The Optimal Design of State-Run Lotteries

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Acknowledgments

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Views here are our own.

Lottery consumption in the U.S.

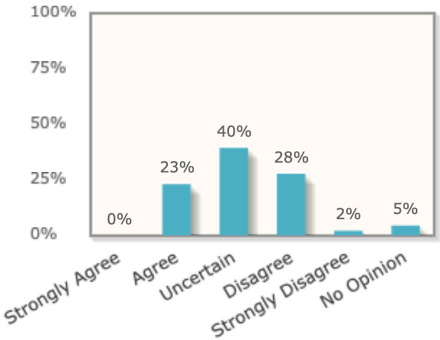
- Americans spend nearly $90 billion each year on state-run lotteries.
  - About $285 per US adult ($\sim$$24 per month)
  - More than on music, sports events, movie tickets, and video games combined.
- Lotteries are administered by 44 state governments.
  - Over $25 billion annually in state public funds.
  - Revenues similar to federal gasoline tax or estate tax.
- Not unique to U.S., of course
  - E.g., National Lottery in the U.K.

A motivating question

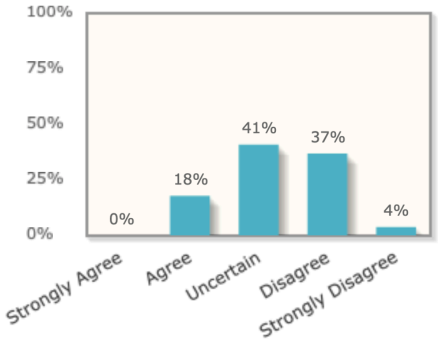
State-run Lotteries

Taking into account the revenues, consumer surplus, purchasing patterns by income, and possible consumer biases, state-run lotteries (such as Powerball and scratch-off games) increase social welfare.

Responses



Responses weighted by each expert's confidence



Are state-run lotteries welfare-enhancing?

"[A lottery] preys upon the hard earnings of the poor; it plunders the ignorant and simple."
– U.S. Supreme Court, in Phalen v. Virginia (1850)

"In our stressful world, the ability to dream is well worth the price of a lottery ticket ... The lottery is simply a form of entertainment that happens to benefit your state."
– National Association of State and Provincial Lotteries (2021)

"Since this regressive, addictive, partially hidden tax is here to stay, might a little improvement still be conceivable? ... Here's a modest suggestion: States should consider reducing their skim of the wagers."
– Former Indiana governor Mitch Daniels (2019)

Are state-run lotteries welfare-enhancing?

Our view

This is fundamentally a question for behavioral public economics.

- Lotteries are a heavily taxed, regulated product.
  - Explicit + implicit taxes over 50%.
- Distributional concerns
  - Regressive tax on low-income, low-education consumers?
- Behavioral biases
  - Gambling considered a classic “sin good.”
  - Misperception? Overoptimism? Self-control problems?

This project

Part 1: Model of Optimal Lottery Design

- New sufficient statistics formula for optimal attributes of a publicly supplied good.

Part 2: Empirical evidence

- Estimates from observational data of key elasticities that govern optimal policy.
- New large-scale survey of lottery demand and behavioral biases.

Part 3: Structural estimation and welfare

- Address policy questions: Are lotteries welfare enhancing? What is optimal tax treatment?

Related literature

State-run lotteries

- Overviews: Clotfelter & Cook (*JEP* 1990), Clotfelter, Cook, Edell & Moore (1999)
- Revenue and efficiency considerations: a “voluntary tax”? Brinner & Clotfelter (NTJ 1975), Clotfelter & Cook (1987), Morgan (2000), Lange, List, & Price (2007)
- “Regressivity:” Price & Novak (2000), Oster (2004), Barnes et al. (2011)
- Consumption patterns and demand: Kearney (2005), Guryan & Kearney (2008), Haisley, Mostafa, and Loewenstein (2008)
- Large literature on prize-linked savings and other behavior change

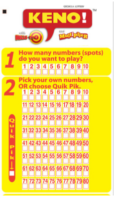
Risk preferences

- Prospect theory and probability weighting: Kahneman & Tversky (1979), Tversky & Kahneman (1992), Bernheim & Sprenger (2019), many more; overview in O’Donoghue & Somerville (2018)

Optimal taxation with redistributive and corrective motives

- Sin taxes: O’Donoghue & Rabin (2006); Farhi & Gabaix (2019); Allcott, Lockwood, & Taubinsky (2019).
- Redistributive commodity taxes: Atkinson & Stiglitz (1976); Saez (2002).

Primer on U.S. lotteries



"Lotto" style games

- Powerball, Mega Millions (+ others)
- Tickets cost $1 or $2.
- Player preselects 5+1 numbers; winners chosen in bi-weekly drawing.
- Parimutuel jackpot: accumulates until won.

Instant games

- "Scratch tickets," typically cost $1 to $20+.

Other games

- Video lottery terminals, Keno.

Model

Model setup

Publicly provided good s

- Price p and a set of attributes, a_1, a_2, a_3, etc., set by government.
- Example: lottery attributes include prizes (w_k), probabilities (π_k), other aspects of games.

Consumers

- Heterogeneous income-earning ability and preferences; types indexed by θ.
- Maximize *decision utility* $U \Rightarrow$ average demand $s(p, a_1, a_2, \ldots; \theta)$.
- Realize *normative utility* $V \Rightarrow$ latent "debiased demand" $s^V(p, a_1, a_2, \ldots; \theta)$.
- Money-metric bias $\gamma(\theta)$: "price reduction that would cause debiased θ to buy $s(\theta)$."

Policymaker

- Utilitarian, inequality averse; applies welfare weights $g(\theta)$, declining with income.
- Sets price and attributes for good s; incurs cost $C(s, a_1, a_2, \ldots)$.

Optimal price condition

- Can write first-order condition for optimal price as markup, like optimal tax. (See Allcott, Lockwood, Taubinsky 2019).

- Increases with **corrective motive**, decreases with **redistributive motive**:

$$p - MC = \bar{\gamma}(1 + \sigma) - \frac{Cov\left[g(\theta), s(\theta)\right]}{\bar{s}|\bar{\zeta}_p|}$$

MC: marginal cost of s to government (for lottery: expected payout + overhead)
$\bar{\gamma}$: money-metric bias (avg. weighted by demand response)
$\zeta_p(\theta) = \frac{d \ln s(\theta)}{dp}$: semi-elasticity of demand with respect to price (avg: $\bar{\zeta}_p$)
$\sigma = Cov\left[g(\theta), \frac{\gamma(\theta)}{\bar{\gamma}} \frac{\zeta_p(\theta)}{\bar{\zeta}_p} \frac{s(\theta)}{\bar{s}}\right]$: bias correction progressivity

- With income effects, use s_{pref} from preference heterogeneity (vs. causal income effects).

Intuition for regulating attributes

- Now consider optimal choice of some (continuous) attribute *a*.E
  - Example: lottery jackpot.
- **Like a tax**, the change Δa affects demand.
  - This has a corrective benefit proportional to $\gamma(\theta) \times$ change in demand.
- **Unlike a tax**, Δa isn't a mechanical transfer of $ from inframarginal consumers to gov't.
  - Consumers may have WTP for Δa that differs from marginal cost.
  - That WTP for Δa may be *biased*.
- We formalize these complications to characterize optimal *attribute* regulation.

