A Measure of Faith

Probability in Religious Thought

Dr. Brian S. Blais

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Dedicated to E. T. Jaynes who first gave me the appreciation for the mathematics of rational thinking.

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Chapter 1

Introduction

> *Life's most important questions are, for the most part, nothing but probability problems.* - Laplace

1.1 Why write a book like this?

I write this book to convey a way of thinking about the world – one that has a demonstrated history of success. This way of thinking forms the basis for all of scientific progress and is the foundation of *all* rational thought. You use it, in one form or another, when you buy a used car or decide to bring an umbrella when you leave the house, when you vote for president or choose to get vaccinated. From the trivial to the monumental, many of our decisions are governed by – or should be governed by – rational thought. However, people are a collection of inconsistencies and biases combined with the ability to carefully analyze a situation and its consequences. Humans have developed on this planet to survive and our mental faculties have a number of short-cuts which allow us to very rapidly come to "pretty-good" solutions. Although these solutions can be quite useful, we can transcend them by the proper and careful application of evidence and probability.

Some of the short-cuts we have developed have been *useful* and some of them may *feel good* to us, but it is my contention that *the truth matters more.* I write this book for those who agree with this basic premise. I find that the pursuit of truth is the only way to be completely honest with myself and others.

> *I want to believe as many true things and as few false things as possible.* – Matt Dillahunty

Where we don't follow the truth, our actions are at best sub-optimal and at worst dangerous. I have found that the pursuit of the truth is a challenging yet rewarding activity. One of my goals is to describe a framework you can apply to any situation. I have found it useful to practice this framework on small, trivial situations first and build up to more consequential ones. In this way one can strengthen, like a muscle, one's ability to pursue the truth, to see where someone is not being entirely honest with themselves or others, and to more fully understand the consequences of our actions.

1.2 It's not about the math

Although "probability theory" is a mathematical one, the process is not entirely about the math. Writing the methods of thinking properly can involve equations; it can also involve pictures and diagrams. While we reason every day without explicitly placing numbers on our beliefs, we do weight certain possibilities as "more likely" than others. In this way, we have the relationships *between* possibilities in our heads and we re-weight them as we experience the world. If we use the methods of probability properly, then we will orient ourselves toward the truth. If, however, we merely follow our intuitions, our emotions, or our history, then we open ourselves up to the biases that we all have. It is with the disciplined use of probability that we can hope to avoid these biases.

Unlike English, mathematics is unambiguous. For this reason, it is the language that must, at base, be the appropriate platform for all discussions of this sort. However, while there can be a clarity achieved through the terseness and concreteness of mathematics, there are some dangers as well. Mathematics can be *too abstract*, which can make many people disconnect from the discussion and not see the relevance to the real world. Mathematics can also be *overly formal*, which can hide mistakes of reasoning behind a veneer of objectivity. For these reasons I plan on developing the ideas in a couple of ways *in parallel*. For those comfortable with the mathematics, I will present the equations and the analysis. The math, although appearing cryptic at first, is nothing more than arithmetic and can be understood through specific examples. For those who are less comfortable with the mathematics, I will also present the ideas wherever possible with pictures and diagrams. In this way, I hope to make the structure of rational thought to be accessible to as many people as possible.

A further benefit to describing the thought process using probability is to provide a unifying vocabulary to facilitate discussions. This vocabulary helps to describe our cognitive biases and recognize our prejudices. By casting our thinking in a somewhat more formal way, we have a method for addressing and avoiding these biases and prejudices.

1.3 Organization

I begin the book with an elementary introduction to probability. Those familiar with it can skip that coverage. I then move on to applications of this way of thinking to topics common in religion, specifically the Christian religion with which I was brought up and have had an interest in for many years. I cover terms including *belief*, *faith*, *miracles*, *evidence*, *proof* and *knowledge* — terms we commonly use but are often clouded by the fuzzy use of language. The use of mathematics brings a concreteness and lucidity to more verbose and *murky* approaches. These methods force the discussion to be both *specific* and *complete*.

1.4 Who is this for?

Although I intend this book to be technical, I do not want to scare away those who are less mathematical. In fact, I would consider it a success if someone who is not particularly inclined in the mathematical arts would be able to get a new appreciation for these topics upon reading this book. Thus I will use pictures and diagrams to augment the quantitative analysis, and I will rely heavily on specific examples at all times.

Chapter 2

Probability is not just about the math

When we speak about probability, we speak about a percentage chance (0%-100%) for something to happen, although we often write the percentage as a decimal number, between 0 and 1. If the probability of an event is 0 then it is the same as saying that *you are certain that the event will never happen.* If the probability is 1 then *you are certain that it **will** happen.* Life is full of uncertainty, so we assign a number somewhere between 0 and 1 to describe our state of knowledge of the certainty of an event. The probability that you will get struck by lightning sometime in your life is $p = 0.0002$, or 1 out of 5000. Statistical inference is simply the inference in the presence of uncertainty. We try to make the best decisions we can, given incomplete information.

Pierre-Simon Laplace, who first formalized the mathematics of probability, spoke of an agent with perfect knowledge. This agent, Laplace claimed, would not need probability at all.

> *We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature*

> *in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.* [Laplace, 1814 English Edition 1951]

E.T. Jaynes describes it in much the same way. He says that we label something "random" due to our ignorance of the system not to any intrinsic randomness. He calls this labeling the mind-projection fallacy [Jaynes, 2003], where you misattribute the unpredictable behavior of a system as a product of the system itself. A rolled die is following the laws of physics, deterministically, and detailed knowledge of the die, the roll, and the surface should allow you to predict 100% of the time what it will do. We lack that knowledge, thus the behavior becomes unpredictable. We often then attribute that unpredictable behavior as a "random die", as if it were the die that contains the randomness and not our own state of knowledge.

2.0.1 The basic rules of probability

For a complete description of the rules of probability, and their application in general statistical inference there are several books available, one of which from the present author [Blais, 2014]. We will need to establish a basic set of notation and mathematics in order to address the concepts. This notation will in some cases make clear and condensed (due the the terseness of mathematics) much longer expositions of the same concepts in English. In other cases, it will provide a systematic framework for exploring disparate problems, in order to see the connection to all of rational thought. We begin by describing the rules of probability, and some of their consequences.

When we write $P(A)$ the "P" stands for "probability" and "A" is some proposition or claim. We will be in the habit of naming sentences or statements with a short-hand of a single letter, like $A \equiv$ "*I draw an Ace from a well-shuffled, typical 52-card deck of cards*"[1].

We then can talk about the probability of this statement being true with $P(A)$, which is a summary of "*the probability that if I draw a card from a well-shuffled, typical 52-card deck of cards that I will draw an Ace.*" The shorthand allows us to write some general statements in a small amount of space.

Instead of using real-life examples to start, I prefer to use examples from simple card games. Using playing cards has several distinct advantages:

1. most people are comfortable with the concept of uncertainty in card games
2. a deck of cards is a *small* system
3. the *probabilities* are a direct parallel with *fractions* of card counts, making the subject more intuitive
4. a deck of cards is amenable to pictures

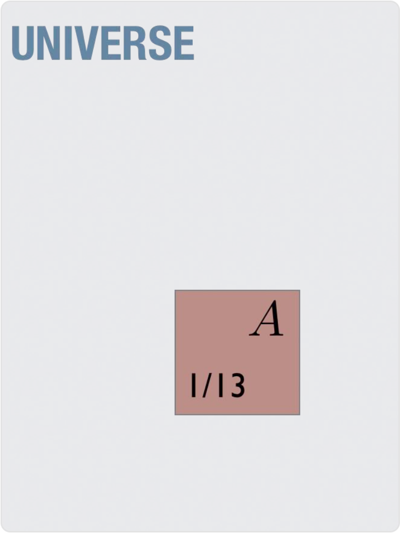
2.0.1.1 Rule 1 (Definition rule):

The probability of a proposition or statement, e.g. $P(A)$, is a number between 0 and 1, representing the strength of belief in a statement, A.

$$\begin{aligned} P(A) &= 0 \text{ certainly false} \\ P(A) &= 1 \text{ certainly true} \\ 0 < P(A) &< 1 \text{ degrees of belief} \end{aligned}$$

The following picture shows a way to visualize this, with a "Universe" of all possibilities and the subset, which is our particular statement as a fraction of the total.

[1] We use the symbol "$\equiv$" to denote a *definition*, not just the equality relationship denoted by the "$=$".



Although this probability is *estimated* by the fraction of Aces in a deck of cards, whenever we write probabilities we are referring to a measure of the strength of one's belief in the statement. Thus, when we write that $P(A) = 4/52 = 0.077$ this probability means that you believe it to be unlikely — but not extremely unlikely — that you draw an Ace from a well-shuffled deck (see the Rough guide for the conversion of qualitative labels to probability values in Table 1 at the end of the chapter). This belief, as we will see, is not just a *guess* but is something arrived at through proper rational processes, or in other words, by adhering strictly to the rules of probability.

2.0.2 A bit more about the deck-of-cards analogy

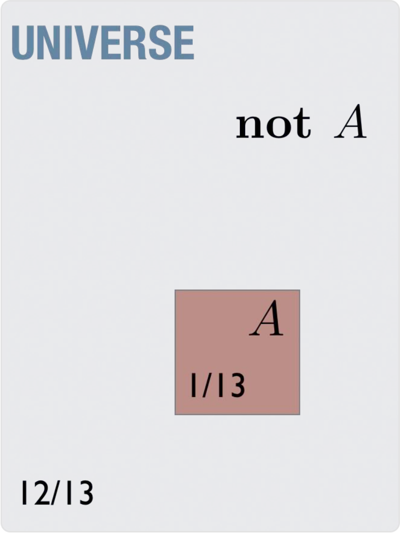
We are using the deck of cards to be analogous to the real world. While there are things we know about the real world there is a lot of uncertainty as well, just like the uncertainty of the deck of cards. We can gather evidence in the real world, to better know what the truth is, and analogously we can draw cards from the deck to better know what the properties of the deck is, i.e. is it well-shuffled, is it a standard 52-card deck or a deck of Tarot cards, etc... As in the real world, we can propose *models* or simplified descriptions of what we think the deck is, and we can perform tests of these models by drawing from the deck and comparing to what we expect from the models. The methods of science do this with nature itself, by arranging situations where the observations will possibly rule out some models in favor of others, so that we get closer to the truth. The same rules of probability apply in all of these cases, but given the intuitive nature of a deck of cards, it is easier to see them applied in this simple system.

2.0.2.1 Rule 2 (Negation rule):

The negation rule states that a proposition is either true or its negation (i.e. direct opposite) is true.

$$P(A) + P(\textbf{not } A) = 1$$

The following picture shows a universe of all possibilities and the subset (A) which is our particular statement as a fraction of the total, along with the *opposite* of this subset (**not** A) which adds up to the total.



In other words, either a statement is true or its negation is true.

$$\begin{aligned} A &\equiv [\text{“you will draw an Ace from a deck of cards”} \\ P(A) + P(\textbf{not}\, A) &= 1 \\ \frac{1}{13} + \frac{12}{13} &= 1 \end{aligned}$$

means that you can be *certain* (i.e. probability equal to 1) that when you draw a card from a deck, it will either be an Ace or it won't be an Ace. This all seems rather obvious, and you may be wondering why we even bring it up. Surprisingly, this "*obvious*" property becomes a source of one of the most common logical fallacies - the either-or fallacy.

Notice how this occurs. The following is correct logical inference:

$$\begin{aligned} B &\equiv [\text{“a playing card drawn from a deck is black”} \\ P(B) + P(\textbf{not}\, B) &= 1 \end{aligned}$$

means that, if you draw a playing card from a deck, you can be *certain* that it is either black or it is not-black. This is true no matter what kind of deck of cards you are dealing with, even if it contains no black cards! The following, however, is *not* a correct logical inference:

$$\begin{aligned} B &\equiv [\text{"a playing card drawn from a deck is black"} \\ R &\equiv [\text{"a playing card drawn from a deck is red"} \\ P(B) + P(R) &= 1 \leftarrow \text{this is incorrect} \end{aligned}$$

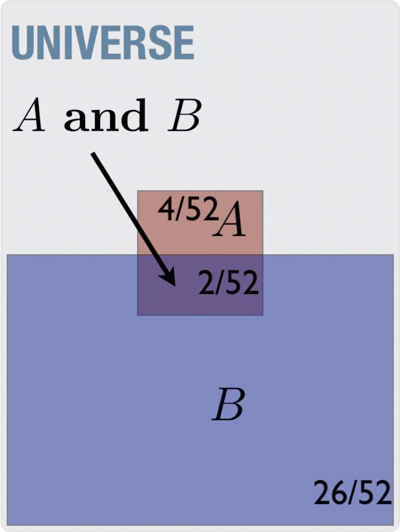
The key point here is that "not-black" is not the same as "red" except in those cases where you can be *certain* that there are only those two possibilities. One has to be on the lookout for hidden possibilities - perhaps one has a *Five Crowns* deck which has green and yellow cards as well? Failure of imagination can easily lead to accidental either-or logical failures. Statements like "You're either with us or against us!" (no third option?) and "If you don't take a stand against a political candidate then you must be supportive of her." (again, no other options?) serve as a reminder to recognize this lack of imagination on our part.

2.0.2.2 Rule 3 (Conjunction rule):

The conjunction rule defines how we handle two propositions being true at the same time, relating them to the probabilities of the individual propositions *alone* and the probabilities of each one assuming the other is true, also called the *conditional* probability. The latter probability captures how the two propositions are related.

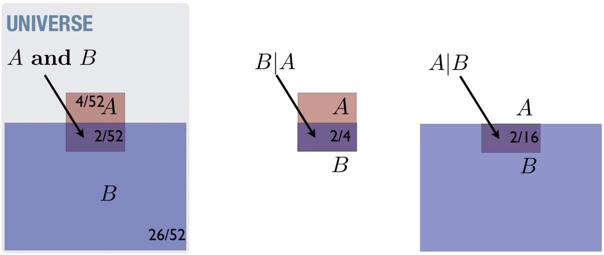
$$P(A\,\mathbf{and}\,B) = P(B|A)P(A)$$

which is the probability of two statements both being true, A **and** B. We define a new symbol, |, which should be read as "given." When there is information given, we call this probability *conditional* on that information. We can represent the conjunction as the *overlap* in the following picture:



The numbers in the picture are coming from the propositions $A \equiv$ "you will draw an Ace from a deck of cards" and $B \equiv$ "a playing card drawn from a deck is black"

The conditional statements are like restricting the universe to the small part of what you're "given" (i.e. what's on the right-hand side of the bar symbol, |). For example, $P(A|B)$, is the probability of A if you restrict your cases to those satisfying B (see the right-hand rectangle in the following picture) and $P(B|A)$, is the probability of B if you restrict your cases to those satisfying A (see the middle rectangle in the picture):



The conjunction rule in this case means that the probability of you drawing a black Ace is related to the probability of you drawing an Ace from a collection of black cards (i.e. "given that the card is black", $P(A|B)$) and the probability that you will draw a black card at all ($P(B)$) or equivalently, the probability of you drawing a black card from a collection of Aces (i.e. "given that the card is an Ace", $P(B|A)$) and the probability that you will draw an Ace at all ($P(A)$).

Numerically we have

$$
\begin{aligned}
P(A) &= \frac{4}{52} && \text{(probability of Ace out of all cards)}\\
P(B) &= \frac{1}{2} && \text{(probability of black out of all cards)}\\
P(A|B) &= \frac{2}{26} && \text{(probability of Ace out of black cards)}\\
P(B|A) &= \frac{2}{4} && \text{(probability of black out of Ace cards)}
\end{aligned}
$$

From which follows the probability of having a black Ace in two mathematically equivalent ways,

$$P(A\,\textbf{and}\,B) = P(A|B)P(B) = \frac{2}{26} \times \frac{1}{2} = \frac{1}{26}$$

and

$$P(A\,\textbf{and}\,B) = P(B|A)P(A) = \frac{2}{4} \times \frac{4}{52} = \frac{1}{26}$$

Mathematically, it can be seen that $P(A\,\mathbf{and}\,B)$ is always lower than $P(A)$ unless we are *certain* that the other statement, B, is true — the conjunction of two things is inherently (and mathematically) less probable than the individual components. Failure to recognize this leads to the *conjunction fallacy.* The most common example given is as follows (from https://en.wikipedia.org/wiki/Conjunction_fallacy),

> *Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.*
>
> Which is more probable?
>
> 1. Linda is a bank teller.
> 2. Linda is a bank teller and is active in the feminist movement.

Most people lead towards (2), but the actual answer is (1) because the combination of two things (a bank teller *and* is a feminist) is a smaller subset and is thus less likely[2].

There are a few points to be made about the approach we've been using so far, which become important in later examples.

- Notice how concise the description is — the math can summarize the relationship between several concepts with few words or symbols.
- As we've seen, the conjunction of two things is inherently (and mathematically) less probable than the individual components, sometimes with unintuitive consequences.

[2] this may be partly due to English being sloppier than math. In choices like the Linda problem, there may be an implied "and is *not* a feminist" in option (1) in common usage but is not strictly present.

- If there is more than one way to reason properly to an answer, those different ways must come to the same answer. This is a good check to see that you are thinking properly when you see the same answers, but it is also a way to distinguish two methods that are in fact not equivalent even if they seem to be — they come to different answers.

- Two statements are considered *independent* if knowledge of one gives you no more information about the other. In probabilistic terms, this means that $P(B|A) = P(B)$ — knowing A is true doesn't make the probability of B any more or less. Flipping a coin a second time is still going to be 50-50 heads-tails whether you flipped a heads the first time or not. Drawing a second card from the top of a deck is a little less likely to be a black card if you draw a black card on the first time — knowledge of previous cards drawn tells you information about the probabilities for the second. Independence becomes important in the evaluation of evidence because it changes how evidence can be accumulated.

