stationary formulation (no unit root dynamics)

Detection

- Tests for stable seasonality → Inadequate (in theory)
- Pragmatic solution → Established seasonality tests on suitable subspan (e.g. last 8 years)
- Advanced solution → Tailored seasonality tests

Flip side



Gaussian-kernel **density estimates for ACF(12)** with average **Bartlett S.E.** (*shaded area*) obtained from differenced logged seasonally adjusted **industrial production** ($n = 301$), **orders received** ($n = 319$) & **turnover in industry** ($n = 567$)

Key messages

Main questions

- ☞ SA data → No seasonality?
- ☞ SA adequacy → Over- or under-adjustment?

Tools

- ☞ Seasonality tests → Only dynamic seasonality
- ☞ Descriptive statistics
  - ACF, PACF → No peaks at seasonal lags
  - Estimated spectral density → No peaks at seasonal frequencies
- ☞ Approach-specific diagnostics
  - X-11 heuristics → M- & Q-statistics $\in [0, 1]$
  - AMB diagnostics → UC estimators & estimates in line

References (I/II)

- B Chen, T S McElroy & O C Pang (2022), Assessing Residual Seasonality in the U.S. National Income and Product Accounts Aggregates, Journal of Official Statistics 38 (2), 399–428.
- D F Findley & C C Hood (2000), X-12-ARIMA and Its Application to Some Italian Indicator Series, Annali Di Statistica: Serie X 20, 249–269.
- J Lothian & M Morry (1978), A Set of Quality Control Statistics for the X-11-ARIMA Seasonal Adjustment Method, Research Paper 78-10-005E, Seasonal Adjustment and Time Series Analysis Staff, Statistics Canada.

References (II/II)

- T S McElroy (2021), A Diagnostic for Seasonality Based Upon Polynomial Roots of ARMA Models, Journal of Official Statistics 37 (2), 367–394.
- T S McElroy & S H Holan (2009), A Nonparametric Test for Residual Seasonality, Survey Methodology 35 (1), 67–83.
- T S McElroy & A Roy (2022), A Review of Seasonal Adjustment Diagnostics, International Statistical Review 90 (2), 259–284.
- M Morry & J Lothian (1978), A Test for the Presence of Identifiable Seasonality When Using the X-11-ARIMA Program, Research Paper 78-10-002, Seasonal Adjustment and Time Series Analysis Staff, Statistics Canada.

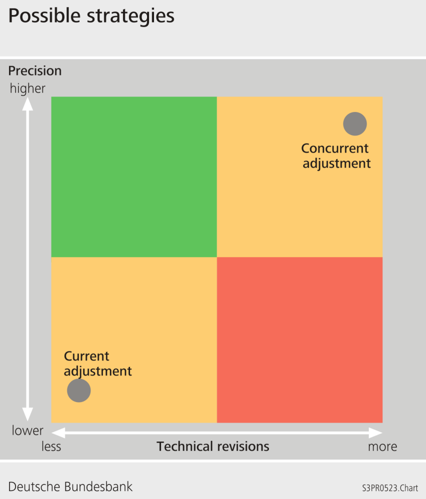
Revisions
Sources of revisions I

Revisions
Sources of revisions II

Concurrent adjustment (C)
Re-identification of model, filters and outliers and re-estimation of parameters and components every time
new or revised data become available

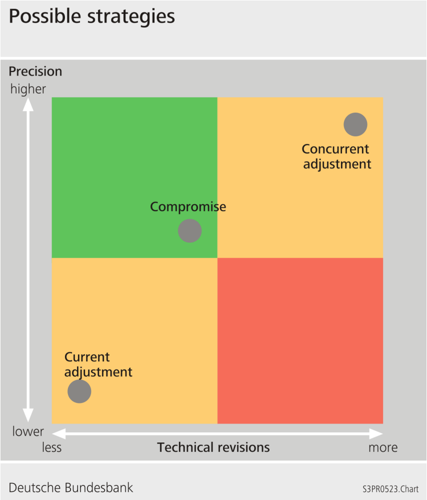
Current adjustment (B)
Re-identification and re-estimation once a year (revision period) and usage of forecasted components in the meantime

Revisions
Revision policies ESS Guidelines II



Revisions

Recommended approaches ESS Guidelines I/II



Revision policies

Recommended approaches ESS Guidelines II/II

Partial concurrent adjustment (A)

Re-identification once a year, partial re-estimation every time new or revised data become available

Controlled current adjustment (A)

Current adjustment combined with an internal check against the results of the partial concurrent adjustment