Optimal attribute condition

$$p - MC = \bar{\gamma}(1+\sigma_a) - \frac{Cov\left[g(\theta), \kappa(\theta)-\rho(\theta)\right] + \overbrace{\bar{\kappa} - \bar{\rho} - \frac{dAC}{da}\bar{s}}^{\text{Avg. mech. effect}}}{\bar{\zeta}_a \bar{s}}$$

- Modifications affect mechanical term.
  - Effect of Δa on average cost, $\frac{dAC}{da}$. Example: raise jackpot by Δw_1, then AC rises by $\Delta w_1 \cdot \pi_1$.
  - Demand response to Δa: semi-elasticity $\zeta_a(\theta) := \frac{d \ln s(\theta)}{da}$
  - Consumers' WTP for Δa. We can estimate this with $\kappa(\theta) := \frac{\zeta_a(\theta)}{\zeta_p(\theta)} \cdot s(\theta)$.
  - How much of that WTP is due to bias? $\rho(\theta)$
- Optimal price is special case: $\frac{dAC}{dp} = 1$ and normative WTP $(\kappa - \rho)$ for Δp is just $\Delta p \cdot s(\theta)$

Empirical agenda

Optimal attribute regulation formula

$$p - MC = \bar{\gamma}(1 + \sigma_a) - \frac{\bar{\kappa} - \bar{\rho} + Cov\left[g(\theta), \kappa(\theta) - \rho(\theta)\right] - \frac{dAC}{da}\bar{s}}{\bar{\zeta}_a \bar{s}}$$

Empirical estimation

Formula motivates empirical questions of interest:

1. $s(\theta)$: What is profile of lottery spending across income distribution?
2. $\gamma(\theta)$: What is money-metric bias in lottery consumption, across incomes?
3. $\bar{\zeta}_p$: What is price elasticity of lottery demand?
4. $\bar{\zeta}_1$: What is elasticity of lottery demand with respect to jackpots?
5. $\bar{\zeta}_{2+}$: What is elasticity of lottery demand with respect to smaller prizes?

Empirical Evidence

Road map: Empirics

I. Observational evidence: characteristics of lotteries that affect demand

1. What is the elasticity of lottery demand with respect to jackpots? $\bar{\zeta}_1$
2. What is the elasticity of lottery demand with respect to smaller prizes? $\bar{\zeta}_{2+}$
3. What is the price elasticity of lottery demand? $\bar{\zeta}_p$

II. Survey evidence: characteristics of people who buy lotteries

1. How does lottery spending vary across incomes?
2. How does lottery spending vary with behavioral biases?

Data: observational evidence

Primary data

- Powerball and Mega Millions biweekly prizes and ticket sales, 2010–2020
  - Exploit high-frequency jackpot variation + occasional format changes for identification.
- Jackpot, ticket sales data from LottoReport.com
- Second prize amounts (in CA) from California Lottery website

Supplemental data

- Lottery ticket sales by week $\times$ state $\times$ game since 1994
- Useful for cross-game substitution analysis
- Purchased from La Fleur's industry research group

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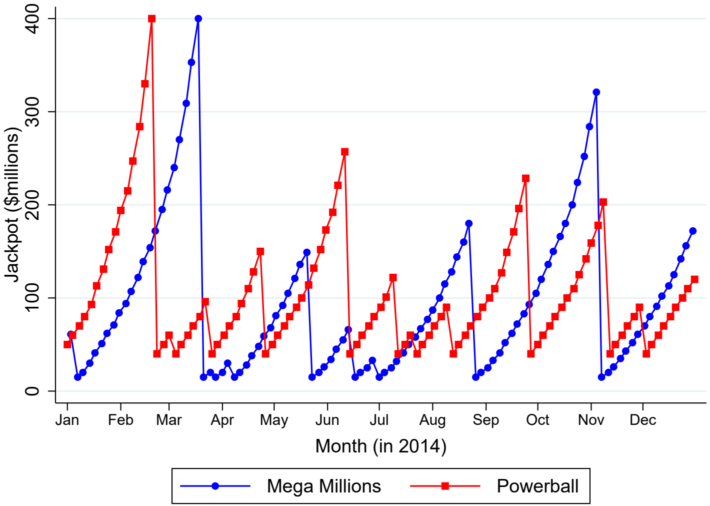
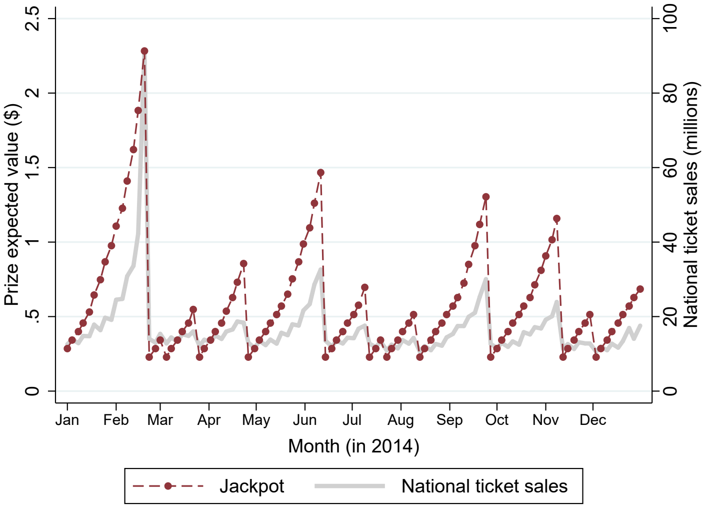
Large variation in lotto jackpots over time



- “Parimutuel” jackpots: pool accumulates until won.
- Here: Mega Millions and Powerball, 2014.

Sales covary with jackpot



- Powerball jackpot expected value and sales, 2014.
- Results are very similar if we formally account for jackpot splitting.

Estimation strategy for $\bar{\zeta}_1$: semi-elasticity of demand with respect to jackpot

Regression framework

$$\ln \bar{s}_{jt} = \bar{\zeta}_1 \pi_{1j} w_{1jt} + \boldsymbol{\beta}_H \boldsymbol{H}_{j,t-1} + \boldsymbol{\xi}_{jt} + \epsilon_{jt}$$

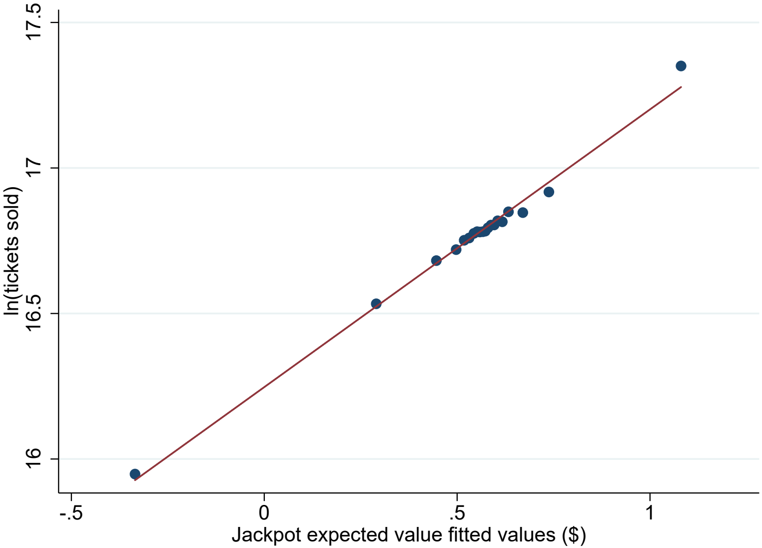
$\bar{s}_{jt}$ = ticket sales for game j in half-week t $\Rightarrow$ $\bar{\zeta}_1$ = jackpot EV semi-elasticity from prize variation

$\boldsymbol{H}_{j,t-1}$ = flexible controls for history of ticket sales, prizes

$\boldsymbol{\xi}_{jt}$ = fixed effects: game×format, game×regional-coverage, game×quarter-of-sample, game×weekend

- Challenge: jackpot w_{1jt} depends mechanically on current sales, and (through rollover probability) on previous sales.
- Strategy: exploit pure randomness in lotto drawing. Instrument for w_{1jt} with jackpot forecast innovation based on random rollover realization, conditional on previous sales. [Details]

Sales vs. jackpot expected value (instrumented)



- Visual representation of jackpot elasticity estimation.
- Absorbs controls in regression equation.
- Linearity suggests constant semi-elasticity (structural model will respect this).