- In the case of *independent* statements, the conjunction rule simplifies to

$$P(A\,\mathbf{and}\,B) = P(A) \cdot P(B) \quad \text{(independent)}$$

- The *general case* of the conjunction rule looks like

$$\begin{aligned} P(A\,\mathbf{and}\,B\,\mathbf{and}\,C\,\mathbf{and}\,D) =& P(A) \times P(B|A) \times P(C|A\,\mathbf{and}\,B) \times \\ & P(D|A\,\mathbf{and}\,B\,\mathbf{and}\,C) \end{aligned}$$

2.0.2.3 Rule 4 (Bayes' rule)

This rule is perhaps the most obtuse to see for the first time, but is by far the most important rule of them all, so it is worth the effort.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Mathematically, it is just a rewriting of the conjunction rule above. Its deeper meaning can be seen when rewritten in a somewhat more elaborated form describing our belief in an explanation given some data,

$$P(\text{explanation}|\text{data}) = \frac{P(\text{data}|\text{explanation})P(\text{explanation})}{P(\text{data})}$$

where each term is described more fully as

- $P(\text{explanation})$ —the probability the explanation is correct *prior* to seeing the data. The term itself is often called the *prior*, and represents your beliefs before you see the data. Typically, more complex explanations are less likely a-*prior*i than simpler ones. I will use the term *model* in place of explanation in most of this book, but it means the same thing.

- $P(\text{explanation}|\text{data})$ — the probability the explanation is correct *after* seeing the data (*a-posteriori*). The term itself is often called the *posterior* for this reason, and represents your updated beliefs once you have data. Thus, Bayes' rule is a mathematical expression of *learning* from evidence.

- $P(\text{data}|\text{explanation})$ — the probability that the data can be explained with *this particular explanation*. The term itself is often called the *likelihood*, and can be thought of as a measure of how well the explanation fits the data. If the explanation fits the data well, this number will be high, for example. If it fails to explain the data, this number will be low. Although related to our final (*posterior*) belief, it is not equivalent. An explanation that fits the data well may be very unlikely as our best explanation just because that explanation was extremely unlikely in the first place (i.e. before we saw the data, aka the *prior* was low).

- $P(\text{data})$ — the total probability of the data, regardless of the explanation. It is easiest to understand this term with an examples below.

Imagine we are playing a game with several small decks of cards, defined here:

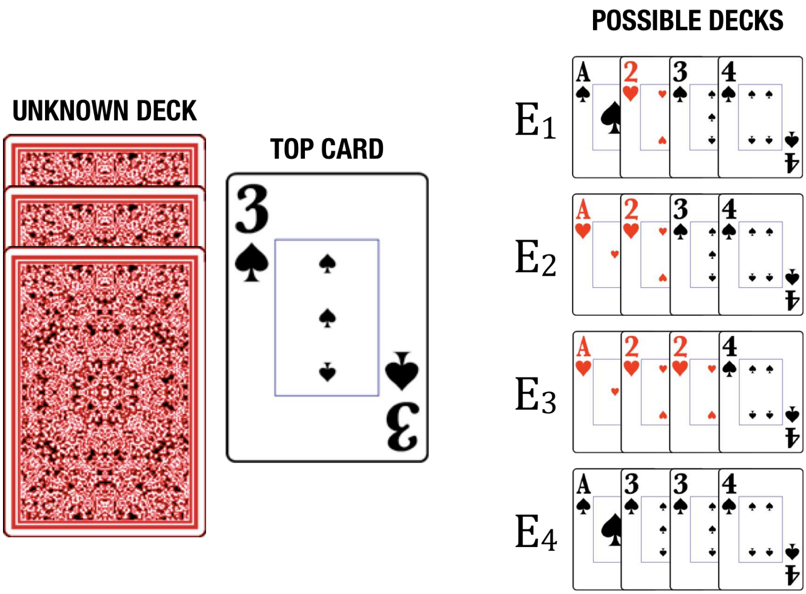
- $E_1 : A\spadesuit, 2\heartsuit, 3\spadesuit, 4\spadesuit$

- $E_2 : A\heartsuit, 2\heartsuit, 3\spadesuit, 4\spadesuit$

- $E_3 : A\heartsuit, 2\heartsuit, 2\heartsuit, 4\spadesuit$

- $E_4 : A\heartsuit, 3\spadesuit, 3\spadesuit, 4\spadesuit$

Where E is denoting a deck or, more generally, an *explanation* for the data we will collect by drawing cards. In the game, someone has handed us one of the decks (i.e. E_1, E_2, E_3 or E_4) but we don't know at all which one it is. The analogy here is that the universe is set up with a set of rules that we are trying to determine. Thus, deciding on which deck we are holding is analogous to deciding which universe we are actually in, given our observations of the universe. In other words, providing an *explanation* of the data is really about determining which of the many possible universes we are in.



We then draw the top card, observe that it is a 3♠, and see if we can reason about which deck is likely to be the one we are holding. We choose such a small, simple system because it is easy to intuit the answers without the math. This intuition can provide a scaffold for understanding the mathematics, which can be used in more complex examples where one *doesn't* have a strong intuition. It is therefore worth going through at least one example in detail.

To begin, we need to assign the probabilities of the four cases *prior* to the data. Given total ignorance of which deck was chosen— we know that there are 4 possibilities but have no idea about anything more about the selection process — we assign equal probabilities to the four cases[3]

[3]not all cases will lead to equal probabilities of the outcomes, because we almost always have *some* knowledge to go on.

$$\begin{aligned} P(E_1) &= \ 1/4 \\ P(E_2) &= \ 1/4 \\ P(E_3) &= \ 1/4 \\ P(E_4) &= \ 1/4 \end{aligned}$$

Note that in this example we know what cards are in each deck – we know, for example, that there are no spades in E_3 – we just don't know which deck we were given. Here we describe our intuitions, with the mathematics in parallel below. Since we drew a 3♠, our intuition says that this should rule out E_3 altogether. Further, it says E_4 should be more likely than the other remaining two because it contains the observed card, 3♠, more than one time – it is easier to get that particular card from the fourth deck than the others.

The mathematics would look like this

$$\begin{aligned} P(E_1|3\spadesuit) =& \frac{P(3\spadesuit|E_1)P(E_1)}{P(3\spadesuit)} \quad \leftarrow \text{this term the same in all} \\ P(E_2|3\spadesuit) =& \frac{P(3\spadesuit|E_2)P(E_2)}{P(3\spadesuit)} \\ P(E_3|3\spadesuit) =& \frac{P(3\spadesuit|E_3)P(E_3)}{P(3\spadesuit)} \\ P(E_3|3\spadesuit) =& \frac{P(3\spadesuit|E_4)P(E_4)}{P(3\spadesuit)} \end{aligned}$$

where we already have

$$P(E_1) = P(E_2) = P(E_3) = P(E_4) = 1/4$$

Further, we have

$$P(3\spadesuit|E_1) = 1/4$$

because one card out of 4 in the first deck is the 3♠. Likewise, we have

$$P(3♠|E_2) = 1/4$$
$$P(3♠|E_3) = 0$$
$$P(3♠|E_4) = 2/4$$

Finally we have[4]

$$P(3♠) = 4/16$$

because there are four "3♠" cards out of all the 16 in the game. Plugging these numbers into the above equations, and performing the arithmetic, we have

$$P(E_1|3♠) = \frac{(1/4) \times (1/4)}{(4/16)} = 1/4$$
$$P(E_2|3♠) = \frac{(1/4) \times (1/4)}{(4/16)} = 1/4$$
$$P(E_3|3♠) = \frac{(0) \times (1/4)}{(2/12)} = 0$$
$$P(E_4|3♠) = \frac{(2/4) \times (1/4)}{(4/16)} = 1/2$$

which perfectly matches our intuition — $E3$ is certainly false, and E_4 is more likely than the other two. Notice further that $P(3♠|E_1)$ is another way of saying "how well is the observation of a 3♠ explained by the idea that we're holding the first deck?" The entire process can then be thought of as updating our initial beliefs with the new evidence.

Any process of reasoning, in any field whatsoever, is either consistent with this process of calculation or it is not rational.

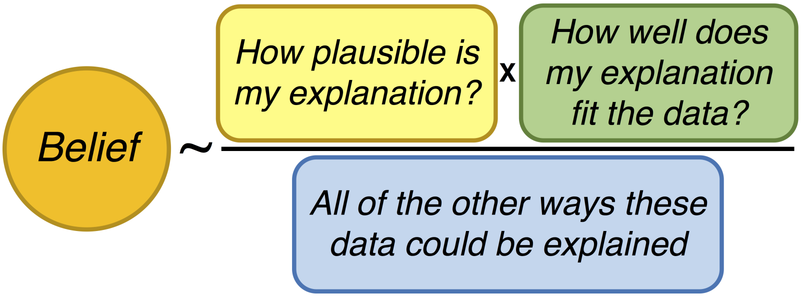
[4]The reader might be thinking at this time, "why do we have to do all this? Seems complicated!" I address this shortly, so please bear with me.

It is for this reason that we explore this process in such detail.

On another front, an alternate way to have calculated the shared bottom term, $P(3\spadesuit)$, is the following totally long-winded and complicated way

$$\begin{aligned}
P(3\spadesuit) =& P(3\spadesuit|E_1)P(E_1) + P(3\spadesuit|E_2)P(E_2) + \\
& P(3\spadesuit|E_3)P(E_3) + P(3\spadesuit|E_4)P(E_4) \\
=& (1/4) \times (1/4) + (1/4) \times (1/4) + (0) \times (1/3) + (2/4) \times (1/4) \\
=& 4/16
\end{aligned}$$

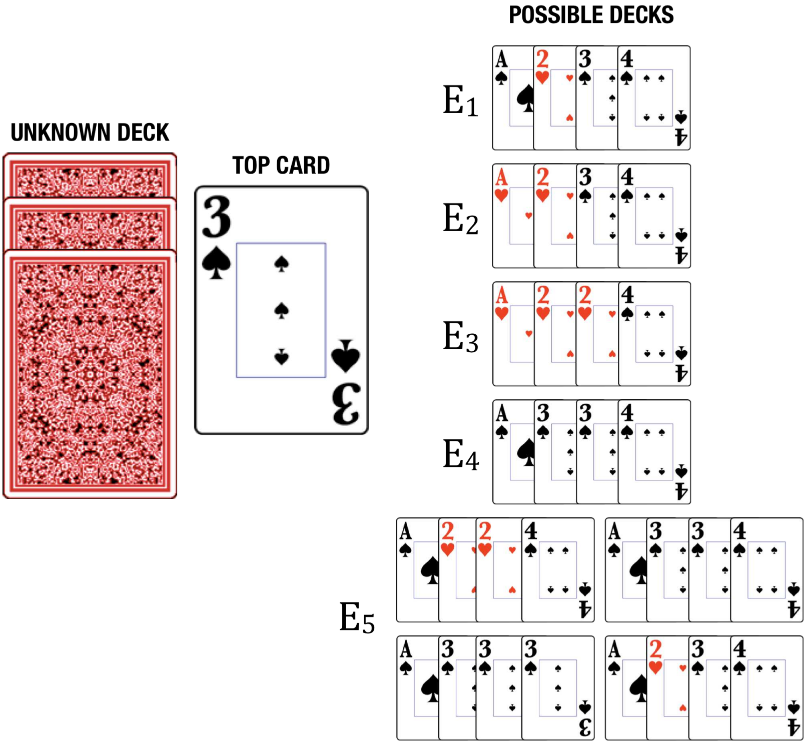
Why would one write it in this seemingly over-complex fashion? Because it makes it easier to say, in words, what this term is doing. It is the sum of all of the probabilities for how well each explanation accounts for the data scaled by how likely that explanation was before seeing the data. In other words, proper rational inference requires that you re-weight the strength of your beliefs in an explanation not just by how well that explanation describes your observations, but also by how intrinsically likely that explanation is before your observations and how well all of the *alternatives* perform on those same observations.



An observation can be very well explained by a particular explanation, but if it can be equivalently explained by other, simpler, explanations, then your belief in that more-complex explanation may in fact *weaken* with the new observation (i.e. its probability could go down).

2.1 On simplicity

Ockham's razor, which is the philosophical idea that simpler theories are preferred, is a consequence of Bayes' rule when comparing models of differing complexity [Jefferys and Berger, 1991]. We can see this by extending the card game example with a fifth possibility.



Instead of giving the specific cards in this deck, we are simply told

- E_5 : the deck can have anywhere from zero to three 3♠, and enough other cards to make a total of four cards

This explanation of the game is what is called *plastic*[5] - a value in the

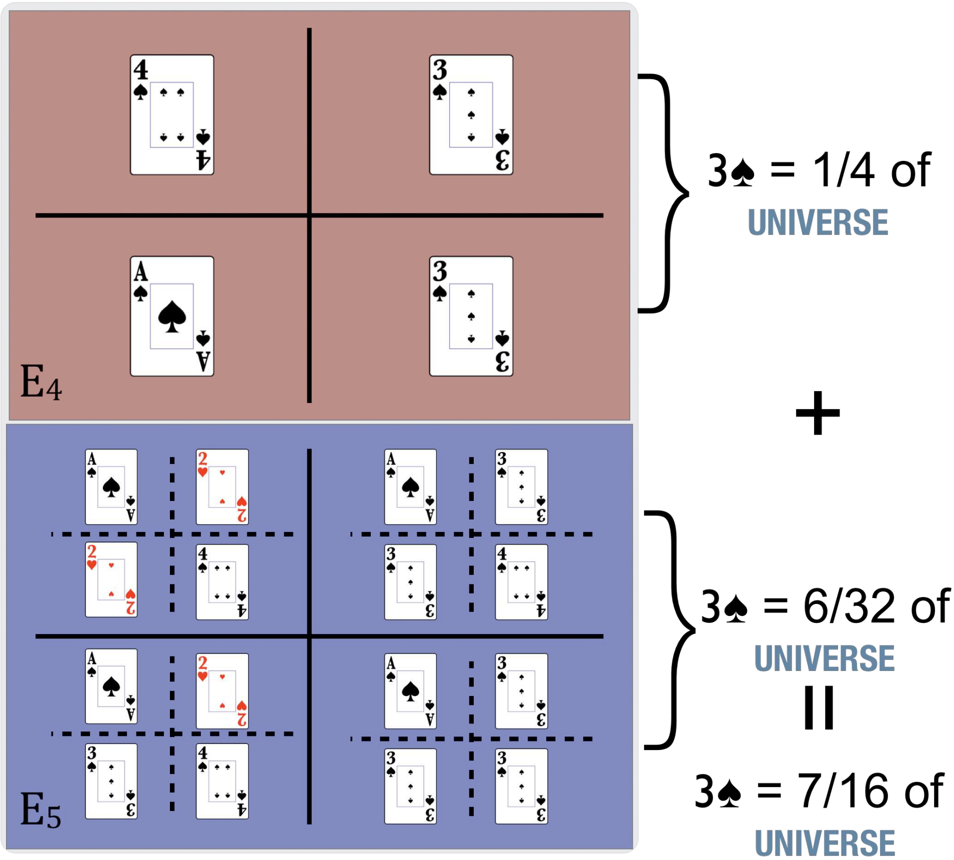
[5]In mathematical models, this is often referred to as having an *adjustable parameter*

model that is not specified ahead of time, but can be *fit* to the data, and an optimum value found. We could potentially think like this, "depending on the data, we may infer a different value for the number of 3♠ in this deck. It may be heavily loaded toward 3♠, which would make E_5 explain the data very well; however it may have none, and not explain the data at all. Clearly, once you observe a 3♠, the"best" value for this deck is to have three of them out of the four cards - making it more likely than the previously best explanation, E_4, which only had two out of four."

However, this process of reasoning violates the laws of probability by not taking our uncertainty of this parameter (i.e. the number of 3♠ in the deck) into account. For simplicity, let's just consider the two decks in question, E_4 and E_5, and play the game with them (again, as before, drawing a 3♠ from the top).

- E_4: $A\heartsuit$,3♠, 3♠, 4♠

- E_5: the deck can have anywhere from zero to three 3♠, and enough other cards to make a total of four cards



Having a more complex model (E_5) divides the space of options into more areas, and thus makes the model less likely. A more complex model needs to explain the data *even better* than a less complex one.