Revisions
Available revision policies in JD+

Option	Meaning
Current adjustment (AO approach)	Fix model (including all parameters), handle new observation as AO
Fixed model	Re-estimation, with all parameters fixed
Estimate regression coefficients	Re-estimation of the regression coefficients
+ Arima parameters	and Re-estimation of the parameters of the ARIMA model
+ Last outliers	and Re-estimation of the outliers of the last year only
+ All outliers	and all the outliers
+ Arima model	and Re-estimation of the ARIMA model.
Concurrent	The reference specification is used

Revisions
Extension of JDemetra+ for Controlled Current Adjustment

Basic idea

- Decision whether current factors are still appropriate
- Check current results against partial concurrent

Graphical tools

- Seasonally adjusted series, seasonal components, S-I-ratios

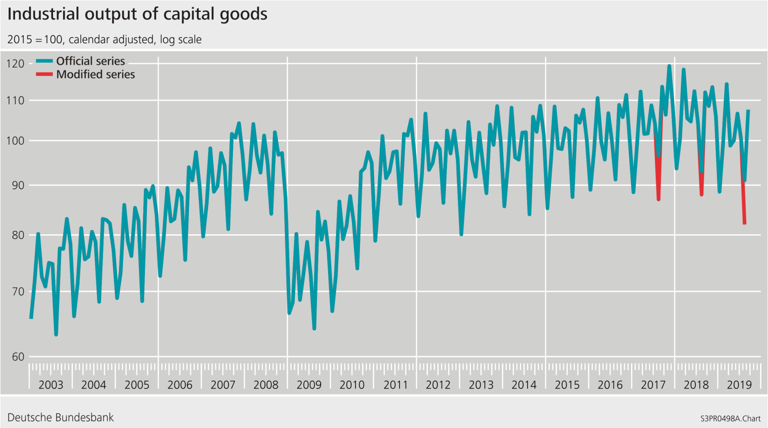
Pre-defined statistics

- Final filters, MSR and I/C-ratio,
- D8B with current and new D10

HTML-Report

- Summary of figures, tables and statistics (including pre-processing)

New SA-View
Example: CompRes



New SA-View
Example: CompRes

D8B

	jan	feb	mar	apr	may	jun	jul	aug	sep	oct	nov	dec
2009									* 113,514	105,419	109,056	100,333
2010	86,208	* 92,39	108,536	97,848	99,779	105,759	98,139	86,703	107,074	105,977	108,178	* 104,099
2011	85,813	95,193	108,705	97,604	98,272	* 101,844	101,257	89,013	* 104,896	104,546	108,718	99,482
2012	86,592	95,463	110,575	* 96,735	98,134	102,817	101,445	89,613	106,958	102,077	108,426	101,52
2013	* 84,964	95,861	110,026	* 100,193	* 95,983	106,139	98,218	* 91,108	106,535	* 100,893	110,041	100,826
2014	86,512	95,568	109,807	98,054	97,891	104,515	* 104,531	85,794	107,512	103,228	109,412	101,957
2015	85,678	96,053	109,171	98,73	98,189	102,78	101,847	87,139	105,832	104,225	107,573	99,871
2016	* 88,636	96,83	109,673	98,872	* 94,575	105,251	98,061	* 89,352	106,527	103,646	109,504	* 98,078
2017	87,097	97,17	109,55	98,67	98,458	105,045	100,401	83,572	108,474	* 100,748	* 112,179	101,102
2018	87,12	* 93,418	110,126	98,487	98,281	106,222	98,963	83,881	107,142	103,881	108,871	101,794
2019	85,635	96,551	111,541	* 96,894	98,693	106,027	100,799	82,864				

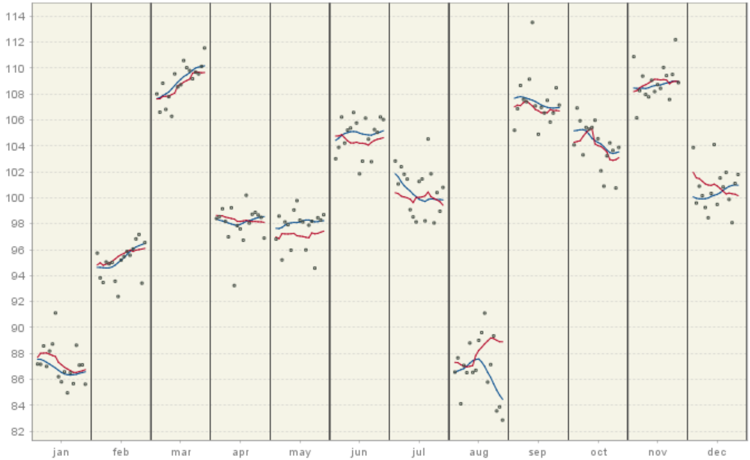
Seasonal Factor

	jan	feb	mar	apr	may	jun	jul	aug	sep	oct	nov	dec
2018									106,942	103,536	108,95	100,961
2019	86,585	96,477	110,165	98,548	98,233	105,167	99,835	84,47				

Seasonal Factor (current)

	jan	feb	mar	apr	may	jun	jul	aug	sep	oct	nov	dec
2018									106,696	103,088	108,963	100,164
2019	86,751	96,099	109,649	98,107	97,426	104,627	99,432	88,9				

New SA-View
Example: CompRes



How to use CompRes?