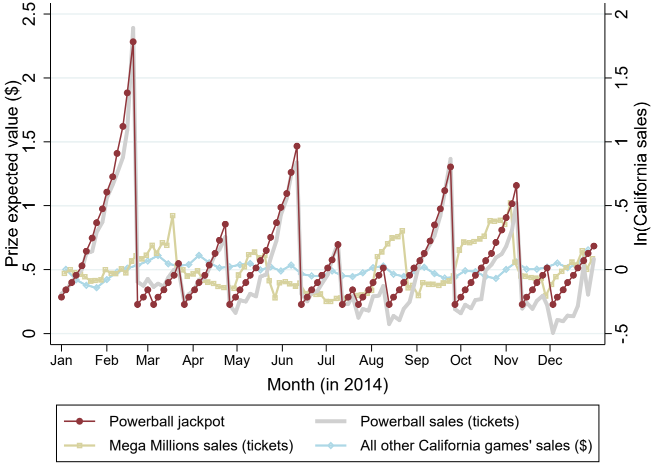
Results for $\bar{\zeta}_1$: semi-elasticity of demand with respect to jackpot

	(1) OLS	(2) IV	(3) IV
Jackpot expected value ($)	0.9984*** (0.0309)	0.9437*** (0.0383)	0.9543*** (0.0374)
Lags in $\boldsymbol{H}$	0	1	4
Quadratic terms in $\boldsymbol{H}$	No	No	Yes
Observations	2,035	2,035	2,035

$$\ln \bar{s}_{jt} = \bar{\zeta}_1 \pi_{1jt} w_{1jt} + \boldsymbol{\beta}_H \boldsymbol{H}_{j,t-1} + \boldsymbol{\xi}_{jt} + \epsilon_{jt}$$

- Point estimate for $\bar{\zeta}_1$: 10 cent increase in jackpot EV raises sales by 9.5%.
- In paper: details on instrument and variation from jackpot evolution vs. format changes.
- Note: theory depends on long-run, all-lottery demand response. Next up: check for cross-game and cross-time substitution.

Negligible substitution across games or across time



- Can use same jackpot variation. When $w_{1jt} \uparrow$, do we see $\bar{s}_{-jt} \downarrow$? (Employ La Fleur's data for non-MM/PB games.)
- Finding: very little cross-game substitution, precisely estimated. [Regression]
- For cross-time substitution: measure response to lagged jackpots, and aggregate up to "jackpot spells" between rollovers. Results are statistically indistinguishable from our primary estimate. [Regression]

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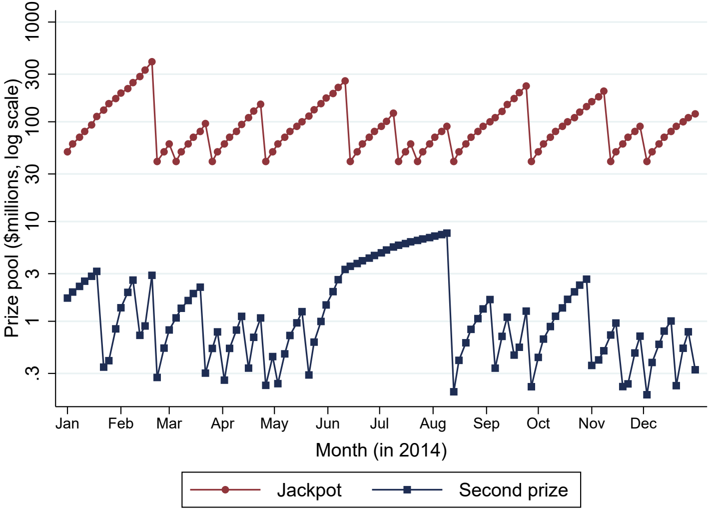
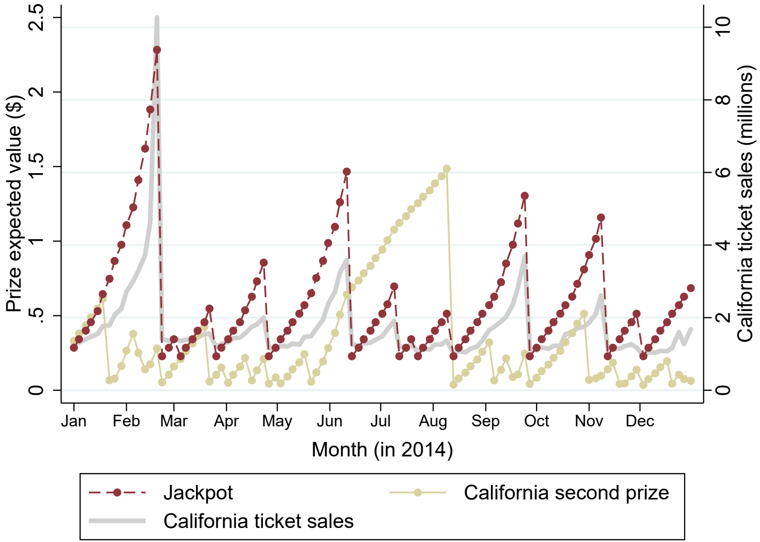
In California: jackpot and 2nd prize pools vary independently



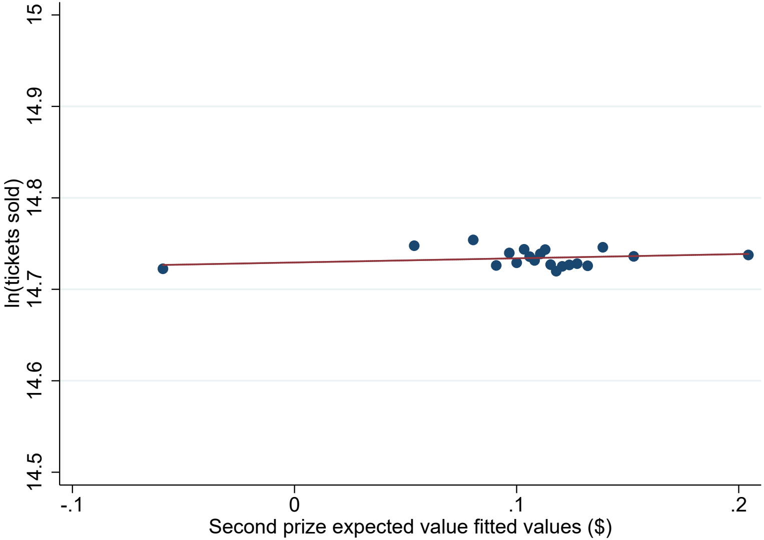
- In CA: unusual legal requirement that *all* prize levels have parimutuel structure.
    - 1996 CA Supreme Court ruling disallowed house-banked games, requiring “lotteries” to be fully parimutuel.
- Powerball and Mega Millions have CA-specific parimutuel sub-jackpot prizes.

Sales are highly responsive to jackpot EV, not to second prize EV



- June – July: ticket EV mostly from large 2nd prize pool.
- Visually: demand highly responsive to jackpot EV, but not to 2nd prize.

CA sales vs. 2nd prize expected value (instrumented)



- Same IV regression specification as before, using CA data, and including w_{2jt}.

Key statistic $\bar{\zeta}_2$: semi-elasticity with respect to second prize EV

	(1) OLS	(2) IV	(3) IV
Jackpot expected value ($)	1.0764***	0.9972***	1.0191***
	(0.0418)	(0.0470)	(0.0494)
2nd prize expected value ($)	0.0441	0.0578	0.0457
	(0.0390)	(0.0744)	(0.0633)
Lags in $\boldsymbol{H}$	0	1	4
Quadratic terms in $\boldsymbol{H}$	No	No	Yes
Observations	1,705	1,701	1,695

$$\ln \bar{s}_{jt} = \bar{\zeta}_1 \pi_{1jt} w_{1jt} + \bar{\zeta}_2 \pi_{2jt} w_{2jt} + \boldsymbol{\beta}_H \boldsymbol{H}_{j,t-1} + \boldsymbol{\xi}_{jt} + \epsilon_{jt}$$

- Second prize semi-elasticity is quantitatively small, statistically insignificant.
- Point estimate: 10 cent increase in 2nd prize EV raises sales by 0.46%.
- Implication: $\bar{\zeta}_2 \ll \bar{\zeta}_1$. Possible explanations: less advertising, probability weighting. (More on this later.)