We set up the calculation as before,

$$P(E_4|3\spadesuit) = \frac{P(3\spadesuit|E_4)P(E_4)}{P(3\spadesuit)}$$
$$P(E_5|3\spadesuit) = \frac{P(3\spadesuit|E_5)P(E_5)}{P(3\spadesuit)}$$

We defer the calculation of the shared term, $P(3\spadesuit)$, and focus on the numerators of both calculations.[6] First the one for E_4 (and remember, we have only two decks here, so we'll have a prior of $P(E_4) = P(E_5) = 1/2$),

$$\begin{aligned} P(E_4|3\spadesuit) \sim& \quad P(3\spadesuit|E_4)P(E_4) \\ =& \quad (2/4) \times (1/2) = 1/4 \end{aligned}$$

Next with the E_5 deck,

$$\begin{aligned} P(E_5|3\spadesuit) &\sim P(3\spadesuit|E_5)P(E_5) \\ &= P(3\spadesuit|E_5) \times (1/2) \end{aligned}$$

where the term $P(3\spadesuit|E_5)$ is arrived at by breaking it into the four possibilities, e.g. from zero to three 3♠. Each of the possibilities (all equally likely, because we are given no other information) has the form of the fraction of ♠ for that possibility times 1/4 because there are 4 total possibilities to consider, for example

$$\begin{aligned} P(3\spadesuit|E_5 \text{with zero } 3\spadesuit s)P(\text{zero } 3\spadesuit s|E_5) &= \underbrace{(0/4)}_{\text{zero } \spadesuit s} \times (1/4) \\ P(3\spadesuit|E_5 \text{with one } 3\spadesuit)P(\text{one } 3\spadesuit|E_5) &= \underbrace{(1/4)}_{\text{one } \spadesuit} \times (1/4) \\ &\vdots \end{aligned}$$

Doing the same for all the possibilities, we get for the E_5 numerator,

$$\begin{aligned} P(3\spadesuit|E_5)P(E_5) &= [(0/4) \times (1/4) + (1/4) \times (1/4) + \\ &\quad (2/4) \times (1/4) + (3/4) \times (1/4)] \times (1/2) \\ &= 3/16 \end{aligned}$$

Finally, we can get the shared term,

[6] Once we have the numerators, we can add them up to get the shared term $P(3\spadesuit)$

$$P(3\spadesuit) = 1/4 + 3/16 = 7/16$$

and the probabilities of each of the decks, given the observation of a $P(3\spadesuit)$,

$$\begin{aligned} P(E_4|3\spadesuit) = &\ \frac{1/4}{7/16} = 4/7 \\ P(E_5|3\spadesuit) = &\ \frac{3/16}{7/16} = 3/7 \end{aligned}$$

This means that, although E_5 contains the *possibility* of a better fit to the data, it is less *probable* because it has a flexible parameter that is unspecified *before* the data. Even more interesting, is that E_5 actually *contains* E_4 as a possibility! In other words, if you have an explanation which is not specific but can adjust to the data only after seeing it, there is a danger in accepting that explanation over one that is specific and doesn't change its prediction after the data. The proper use of the rules of probability help guard against this danger. In religious contexts, an example of this happens when addressing the efficacy of prayer. Someone might reason as follows, "if the prayer works then God chose to act, otherwise God had a reason to withhold action". This is a poor explanation, because it is not specific and adjusts to the data only after the fact – only couldn't make a prediction with this model of the world.

When we prefer a "simpler" model with Ockham's razor, simpler means *fewer adjustable parameters.* It also means that the predictions are both *specific* and not *overly plastic.* For example, a hypothesis which is consistent with the observed data, but also would be consistent if the data were the opposite would be overly plastic. An example of an infinitely plastic "explanation" is "*magic did it.*" Because it can "explain" anything (given that it is consistent with any possible observation), it thus explains nothing. Conspiracy theorists suffer from this problem, because any counter-evidence provided get lumped into the conspiracy. If you claim that we staged the moon landing, and someone provides photographs of the astronauts on the moon, then you can just say that the astronaut and the photographer are "in on the conspiracy" – the model gets adjusted after the fact and is *overly plastic*.

Notice also that *simpler* does not necessarily mean having fewer parts. The explanation E_5 actually has fewer parts than E_4 because I don't specify any of the other cards, only the possible number of 3♠. The complexity of the explanation comes from its flexibility.

2.2 On belief, knowledge, and proof

Clearly the terms belief, knowledge, and proof are related but it is the framework of probability that helps us be specific about their differences. We can summarize the situation with the following,

- *Belief* is measured by probability. Higher probability is equivalent to stronger belief, and likewise, lower probability is equivalent to weaker belief.
- *Knowledge* is a subset of belief, described more specifically below.
- *Proof* refers specifically to the result of a *deductive* process. In this process, one starts with some (given) axioms, and can demonstrate *with certainty* (i.e. prove) a number of theorems (i.e direct consequences) from those axioms. Thus, the word proof should only be used in situations involving *certainty*, or in other words, probabilities of exactly 1 or exactly 0 only.

2.2.1 Belief

> We say we believe a proposition A when $P(A) > 0.5$. We say we believe *strongly* in a proposition A when $P(A) > 0.95$ or some other, somewhat arbitrary, high number. The strength

> of a belief is a *scale* measured by the probability assigned to that proposition.

Everything we have covered so far in this book relates to belief. Notice that someone can have a *false* belief, if the information they are provided is incorrect. If we want to believe as many true things and as few false things as possible then we should look for ways to test our beliefs and challenge the information we are given. Belief, as used here, is different from *opinion*. When we use the word *opinion* we can simply mean *preference* (e.g. I like vanilla more than chocolate) which is not a belief. However I can also say I have an *opinion* about whether a political candidate is being honest, which is a usage equivalent to *belief* as long as the *opinion* is based on evidence and not just preference.

2.2.2 Knowledge

What is knowledge? Plato gave the following definition of knowledge:

> Knowledge is justified true belief. [Fine, 2003]

This is an unsatisfying definition because, it seems, in order to justifiably label anything as *knowledge* with this definition we'd need to be able to independently determine that the statement is true. This presupposes that there is some "outside" knowledge, but we have no access to that. I believe there is likely to be a truth to be known, but that we can never truly know what it is for certain — but this is not a problem. It is a red herring to bring up 100% certainty for knowledge, because it is never achievable, and isn't what we practically call knowledge. I prefer a definition inspired by Stephen J Gould:[7]

> In science, '*fact*' can only mean confirmed to such a degree that it would be perverse to withhold provisional assent. 'I

[7] In this way, knowledge is a subset of belief.

> suppose that apples might start to rise tomorrow, but the possibility does not merit equal time in physics classrooms.' [Gould, 1981]

Where it says "fact," read "knowledge." Where it says "science" read "life." The fact, or knowledge, that the Sun rises in the east and sets in the west is not due to a formal *proof* that this is always true, or will continue indefinitely into the future. It is an admission that the probability is so outrageously high, given the evidence of every other sun rise and sun set observed, and the further confirmation of models of the solar system, that it would be "perverse to withhold provisional assent." We accept the claim as a practical matter, despite not being 100% certain, because of the overwhelming probabilities. This we call knowledge. It is thus never a problem to admit lack of certainty in knowledge, and it can be seen as an obvious diversion if anyone tries to argue in a manner that suggests it's a problem.

Mathematically, we might write it as the probability of proposition "A" is very close to certain, or $P(A) > 0.9999 \approx 1$ where "$\approx$" means *approximately*. Notice that we don't need 100% certainty to claim knowledge, and that it is possible for the "knowledge" to be wrong (although, by definition, it is highly unlikely for this to be the case).

We can ask the question, how did we come to this knowledge? The answer is simply, by applying the rules of probability! According to Bayes' rule, we update our probabilities given the evidence. This can lead us to approach, but never equal, probability of 1. We can approach, but never achieve, complete certainty of any claim. Rationality only insists that we apply the rules of probability systematically.

2.2.3 Proof

Do *proof* and *evidence* mean the same thing? No, they don't. There are two primary processes for rational inference — *deductive* and *inductive*. In *deductive* reasoning, one starts with some (given) axioms, and can demonstrate *with certainty* (i.e. prove) a number of theorems (i.e direct consequences) from those axioms. In *inductive* reasoning, one presents evidence or data which makes certain models either more or less likely — never to absolute certainty. Thus, the word "*proof*" should only be used in situations involving *certainty*, derived from axioms with logic. In this way, *deductive* reasoning is a subset of *inductive* for those cases where the probabilities are *exactly* 1 or *exactly* 0. *Induction* is just another name for the application of the rules of probability, so we have been doing that from the beginning of the book.

I realize that I'm being pedantic here because in common speech we use the word "proof" a little more loosely. We speak about "scientific proof" or "I won't believe this without proof". However, it is important to be specific about one means here and I will insist though this book that the word "proof" be used only in the more restrictive sense. Everything else is just probabilities approaching proof. As a result, we have the following maxim,

> *Proof* does not exist in science, only in math and philosophy.

The only place you can have proof is where you have *axioms* (i.e. unprovable statements), and can then *prove* a number of consequences of those axioms. We can prove, for example, that the sum of the angles of a triangle is 180 degrees, if we start with the Euclidean axioms of geometry. Science doesn't have axioms, and thus there are no proofs - there is only evidence. We sometimes hear the term "proven scientifically," even from people who should know better.[8]

As it applies to science, it has the following consequence

> All of the evidence in the universe cannot bring the probability of a scientific claim to certainty.

[8] An example of someone who should know better is Richard Carrier in his "Is Philosophy Stupid" talk., his books, and his articles.

To see this directly, imagine we have two (and only two) hypotheses for the earth - flat earth and round earth. We can write Bayes' rule for the probability of each given the data. Note that these data includes things like the experience of airplane flight, Magellan's trip, pictures from space - all of which could be faked! The flat earth hypothesis is not *logically* impossible, it's just been overwhelmed by the evidence.

$$P(\text{round}|\text{data}) = \frac{P(\text{data}|\text{round})P(\text{round})}{P(\text{data}|\text{flat})P(\text{flat}) + P(\text{data}|\text{round})P(\text{round})}$$

$$P(\text{flat}|\text{data}) = \frac{P(\text{data}|\text{flat})P(\text{flat})}{P(\text{data}|\text{flat})P(\text{flat}) + P(\text{data}|\text{round})P(\text{round})}$$

Notice that, no matter how well the round earth hypothesis explains the data (i.e. $P(\text{data}|\text{round}) \approx 1$) and how unlikely you believe it is that the Earth is flat even *before* the data, (i.e. $P(\text{flat}) \ll 1$) as long as there is some *possible* (even if seriously contrived) way to explain the data with the flat earth hypothesis (i.e. $P(\text{data}|\text{flat}) \neq 0$) it is *mathematically impossible* to make the round earth hypothesis certain. All of the terms in both equations above are greater than 0 and less than 1 and thus the round-earth model isn't 100% certain (i.e.$P(\text{round}|\text{data}) < 1$)

This does not mean that we can't be *confident* of claims, only that we cannot have *absolute certainty of anything* in science (and therefore, in life in general). Anyone who doesn't understand that does not understand science.

For it to be *reasonable to believe in something*, it must rise to a level of probability that you would label it as *belief*. Does this ever happen, or should this ever happen, with untrue things? Certainly. Here are a few that come to mind.

- the world is flat - as long as you are constrained to not live near the shore, or see a lunar eclipse

- life is designed - before the advent of Darwin's theory of natural selection
- the Sun, and the stars, all go around the Earth - until the maturation of astronomy and physics

In each of these cases, *at the time* there was in fact strong evidence for the (false) claims, and against the counter claims making it reasonable to believe them. It is no longer reasonable to believe these claims - the process of reason forces one to adjust the probabilities of the hypotheses given new evidence, and to discard those hypotheses that become too improbable.

2.3 An example of independence

The nature of independence comes up in many places so it is important to understand how it can affect inference. Recall that two statements are considered *independent* if knowledge of one gives you no more information about the other. In probabilistic terms, this means that $P(B|A) = P(B)$ — knowing A is true doesn't make the probability of B any more or less. In the case of cards, if I constantly reshuffle the deck after each draw then knowing one card will not help you with the next.

I can easily make certain cards more or less likely by changing the process of drawing. If I always place cards the I draw from the top of the deck back at the bottom of the deck, then knowing I've drawn two Aces so far makes the drawing of Aces *less likely* in future draws. If however I place the cards back near the top of the deck (e.g. throwing them roughly into a pile that I draw from), then knowing I've drawn two Aces so far makes the drawing of Aces *more likely* in future draws.

If you are assuming (for simplicity) that information you're getting from two different people is *independent* and it turns out that it isn't, then you will not reach proper conclusions. It might be that the information is less reliable than you thought — that the second person only heard the information from the first, so you're effectively getting it from one person. The equations would look like (focussing on the numerator of Bayes' Rule)

$$\begin{aligned} P(\text{claim}|A\,\textbf{and}\,B) &\sim P(A\,\textbf{and}\,B|\text{claim}) \\ &= P(A|\text{claim})P(B|A,\text{claim}) \end{aligned}$$

The key part is $P(B|A,\text{claim})$ — how likely is the testimony from B given the testimony from A? If it is independent, then one gains relevant information from the testimony. If B is parroting A, then this term is nearly 1 and you will effectively only have $P(A|\text{claim})$. If B typically avoids agreeing with A, and has heard A giving the testimony, then the probabilities are shifted in the opposite direction. One can see that it becomes critical to know whether the parts of the process that provided your evidence are all independent.

Scientists design experiments to make sure that the evidence is as informative as it can be — that two measurements of the same thing giving the same answer are doing so not because they correlate but because they independently came to the same answer so we can be confident in it.

2.4 Lessons from probability

The mathematics of probability theory is the gold standard for all statistical inference. It structures all inference in a systematic fashion. However, it can be used without doing any calculations, as a guide to qualitative inference. Some of the lessons that are consequences of probability theory are listed here, and will be noted throughout this text in various examples.

- Confidence in a claim should scale with the evidence for that claim - the more evidence the higher confidence.

- Ockham's razor, which is the philosophical idea that simpler theories are preferred, is a consequence of Bayes' Rule when comparing models of differing complexity that explain the data equally.
- A complex model must explain the data *even better* than a simple model to be preferred.
- Simpler means *fewer adjustable parameters* not fewer parameters or parts.
- Simpler also means that the predictions are both *specific* and not *overly plastic*. For example, a hypothesis that is consistent with the observed data, but can also be consistent if the data were the opposite, would be overly plastic.
- Your inference is only as good as the hypotheses (i.e. models) that you consider.
- Extraordinary claims require extraordinary evidence. [Sagan, 1996]
- It is better to explicitly display your assumptions rather than implicitly hold them.
- It is a *good thing* to update your beliefs when you receive new information.
- Not all uncertainties are the same.

There is not a universal agreement for the translation of numerical probability values to qualitative terms in English (i.e. highly unlikely, somewhat unlikely, etc…). One rough guide is shown in Table 1. I will be following this convention throughout the book, but realize that the specific probability distinctions are a bit arbitrary.

2.4.0.1 Table 1

Rough guide for the conversion of qualitative labels to probability values.

term	probability
term	probability
virtually impossible	1/1,000,000
extremely unlikely	0.01 (i.e. 1/100)
very unlikely	0.05 (i.e. 1/20)
unlikely	0.2 (i.e. 1/5)
slightly unlikely	0.4 (i.e. 2/5)
even odds	0.5 (i.e. 50-50)
slightly likely	0.6 (i.e. 3/5)
likely	0.8 (i.e. 4/5)
very likely	0.95 (i.e. 19/20)
extremely likely	0.99 (i.e. 99/100)
virtually certain	999,999/1,000,000

Chapter 3

Belief and evidence

3.1 Belief, non-belief, and disbelief

Moving beyond the cards analogy, imagine that we have someone claiming

> There are an even number of stars in the Galaxy.

how should one evaluate this claim? Let's see what we know.

1. There are trillions of stars in the Galaxy
2. By Rule 2 (Negation rule) the actual number of stars *must* be either even or "not-even"
3. The property of numbers themselves states that if a whole number is "not-even" then it must be odd — there are no alternatives
4. Given no obvious way to estimate the number of stars within an accuracy of 2 stars, we assign equal probabilities to each,

$$P(\text{even}) = 0.5$$
$$P(\text{odd}) = 0.5$$

With no more information than this, I would not believe the original claim. Anyone making the strong claim that, for example, $P(\text{even}) > 0.95$ would have to present compelling evidence. Note, however, that me saying, “I don’t believe your claim” or “I am not convinced by the evidence you have given” does **not** mean that I think the claim is false. Put another way, if we’re having a conversation and it goes like:

- Bob: I think there are an even number of stars in the Galaxy.
- Me: I don’t believe you can support that claim.
- Bob: So you think there is an *odd* number of stars in the Galaxy? (note to reader: see the either-or fallacy here?)
- Me: No, I have no idea whether there is an even or odd number - it seems like a coin flip to me. I just don’t believe someone can make the positive claim that there is an even number. I also don’t believe someone can make the positive claim that there is an odd number.

Put another way, in a court of law there is a difference between a verdict of “not-guilty” and one of “innocent.” Juries are only allowed to give a verdict of “not-guilty” — that the prosecution (i.e. the one making the positive claim) has not given persuading evidence to convict.

Given that believing a claim, not-believing the claim, and believing the opposite of the claim are all distinct perspectives one can have for any claim and given further that these can get confusing when you talk about them, it is no wonder that people get hung up on definitions! Seen in terms of math, we can at least get a little bit of clarity.

Given the claim

> There are an even number of stars in the Galaxy.

we can have the following perspectives (and more!):

- $P(\text{even}) > 0.95$ (which implies $P(\text{odd}) < 0.05$) — I am confident the claim is true
- $P(\text{even}) \approx 0.7$ (which implies $P(\text{odd}) \approx 0.3$) — I believe the claim, but not strongly
- $P(\text{even}) \approx 0.5$ (which implies $P(\text{odd}) \approx 0.5$) — I have no idea whether the claim is true, but I disagree with anyone who is confident the claim is true or false
- $P(\text{even}) \approx 0.3$ (which implies $P(\text{odd}) \approx 0.7$) — I believe the claim is *false*, but not strongly
- $P(\text{even}) < 0.05$ (which implies $P(\text{odd}) > 0.95$) — I am confident the claim is *false*

Using these simple numbers it becomes easier to separate the different viewpoints, and to admit others.

3.1.1 Theism and atheism

In religious circles, the first claim we look at is

> G: God exists.

In shorthand we'll call this claim "G," so we can talk about the probability of God existing as $P(G)$ without having to write many words. Those who are religious generally would set a high value for $P(G)$. We would call them *theists*. In the section On Belief, Knowledge, and Proof we define knowledge as a high-probability belief. *Theism* typically refers to *belief* (of any magnitude) , *gnosticism* refers to knowledge. Thus, you can be a *gnostic theist*.

In parallel with the stars example above, we can have the following definitions:

- $P(G) > 0.95$ — I am confident God exists. I am a *gnostic theist*.
- $P(G) \approx 0.7$ — I believe that God exists claim, but not strongly. I am an *agnostic theist*.
- $P(G) \approx 0.5$ — I am unconvinced by anyone who is confident God exists or God doesn't exist. I am an *agnostic atheist* or for an *atheist*.
- $P(G) \approx 0.3$ — I believe that God *doesn't* exist, but not strongly. I am an *agnostic anti-theist*.
- $P(G) < 0.05$ — I am confident that God *doesn't* exist. I am an *gnostic anti-theist*.