Install plug-ins:

- CompRes-1.3.2.nbm
- CompResReport-1.3.2.nbm

Save to workspace

- Calendar Factor
- Seasonal Factor

Create HTML

- Summary Report for CompRes

How to use CompRes?

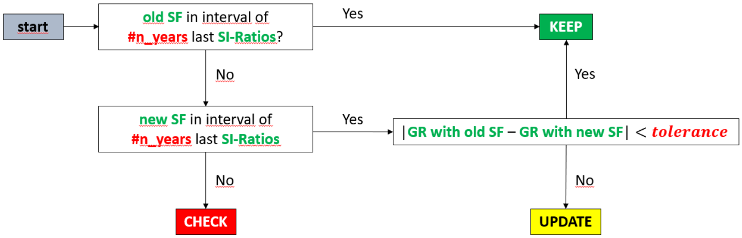
The Recommendation functionality by the CompRes plugin helps with the decision of whether to keep or not to keep using the previously forecast seasonal component.

Recommendation	Meaning
Check	Check the specification of the SA-Item.
Update	Switch to the new components.
Keep	Continue with the current seasonal and calendar components

How to use CompRes?

Recommendation	Meaning
Check	Check the specification of the SA-Item.
Update	Switch to the new components.
Keep	Continue with the current seasonal and calendar components

Simplified decision tree of CompRes?



Variables:

- new SF: most recent value in D10 table
- old SF: most recent value in D10 table estimated previously
- SI-Ratios: D8B without extreme values
- GR with new SF: most recent growth rate of D11
- GR with old SF: most recent growth rate of D11 when using old SF instead of new SF