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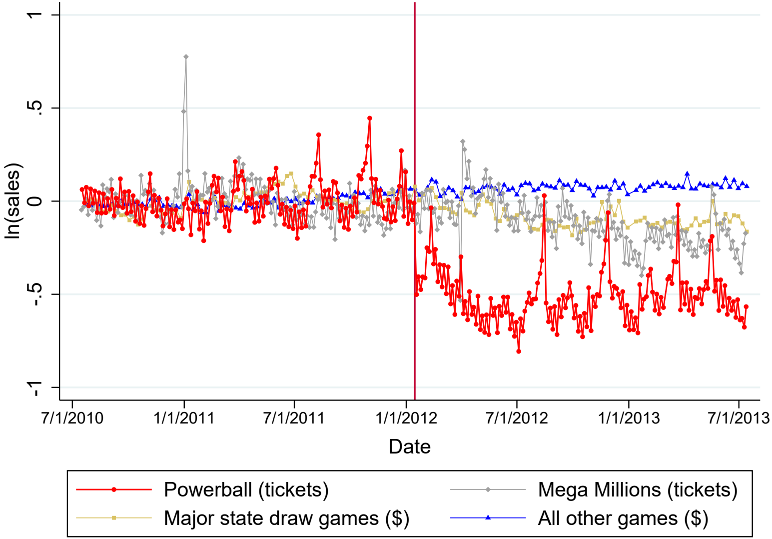
Price elasticity: estimation strategy

- Challenge: unlike prizes, prices (and probabilities) generally constant over time.
- Two key exceptions:
  - January 2012: Powerball ticket price increased \$1 $\rightarrow$ \$2.
  - October 2017: Mega Millions ticket price increased \$1 $\rightarrow$ \$2.

Powerball price change 2012



- Powerball price increased \$1 $\rightarrow$ \$2 in January 2012.
- Here: ticket sales in estimation window around event, residualizing predicted jackpot effect.

Key statistic $\bar{\zeta}_p$: semi-elasticity of lottery demand with respect to price

	(1) Pooled	(2) Powerball	(3) Mega Millions
Price $\times$ 12-month window	-0.4958***	-0.5588***	-0.4305***
	(0.0435)	(0.0573)	(0.0568)
12-month window	0.7412***	0.8565***	0.6223***
	(0.0745)	(0.0971)	(0.0954)
Jackpot expected value ($)	0.8770***	0.8412***	0.9019***
	(0.0459)	(0.0406)	(0.0777)
Observations	625	312	313

$$\ln \bar{s}_{jt} = \bar{\zeta}_p p_{jt} W_{jt} + \beta_1 W_{jt} + \beta_2 \pi_{1jt} w_{1jt} + \xi_j + \epsilon_{jt}$$

- Point estimate: 10 cent rise in price reduces sales by 5.0%.
- W_{jt} = binary indicator for estimation window around price change.

Taking stock: observational evidence

Semi-elasticity point estimates

$$\bar{\zeta}_1 = 0.95, \quad \bar{\zeta}_2 = 0.05, \quad |\bar{\zeta}_p| = 0.50$$

- If consumers are risk-neutral, these semi-elasticities should be about equal.
- If risk-averse, should be more responsive to higher-probability EV changes: $|\bar{\zeta}_p| > \bar{\zeta}_2 > \bar{\zeta}_1$.
- $\bar{\zeta}_1 > |\bar{\zeta}_p|$ suggests substantial “overweighting” of the jackpot probability.
    - Qualitatively consistent with the probability weighting proposed by Kahneman and Tversky (1979) and many others.
    - Yet $\bar{\zeta}_1 \gg \bar{\zeta}_2$ is quantitatively inconsistent with typical probability weighting functions estimated in the lab. (More on this later.)

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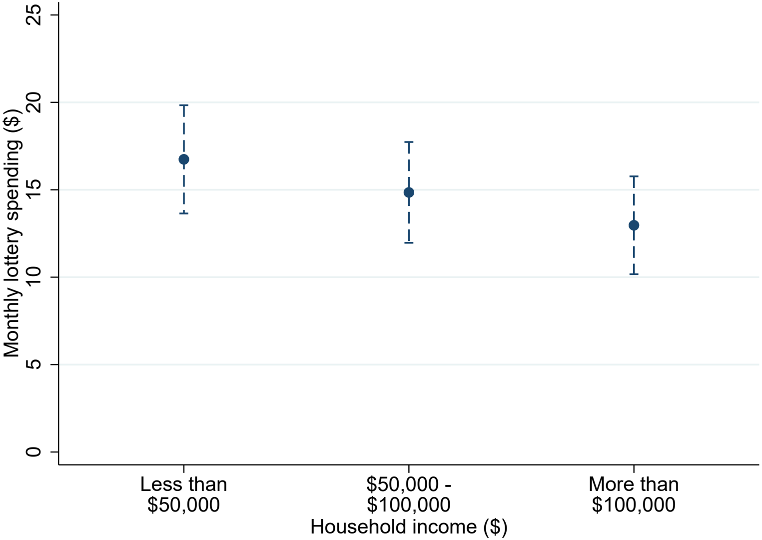
Data: new survey of lottery consumption

	Mean	Std. dev.	Min	Max
Household income ($000s)	72.12	53.08	5	250
Years of education	14.32	2.26	4	20
Age	48.82	16.79	18	91
1(Male)	0.50	0.50	0	1
1(White)	0.66	0.47	0	1
1(Black)	0.11	0.31	0	1
1(Hispanic)	0.16	0.36	0	1
Household size	3.04	1.62	1	6
1(Married)	0.53	0.50	0	1
1(Employed)	0.63	0.48	0	1
1(Urban)	0.83	0.37	0	1
1(Attend church)	0.36	0.48	0	1
Political ideology	3.83	1.59	1	7

- 2,879 respondents from AmeriSpeak survey panel, fielded early 2020.
- Resampled in early 2021 to understand test-retest reliability.

Key statistic $s(\theta)$: lottery spending across incomes



- Spending declines modestly as income rises.
- Heavily skewed: top 10% of spenders account for 56% of spending.
- Consistent with 1998 NORC survey of gambling consumption.
- Two measures of causal income effects, do not explain profile.

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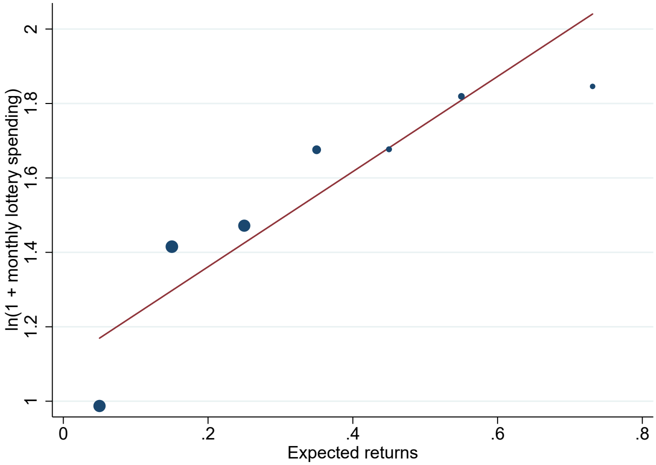
Survey questions to assess bias