3.1.2 A warning about labels

Too many discussions get hung up on labels. Some insist that *atheism* is the lack of a belief, that atheists are simply not convinced by the positive claim. Others use the same term to denote the *belief* that no God exists. We can see that these are different uses of the same term "*atheism*," and if we are to make any headway in discussions we should either agree to use the same labels for the same things or dispatch with the labels and talk directly about claims and their probability. Otherwise, everything is so ambiguous that the discussion falls apart.

For myself, when I use the term *atheist* I am referring to someone who doesn't believe in a God - that they are not convinced by the positive claim. I reserve the use of the term *anti-theist* for the one who promotes the positive claim that God does not exist. At no point, for any of these claims, is anyone 100% certain of their side. That is too high a bar to attain and is not useful. As a result, to counter a claim with "you're not 100% sure, are you?" is not an actual argument because no one — neither theist nor atheist — is claiming surety.

3.1.3 Claims and definitions

When we go back to the original theistic claim,

> G: God exists.

we immediately run into the problem of definitions. What do these words mean? Probability theory only works for *well-defined* statements. I can't talk about $P(\text{grue exists})$ without knowing what a *grue* is. Further, when speaking about potentially supernatural things, even the word "*exists*" has to be examined — what does it mean to "exist" if you might not be talking about energy, matter, and other things that we can directly measure? In what way do concepts like the number "2" or "liberty" exist? I'm not going to explore this much, but we need to make sure that our probability assignments don't accidentally shift under ill-defined terms in the discussion.

Even with the claim that God exists, people have widely varying beliefs concerning the specifics. The following provides some of the common terminology which will help the discussion.

- *theism* — the belief in one or more gods, which includes
  - *monotheism* — the belief in one God (e.g. Judaism, Christianity, Islam)

  - *polytheism* — the belief in many gods (e.g. early Judaism, modern Hinduism, Cthulhu cults)

- *deism* — the belief in a creator that does not currently act in the universe

- *pantheism* — the belief that God is equivalent to Nature (e.g. Spinozism, some Wicca)

- *classical theism* — a form of *monotheism* which further states that the one God is the all-powerful, all-knowing, creator of the universe beyond space and time.

There are several reasons why these terms are helpful to us. When setting up the evidence for God, both Bayes' rule and the Conjunction rule require spelling out all of the alternatives. To avoid either-or fallacies one must be vigilant to know these various alternatives. One also has to know what any particular argument or case is trying to present. An apologist might have in mind that they are presenting evidence for the Christian God, but the actual evidence they are using could apply to many others, or even to the nearly empty concept of deism. Keeping this in mind helps us avoid logical fallacies.

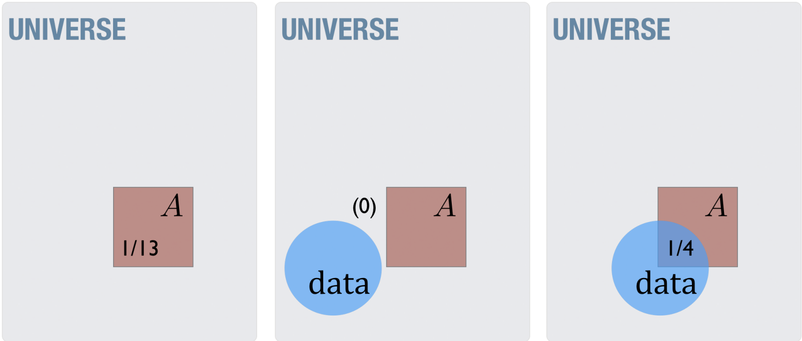
3.2 Evidence

3.2.1 What is evidence?

One definition of evidence for a claim is any data which makes the posterior probability of a claim higher than the prior. Something like,

$$P(A|\text{data}) > P(A)$$

When does this happen?



We can see it that, of all of the possible events in the universe, A occupies a small fraction. However once we observe data then we are restricting the number of events to those that are consistent with those observations so the fraction occupied by A can be larger or smaller, depending on the data and the claim.

Unfortunately, it can sometimes occur that people will use the shorthand "there is no evidence for X" when what they really mean is "there is not good evidence for X" or "not sufficient evidence." The different is in magnitude. "No evidence" would mean that there is no data which would make the probability of the claim go up — even by a minuscule amount. "Not sufficient evidence" would mean that the data may increase the probability of the claim, but not up to the level of belief, so

$$P(A|\text{data}) < 0.5$$

Although a bit shorter and punchier, the term "*belief without evidence*" is misleading, even if you know you are focussing on the probability side of the analysis. The more honest phrase would be "*belief without sufficient evidence*." When people say there is no evidence for something (like God, UFOs, astrology, psychic phenomena, etc...), they really mean that there is terrible evidence for something. Even in the case of something so poorly supported as astrology, there is *some* evidence for its claims - the probability is not *zero*. It may be very small, but one could imagine evidence (in principle) that would convince you, which means that the probability is indeed non-zero. The exaggerated, more simple, phrase of "*belief without evidence*" is counterproductive, especially when the more accurate phrase, "*belief without sufficient evidence*," is nearly as simple.

3.2.2 Evidence in the sciences

In the sciences, one is always comparing multiple claims or *models*, and constructing experiments to distinguish one model from another. Experiments may not be "in the lab." They may refer to computer simulations or methods of observation like measuring wavelengths of light coming from stars even if you can't bring stars into the lab. The goal of the analysis is to take, say, two models M_1 and M_2 and to find scenarios where we can discern which ones to trust. This is done by *falsification* or *disconfirmation* — showing that one of the models is *unlikely* — not by confirming one of the models. Why is that? Confirmation of a model would be to look for data which is likely if the model is true, or $P(\text{data}|M_1)$ is high. However, what we really want to have is data which makes the *model* likely, or $P(M_1|\text{data})$ is high. In order to do that we apply Bayes' Rule,

$$P(M_1|\text{data}) = \frac{P(\text{data}|M_1)P(M_1)}{P(\text{data}|M_1)P(M_1) + P(\text{data}|M_2)P(M_2)}$$

if we simplify with equal *priors*, $P(M_1) = P(M_2) = 1/2$,

$$P(M_1|\text{data}) = \frac{P(\text{data}|M_1)}{P(\text{data}|M_1) + P(\text{data}|M_2)}$$

we will notice that the only way to have the *posterior*, i.e. $P(M_1|\text{data})$, to be high is to both have the model confirmed, i.e. $P(\text{data}|M_1)$ is high, but also the alternate model *disconfirmed*, i.e. $P(\text{data}|M_2)$ is low. Otherwise, if we don't disconfirm alternatives we can not demonstrate any confidence in the model.

Because of this, we can never be confident in a claim or model that is not able to be — in theory — falsified. Invisible dragons that elude detection [Sagan, 1996] cannot be used to explain anything because there is not any way that one could — even in principle — show it to be false. Another way to put it, these unfalsifiable claims are consistent with *every* possible experiment.

3.2.3 Burden of proof

Although I refrain from using the word "proof" in a discussion of evidence, choosing only to use it in the cases of *absolute* certainty (e.g. geometrical proofs, etc...), the term "burden of proof" refers to the status of someone making an argument to support a claim. The person making the *positive claim* is the one that is required to support it. In the Section Belief, Non-Belief, and Disbelief the person making the positive claim "there are an even number of stars" must bring the evidence to support it — they have what is called the "burden of proof." The person not convinced by the claim need not bring evidence — they are just not convinced the positive claim — and thus do not have the "burden of proof."

Establishing who has the burden of proof is critical in any argument, because it establishes where the evidence must come from. A common mistake made by someone making the positive claim is to try to put the burden of proof on the other person, asking for a justification for the negative claim where one is not needed. What makes the situation more complicated is that most claims have multiple components, so an individual person can have positive, negative, and indifferent perspectives on the claims. One can be simultaneously indifferent to the generic claim of a deistic creator, not be convinced of theistic claims, and actively disbelieve in classical theistic claims. It is therefore advantageous to break up complex claims into a collection of simple claims and tackle one simple claim at a time.

Technically, "burden of proof" is only required in a court of law, it is still important to recognize that it applies to any form of logical argument. Mostly it is important to be clear about what is specifically being claimed and what evidence there can be for that specific claim. Much energy is often wasted arguing for an ill-defined claim.

3.2.4 Utility - probability and action

Probability relates to belief, a measure of state of knowledge about a claim or set of claims. We can use the mathematics of probability to determine the most likely claim, and use it to inform our actions, but it isn't enough to truly determine the best course of action. For that, we need to extend the mathematics to include the notion of *utility*, an extension commonly referred to as *decision theory*. Because faith seems to involve action or potential actions, it will need to be formulated in this way.

Decision theory uses the idea of expected value to aid in making decisions, defined as [Wikipedia, 2015a]

> The idea of expected value is that, when faced with a number of actions, each of which could give rise to more than one possible outcome with different probabilities, the rational procedure is to identify all possible outcomes, determine their values (positive or negative) and the probabilities that will result from each course of action, and multiply the two to give an expected value. The action to be chosen should be the one that gives rise to the highest total expected value.

The utility values, costs and benefits, could be written in monetary terms, but need not. One needs only to have a scale to represent how good or bad an outcome is. The total expected value, also called the average utility, is just the sum of the individual costs and benefits associated with possible outcomes, weighted by their probability - more likely outcomes are weighted more than less likely ones. An example will help.

The example here is called the "farmer's dilemma" [Jordaan, 2005], concerns a farmer who can plant one of three crops (labeled A, B, and and C) with the possibility of three different environments out of the farmer's control, *perfect weather*,*fair weather*, and *bad weather*. Each of the three crops fare differently in different weather, and thus provide different costs and benefits to the farmer depending on the environment. Crop A, for example, does very well in good weather but very badly in bad, whereas crop C doesn't do as well but is more consistent, with crop B in between. This can be summarized by the following table of utilities showing benefits (positive) and costs (negative) for each possible combination:

	perfect weather	fair weather	bad weather
plant crop A	11	1	-3
plant crop B	7	5	0
plant crop C	2	2	2

Any decision the farmer makes must include the probabilities (his state of knowledge) of the weather environments. At the extremes, it is easy to see this. If perfect weather is nearly guaranteed, for example, then planting crop A is of course the best option, whereas if bad weather is guaranteed, then planting crop C is the best. How does one handle the decision away from the extremes? This is done by asking, what is the average utility (benefit or cost) for each action, and then choosing the action which maximizes this. Imagine that the farmer consults a meteorologist, and they determine the following probabilities for the weather environment

$$\begin{aligned} P(\text{perfect weather}) &= 0.1 \\ P(\text{fair weather}) &= 0.5 \\ P(\text{bad weather}) &= 0.4 \end{aligned}$$

The average utility[1] for each action is simply the sum of the probabilities of each environment times the utility for that environment given the action, such as

[1] Average, or expected, value of a variable U is denoted with angle brackets, $\langle U \rangle$.

$$\begin{aligned}\langle U_A \rangle =& P(\text{perfect weather}) \times \text{U}(\text{perfect weather}|\text{plant crop } A) + \\ & P(\text{fair weather}) \times \text{U}(\text{fair weather}|\text{plant crop } A) + \\ & P(\text{bad weather}) \times \text{U}(\text{bad weather}|\text{plant crop } A) \\ =& 0.1 \times 11 + 0.5 \times 1 + 0.4 \times (-3) = 0.4\end{aligned}$$

Performing the same calculation for all of the actions yields,

$$\begin{aligned}\langle U_A \rangle &= 0.1 \times 11 + 0.5 \times 1 + 0.4 \times (-3) = 0.4 \\ \langle U_B \rangle &= 0.1 \times 7 + 0.5 \times 5 + 0.4 \times 0 = 3.2 \\ \langle U_C \rangle &= 0.1 \times 2 + 0.5 \times 2 + 0.4 \times 2 = 2.0\end{aligned}$$

and the best action would be planting crop B, because it has the highest expected utility.

The primary point about this process that is relevant to our discussion of faith is that the process involves two separate entities - the probability of various states and the value those states are to us given our actions. This maps directly to the concepts of *belief* and *trust*, respectively. One can focus on each one individually, but it is the combination that is important.

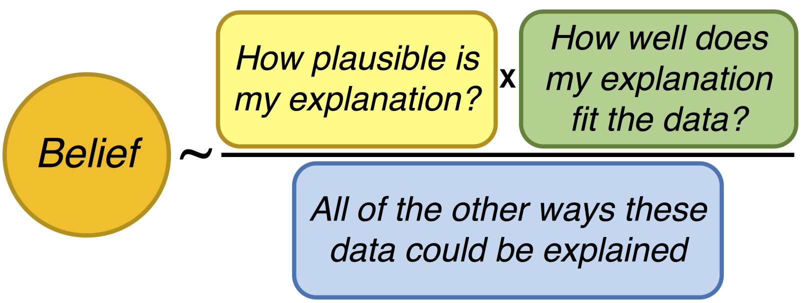
Although there are cases where the average utility is a bad guide, it is a very useful framework to structure a problem. It should be noted that *utility* does not need to be restricted to money, but can include things like comfort and discomfort. For example, someone could be *risk averse*, which means that they put a utility on some psychological comfort at the expense of some monetary loss. Thus the utility value would combine both.

Chapter 4

The case for God

4.1 Some first thoughts

I want to remind you of the structure of the probability of belief. Ones belief in an explanation should increase with how well that explanation fits the data and how plausible it is before the data, but it should also *decrease* with all of the other ways the data can be explained by other descriptions.



Arguments fail primarily for a few reasons:

1. lack of imagination in constructing alternative explanations

2. ill-formed hypotheses where initial plausibilities can't be established

3. hypotheses that are too flexible and can fit any kind of data

A specific example that arises due to point (1) is the false-binary construction of the argument. You can see this when the argument is built to compare $P(A)$ vs $P(\textbf{not } A)$ where A is the model the arguer wants to support. Framed in this way, many other alternatives that can push the probabilities up and down are hidden in "**not** A."

Watch for these points of failure as I explore a number of apologetic arguments that have been put forward to support religious concepts.

4.2 Swinburne

In his book "The Existence of God" [Swinburne, 2004], Richard Swinburne puts forward an argument for the existence of God using probability theory. We are in a good position now to examine this argument in detail and to see where it falls short of making the case. I summarize his definitions and arguments first, hopefully not leaving anything out that he would consider crucial. Whenever I analyze a particular case I prefer to "*steelman*"[1] the arguments as a matter of good practice.

The argument he describes tries to establish

$$P(G|\text{data})$$

where the "data" refers to the observations of the universe as we know from science[2]. I anticipate his narrow definition of "theism" to refer to

[1] "The steel man argument (or steelmanning) is the opposite of the straw man argument. The idea is to find the best form of the opponent's argument to test opposing opinions." (Wikipedia, Steelman. https://en.wikipedia.org/wiki/Straw_man#Steelmanning)

[2] To simplify the discussion we change the notation of Swinburne [Swinburne, 2004]. He writes $P(h|e\&k)$ for our $P(G|\text{data})$

the narrow use of "classical theism," so the proposition G will refer to this definition.

4.2.1 Summary of the argument

Swinburne starts with a definition of theism,

> G: "there exists necessarily a person [mind] without a body (i.e. a spirit) who necessarily is eternal, perfectly free, omnipotent, omniscient, perfectly good, and the creator of all things" [clarification added]

with the data referring to the various data of natural theology [Swinburne, 2003]:

1. the existence of a complex physical universe
2. the (almost invariable) conformity of material bodies to natural laws
3. those laws together with the initial state of the universe being such as to lead to the evolution of human organisms
4. these humans having a mental life (and so souls)
5. these humans having great opportunities for helping or hurting each other
6. these humans having experiences in which it seems to them that they are aware of the presence of God.

Swinburne then spends time describing the simplicity of the God hypothesis.

4.2.1.1 Simplicity

Swinburne states that this explanation is *simplest* for a number of reasons,

> I stress the enormous importance of the criterion of simplicity, an importance that is not always appreciated.
>
> A scientific explanation, will have to postulate as a starting point of explanation a substance or substances that caused or still cause the universe and its characteristics. To postulate many or extended such substances (an always existing universe; or an extended volume of matter energy from which, uncaused by God, all began) is to postulate more entities than theism.
>
> To start with, theism postulates a God who is just one person, not many. To postulate one substance is to make a very simple postulation. He is infinitely powerful, omnipotent. This is a simpler hypothesis than the hypothesis that there is a God who has such-and-such limited power (for example, the power to rearrange matter, but not the power to create it)....But, if God's essence is an eternal essence, then any complete explanation of phenomena in terms of God's agency is also an ultimate explanation. For God's existence at a time is entailed by his existing at all, and does not require to be explained in terms of his previous existence and previous choices. So the simplest kind of God is a factually necessary one, in the sense defined earlier.

So God being one thing (not many), and infinite in various attributes, and actually exists constitutes the simplest sort of explanation to Swinburne. He then spends some time describing the properties of omnipotence, omniscience, and omni-benevolence which we won't go into detail here - please read his book of the details.

4.2.1.2 Problem of evil

Swinburne recognizes that the existence of evil in the universe makes it less likely that a God would have made it.

> Although much evil is necessary for the attainment of many of these purposes, there is a limit to the amount of evil that God ought to allow humans (and animals) to suffer for the sake of the good that it makes possible. Even so, if he allows us to suffer as much as we do, he would need to provide a compensatory period of afterlife for any who suffer too much and perhaps to become incarnate to share our suffering. To add to the hypothesis of theism that he does these things complicates it, but not very much. In the absence of this extra evil, the probability that God would create a universe such as I have described would be, I claimed, an equal best kind of act, and so there would be a probability of 1/2 that he would do so. But, if we need to complicate theism somewhat in order to account for the amount of evil, we must put the probability that God would bring about our kind of universe a bit lower.

He then settles on $P(\text{data}|G) = 1/3$.