CompRes with Excel

Additional plug-in

- ExcelAdapter-1.2.nbm

DataSourceUpdateOption

- de.bundesbank.jdemetra.exceladapter.excelConnection

Save to Excel

- Calendar factor
- Seasonal factor
- Forecast

Advantage: use the results without JD+

Useful links

Guidelines

ESS Guidelines

Plug-ins

https://github.com/bbkrd/CompRes

https://github.com/bbkrd/ExcelAdapter

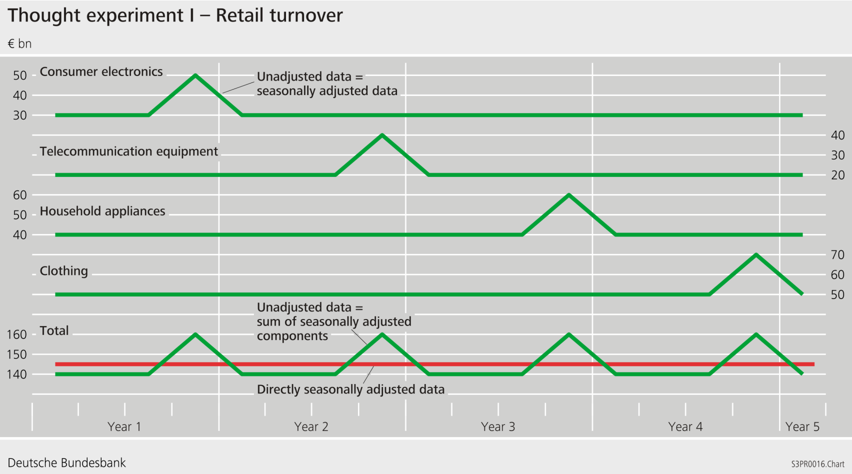
Documentation

https://bbkrd.github.io/pages/compres/

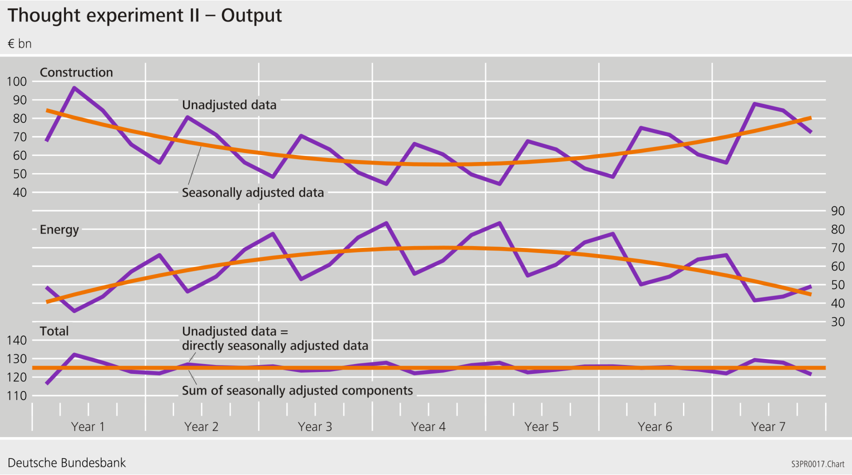
Two approaches & one question

$$
\begin{tikzcd}
\text{Unadjusted components} \arrow[r] \arrow[d] & \text{Unadjusted composite series} \arrow[d] \\
\text{Seasonally adjusted components} \arrow[r] & \text{Seasonally adjusted composite series} \arrow[d] \\
 & \text{Direct or indirect?}
\end{tikzcd}
$$

Direct approach



Indirect approach



Advantages

Direct approach (aggregation precedes adjustment)

- Residual seasonality → Less likely
- Component irregularities → Partial cancellation (given no positive crosscorrelation)

Indirect approach (adjustment precedes aggregation)

- Misinterpretation of moving seasonality as irregularity → Less likely
- Seasonality, calendar effects in components → More “precise” identification/estimation

References (I/II)

- Deutsche Bundesbank (2010), The whole and its parts: problems with the aggregation of seasonally adjusted data, Monthly Report 62 (6), 59–67.
- J Geweke (1978), The Temporal and Sectoral Aggregation of Seasonally Adjusted Time Series, in: A Zellner (ed), Seasonal Analysis of Economic Time Series, U.S. Department of Commerce, Bureau of the Census, Washington, D.C., 411–427.
- A Maravall (2006), An Application of the TRAMO-SEATS Automatic Procedure; Direct Versus Indirect Adjustment, Computational Statistics & Data Analysis 50 (9), 2167–2190.

References (II/II)

- T S McElroy, O C Pang & B Chen (2025), Mitigating Process Distortion While Preserving Accounting Relations in Hierarchical Time Series, Journal of Business & Economic Statistics, forthcoming.
- E Otranto & U Triacca (2002), Measures to Evaluate the Discrepancy Between Direct and Indirect Model-Based Seasonal Adjustment, Journal of Official Statistics 18 (4), 511–530.

- Motivation
- Introduction to JD+
- Seasonality diagnostics
- RegARIMA pretreatment
  - Regression equation
  - TransReg plug-in
  - ARIMA equation
- X-11 approach
  - Basic principle
  - Further issues
- ARIMA model-based approach
- Quality assessment
- Revision policies
- Composite time series
- Summary

Seasonal adjustment

Goal

☞ Data → Extraction of news

Seasonally adjusted (SA) data

☞ Components → Trend-cycle, irregular

☞ Calendar effects → Permanent elimination

☞ Outliers → Temporary removal (except for seasonal outliers)

Cautionary remarks

☞ SA effects → Non-seasonal data dynamics

☞ SA data → Careful use in econometric models

- Non-invertible estimators versus invertible models (e.g. VAR)
- Business cycle analysis, cointegration & unit root tests

Visibility in seasonally adjusted data

Seasonal influences

- ☞ Deviation → Normal seasonal average
- ☞ Extreme weather conditions, atypical holiday constellations, etc.

Random disruptions, unusual movements

- ☞ Understandable → Economic terms
- ☞ Large-scale orders, strikes, etc. → Outliers

Revisions

- ☞ Unadjusted data → Corrections, new observations
- ☞ SA approach → Methodological changes

Data errors

Seasonal adjustment in practice (I/II)

Step 1: look at your data

- ☞ Time series plot → Visual outliers & seasonal behaviour
- ☞ Seasonality diagnostics → ACF/PACF, spectral graphs, tests

Step 2: prepare your data for seasonal adjustment

- ☞ Log transformation, outliers, calendar effects → Smoothing ("linearisation")
- ☞ ARIMA model → Identification
- ☞ First guess → Automatic procedures
- ☞ Second guess → Expert knowledge (especially outliers & calendar effects), regARIMA residual analysis

Seasonal adjustment in practice (II/II)

Step 3: seasonally adjust your pretreated data

- ☞ Specification → "Best" parameters
- ☞ X-11 approach → Trend & seasonal filters, extreme SI ratio detection

Step 4: check your seasonally adjusted data

- ☞ Residual seasonality → Absence
- ☞ Quality diagnostics → See Step 1

Outlook: companion level II course

Advanced topics in seasonal adjustment

- Date → 3–7 November 2025
  - Application deadline → 22 August 2025
- Next chance → Probably Q4 2027

Selected topics

- Outliers → Modelling during strong economic changes
- Calendar effects → User-defined regression variables
- Seasonal adjustment → ARIMA model-based approach, composite time series, daily data, revision policies
- JD+ → Access via R