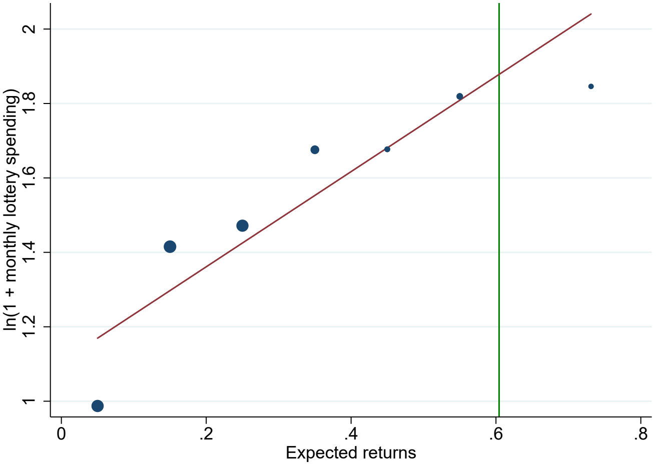
- **Expected returns**: *What percent of the total spending on lottery tickets do you think is given out in prizes?*
- **Self-control**: *Do you feel you should play the lottery less/same/more than you do now?*
- **Financial literacy**: share of correct answers to set of standard financial literacy questions
- **Statistical mistakes**: gambler's fallacy, law of small numbers, expected value calculation
- **Overconfidence**: *"For every $1000 you spend, how much do you think you would win back in prizes, on average?"* vs. *"How much would average player win back?"*
- **Predicted life satisfaction**: *How much do you think $100k more in winnings raised reported well-being?*

Lottery expenditures across perceived returns to lottery



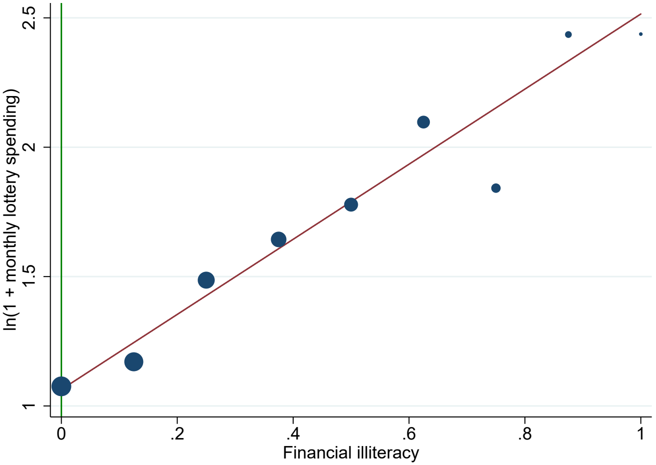
- Plot expenditures across bias proxy.

Lottery expenditures across perceived returns to lottery



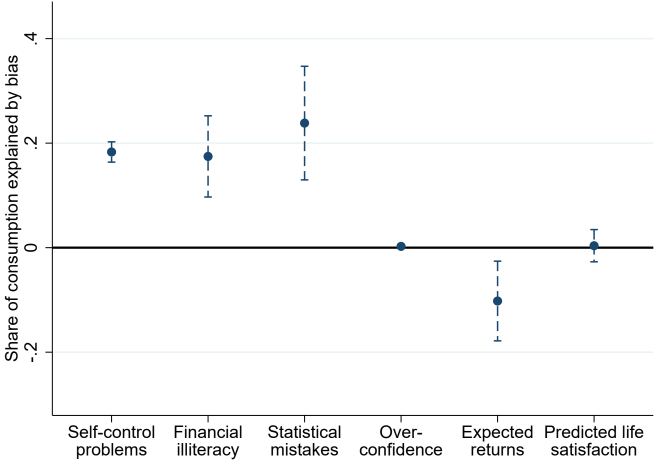
- Plot expenditures across bias proxy.
- Green line indicates "normative" (unbiased) response.
- On average people substantially *underestimate* payout: unlikely source of overconsumption bias. (See also Clotfelter & Cook 1999)

Lottery expenditures by financial illiteracy



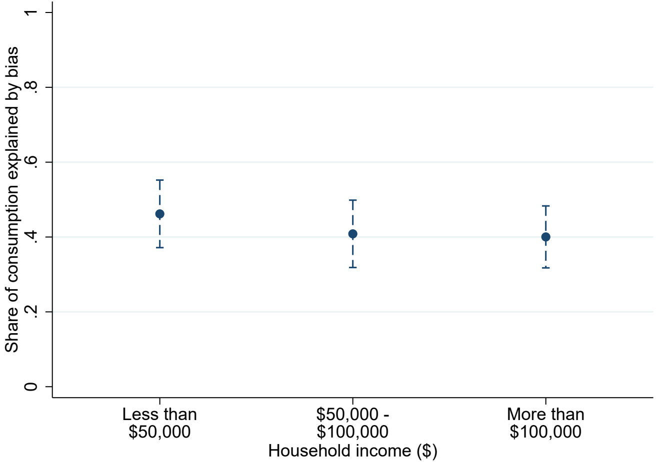
- In contrast, financial illiteracy suggests substantial overconsumption.
- Robust relationship, quantitatively important.
- Substantial heterogeneity in population.

Biases contributing to overconsumption



- Compute counterfactual spending for each consumer if they were unbiased on each dimension, while retaining own demographics, normative preferences.
- Financial illiteracy and statistical mistakes are primary drivers.
- Can use these to predict latent “unbiased consumption” for each consumer. (Caution: causal interpretation!)

Key statistic: quantity effect of bias



- Average person overconsumes lotteries by 43% due to bias.
- Using price elasticity estimate, can convert this *quantity effect* to money-metric bias estimate.

Empirics: summing up

I. Observational evidence: characteristics of lotteries that affect demand

- $\bar{\zeta}_1 = 0.95, \ \bar{\zeta}_2 = 0.05, \ |\bar{\zeta}_p| = 0.50$
- Indicates substantial overweighting of jackpot probability, but not second prize.
- Nonparametrically identifies willingness to pay for higher jackpot.

II. Survey evidence: characteristics of people who buy lotteries

- Lottery purchases are modestly declining with income.
- Likely that some (but not all) probability weighting is due to innumeracy, confusion, etc.

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Up next

- Structural estimates of probability weighting
- Optimal policy and welfare implications (using theory formulas)

Structural Estimation

Conceptual framework

Demand comes from aggregating discrete decisions (narrow bracketing), with iid taste shocks ε_t:

$$u(p, w_k, \pi_k) = \sum_{k=1}^{K} \Phi_k m(w_k) - \varepsilon_t$$

$$s(p, w_k, \pi_k) = F_\varepsilon \left[\sum_{k=1}^{K} \Phi_k m(w_k) - p \right]$$

- Flexible decision weights on prizes $k \Rightarrow$ nests [cumulative] prospect theory, expected utility, ...
- Concave value function $m(\cdot)$ is normalized by marginal utility of consumption $\Rightarrow$ money metric. (Baseline: CRRA log utility of wealth.) [Details]
- Useful feature: ratios $\bar{\zeta}_1/\bar{\zeta}_p$ and $\bar{\zeta}_2/\bar{\zeta}_p$ identify Φ_1 and Φ_2.
    - Interpreted as probability weighting function $\Phi(\pi)$, these are two points for calibration.
    - Inconsistent with standard continuous parameterizations; consistent with Chateauneuf et al. (2007): $\Phi(\pi) = b_0 + b_1\pi$ [Details]

Details of structural model

$$s(p, w_k, \pi_k) = F_\varepsilon \left[\sum_{k=1}^{K} \Phi(\pi_k) m(w_k) - p \right]$$

Structural assumptions

- CRRA utility over wealth (baseline = log), $m(\cdot)$ normalized by marginal util. of consump.
- Representative lottery: current Powerball. Overhead = \$0.20/ticket. Winnings taxed 30%.
- Chateauneuf et al. (2007) weighting function: $\Phi(\pi) = b_0 + b_1\pi$
- Distribution of F_ε chosen so semi-elasticity of demand w.r.t. jackpot is constant.
- Share χ of overweighting $\Phi(\pi_k) - \pi_k$ attributed to bias, calibrated to fit quantity effect of bias.
- With heterogeneity: 3×3 discretized grid of income, lotto consumption.
  - Welfare weights declining with income: $g(\theta) \propto 1/c(\theta)$.
  - Heterogeneous $s(\theta)$, $\chi(\theta)$ from survey analysis; homogeneous $\bar{\zeta}$.