4.2.1.3 Balance of probability

At the end of his book, Swinburne outlines the mathematical form of his argument.

$$P(G|\text{data}) = \frac{P(\text{data}|G) \cdot P(G)}{P(\text{data}|G) \cdot P(G) + P(\text{data}| \sim G) \cdot P(\sim G)}$$

where G refers to "the classical theism God exists" and $\sim G$ refers to "the classical theism God *does not* exist." He then breaks down the terms with $\sim G$ into several different hypotheses, which we can label H_1, H_2, H_3, etc... In the text, he limits this to three hypotheses,

$$\begin{aligned} P(\text{data}|\sim G)P(\sim G) = & P(\text{data}|\sim H_1)P(H_1)+ \\ & P(\text{data}|\sim H_2)P(H_2)+ \\ & P(\text{data}|\sim H_3)P(H_3) \end{aligned}$$

where

- H_1: "there are many gods or limited gods"
- H_2: "there is no God or gods but an initial (or everlasting) physical state of the universe, different from the present state but of such a kind as to bring about the present state"
- H_3: "there is no explanation at all (the universe just is and always has been as it is)"

The probabilities for these hypotheses are set quite low, given the following,

- H_1: "hypothesis of theism is a very simple hypothesis indeed, simpler than hypotheses of many or limited gods"
- H_2: "But there is no particular reason why an unextended physical point or any of the other possible starting points of the universe, or an everlasting extended universe, should as such have the power and liability to bring about all the features that I have described. [...] It will only become at all probable that there will be a universe of our kind if we build into the hypotheses an enormous amount of complexity."
- H_3: "And that our universe should have all the characteristics described (above all, the overwhelming fact that each particle of matter throughout vast volumes

> of space should behave in exactly the same way as every other particle codified in 'laws of nature') without there being some explanation of this is beyond belief. While $P(\text{data}|H_3) = 1$ (the universe being this way unexplained entails it being this way),$P(H_3)$ is infinitesimally low."

He concludes with

> And so, $P(G|\text{data})$, the posterior probability of theism on the evidence considered so far, will not be less than 1/2.

with the following caveat,

> I stress again that it is impossible to give anything like exact numerical values to the probabilities involved in these calculations. I have attempted to bring out the force of my arguments by giving some arbitrary values that do, I hope, capture within the roughest of ranges the kinds of probabilities involved. But in reality all that my conclusion so far amounts to is that it is something like as probable as not that theism is true, on the evidence so far considered.

Finally, he adds that the role of religious experience has the effect of taking the modest probability of $P(G|\text{data}) \sim 1/2$ and increasing it substantially,

> unless the probability of theism on other evidence is very low, the testimony of many witnesses to experiences apparently of God suffices to make many of those experiences probably veridical. That is, the evidence of religious experience is in that case sufficient to make theism overall probable.

4.2.2 Problems with the argument

There are several problems with the argument as described, broken down into a *mistaken understanding simplicity*, a number of *ill-defined concepts*, a *lack of imagination*, and an *arbitrary assignment of values.*

4.2.2.1 Simplicity

We saw in [an earlier section][#on-simplicity] that *simplicity* is not about the *number* of parts in an explanation but the *flexibility* of those parts. Saying that one God is simpler than many gods is not correct, unless that God is equivalent to one of the many gods. Since Swinburne is proposing a God with infinite faculties compared to many gods with limited faculties the comparison is not equivalent. A God with infinite faculties also has *infinite flexibility* whereas many gods may not, and thus may be simpler than the one. The proposal of an infinite God to "explain" the universe has the same content as "magic did it" because both are

1. single things

2. infinite (or undefined) abilities

Since Swinburne's primary arguments against alternative hypotheses hinges on the simplicity of his proposal, this one critique is enough to call it into question. A good discussion of these matters of simplicity and models of the world as they apply to Newton's Laws and relativity can be found in [Jefferys and Berger, 1991].

4.2.2.2 Ill-defined concepts

We start with one of the alternative hypotheses,

- H_2: "there is no God or gods but an initial (or everlasting) physical state of the universe, different from the present state but of such a kind as to bring about the present state"

I am not sure if H_2 is well-defined, but being charitable we can read it as simply vague. In science we would break H_2 into a number of *specific* models of the universe which make *specific* prediction for quantities like the initial entropy of the universe, the mass distribution of the early universe, the speed of the expansion, etc... The models that are most probable use a modest number of parts to explain *in detail* the present and past state of observable universe. Does the God-hypothesis, G, make *any* of these predictions? No — it doesn't. Equivalent to "magic" God-hypothesis is not well-defined enough to make such predictions.

4.2.2.3 Imagination

The biggest problem when dealing with the probabilities of claims arises with a *lack of imagination.* We are limited in the number of alternatives that we can consider, so if we cannot *logically* determine that there is a limited set to choose from (e.g. even vs odd whole numbers) then we must be honest about the fact that we probably don't have all of the alternatives listed. This comes up most clearly in Swinburne with his three alternative hypotheses:

- H_1: "there are many gods or limited gods"
- H_2: "there is no God or gods but an initial (or everlasting) physical state of the universe, different from the present state but of such a kind as to bring about the present state"
- H_3: "there is no explanation at all (the universe just is and always has been as it is)"

how about,

- H_4: *Stephen Law's Evil-God* [Law, 2010]
- H_5: *Greek Pantheon*, or any number of other mythos, exists. This is H_1 broken up into specifics
- H_6: *Multiverse* models (there could be many)
- $\vdots$
- etc...

In short, one can come up with any amount of alternative models which explain the base-level data that Swinburne wants to handle. These data don't include specific observations like the expansion rate of the universe, or the fraction of hydrogen and helium in the universe, or the balance between matter and anti-matter, or really anything specific so the number of compatible models is enormous. The only limit is our imagination.

Once we decide that specific observations are important then it is up to the investigator to propose methods for testing their model (or models) directly agains those observations. This is true for the theist and atheist alike.

4.3 The Kalam cosmological argument

4.3.1 Summary of the argument

One of the most popular arguments for God is what is called the Kalam Cosmological Argument popularized by William Lane Craig [Swinburne and Craig, 1979]. The argument, presented in numerous debates and articles, is the following (more detail here),

1. Whatever begins to exist has a cause

2. The universe began to exist therefore
3. The universe has a cause.

He then adds that, for the *cause* to be able to cause the universe, it must be

> timeless, spaceless, immaterial, and personal

4.3.2 Problems with the argument

The clearest exposition of the problems with this argument I have found is from Sean Carroll, in his debate with William Lane Craig. The problems with the argument are both from the argument itself and from the scientific support for its premises.

The first premise assumes that our everyday experience of causation applies in all cases, even well outside our everyday experience. We know from physics that this isn't true — our intuitions fail at the very small, the very large, and the very fast at the least. On the small scale, there are (to our best knowledge) *uncaused* events such as radioactive decay and the production of virtual particles in the vacuum. To imagine that our notions of causation should apply to entire universes is naive.

The second premise is often supported by referring to the Big Bang theory, and such theorems as the Borde-Guth-Vilenkin theorem which William Lane Craig interprets as "the universe must have had a beginning" although Carroll corrects him by saying that the theorem implies that the universe *might* have a beginning but that it only requires that the *expansion* had a beginning.

Finally, even if our universe had a beginning, its cause could be some other natural process. Perhaps an outcome of multiverse models, at which one would have to back the argument up to support that the multiverse has a beginning. However, once this is admitted, we have to come to terms with the fact that we have no evidence about the beginning — or not — of the multiverse and the entire argument collapses.

4.4 The fine tuning argument

4.4.1 Summary of the argument

There are a several arguments which go by the name of Fine Tuning but they all stem from the same idea. The basic idea comes from scientific observations of the sensitivity of certain physical systems to change. On the largest scale, the observation is that the conditions in the universe for the development of atoms, molecules, and life itself depend critically on certain *arbitrary* physical constants [Craig, 2018] such as the speed of light, strength of gravity, and the expansion rate of the universe [Rees, 2008]. By critically, we have examples like the gravitational constant needs to be the value it currently is by 1 part in 10^{34} (1/10000000000000000000000000000000000 or 1 followed by 34 zeros) or stars wouldn't form, and the expansion rate of the universe within 1 part in 10^{55} or the universe would not have lasted long enough for life to form or expanded too quickly to allow for planets to form.

Similar fine-tuning observations have been suggested more locally [Wallace, 2015], like the mass of our Moon and the location of the Earth relative to our Sun. The argument is the same — if these properties were at all different then life, and thus humans, would not be here.

These observations are coupled to the argument in the following way. If we observe some property, then its value must be the result of

1. random chance
2. necessity (i.e. a product or the laws of physics)

3. design

If it is extremely unlikely (i.e. not random chance) and there is no particular reason from physics for it (i.e. not of necessity) then it must be design.

4.4.2 Problems with the argument

The clearest exposition of the fine tuning argument I have found is from Sean Carroll, especially in his debate with William Lane Craig.

The problem is that, despite the confident claims of many theists, the physicists are not at all clear how much fine tuning there is or if there is any at all. The way fine tuning is demonstrated starts with noticing that there are some *seemingly arbitrary* constants in our physical laws (e.g. speed of light, expansion rate of the universe, etc...). These constants if *varied independently* lead to conditions where life *as we know it* couldn't form. The emphasis here is to point out the problems with making confident claims about these observations.

1. The constants may be determined by other laws we don't know right now. The original fine tuning evidence from the expansion rate of the universe used *classical gravity equations* because it was an easier calculation to do. However, when the more difficult but correct calculations were derived from Einstein's equations it was found that the expansion rate *had to be the particular value* and is thus not fine-tuned. As our understanding increases, it may happen with some of the other claimed fine-tunings.

2. The constants may not be independent, so changing one may necessitate changing another by a specific amount negating the effect

3. We don't understand the conditions *necessary* for life

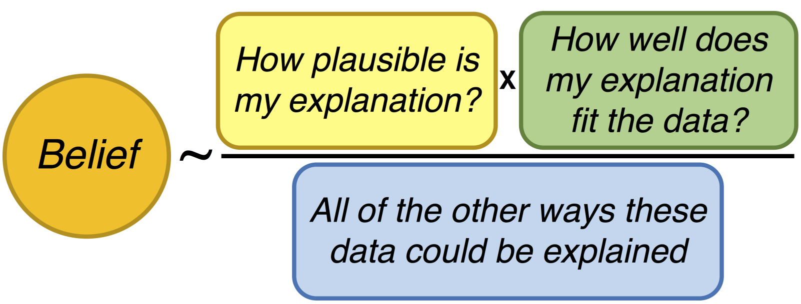
With these objections, it seems premature to make any strong statements about fine tuning even if you are convinced it is there.

Chapter 5

The case for Jesus

5.1 Some first thoughts

I repeat the same thoughts with which I started last chapter, at the danger of being too repetitive, to remind you of the structure of the probability of belief. It's good to keep looking at the big-picture of the structure of probability. Ones belief in an explanation should increase with how well that explanation fits the data and how plausible it is before the data, but it should also *decrease* with all of the other ways the data can be explained by other descriptions.



Recall that arguments fail primarily for a few reasons:

1. lack of imagination in constructing alternative explanations
2. ill-formed hypotheses where initial plausibilities can't be established
3. hypotheses that are too flexible and can fit any kind of data

And recall that the false-binary construction of an argument can hide many other alternatives in "**not** A" part of the equations. Watch for these failures of proper reasoning!

Following up with his argument for the existence of God, Swinburne presents a probabilistic argument for the Resurrection.

5.1.1 Summary of the Argument

> Now at the end of the day this book is interested in $P(h|e\&k)$ —the probability that Jesus was God Incarnate who rose from the dead (h), on the evidence both of natural theology (k) and of the detailed history of Jesus and of other human prophets (e)

For clarity, I prefer to use the letter R for the primary claim that Jesus was risen from the dead, rather than h. Since Swinburne is assuming the "evidence of natural theology" through the entire argument, we don't need the extra symbol k — we can subsume it into the "evidence." Finally, in Swinburne's argument, he brings in two extra symbols: c for the claim that God became incarnate and f that "we have *claims* that there is evidence of the strength given by e." Although I agree that we should always focus on what evidence we have (i.e. we have claims of events), Swinburne makes no use of these two extra symbols because near the end of the argument he makes the following two statements (in his notation):

1. $P(h|e\&k)$ will not be very different from $P(c|e\&k)$

2. It cannot make any difference to the probability that f (with k) gives to c, if we add to f [...] the details of the evidence [...] So, $P(c|e\&k) = P(c|f\&k)$ [...]

which effectively makes his two extra symbols redundant: f is equivalent to the evidence e, and c is equivalent to the primary statement under analysis h. After recognizing this, the basic structure of the argument follows the Bayes' Rule,

$$P(R|\text{evidence}) = \frac{P(\text{evidence}|R)P(R)}{P(\text{evidence})}$$

where R is the statement

> R: Jesus was God Incarnate who rose from the dead

where the evidence is

> The various data of natural theology:
>
> 1. the existence of a complex physical universe
> 2. the (almost invariable) conformity of material bodies to natural laws
> 3. those laws together with the initial state of the universe being such as to lead to the evolution of human organisms
> 4. these humans having a mental life (and so souls)
> 5. these humans having great opportunities for helping or hurting each other
> 6. these humans having experiences in which it seems to them that they are aware of the presence of God.

> The detailed historical evidence, consisting of a conjunction of three pieces of evidence
>
> 1. the evidence of the life of Jesus
> 2. the detailed historical evidence relating to the Resurrection
> 3. the evidence that neither the prior nor the posterior requirements for being God Incarnate were satisfied in any prophet in human history in any way comparably with the way in which they were satisfied in Jesus.

The math follows straightforwardly by addressing each term of the equation above,

The probability that if God became incarnate/was resurrected we would have the evidence we have, Swinburne suggests the "fairly low" number of

$$P(\text{evidence}|R) = \frac{1}{10}$$

The prior probability of the resurrection is broken up into two parts,

$$P(R) = P(R|G)P(G)$$

where the probability of God, $P(G) = 1/2$, is the "modest" value taken from the previous work. The first part, $P(R|G)$, is an attempt to divine the motivations of God,

> Then let us represent by c [in our simplified notation, R] the claim that God became incarnate among humans at some time with a divided incarnation, a more precise form of the way described by the Council of Chalcedon. I suggested that it was 'as probable as not' that he would do this and so in numerical terms the probability of his doing it is 1/2.

Thus, Swinburne arrives at the prior probability of the resurrection,

$$P(R) = P(R|G)P(G) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

and by the negation rule,

$$P(\textbf{not}\, R) = 1 - P(R) = \frac{3}{4}$$

The evidence terms are handled as

> How probable, then, is it that if God does become incarnate (into a human race sinning and suffering), we would have evidence of the strength described, connected with one and only one prophet? Let me not exaggerate my case and suggest (despite my strong feeling that this value should be higher) that we give it a fairly low value and put it provisionally at 1/10

and

> But it would be immensely unlikely that there would be evidence of these degrees connected with the same prophet unless God so planned it. It would have been deceptive of God to bring about this combination of evidence (or permit some other agent to do so), unless he had become incarnate in this prophet; and so God would not have brought this about. So let's say 1/1000.

which in our notation is

- $P(\text{evidence}|R) = 1/10$
- $P(\text{evidence}|\textbf{not}\, R) = 1/1000$

We now have all the pieces to arrive at Swinburne's final answer,

$$\begin{aligned}
P(R|\text{evidence}) &= \frac{P(\text{evidence}|R)P(R)}{P(\text{evidence})} \\
&= \frac{P(\text{evidence}|R)P(R)}{P(\text{evidence}|R)P(R) + P(\text{evidence}|\textbf{not } R)P(\textbf{not } R)} \\
&= \frac{\frac{1}{10}\cdot\frac{1}{4}}{\frac{1}{10}\cdot\frac{1}{4}+\frac{1}{1000}\cdot\frac{3}{4}} \\
&= \frac{100}{103} \sim 97\%
\end{aligned}$$

5.1.2 Problems with the argument

The overall structure of the argument is, of course, sound. Bayes' Rule will work but is only as good as the probabilities we assign. The main problems with Swinburne's argument are that he *attempts to assign probabilities to the actions of God* and he *lacks imagination for alternatives*, thus skewing any probability assignments.

5.1.2.1 The mind of God

For example,

> Then let us represent by c [in our simplified notation, R] the claim that God became incarnate among humans at some time with a divided incarnation, a more precise form of the way described by the Council of Chalcedon. I suggested that it was 'as probable as not' that he would do this and so in numerical terms the probability of his doing it is 1/2.

Swinburne puts great store in the idea that God would do things the Christian way. He has entire chapters littered with phrases starting with

"God will want to…," "…reason God would have for…," "…reasons which God might have for…,"etc… We are told that the mind of God is inscrutable, so I find it unconvincing that Swinburne can read it so clearly and narrowly. How do we know that an incarnate God is the best one? Perhaps it would have been better if he had come down looking like an alien? Why the particular miracle of raising from the dead? Why not glowing skin, flying, or invincibility? Although my suggestions sound suspiciously like Superman or someone from the X-Men, other than 20-20 hindsight[1] there really is no way to tell what the reasoning should be, thus making all probabilities based on that to be arbitrary. As a game, try to come up with a rational argument for why it would be in God's plan to send Superman. God would want an alien to stress the Fall of mankind, but would want the alien to appear human in all obvious ways. God would want this representative to be invincible to give good evidence for the omnipotence of God himself, but be limited in space and time to give humans the free will to work on problems on their own. One can keep going with this line of argument because it is completely unconstrained by any possibility of confirmation or disconfirmation.