Parameter estimates

Representative agent model						
	b_0	b_1	χ			
	1.12×10^{-6}	0.27	0.41			
Heterogeneous agent model						
	Below median s			Above median s		
	b_0	b_1	χ	b_0	b_1	χ
Low incomes	2.16×10^{-6}	0.36	0.31	2.16×10^{-6}	0.36	0.42
Middle incomes	1.08×10^{-6}	0.27	0.21	1.08×10^{-6}	0.27	0.4
High incomes	5.8×10^{-7}	0.23	0.2	5.8×10^{-7}	0.23	0.43

- Probability weights $\Phi(\pi) = b_0 + b_1\pi$
- Recall that jackpot odds are $\sim 3 \times 10^{-8}$
- χ is share of $\Phi(\pi) - \pi$ due to bias

Optimal Policy and Welfare

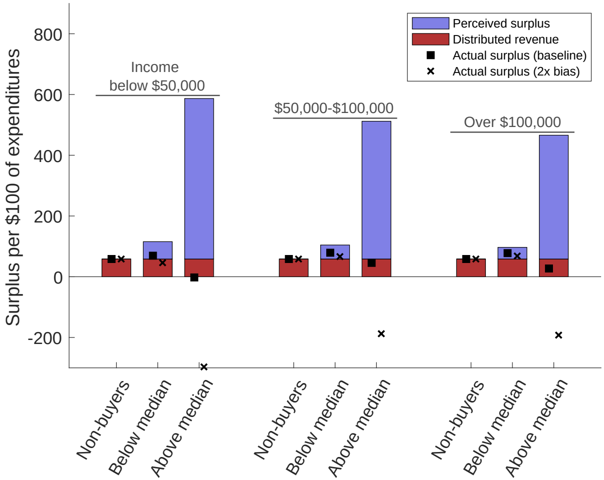
Two ways to compute optimal policy

1. Using sufficient statistics formulas.
   - Pros: Easy and transparent.
   - Cons: Requires approximations (elasticities do not vary across the policy space).
2. Using structural model.
   - Pros: Don't need to restrict to "local" policy perturbations.
   - Cons: Need to believe all of our structural assumptions.

Luckily, answers are very similar both ways.

Today will just show you a few numbers from simulations with structural model.

Are lotteries welfare-enhancing?



- Estimated surplus per $100 of lotto spending, assuming revenues evenly distributed.
- Substantial perceived surplus, most among heavy consumers.
- Under our estimated bias, average surplus modestly positive.
- Negative surplus if bias were 2x our estimate.
- Suggests alternative policy instruments like quantity caps may be beneficial.

Optimal lottery structure

- Optimal implicit+explicit tax rate: 72% (compare to $\sim$60% for PB, MM; lower for other games)
- Price: \$4.73 (compare to \$2)
- Jackpot expected value: \$0.92 (compare to \$0.63, net-of-tax)
- Intuition: substantial normative value from jackpot, relative to price.

Sensitivity

	Ticket price ($)	Average jackpot expected value ($)	Effective tax rate
1. Baseline	4.73	0.92	0.72
2. Completely unbiased	1.97	1.58	-0.01
3. 50 percent more biased	5.16	0.56	0.81
4. CRRA = 0.8	3.89	0.92	0.65
5. CRRA = 1.5	9.59	0.87	0.87
6. Weaker redistribution	4.75	0.94	0.71
7. Stronger redistribution	4.70	0.88	0.72
8. Higher value of $\bar{\zeta}_2 = \bar{\zeta}_1$	5.05	0.96	0.73
9. All bias is on jackpot	4.76	0.92	0.72
10. Variable jackpot	4.47	0.73	0.72
11. Measurement error correction	4.86	0.86	0.74
12. Same bias share across incomes	4.70	0.92	0.71
13. Same bias share for everyone	3.64	1.22	0.55
14. Steeper decline across incomes	4.62	0.88	0.72

Conclusion

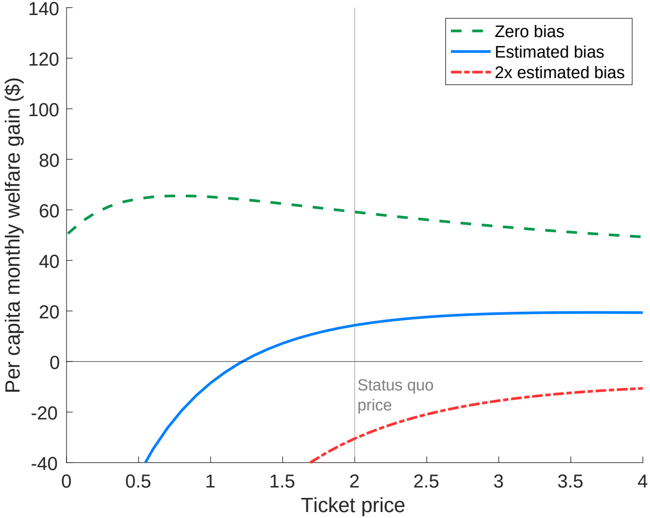
Recap

1. Derivation of new "optimal attribute" formula and application to lotteries.
   - Extends behavioral public economics policies to non-price attributes.
2. New descriptive evidence on lottery consumption, elasticities, and behavioral biases.
   - New estimates of demand elasticities w.r.t. price and key prize attributes.
   - Bias explains modest share of total consumption.
   - Consumption mildly declining with income $\Rightarrow$ lottery sales are not particularly regressive.
3. Calibrated model to explore welfare and policy counterfactuals.
   - Lotteries raise welfare on average in baseline, but little/no surplus for heavy consumers.
   - Optimal implicit+explicit tax is in the neighborhood of status quo for current multi-state games.
   - These findings may motivate alternative instruments, like quantity limits.

Thank you!

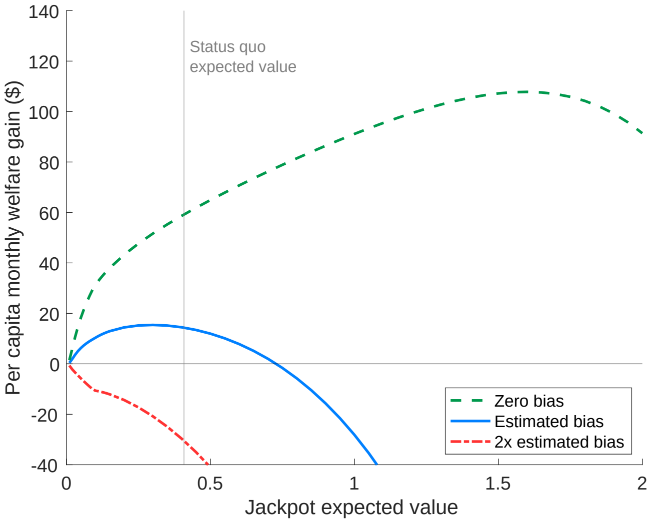
Appendix

Welfare gains across price



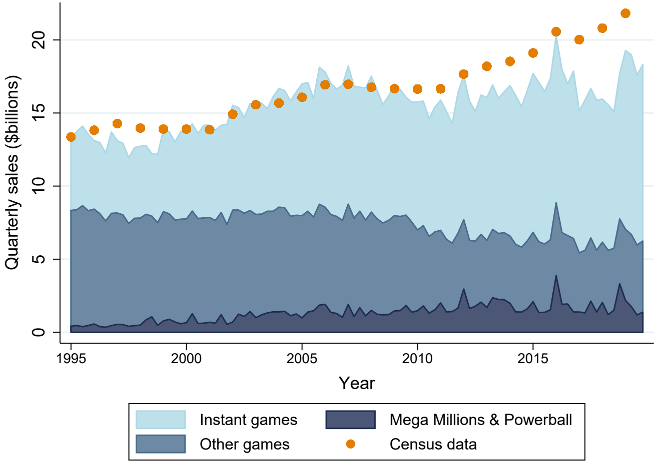
- Welfare gain across p (fixing w_k, π_k)
- If unbiased, $p^* \approx$ marginal cost (no implicit tax)
- With estimated bias: $p^* > MC$
- Large bias: low prices are welfare-reducing.