5.1.2.2 The nature of the evidence

Another problem with Swinburne's argument is the selection of the evidence. He claims that, if God doesn't exist, then the evidence we see would be highly unlikely. Focussing on the Resurrection specifically,

> So, while the fact that the universal agreement on the fact of Jesus' Resurrection, and that there were many witnesses thereof, is good evidence of its truth, the existence of somewhat different versions of to whom he appeared, and when, do little to lessen the force of that evidence unless they are too much at odds with each other. The band of followers of Jesus, routed by his Crucifixion, had become enthusiastic

[1]which in philosophical circles is called post-hoc or "after-the-fact" reasoning.

> missionaries for a Gospel centred on the Resurrection of Jesus. This requires explanation.

He admits that there may be some detailed alternative story,

> But such a story, even if it is made so elaborate as to make the evidence probable, becomes thereby highly complicated and so highly improbable a priori. Only if any other hypothesis were even more improbable should we adopt this one.
>
> The traditional account does not have these disadvantages of complexity. If Jesus did indeed rise, then this one action would lead us to expect to find the data which I have been discussing with no very great improbability. If the traditional account is improbable overall, that is because of its prior improbability, involving a massive violation of laws of nature, and just how improbable that makes it depends on the worth of natural theology;

But the "worth of natural theology" in his argument comes in the form of reading God's mind, and is thus arbitrary. Further, he never considers evidence which we *don't* have but should be observable and recorded if the actual events transpired as claimed. For example, the Romans should have had trials for grave robbing — whether Jesus was resurrected or no — but these are not mentioned by other historians of the time at all, even while other Roman trials of the early Christians are discussed in great detail. Any mention of the temple veil ripping, the dead saints walking around Jerusalem, and the purported earthquake are also never mentioned. Any probabilistic treatment needs to handle these cases. The part of Bayes' Rule which describes how well the claim explains the evidence can be broken up into separate parts of the evidence,

$$P(\text{evidence}|\text{claim}) = P(e_1\,\textbf{and}\,e_2|\text{claim}) = P(e_1|e_2,\text{claim})P(e_2|\text{claim})$$

The evidence, for example, could be e_1: reported sightings of Jesus, e_2: reported Roman trials. If we consider, for simplicity, two claims

C_1: Jesus actually rose from the dead and C_2: Jesus was seen by his disciples as visions, then it is clear that we'd have the following,

- C_1: e_1 is likely and e_2 is also likely
- C_2: e_1 is likely but $e_2$is *unlikely*

If we observe e_1 but not e_2 then that makes $P(\textbf{not}\, e_2|C_1)$ quite small, and thus the final answer is diminished and our confidence in the resurrection is reduced.

It is therefore important to not only come up with many alternatives, but to recognize that both positive and *negative* evidence must be used.

5.2 McGrew on the Resurrection

In "The Argument from Miracles" [McGrew and McGrew, 2009], Tim and Lydia McGrew offer a probabilistic argument for the *cumulative* case for the resurrection.

5.2.1 Summary of the argument

The primary assumptions of the approach are stated up-front,

> Our argument will proceed on the assumption that we have a substantially accurate text of the four gospels, Acts, and several of the undisputed Pauline epistles (most significantly Galatians and I Corinthians); that the gospels were written, if not by the authors whose names they now bear, at least by disciples of Jesus or people who knew those disciples – people who knew at first hand the details of his life and

> teaching or people who spoke with those eyewitnesses – and that the narratives, at least where not explicitly asserting the occurrence of a miracle, deserve as much credence as similarly attested documents would be accorded if they reported strictly secular matters. Where the texts do assert something miraculous – for example, Jesus' post-resurrection appearances – we take it, given the basic assumption of authenticity, that the narrative represents what someone relatively close to the situation claimed. For the purposes of our argument, we make no assumption of inspiration, much less inerrancy, for these documents, and we accept that there are small textual variations and minor signs of editing, though we do not in any place rely on any passage where the textual evidence leaves serious doubt about the original meaning.

The core of the argument comes down to,

> As a first step, let us consider a single disciple. The best of the available naturalistic explanations, the hallucination theory, requires (if it is to match R in likelihood) an extraordinary level of detailed delusion, seamlessly integrated (so far as he can tell) with his experience of those around him. Such delusions do occur in waking life in those who suffer from severe mental illness, but such illness is mercifully rare and is accompanied by other noticeable conditions that were absent in the case of the disciples. The other naturalistic hypotheses have higher prior probabilities, perhaps as high as 0.001, but they do not come close to matching the explanatory power of R; their contribution to the likelihood $P(D_i| \sim R)$ is negligible even by comparison to the hallucination theory.
>
> [...]
>
> But having assigned a single factor, we must ask what happens when we take into account the fact that there were

> thirteen such disciples. We can get a first approximation to the result by assuming independence. [. . .] So under the assumption of independence, the Bayes factors for each of the thirteen D_i must be multiplied, which yields a staggering combined factor
>
> $$\begin{aligned} P(D|R)/P(D|\sim R) =& P(D_1|R)/P(D_1|\sim R)\times \\ & P(D_2|R)/P(D_2|\sim R)\times\cdots\times \\ & P(D_{13}|R)/P(D_{13}|\sim R) \\ =& \underbrace{\frac{1}{0.001}\times\frac{1}{0.001}\times\cdots\times\frac{1}{0.001}}_{13 \text{ times}} \\ =& 10^{39} \end{aligned}$$

by including two other pieces of evidence (e.g. the testimony of the women which they call W and the conversion of Paul which they call P), they arrive at

> But our estimated Bayes factors for these pieces of evidence were, respectively, 10^2, 10^{39}, and 10^3. Sheer multiplication through gives a Bayes factor of 10^{44}, a weight of evidence that would be sufficient to overcome a prior probability (or rather improbability) of 10^{-40} for R and leave us with a posterior probability in excess of 0.9999.

McGrew anticipates the challenge of the independence assumption and provides a discussion and justification.

> the invocation of independence assumptions at several points is contestable; in fact, we believe that in the case of the calculation for D the independence assumption almost certainly breaks down. Surprisingly, however, this fact does

> not necessarily lessen the strength of the argument. Everything depends on the balance of considerations regarding the direction and extent of the breakdown of independence under R and under $\sim R$.

They describe the problem with the independence assumption

> If three men accused of committing a crime all give, in essentially the same words, the same innocent explanation of their actions, the plausibility of the claim that they are conspiring to give themselves an alibi undermines the force of their combined testimony. Even when there is no definite intent to deceive, witnesses may influence one another's testimony causally in a way that would obtain even if the event had not happened, or had not happened in the way that they are saying it did.

First, with the three pieces of evidence under consideration

> First, let us consider the independence of the strands of argument which we have labeled W, D, and P. [...] The testimony of the women to the empty tomb and to the appearances of Christ are independent, obviously, of Paul's conversion [...] The women's testimony is essentially independent of that of the thirteen male witnesses. [etc...]
>
> But the assumption of independence *among* the thirteen male witnesses raises greater difficulties. [emphasis original]

Then the argument moves to the idea of collusion in early Christianity,

> When people are claiming to be eyewitnesses to some event (in this case, the appearance of the risen Jesus), and when

> they are in danger of an unpleasant fate for making the claim in question, their believing and having better evidence for this claim is a better explanation of positive dependence among their accounts – their being able to encourage one another to continue making their testimony – than their not believing the claim or having worse evidence for it. [...] If any one of the witnesses in question had not actually had clear and realistic sensory experiences just as if Jesus were physically present, talking with them, eating before them, offering to let them inspect his hands and side and the like, it is not credible that he would listen to the urging of his fellows to remain steadfast in testifying to such experiences.

So, in sum, the McGrews argue that either the testimony of the disciples is independent or when it is not independent, the independence is broken in such a way that *favors* the resurrection claim. This is due to the idea that the existence of thirteen claims to the resurrection — under penalty of death — is much less likely than would be expected if the event didn't happen than if it did.

5.2.2 Problems with the argument

The problems with this argument stem from a *lack of imagination* of alternatives (as we've seen before) and a *lack of skepticism* about the independence of the sources (which somewhat follows from the lack of imagination).

5.2.2.1 Lack of imagination

My first objection comes from the assumptions about the text — this already gives too much ground. I don't have a problem with "substantially accurate text" because I don't think the primary issues come from translation or copying problems. Quotes like "people who knew at first hand the details of his life and teaching or people who spoke with those eyewitnesses" and "basic assumption of authenticity, that the narrative represents what someone relatively close to the situation claimed" are not well-supported historically. I think this can be handled in the present analysis by rolling these assumptions into the primary claim, R, and adding the contrary claims into the alternative, $\sim R$. By restricting the analysis to only those that, effectively, assume eye-witness testimony already favors the primary claim unduly.

For an example, it helps to recall that what we actually have for evidence are *claims* written decades later which all show theological embellishment and literary structures. If you see the Gospel of Mark as a story about how the "last shall be first," a response to the Roman destruction of the Jewish temple, then many of the claims collapse as possible literary constructions. The testimony of the women is easily understood in this context and can be further understood as a way of Mark dealing with the possible fact that the entire story is a late creation — those "unreliable women" didn't get the message out until now. In fact Mark, our earliest Gospel, doesn't even mention any of the testimony from the disciples about the resurrection and the letters of Paul never even mention disciples. . . at all.[2]

A similar line of argument can be held for the writings of Paul. I'll admit that the conversion of Paul itself would be a rare event, but not unheard of. I'll accept the McGrew estimate of 1 out of a 1000 for it, but how many early religious sects were there in the Roman empire at that time? If you play the lottery enough, as we've seen, you will invariably have rare events happening frequently. Also, whichever sects were successful would have to have had a successful group of founders with a compelling (and flexible) story. Even Paul's conversion fits the "last shall be first" theme which would be a compelling narrative for the early Christians regardless of the truth value of any of the claims. The point here is not to suggest any particular way that Christianity began, but to point out that there are numerous, unremarkable pathways which are still consistent with a rare conversion.

[2] Paul mentions apostles but never mentions anyone actually knowing Jesus personally, listening to his ministry, or being with him during the crucifixion.

5.2.2.2 Lack of scepticism

The McGrews provide a quote which they should follow themselves,

> "William Kruskal sums up his detailed discussion of independence in the combination of testimony with a succinct cautionary moral: 'Do not multiply lightly' (Kruskal, 1988, p. 929)."

When your calculation *critically* depends on the independence assumption and there is a chance — even a small one — that the sources were not independent, especially in the direction against your claim, then you need to establish that this chance is less likely than the claim you're using independence to support. Otherwise you are biasing the calculation towards your claim.

Other than collusion, what other mechanisms are there? Recall that all the evidence we have is from *literary* constructions decades after the events. You could have *literary* "collusion." Say, only 5 disciples independently made the original claims, but for *literary* reasons you like the number 12 or 13, you might have expanded those claims to include all the disciples. Because of the gap between the original events and the writings it would be challenging to rule out such processes, yet the calculations presented depend *critically* on those processes not happening.

5.2.3 How should the argument be done?

If you're really trying to structure the

5.3 Minimal facts argument

One of the more recent arguments for resurrection comes in the form of the so-called "Minimal Facts Approach" [Habermas et al., 2004], introduced by Gary Habermas and Michael Licona and popularized by William Lane Craig. In this approach a core handful of claims are supposedly supported even by secular scholars, and that this core inevitably leads directly to supporting the resurrection. Different people have somewhat different lists of these minimal facts[3], but Craig summarizes them[4] neatly as

- Jesus's honorable burial
- his empty tomb
- his post-mortem appearances
- the origin of the disciples' belief in his resurrection.

https://crossexamined.org/the-minimal-facts-of-the-resurrection/

Matthew Ferguson in his essay "Knocking Out the Pillars of the "Minimal Facts" Apologetic" [Ferguson, 2017] has a very detailed historical dismantling of these primary claims and is very good reading.

An immediate rejoinder to the minimal facts approach is the fact that Paul, the earliest Christian writer, doesn't mention most of them - the honorable burial or the empty tomb. Paul also uses the same word for Jesus "appearing" to Peter and James as to "appearing" to himself, which calls into question whether these were actual visitations or just visions, clearly not requiring an empty tomb or any other details of the crucifixion. Further, Paul never talks about any post-resurrection *stories*, only *visions*. This, too, doesn't lend any support to the minimal

[3] The fact that there is disagreement even among apologists about what counts as a "minimal fact" should give one pause and be perceived as a problem for the argument as a whole.

[4] https://www.reasonablefaith.org/media/debates/is-there-historical-evidence-for-the-resurrection-of-jesus-the-craig-ehrman

facts argument because a big part of the post-resurrection appearances the stories. If the stories aren't true then the appearances are called into question too.

Another thing to consider is that if this line of logic were applied to, say, the Roswell Incident it would be quite easy to argue for alien visitation. Given the rarity of alien visitations, as well as supernatural resurrection, I will need more than a few claims about tombs and visions to convince me that something extraordinary occurred.

Chapter 6

Miracles

Perhaps the most famous treatment of the concept of miracles comes from Hume's essay "On Miracles" [Hume, 1748] available online in part 1 and part 2. The critique of this treatment comes from the Stanford Encyclopedia of Philosophy article on miracles [McGrew, 2014] which I will refer to as *SEP*. In the chapter I address this SEP article, and apply the probabilistic thinking that I have been presenting all along.

6.1 Concepts and definitions

The *SEP* article begins by discussing one of Hume's definitions of a miracle as "a violation of the laws of nature." From what I can tell, their main critique is they don't like the connotations of the word "law," a perspective I share — the word "law" has too many alternate meanings to be the foundation of a well-defined argument. Their revised definition is the following:

> A miracle is an event that exceeds the productive power of nature

Perhaps this is the scientist in me, and why I am not a philosopher, but I don't see a striking difference between these two definitions in at least how they are used. So it seems reasonable to adopt this as a good working definition.

SEP goes on to clarify a subset of miracle,

> a religiously significant miracle is a detectable miracle that has a supernatural cause.

This clarification is to deal with the following problem, and I'd agree with at least the sentiment.

> An insignificant shift in a few grains of sand in the lonesome desert might, if it exceeded the productive powers of nature, qualify as a miracle in some thin sense, but it would manifestly lack religious significance and count not be used as the fulcrum for any interesting argument.

I am not sure how, in practice, one would be able to determine a "supernatural cause," let alone establish how an event could be beyond the "productive power of nature" without committing a fallacy of *argument from ignorance*, but let's leave that for now.

6.2 Arguments for miracle claims

This section of *SEP* starts with a quick list of the types of evidence and arguments made for miracles.

> Many arguments for miracles adduce the testimony of sincere and able eyewitnesses as the key piece of evidence on

> which the force of the argument depends. But other factors are also cited in favor of miracle claims: the existence of commemorative ceremonies from earliest times, for example, or the transformation of the eyewitnesses from fearful cowards into defiant proclaimers of the resurrection, or the conversion of St. Paul, or the growth of the early church under extremely adverse conditions and without any of the normal conditions of success such as wealth, patronage, or the use of force. These considerations are often used jointly in a cumulative argument. It is therefore difficult to isolate a single canonical argument for most miracle claims. The various arguments must be handled on a case-by-case basis.

All of these pieces of so-called evidence are the worst kind of evidence, for which there are countless examples of the same, or similar evidence used to shore up the claims of other (presumably false) beliefs. You can think "Mormonism" or "Alien Abductions" for nearly every point listed.

They then outline two types of inductive arguments:

> 1. the conclusion (in this case that the miracle in question has actually occurred) is probable to some specific degree, or at least more probable than not
> 2. the conclusion is more probable given the evidence presented than it is considered independently of that evidence

Point (1) is just specifying either $P(\text{miracle}|\text{data})$ directly or establishing only that $P(\text{miracle}|\text{data}) > 0.5$. Point (2) is $P(\text{miracle}|\text{data}) > P(\text{miracle})$. Point (2) is nearly useless. For example, you could have

$$P(\text{miracle}) = 0.00001$$
$$P(\text{miracle}|\text{data}) = 0.001$$

and still have a seriously unlikely hypothesis, even given a factor of 100 increase in probability of a miracle given the data! Thus the *only* thing that matters must be the actual value of $P(\text{miracle}|\text{data})$.

One such argument for miracles specifies the type of evidence needed to make one confident that one is talking about a miracle. The article calls this a "criteriological" argument, but all of the arguments dealt with are probabilistic. What are the criteria, for example? This one is from Charles Leslie:

> 1. That the matters of fact be such, as that men's outward senses, their eyes and ears, may be judges of it.
> 2. That it be done publicly in the face of the world.
> 3. That not only public monuments be kept up in memory of it, but some outward actions to be performed.
> 4. That such monuments, and such actions or observances, be instituted, and do commence from the time that the matter of fact was done.

One can easily site both the golden plates of Joseph Smith and the events surrounding Roswell that satisfy all of these. Clearly, there is an issue with these "criteria."

6.2.1 Probabilistic arguments

The first form here deals with *testimony*, with the following assumptions and conventions:

1. $T_i \equiv$ the proposition "Witness i testifies to M"
2. $P(T_i, T_j) = P(T_i) \times P(T_j)$: independence
3. $P(T_i|M) = P(T_j|M)$ for all i and j: all testimony is equally informative

We then easily derive:

$$\frac{P(M|T_1, T_2, \cdots, T_n)}{P(\sim M|T_1, T_2, \cdots, T_n)} = \left(\frac{P(T_1|M)}{P(T_1|\sim M)}\right)^n \times \frac{P(M)}{P(\sim M)}$$

The *SEP* article then spins this in a positive way toward miracles:

> [I]f independent witnesses can be found, who speak truth more frequently than falsehood, *it is ALWAYS possible to assign a number of independent witnesses, the improbability of the falsehood of whose concurring testimony shall be greater than the improbability of the alleged miracle.* (Babbage 1837: 202, emphasis original; cf. Holder 1998 and Earman 2000)

However, comparing with Hume, it becomes obvious why this spin fails:

> When anyone tells me, that he saw a dead man restored to life, I immediately consider with myself, whether it be more probable, that this person should either deceive or be deceived, or that the fact, which he relates, should have really happened. I weigh the one miracle against the other; and according to the superiority, which I discover, I pronounce my decision, and always reject the greater miracle. If the falsehood of the testimony would be more miraculous, than the event which he relates; then, and not till then, can he pretend to command my belief or opinion. (Hume)

The first quote implies that the terms $P(T_1|M)$ and $P(T_1|\sim M)$ refer to speaking truth vs falsehoods (i.e. lying), as opposed to speaking correctly vs being mistaken. In the latter, it is very easy to see why, for certain types of extraordinary events, we would expect fallible observers to have $P(T_1|\sim M) > P(T_1|M)$ and further that even *if* witnesses were in general slightly more reliable than not, we can't expect the observations to be *independent* in general. In the specific case of the (anonymous) Gospel writers, there is strong evidence of *dependence* in the accounts to make this entire calculation (except in its gross qualitative features) irrelevant.