Welfare gains across jackpot expected value



- Welfare gain across $\pi_1 w_1$ (fixing p).
- If unbiased, raise jackpot so $p^* \approx$ marginal cost (no implicit tax).
- With estimated bias: lower jackpot (raise implicit tax).

Lottery composition over time



- Real lottery expenditures rising over time.
- Market shifting toward instant games.
- Powerball and Mega Millions are a growing share of lotto-style games.

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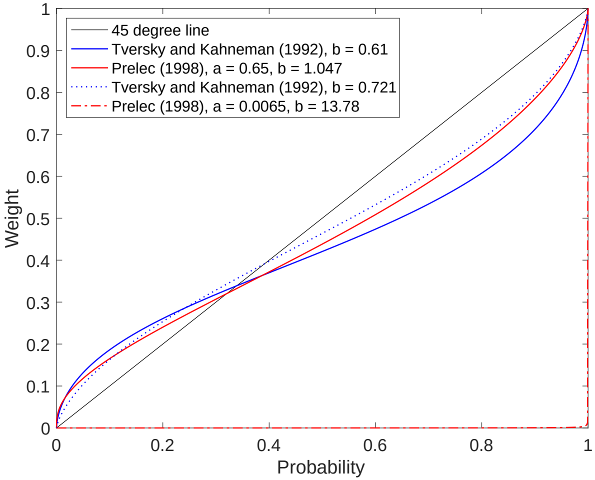
Choice of probability weighting function



- Consider common probability weighting function parameterizations.
- Zoom ($\swarrow$) to see implied points $\Phi(\pi_1)$ and $\Phi(\pi_2)$.
- Incentivized experiments (and Kahneman-Tversky surveys) don't study magnitudes in this range.
- Ranking $\bar{\zeta}_1 > |\bar{\zeta}_p| > \bar{\zeta}_2$ is inconsistent with "standard" parameterizations, but consistent with function in Chateauneuf et al. (2007): $\Phi(\pi) = b_0 + b_1\pi$

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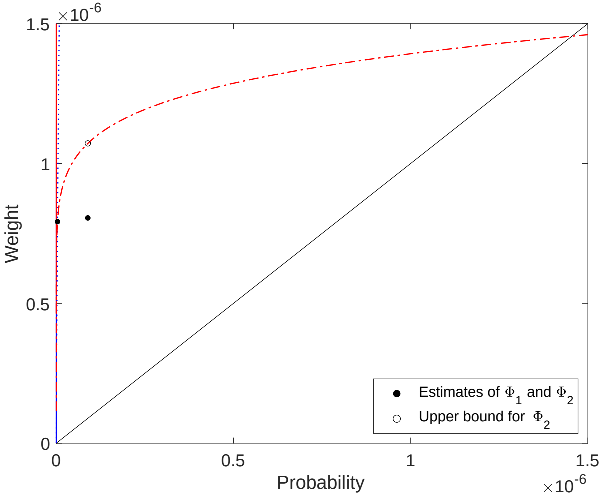

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Powerball format changes

	Powerball Prizes and Odds		
Start date	January 7, 2009	January 15, 2012	October 7, 2015
Ticket price	$1	$2	$2
Format	5/59 + 1/39	5/59 + 1/35	5/69 + 1/26
Jackpot (average)	$66 million	$112 million	$170 million
Reset value	$20 million	$40 million	$40 million
Probability	1/195,249,054	1/175,223,510	1/292,201,338
Expected value	$0.34	$0.64	$0.58
Second prize	$200,000	$1 million	$1 million
Probability	1/5,138,133	1/5,153,633	1/11,688,054
Expected value	$0.039	$0.19	$0.086
Third prize	$10,000	$10,000	$50,000
Probability	1/723,145	1/648,976	1/913,129
Expected value	$0.014	$0.015	$0.055
	. . .	. . .	. . .
Ninth prize	$3	$4	$4
Probability	1/62	1/55	1/38
Expected value	$0.049	$0.072	$0.10
Probability, any prize	1/35	1/32	1/25
Lower prize expected value	$0.17	$0.36	$0.32
Total expected value	$0.51	$1.00	$0.90

- Three Powerball format changes in our data. (Mega Millions has three more.)
- Changes to price, probabilities, and prizes.

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Jackpot elasticity estimation: empirical strategy

- Jackpot size w_{1jt} depends on whether prize rolls over after drawing $t-1$. Define:

$$r_{j,t-1} = \begin{cases} 1 & \text{if lotto } j\text{'s jackpot rolls over at } t-1 \\ 0 & \text{if lotto } j\text{'s jackpot is won} \end{cases}$$

$\Rightarrow \mathbb{E}\left[r_{j,t-1}\right]$ depends on ticket sales $\bar{s}_{j,t-1}$; realization is perfectly random *conditional* on s_{jt-1}.

- Rollover realization is highly predictive of actual jackpot w_{1jt}. Define *jackpot forecast*

$$z_{jt} = \begin{cases} (1+\iota_j)w_{1jt-1} & \text{if } r_{j,t-1}=1, \text{ where } \iota_j \text{ is } j\text{'s average jackpot growth rate} \\ \underline{w}_j & \text{if } r_{j,t-1}=0 \end{cases}$$

Again, $\mathbb{E}\left[z_{jt}\right]$ depends on $t-1$ info, but conditional on that info, realization is perfectly random.

- Assume sales $\bar{s}_{jt}$ are a function of current jackpot EV $\pi_{1j}w_{1jt}$ and historical information:

$$\ln \bar{s}_{jt} = \bar{\zeta}_1 \pi_{1jt} w_{1jt} + \boldsymbol{\beta}_H \boldsymbol{H}_{j,t-1} + \boldsymbol{\xi}_{jt} + \epsilon_{jt}$$

  - $\boldsymbol{H}_{j,t-1}$: vector of lagged ln(sales) and jackpot EV for $l = t-4, \ldots, t-1$, and their squares.
  - Identifying assump.: controls for things predictable about w_{jt} up to random drawing: $z_{jt} \perp \epsilon_{jt} | \boldsymbol{H}_{j,t-1}$.

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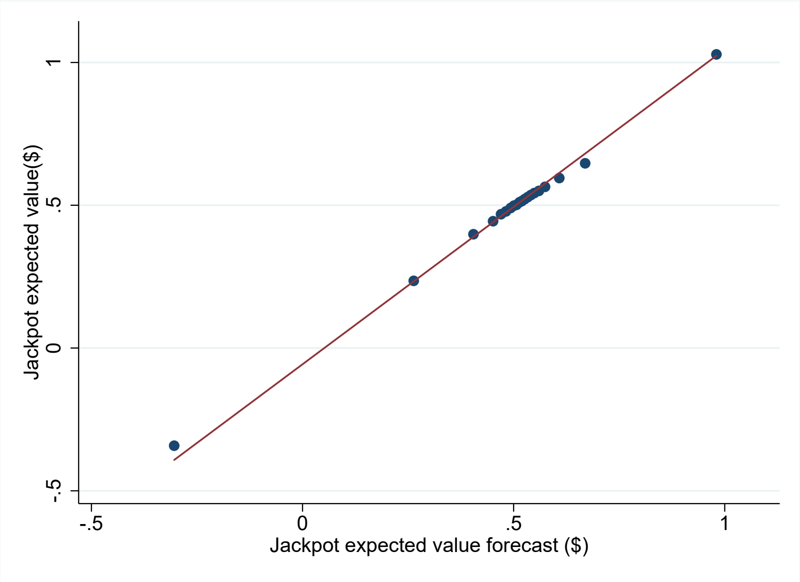
Actual jackpot EV vs. jackpot EV forecast instrument



- Visual “first stage” for IV regression.
- Binscatter (avg within vingtiles) of actual jackpot expected value $\pi_{jt} w_{jt}$.
- Horizontal axis: jackpot expected value forecast $\pi_{jt} z_{jt}$.