6.3 Arguments against miracles

Quoting Hume again,

> The plain consequence is (and it is a general maxim worthy of our attention), "That no testimony is sufficient to establish a miracle, unless the testimony be of such a kind, that its falsehood would be more miraculous, than the fact, which it endeavors to establish: And even in that case, there is a mutual destruction of arguments, and the superior only gives us an assurance suitable to that degree of force, which remains, after deducting the inferior."

This is correct, and is a direct statement of Bayesian reasoning

$$\frac{P(M|E)}{P(\sim M|E)} = \frac{P(E|M)}{P(E|\sim M)} \times \frac{P(M)}{P(\sim M)}$$

where we can use the approximations $P(E|M) \approx 1$ and $P(\sim M) \approx 1$ and achieve

$$\frac{P(M|E)}{P(\sim M|E)} \approx \frac{P(M)}{P(E|\sim M)}$$

The *SEP* article on miracles continues to try to map this to a philosophical structure (needlessly, I'd say), with the following "simple version" of the argument:

> A very simple version of the argument, leaving out the comparison to the laws of nature and focusing on the al-

> leged infirmities of testimony, can be laid out deductively (following Whately, in Paley 1859: 33):
>
> 1. Testimony is a kind of evidence very likely to be false.
> 2. The evidence for the Christian miracles is testimony.
>
> Therefore,
>
> 1. The evidence for the Christian miracles is likely to be false.
>
> This is, however, much too crude an argument to carry any weight, since it turns on a simple ambiguity between all testimony and some testimony. Whately offers an amusing parody that makes the fallacy obvious: Some books are mere trash; Hume's Works are [some] books; therefore, etc.

One reason this fails has to do with the ambiguousness of English vs the math. For example, even if testimony is typically 99% accurate, that means that the probability of getting an incorrect testimony would be 1% ($P(E|\sim M) = 0.01$). The odds for a truth of a rare event, say 1 in a million, based on this testimony then comes down to,

$$\frac{P(M)}{P(E|\sim M)} = \frac{0.000001}{0.01} = 0.00001$$

or a 1 in 10,000 odds *against* the claim. Here again we see the value in not relying on English as the best way to communicate such matters.

The article continues with,

> The presumptive case against the resurrection from universal testimony would be as strong as Hume supposes only if, *per impossible*, all mankind throughout all ages had been watching the tomb of Jesus on the morning of the third day and testified that nothing occurred. Even aside from the problems of time travel, there is not a *single piece* of direct testimonial evidence to Jesus' non-resurrection.

Does anyone seriously think that the case against a claim always (or even usually) takes the form of direct testimony against that claim? Where is the direct testimony that Zeus *didn't* exist?

6.4 Particular arguments

According to the *SEP* article, Hume lists a set of conditions needed to make testimony carry maximum weight:

> [T]here is not to be found, in all history, any miracle attested by a sufficient number of men, of such unquestioned good sense, education, and learning, as to secure us against all delusion in themselves; of such undoubted integrity, as to place them beyond all suspicion of any design to deceive others; of such credit and reputation in the eyes of mankind, as to have a great deal to lose in case of their being detected in any falsehood; and at the same time attesting facts, performed in such a public manner, and in so celebrated a part of the world, as to render the detection unavoidable: All which circumstances are requisite to give us a full assurance in the testimony of men. (Hume 1748/2000: 88)

Essentially, he is saying that the methods of science have never confirmed a miracle. The methods of science help "secure us against all delusion in themselves," remove "suspicion of any design to deceive others," with processes "performed in a public manner" that "render the detection unavoidable."

The SEP article criticizes this by noting that some of these conditions can cut the other way, such as the condition of "credit and reputation,"

> It might have been said with some shew of plausibility, that such persons by their knowledge and abilities, their

> reputation and interest, might have it in their power to countenance and propagate an imposture among the people, and give it some credit in the world. (Leland 1755: 90–91; cf. Beckett 1883: 29–37)

This is, essentially, pointing out fallacy of authority - a good critique. Science, by its processes, attempts to avoid that as well. Of course, Hume predates modern science, so I think we can forgive him some sloppiness or poor choice of terminology.

Hume continues to suggest that the religious context of the miracle claim makes the telling of the miracle story even more likely. This would increase the probability of obtaining the testimony even if no miracle happened - $P(E|\sim M)$ increases - making the probability of a miracle go down. The criticism here? The effect could happen in the other direction:

> But as George Campbell points out (1762/1839: 48–49), this consideration cuts both ways; the religious nature of the claim may also operate to make it less readily received:
>
> > [T]he prejudice resulting from the religious affection, may just as readily obstruct as promote our faith in a religious miracle. What things in nature are more contrary, than one religion is to another religion? They are just as contrary as light and darkness, truth and error. The affections with which they are contemplated by the same person, are just as opposite as desire and aversion, love and hatred. The same religious zeal which gives the mind of a Christian a propensity to the belief of a miracle in support of Christianity, will inspire him with an aversion from the belief of a miracle in support of Mahometanism. The same principle which will make him acquiesce in evidence less than sufficient in one case, will make him require evidence more than sufficient in the other....

I disagree quite strongly with this line of thinking. One of the big problems with pseudoscience is that it promotes poor thinking in other domains. Someone who believes in miracles will not find it hard to believe that the miracle claims of other religions are at least plausible. If you believe in unseen agents, then to move from Christianity to New Age to Scientology isn't that large of a stretch. Often, when ones religion is undermined, the typical response is to switch to another religion! Thus, they are not as opposite as "light and darkness." Poor thinking is poor thinking, regardless of the context.

6.4.1 Argument from parity

Hume brings up miracles in other religions. In a fit of special pleading, the article on miracles retorts,

> All attempts to draw an evidential parallel between the miracles of the New Testament and the miracle stories of later ecclesiastical history are therefore dubious. There are simply more resources for explaining how the ecclesiastical stories, which were promoted to an established and favorably disposed audience, could have arisen and been believed without there being any truth to the reports.

The argument is quite simple - if there are known cases of miracle claims where no miracle actually occurred, that increase $P(E| \sim M)$, making the probability of a miracle go down given testimony. It doesn't matter whether you have good reasons to believe there was no miracle for these cases — it undermines testimony of miracles in general. Add to that the fact that *less data* should yield *less* certainty not more.

6.5 In conclusion

So, as far as I can tell, there is no substantive critique to Hume's statements about miracles. Although he lacks the rigor of the mathematics of probability (through no fault of his own) his wording is so straightforwardly translated to probabilities that I find it difficult to see what the problem is. I also find it ironic that the entire SEP article, which has been pro-miracle throughout its contents, ends with this:

> For the evidence for a miracle claim, being public and empirical, is never strictly demonstrative, either as to the fact of the event or as to the supernatural cause of the event. It remains possible, though the facts in the case may in principle render it wildly improbable, that the testifier is either a deceiver or himself deceived; and so long as those possibilities exist, there will be logical space for other forms of evidence to bear on the conclusion. Arguments about miracles therefore take their place as one piece—a fascinating piece—in a larger and more important puzzle.

This is pretty much exactly what Hume was saying. Given that there is always a non-zero probability of the testifier either lying or being mistaken, one has to establish the evidence for a miracle strong enough to overcome both the negligibly small prior probability of the miracle and this non-zero probability of the testimony being wrong. Since mistakes are a common human trait, and distortions are also common on testimony, evidence for miracles according to probability theory, Hume, and all rational thought have always been found lacking.

Of course, if you could demonstrate it otherwise, please show it! I'd love to believe in miracles. I just have never seen anything even remotely convincing.

Chapter 7

Faith

The word *faith* has many definitions, some which seem conflicting. However, as it is commonly used, the definitions point to a single quantity. It is easiest to see this in the exploration of faith from specific examples.

7.1 Introduction to faith and probability

7.1.1 Statements about faith

It may be helpful to start with the dictionary definition, and then follow with a number of statements about *faith* to see where the common usages fall.

- faith (noun) [Merriam-Webster Online, 2009]. 1. strong belief or trust in someone or something. 2. belief in the existence of God. 3. strong religious feelings or beliefs. 4. a system of religious beliefs.

- "Now faith is confidence in what we hope for and assurance about what we do not see."[1]
- Faith is trusting in, holding to, and acting on what one has good reason to believe is true in the face of difficulties. The difficulties may be where you have to take an action where the outcome is beyond your control. (Tim McGrew) [Brierley, 2014]
- Belief without evidence. Pretending to know things you do not know. (Peter Boghossian) [Brierley, 2014]
- Faith is the excuse people give when they believe something and don't have a good reason. When you believe something and have a good reason, then you give the reason. And in every other instance, you offer something that is faith. - Matt Dillahunty

From these few definitions, we can already see a few things. First, people use the term in different ways, at different times, so it will be critical in any discussion that we agree on what we are talking about. In particular, we don't want to run the risk of changing definitions mid-discussion and we don't inadvertently straw-man anyones argument. Second, the two primary components that seem to make up all definitions of faith are *belief* and *trust*. Faith seems to not be a synonym of either, but a particular combination of them with some restrictions - not all beliefs or acts of trust require faith.

7.1.2 Belief, trust, and faith

We have already seen that *belief* is represented mathematically by *probability*, so where *faith* requires belief we will employ probabilities. *Trust* [Merriam-Webster Online, 2009] is "belief that someone or something is reliable, good, honest, effective, etc." So it would seem that trust is

[1]Hebrews 11:1, New International Version. https://www.biblegateway.com/passage/?search=hebrews+11&version=NIV

a subset of belief, like knowledge is a subset of belief, but pertaining specifically to the reliability or goodness of the thing believed.

Faith, however, seems to go a bit further than trust and involve *action* or the willingness to act. When asked by Peter Boghossian, *"why don't we say that we have faith in the existence of chickens?"*, Tim McGrew replies,

> We are venturing nothing on the existence of chickens. When I believe that chickens exist and I act on that belief I am not taking any step that places outcomes I care about beyond my direct control. [In the case of religion], people are placing the outcome of their eternal soul out of their control. They are taking a risk where the outcomes matter. The decision itself is evidenced but the outcome is uncertain.

7.2 Rollercoasters: faith in everyday life

Words have meaning, and if we are going to communicate with each other we need to make sure to use words as carefully as we can. Otherwise, misunderstandings abound. It seems very common that a word like "faith" is used by different people for different ends, and the definition shifts even within an argument. Take for example, the video by Rich Spear [Spear, 2013]. In it, Spear presents a distinction drawn between "faith" and "belief," using an analogy of a roller-coaster —belief in the ride being safe vs trusting it being safe enough to ride on. Notice that his focus is on trust, and thus on *action*.

It is clear that one must *at least* believe the ride is safe as a prerequisite for trusting it. Since when we say we believe *strongly* in a proposition A when $P(A) > 0.95$ (or some other, somewhat arbitrary, high number), we can map this prerequisite to something like the following

$$P(\text{safe}) = 0.99$$
$$P(\text{unsafe}) = 0.01$$

Once you believe it is safe, do you trust it to ride? This brings in decision theory, where we mix probabilities with utility measures. You could believe it to be safe at the $P\,(\text{safe}) = 0.99$ level, but still not trust it “with your life” because of the *cost* associated with being wrong. A utility table might look like

	safe	unsafe
ride	10	-1000
don’t ride	0	0

Calculating the average utility for each action we get

$$\begin{aligned}\langle U_{\text{ride}}\rangle &= 0.99 \times 10 + 0.01 \times (-1000) = -0.1 \\ \langle U_{\text{don't ride}}\rangle &= 0.99 \times 0 + 0.01 \times 0 = 0\end{aligned}$$

so it is better not to ride.

As a result, *trust* requires both belief and a sufficiently positive net utility. Placed in these terms it is much more clear how the argument is set up.

- when Spear says that “faith” is like “trust,” he is already approaching the problem with strong belief, and is assessing utility - and he rightly claims that belief is not enough.
- when the atheists say that “faith” is “belief without evidence,” they are addressing the strength of the evidence to obtain strong belief in the first place - and claiming that the evidence is not sufficient.
- when Spear and others say that “faith is rational” they are either talking about utility (and not belief) or they are claiming that the evidence is in fact good enough for strong belief, and then consequently high utility.

In all cases, it seems as if for the religious, utility and belief are muddled when using the word "faith." For the atheist, "faith" is always first and foremost about belief, because even the usage involving trust has belief as a prerequisite. Perhaps if we frame the problem in terms of belief (probability) and utility we can clear up the fog surrounding discussions of the term *faith*.

7.3 Faith and trust

We see the same structure occurring in an *Unbelievable* podcast [Brierley, 2014] between theist Tim McGrew and Peter Boghossian. Because they were not thinking in terms of the framework presented here, they talked past each other through most of the episode. In this discussion, Boghossian insists that faith is "*belief without evidence*" or "*pretending to know things you do not know*," and Tim McGrew insists that faith is more like trust. The discussion then devolved into a back and forth with both sides claiming that "all the people I know use my definition," and was generally unproductive. As I've said before, at this point it becomes better to dispense with the terminology and go back to basic concepts where one could agree. It is clear that in the expected utility equation,

$$\begin{aligned}
\langle U_{\text{action } A} \rangle = & P(\text{outcome 1}) & \times U(\text{outcome 1}|\text{action } A) + \\
& P(\text{outcome 2}) & \times U(\text{outcome 2}|\text{action } A) + \\
& & \vdots
\end{aligned}$$

Boghossian is focusing on the probabilities (Poutcome 1, Poutcome 2, etc...) and McGrew is focussing on the utilities (U(outcome 1|action A), U(outcome 2|action A), etc...)

7.4 The discussion

McGrew says that very few Christians (less than 1%) would use Boghossian's definition of faith, "*pretending to know things you do not know*." I agree that no Christian would *articulate* this definition of faith, however they may be *functionally* using it, which I will address later.

McGrew opens with

> Faith is trusting in, holding to, and acting on what one has good reason to believe is true in the face of difficulties. The difficulties may be where you have to take an action where the outcome is beyond your control.

The example he gives is jumping out of an airplane, where one has faith in ones instructor to have packed your parachute properly. Ones act of jumping draws the distinction between faith and *hope* (if you just *hoped* your instructor packed it, you wouldn't jump), and the decision is made in the face of evidence, not despite it or without it.

7.4.1 Addressing the definitions

Clearly claims using expected utility require that probability assignments have already been made, so claims of utility must necessarily be probability claims as well. When translated into these more precise terms, both McGrew's and Boghossian's claims begin to make more sense. It will also show that McGrew is in fact using the Boghossian's definition, in some cases even while denying it.

> You have faith that your instructor packed your parachute, as opposed to Peter packing it. Your act of jumping makes faith more than simply hope (if you just hoped your instructor

packed it, you wouldn't jump), and the decision is made in the face of evidence, not despite it or without it.

The equations are:

$$\begin{aligned}\langle U_{\text{jump}}\rangle =& P(\text{instructor packed}) \times U(\text{instructor packed}|\text{jump})+\\ & P(\text{Peter packed}) \times U(\text{Peter packed}|\text{jump})\\ \langle U_{\text{don't jump}}\rangle =& P(\text{instructor packed} \times U(\text{instructor packed}|\text{don't jump})+\\ & P(\text{Peter packed}) \times U(\text{Peter packed}|\text{don't jump})\end{aligned}$$

with a possible utility table

	Instructor Packed	Peter Packed
jump	100	-10000
don't jump	0	0

For this analysis to work, we would have at least,

- $P(\text{instructor packed}) \sim 1$: one is nearly certain the instructor packed the parachute
- $P(\text{Peter packed}) \sim 0$: one is nearly certain that Peter *didn't* pack the parachute
- $U(\text{instructor packed}|\text{jump}) \gg 1$: good benefit from jumping, with instructor packing the parachute
- $U(\text{Peter packed}|\text{jump}) \ll 0$: large cost from jumping, with Peter packing the parachute
- $U(\text{Peter packed}|\text{don't jump}) = U(\text{instructor packed}|\text{don't jump}) \sim 0$: neutral gain for not jumping in either case

7.4.2 Analyzing the responses

Notice, for McGrew to have "faith in his instructor," two things must be true:

1. $P(\text{instructor packed}) \sim 1$: one is nearly certain the instructor packed the parachute
2. $U(\text{instructor packed}|\text{jump}) \gg 1$: good benefit from jumping, with instructor packing the parachute

McGrew wants to focus on point (2), while Boghossian wants to focus on point (1). Once seen this way, it is very easy to understand the responses to other questions.

For example,

- Boghossian: *Why don't we say that we have faith in the existence of chickens?*
- McGrew: *We are venturing nothing on the existence of chickens. When I believe that chickens exist and I act on that belief I am not taking any step that places outcomes I care about beyond my direct control. [In the case of religion], people are placing the outcome of their eternal soul out of their control. They are taking a risk where the outcomes matter. The decision itself is evidenced but the outcome is uncertain.*

This is because,

$$U(\text{chickens exist}|\text{action}) \sim U(\text{chickens don't exist}|\text{action}) \sim 0$$

for all actions - we have nothing at stake, there is no utility, even if we are confident that chickens exist (i.e. $P(\text{chickens exist}) \sim 1$).