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Substitution across games

	(1) OLS	(2) IV
Own game jackpot expected value ($)	58.00***	50.59***
	(7.83)	(6.40)
Other game jackpot expected value ($)	0.03	-0.03
	(1.31)	(1.25)
Lags in ***H***	0	4
Quadratic terms in ***H***	No	Yes
Observations	2,035	2,035
Dependent variable mean	23.0	23.0

$$\bar{s}_{jt} = \bar{\zeta}_j \pi_{1jt} w_{1,j,t} + \bar{\zeta}_c \pi_{1,-j,t} w_{1,-j,t} + \boldsymbol{\beta}_H \boldsymbol{H}_{j,t-1} + \boldsymbol{\beta}_{-j,H} \boldsymbol{H}_{-j,t-1} + \xi_{jt} + \epsilon_{jt}.$$

- Measures substitution between Powerball and Mega Millions.
- Demand measured in levels, to estimate diversion ratio.
- Negligible demand spillover between games.

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Substitution across games

	(1) Mega Millions & Powerball	(2) Major state draw games	(3) Instant games	(4) Other state-level games
Jackpot expected value ($)	267.90***	0.96***	0.00	-0.62
	(39.63)	(0.17)	(2.67)	(2.93)
Observations	508	508	508	508
Dependent variable mean	108.2	12.4	509.5	669.2

$$\bar{s}_{jt} = \bar{\zeta}_c \pi_{1t} w_{1t} + \boldsymbol{\beta}_H \boldsymbol{H}_{t-1} + \xi_{jt} + \epsilon_{jt}$$

- Measures substitution between (Powerball + Mega Millions) and other games, from La Fleur's data.
- Again, negligible demand spillover between games. (OLS above; IV is noisier, but still rules out diversion ratios > 6%.)

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Substitution across time

	(1) OLS	(2) IV	(3) OLS	(4) IV	(5) OLS	(6) IV
Jackpot expected value ($)	0.8941***	0.8819***	0.9197***	0.8621***	0.9599***	1.0031***
	(0.0427)	(0.0390)	(0.0505)	(0.0393)	(0.1386)	(0.1328)
Jackpot expected value, $t-1$ ($)			0.1516***	0.1111***		
			(0.0373)	(0.0354)		
Jackpot expected value, $t-2$ ($)			0.0870**	0.0635**		
			(0.0356)	(0.0310)		
Jackpot expected value, $t-3$ ($)			0.0477**	0.0523***		
			(0.0196)	(0.0193)		
Lags in $\boldsymbol{H}$	0	1	0	1	0	1
R^2	0.87	0.90	0.88	0.88	0.74	0.80
Observations	193	191	187	184	61	57

$$\ln \bar{s}_{jt} = \sum_{l=0}^{L} \bar{\zeta}_{1l}\pi_{1j,t-l}w_{1j,t-l} + \boldsymbol{\beta}_H \boldsymbol{H}_{j,t-1} + \xi_{jt} + \epsilon_{jt},$$

- Aggregate up to "jackpot spells" between rollovers: no measurable effect on $\bar{\zeta}_1$ estimate.
- Lagged jackpots suggest modest complementarity, no evidence of cross-time substitution.

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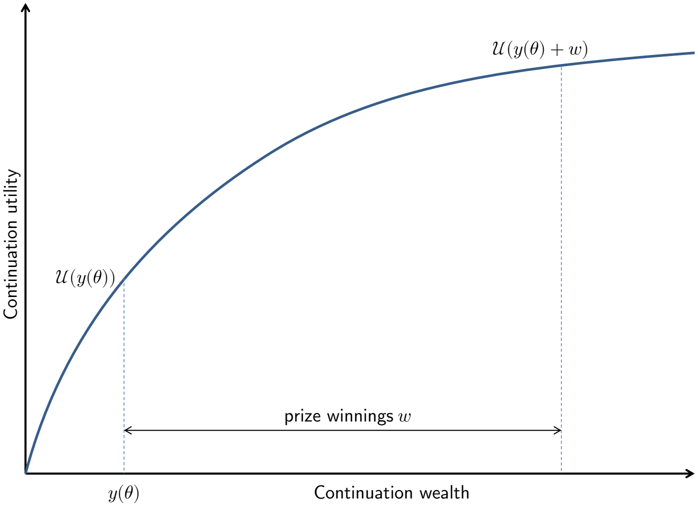
Foundation for value function $m(w)$



- $\mathcal{U}$ represents continuation utility from starting next period with a given wealth.

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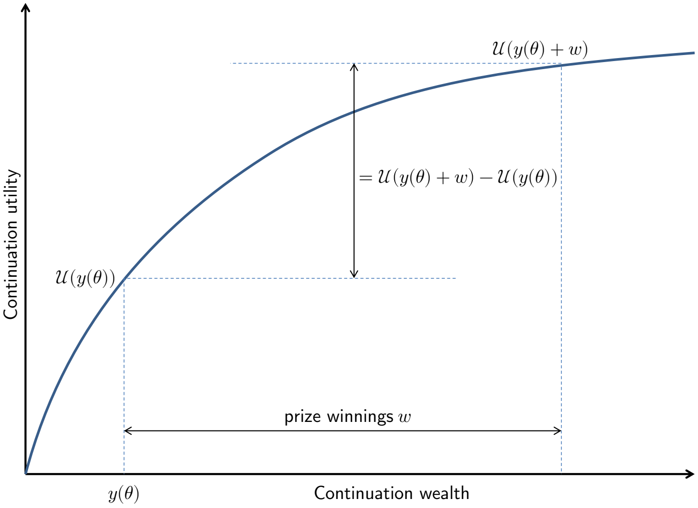

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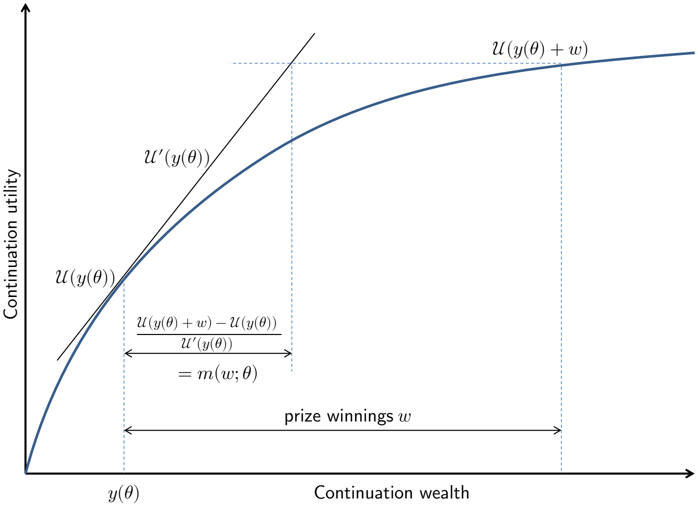

- $\mathcal{U}$ represents continuation utility from starting next period with a given wealth.
- $m(w)$ deflates gain of w due to curvature of $\mathcal{U}$.
- Ex: with log utility, \$1m continuation wealth: $m(\$100m) = \$4.6m$. (Curvature deflates by 95%.)

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Thank you!