Another example,

- Boghossian: *Do you have faith or evidence that Islam is false?*
- McGrew: *Why would I use the word 'faith' when I am venturing nothing on Islam? I am a little bit confused about the framing of the question that way. I think I have evidence that it is false, but since I am not venturing on Islam, I'm not sure why the word faith would come in.*

Does McGrew have have faith or evidence that Islam is false? McGrew claims he has evidence that Islam is false, $P(\text{Islam}|\text{data}) \ll 1$, but is not venturing anything on Islam (or more accurately, on his choice to not follow Islam), $U(\text{not-Islam}|\text{action}) \sim 0$. Again, Boghossian sees the first part (the probabilities), yet ignores the second part (the utilities).

7.4.3 Equivocation

Going back to the exchange, we note, however, that sometimes McGrew is also using Boghossian's definition:

- Boghossian: *what do people mean when they accuse someone of having 'faith in evolution'*
- McGrew: *You're trusting in something that you cannot completely verify because it doesn't lie open to your senses.*

Now I can think of no way to understand this statement from the perspective of McGrew's definition:

> When I act on that belief I am taking some step that places outcomes I care about beyond my direct control.

What outcomes are you placing beyond your control believing in evolution? What obvious utility are you weighing in this case? As far as we can see there is none, and so *faith* in this context is in fact being used here as *belief without sufficient evidence.*

It is called an *equivocation fallacy* to use a word in one way but switch definitions sometime later without pointing that out or noticing it. One of the benefits of mapping statements to mathematical language it to see this explicitly.

7.4.4 Priors and faith

The word *faith* seems to only be used in contexts with low *prior* probability. In the conversation between McGrew and Boghossian they spoke of faith in the context of the supernatural, extreme activities (i.e. jumping out of planes), events beyond our immediate senses — all of which coincide with lower *prior* probability, and need *more* evidence than is typical to overcome them. These examples may, or may not, also have high utility. We don't have faith in the existence of chickens because the existence of chickens has high *prior* probability.

It makes sense then that there will be people who are not convinced by the evidence for things that others have *faith* in. They are not convinced the evidence is of sufficient quantity or quality to raise a low *prior* probability up to a significant *posterior* probability. For those who are convinced by the evidence, it also makes sense that they would then focus on the *utility* of those claims. The problem that arises, however, that the word *faith* refers to two distinct components — the belief and the utility — and the apologist can easily switch between them without even noticing it themselves. Again, we put forward the suggestion to discuss things in terms of decision theory, explicitly outlining the equations in order to avoid words with multiple meanings.

7.5 Does science have faith?

In his talk, "Life: Creation or Evolution" [Miller, 2009], Ken Miller makes the point that science should inform faith and faith should inform science. He cites Paul Davies, a physicist who has an interest in theism, and whose article "*Taking Science on Faith*" [Davies and Tempe, 2007] takes the position that science itself is a faith-based activity. Ken Miller points out, and one can confirm in Paul Davies' article, that there are two tenets in science that are supposedly taken on faith:

1. the universe is ultimately knowable and understandable
2. knowledge is better than ignorance

These concepts, however, are fundamentally different than faith, or even axioms. Even here, it is plain that the claims are referring to the *belief* part of faith, and not the *trust* part of faith. The entire phrase "*taken on faith*" is a signal to the listener that this is so.

The first idea, that the universe is knowable, needs to be a bit more specific: what does it mean to be *knowable*? Prior to 1900, it was believed that the pieces of a physical model, such as the force of gravity, or the electric and magnetic fields of Maxwell were "real": there was one-to-one correspondence between the model components and things in the real world. Thus, it was believed, that knowing the model you would know nature. After 1900, with the advent of quantum mechanics, physical models were evaluated based on their predictive value: those models that predicted well were good models. It was not believed that there was necessarily a correspondence between the model components and the real components in nature. Aspects of the model, such as the wave function in quantum mechanics, were not believed to be real but simply useful in making predictions. To know the world is to be able to predict what would happen.

Let's say we replace "*understandable*" with "*predictable*," a replacement which we think makes practical sense (how else would you determine that you understand something?), and is directly in line with modern physical thinking. Doing this, then tenet (1) ceases to be an axiom, or something we take without sufficient evidence ("*on faith*"), but is observable. If the universe is unpredictable, then all attempts at making prediction will fail. This is not what we observe at all. Surely there are still things that are unpredictable, such as the simultaneous value of the position and momentum of the electron, or the positions of every molecule of air in this room, but even there we can make specific predictions about average quantities or the values of other variables of interest. Practically, the universe has demonstrated itself to be understandable, on the whole. This is not a matter of faith.

The second tenet (2) I would wager is too vague. What does "*better*" mean? Better for whom, or for what? Psychologically, one might argue something akin to "ignorance is bliss," and there might be something to that. If we define, however, "*better*" to be higher standard of living (longer, healthier, more free life) then knowledge can be argued to have a demonstrable benefit over ignorance. The results of science has doubled the life expectancy in the past 100 years, and has allowed us to live more free and healthy lives. As Carl Sagan says, science delivers the goods. Is there any convincing argument that ignorance is better, or that we really can't decide which is better?

There is a danger in using the word *faith* in these contexts. It can communicate to the unaware that there are things that one should justifiably believe on insufficient evidence — a direct violation of the laws of probability. It can also imply, for those who take *faith* to mean *trust*, that the scientists using the term are somehow admitting an agency in the universe that they don't intend. It serves only to propagate sloppy thinking in both the fields of science and religion.

7.6 Another interaction - McGrath and Dawkins

As part of the "Root of All Evil" program, Richard Dawkins conducted many interviews with theists. One in particular, with theologian Alister McGrath, deals with the notion of faith [Dawkins and McGrath]. They start with a discussion about the term *faith*, and McGrath says

> We're dealing with a different situation than, for example, evidence that the moon orbits the earth at a certain distance

and

> There are many possible ways of explaining [the world], and we have to make the very difficult judgement of which is the best of these [explanations] [...] evidence takes us thus far, but then when it comes to deciding between a number of competing explanations, it is extremely difficult to make an evidence-driven argument.

and

> I believe faith is rational, in the sense that it tries to make the best possible sense of things [...] even though we believe this is the best possible sense of things, we cannot *prove* this is the case [...] there is a point where faith goes *beyond* the evidence

By this point, the reader should be able to tell that McGrath is employing the *belief* part of the expected utility form of *faith*. One wonders in these cases why he doesn't simply talk about evidence, and the weight of probabilities? Historians, for example, don't use the word *faith* even though they deal with probabilities, some of which are highly uncertain. Scientists deal with probabilities all the time without invoking the word *faith* in any paper.

Further, McGrath ignores the fact that there already is a proper and rational method to address the "decision between a number of competing explanations," that *doesn't* go beyond the evidence, and doesn't claim more knowledge than is justified. What is this method? It's called the mathematics of probability! So, McGrath is claiming there is a problem that faith solves, which is not a problem at all, and he is using the word faith (at the moment) as synonymous with probability.

Why is he doing this? It seems as if it is because McGrath is holding to a double standard, and shifts the definitions of concepts around whenever pressed. He doesn't like the notion of believing strongly without sufficient evidence (which, as we've seen, is one use of the word faith), so he defines it (at the moment) to be equivalent to probabilities.

7.6.1 Inference to the best explanation

Then McGrath continues to talk about probabilistic reasoning, and says that with faith one is doing *inference to the best explanation*, given a number of competing multiple explanations. As we stated earlier, if all he means is that faith is probabilistic reasoning, then we don't have an argument — except that we think he can make things more clear. We would contend, along with Dawkins, the *vast* majority of people do not take it to mean this — even the notion of *faith* as *trust* isn't the same as this.

However, we'd like to challenge his basic premise: that in dealing with multiple competing explanations that one should try to *infer to the best explanation*, and *believe strongly in that explanation*. A simple example introduced in Section on Belief and Evidence suffices to see this. In this example, we have two explanations of the number of stars, one which says that there is an *even* number of stars and another that says that there is an *odd* number of stars. Pretty much we know that, at any given instant, one of these *must* be true. However strong belief in either one is completely unwarranted - there is simply no way to know. From a probabilistic framework, we express this as

$$
\begin{aligned}
P(\text{odd}) &= 0.5 \\
P(\text{even}) &= 0.5
\end{aligned}
$$

However, it is worse than that. Let's say we had a smidgen of evidence toward the even-star model, such that we had:

$$
\begin{aligned}
P(\text{odd}) &= 0.499995 \\
P(\text{even}) &= 0.500005
\end{aligned}
$$

Even though there is a *best* explanation here (*even* is slightly more probable than *odd*), and we have the exact probabilities, it is *still irrational* to hold strong belief in either explanation. One really does have to look where the weight of the probabilities lay. Inference to the best explanation fails as a guiding principle in the face of uncertainty, and is not well defined in all contexts.

What is happening here is that on the face of it, "inference to the best explanation" sounds like a great thing — something we should always strive for. However, when you look at what it *actually* means, it falls short unless you are in a situation where the best explanation is also very probable. Strong belief is only justified when the claim is very probable, not just that is is the most probable amongst a number of (possibly nearly equivalent) alternatives.

7.6.2 Shifting sands

One of the benefits of seeing these arguments in the light of the framework of probability is that it makes one sensitive to shifting definitions.

We saw that earlier where Tim McGrew seems to change his usage of the term *faith* depending on the response. Here, McGrath does the same thing.

First it was "*faith is reasonable*," based on evidence, going beyond the evidence to the "*inference to the best explanation*" and that as a result one can have a "*reasonable faith in God*." Then, when asked about his belief in a creator, and the evidence for it, despite having difficulty with the implied complexity of such a creator, he says

> I want to go back to CS Lewis who says I believe in Christianity as I believe the Sun has risen, not simply because I see *it* but *by it* I see everything else. Belief in God gives you a way of seeing the world that makes an awful lot of sense of it.

When pressed on what this implies, he says that

> there are many reasons I believe in God and that [origins] is not even the primary one...religion really isn't much about where things came from, about things in the distant past, but really about how things are now. How to live your life, how to be moral, etc...

which then becomes

> the key reason for believing God is Jesus, that there is something [in the Jesus story] that needs explanation.

and then, this becomes that it is not really about the life of Jesus, and his historicity, but how he was perceived by his followers - the significance they saw in the life and teaching of Jesus.

Notice how this keeps shifting? At first, it is about belief, and then it is about significance (which one could argue is a kind of utility). Every time he gets pushed on the specific consequences of his statement, he retreats, redefines, and redirects the conversation.

McGrath doesn't seem to realize that any explanation, even of things currently, entails assumptions that can be tested - perhaps with observations about the past. He can't simply say that religion is "not about where things came from," when they explicitly make statements of origins — statements which have been universally discredited. The atonement, for example, *does* depend critically on the existence of Jesus, the existence the "Fall", and a creator of the universe — for none of which did McGrath provide evidence. If Jesus didn't exist as a real person (or even if he was just an ordinary guy) then it doesn't matter that his followers simply *believed* that Jesus was God incarnate when determining ones belief in the doctrine of salvation. The demonstration of the historicity of the events claimed is *necessary* for the doctrinal belief. If you don't have strong evidence of the former, then you are not rational to believe strongly in the latter - you'd be claiming to know things you could not know.

As a scientist, one takes an idea, and pushes the idea to it furthest consequences to see where it breaks, or to see what it depends on. McGrath seems to change the topic whenever this is done - he does not seem to want to face the very real, specific consequences of his stated beliefs and refuses to see the connections between the things that may be confirmable (apparent design in the biology and the universe itself, historicity of people and events, alleged miracle claims, etc...) and the things that make him feel good, but are unmeasurable (existence of heaven, the atonement of sins, etc...).

7.7 Pascal's wager

Blaise Pascal, a French philosopher, put forward an argument referred to now as "*Pascal's Wager*" [Wikipedia, 2015b] for the religious life .

The argument is based on decision theory, and is one of the first uses of this theory on any topic. In the "Wager," Pascal states that people choose to believe or not believe in God, and the possible environments they find themselves in are the God either exists or doesn't. He then sets up the utility table,

	God Exists	God Does Not Exist
Believe in God	$+\infty$ (infinite gain)	-1 (finite loss)
Don't Believe in God	$-\infty$ (infinite loss)	$1+$ (finite gain)

It is clear then that the best course of action, given this utility table, is to believe. There are many problems with this argument, some which impact the mathematics and others that are theological. An example of the former is the analysis assumes only one possible God - what if you choose the wrong God? Extending the table to include this would make the "best choice" not as clear cut. An example of the latter is the idea that one cannot simple *choose* to believe, and the act of pretending to believe would go against the dictates of God.

Pascal's Wager is, however, a useful starting point for a discussion and highlights some of the issues one faces when applying decision theory too simplistically.

7.8 A more formal exploration

In the philosophical literature, there are more formal explorations of the concept of faith. These explorations can also be helped by casting the ideas into the probabilistic framework. For example, Daniel Howard-Snyder considers what he calls "Propositional Faith" [Howard-Snyder, 2013]. He immediately recognizes, as we do here, that the word *faith* has many usages which he clears away as being not on topic, and focuses on the use of faith in a sentence like "*A wife might have faith that her marriage will survive a crisis*" or "*Frances has faith that her young* *sons will live long and fulfilling lives.*" Each of these cases has the sentence structure of "*A* has *faith that B*." In his covering of other uses, Howard-Snyder removes the following usages,

- Faith as a noun (e.g. "earnestly content for *the faith*")
- Faith as a process (e.g. the process of coming to believe the Gospel as a result of the Holy Spirit)
- Taking something *on faith* (i.e. taking on authority or testimony)
- Faith as assent to a proposition with certainty
- Faith as a kind of knowledge

It is interesting to see what Howard-Snyder considers propositional faith, and some of the considerations around it. We believe he is still using the term inconsistently, in two ways - one which matches the structure we've been exploring in this book, and the other as a direct synonym for *hope*. Consider the following case from his paper.

> Propositional faith does not require 'certainty', without any hesitation or hanging back. A wife might have faith that her marriage will survive a crisis, while harboring doubts about it. Indeed, propositional faith is precisely that attitude in virtue of which she might possess the inner stability and impetus that enables her to contribute to the realization of that state of affairs, despite her lack of certainty.

This case matches decision theory quite nicely. The equations are:

$$\begin{aligned}\langle U_{\text{action}}\rangle = \\ &P(\text{good marriage}|\text{action}) \times U(\text{good marriage}|\text{action}) + \\ &P(\text{failed marriage}|\text{action}) \times U(\text{failed marriage}|\text{action})\end{aligned}$$

$$\begin{aligned}\langle U_{\text{inaction}}\rangle = \\ &P(\text{good marriage}|\text{inaction}) \times U(\text{good marriage}|\text{inaction}) + \\ &P(\text{failed marriage}|\text{inaction}) \times U(\text{failed marriage}|\text{inaction})\end{aligned}$$

where the probabilities of the outcomes, as well as the utilities, clearly change with the possible actions. For example, we expect action to have a positive effect on the probability of a successful marriage, $P(\text{good marriage}|\text{action}) > P(\text{good marriage}|\text{inaction})$.

A possible utility table, which reflects the idea that, if the marriage is successful without action then there was time and resources saved, but if the marriage failed without action there is a penalty in the form of guilt over lost opportunity. Obviously there are many complications that this table overlooks, and should be seen as a basic approximation.

	good marriage	failed marriage
action	90	-100
inaction	100	-110

The high positive utility that the wife puts on her successful marriage, and high negative utility to its failure, directs her to make actions in her marriage's favor despite having lower probability of its success. This is the notion of *faith* worked out.

Another case is

> But one can have faith that something is thus-and-so without entrusting one's welfare to it in any way, as when I have faith that Emily will survive breast cancer but I do not entrust my well-being to her or her survival

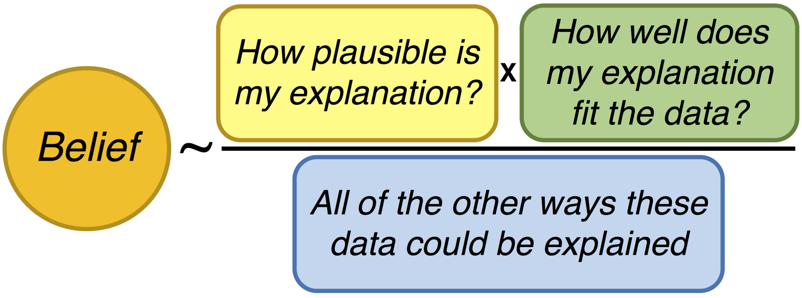
where, as far as we can see, the use of *faith* here is completely indistinguishable from *hope*. There is no utility explicit in the statement, and the probability is presumed to be low.

Chapter 8

Conclusions

Throughout this book I have tried to impart a way of thinking on the reader, something that can be applied to any situation. This way of thinking allows one to spot points where a claim is not supported properly, where someone is using overly formal language to hide an incomplete argument, and to define key concepts like *belief*, *simplicity*, *justification*, and *faith* in a way that is not so slippery. The primary method for proper thinking comes down to,

> Ones belief in an explanation should increase with how well that explanation fits the data and how plausible it is before the data, but it should also *decrease* with all of the other ways the data can be explained by other descriptions.



We saw how arguments fail primarily for a few reasons:

1. lack of imagination in constructing alternative explanations, including framing arguments in either-or terms
2. ill-formed hypotheses where initial plausibilities can't be established
3. hypotheses that are too flexible and can fit any kind of data

Although most of this book concerns the application of these ideas to topics in *religious* thought, they can be equally applied in areas of *health*, *politics*, and even *relationships*. In short, the ideas provide a framework to view all of the claims one encounters in life, from buying a car to voting for a representative to deciding what medical procedures to follow.

It is my hope that, by seeing these methods applied to specific examples in this book, that you will be able to improve your life through improving your approach to evaluating claims of all kinds.

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