ReadMe

We simulate optimal nonlinear tax policies for the United States under various redistributive preferences using the fixed-point algorithm detailed below. We implement the algorithm in MATLAB using the files detailed in Section 2.

1 Details of the Fixed-Point Algorithm

The algorithm is implemented in the following general steps, with further details provided below:

1. Import income and consumption distribution data to infer an underlying income-earning ability distribution, calibrate the status quo tax system, and determine the policymaker's revenue requirement. Initialize the income and consumption distributions at the status quo and compute the marginal social welfare weight for each ability type under the redistributive preferences of the policymaker.

2. Update the marginal tax rate schedule with an alternative tax schedule obtained from the policymaker's first-order condition.

3. Update the income distribution with the optimal labor supply choice under the updated marginal tax rate schedule using the first-order condition in the individual's utility maximization problem. Update the lump-sum grant and consumption distribution.

4. Repeat steps 2 and 3 until the updating of the marginal tax rate schedule and the difference between the proposed alternative tax schedule and the current tax schedule are trivially small.

5. Check the second-order condition that income is non-decreasing in income-earning ability at the fixed point.

The resulting fixed-point marginal tax rate schedule satisfies the necessary first-order condition for optimality. This process does not guarantee convergence to a fixed-point tax schedule. However if it does converge and the second-order condition is satisfied, this represents the optimal tax schedule. This result depends on the fact that the individual utility functions satisfy the single-crossing property.

1.1 Step 1: Initialize economy at the status quo and compute marginal social welfare weights

We begin by importing the discrete status quo U.S. market income distribution $H_{T_{US}}(y)$, consumption distribution, and corresponding probability mass function $f(\theta)$ for unidimensional types $\theta \in \Theta \subset \mathbb{R}^{+}$ from Piketty, Saez, and Zucman (2018).[1] We assume a one-to-one mapping between θ and y. We adjust $H_{T_{US}}(y)$ by imposing a Pareto tail above the 90th percentile income,[2] use linear interpolation to adjust consumption accordingly for income-earners above that threshold, and generate a cumulative distribution function of types $F(\theta)$. Next, we calibrate the current U.S. tax system. We obtain the status quo marginal tax rate schedule $T'_{US}(\theta)$ by mapping the empirical income and consumption distributions to implied marginal tax rates. The methodology in Piketty, Saez, and Zucman (2018) ensures that pre-tax and post-tax national income are equal, in order to reconcile with national accounts. As a result, under this calibration we have an exogenous revenue requirement of $R = 0$.[3]

We initialize the income and consumption distributions and the marginal tax rate schedule at the status quo. We then use $H_{T_{US}}(y)$, $T'_{US}(\theta)$, and the structural elasticity parameter σ to compute an underlying implied income-earning ability distribution using the first-order condition from the individual's utility maximization problem. We assume a homogenous value of 0.33 for the parameter σ across the income-earning ability distribution,[4] based on the estimate of Chetty (2012) of the compensated intensive margin labor supply elasticity. We assume the individual utility function takes the Type 1 form from

[1] We convert the income and consumption distributions from 2014 to 2020 dollars using the Consumer Price Index for all urban consumers from the U.S. Bureau of Labor Statistics.

[2] The Pareto parameter is chosen such that the local parameter just below the 90th percentile income is equal to the constant parameter above the 90th percentile income. We gradually transition from the empirical income below the 90th percentile to the Pareto tail over three income cells. We also modify $f(\theta)$ along the Pareto tail by defining additional types at low and middle incomes and fewer types at high incomes to facilitate convergence to the fixed-point tax schedule.

[3] To enforce this equality in the presence of our numerical integration, which generates approximations, we rescale status quo consumption by a constant factor (about 1.01) so national income and consumption are exactly equal in the status quo. This does not preclude deficit spending; rather, it requires deficits to be accounted for in income. Piketty, Saez, and Zucman (2018) include deficit-funded transfers in consumption and adjust incomes by "allocat[ing] 50% of the deficit proportionally to taxes paid, and 50% proportionally to government spending received."

[4] See equation (4) for the precise relationship between the parameter σ and the elasticity of taxable income ζ.

Saez (2001) with the logarithmic transformation:

$$U\left(c,y;\theta\right)=\ln\left(c-\frac{1}{1+1/\sigma}\left(\frac{y}{w(\theta)}\right)^{1+1/\sigma}\right). \tag{1}$$

For type θ with status quo income $y_{US}(\theta)$, the implied ability is

$$w(\theta)=\left(y_{US}(\theta)(1-T'_{US}(\theta))^{-\sigma}\right)^{\frac{1}{1+\sigma}}. \tag{2}$$

Finally, we compute a fixed marginal social welfare weight for each type as $g(\theta)=\frac{\Phi'(U(c_{US}(\theta),y_{US}(\theta);\theta))}{\lambda}$, where $c_{US}(\theta)$ is type θ's status quo level of consumption, $\Phi(x)=x$ in our baseline specification, and λ is the marginal value of public funds, with $\lambda=\int_{\Theta}\Phi'(U(c_{US}(\theta),y_{US}(\theta);\theta))dF(\theta)$. [5,6] In the specifications where the policymaker has stronger and weaker redistributive preferences, $\Phi(x)=x^{-4}$ and $\Phi(x)=x^{-1/4}$, respectively. In the specification where the policymaker's redistributive preferences rationalize the current U.S. tax system, Φ is a function of the status quo income distribution, marginal tax rate schedule, and elasticity parameter σ.

1.2 Step 2: Update the marginal tax rate schedule

We now employ the result that the optimal nonlinear tax satisfies:

$$\frac{T^{*\prime}(\theta)}{1-T^{*\prime}(\theta)}=\frac{\int_{s=\theta}^{\infty}\left(1-g(s)\right)h_T(y(s))dy(s)}{y(\theta)\zeta(y(\theta);\theta)h_T(y(\theta))}, \tag{3}$$

with endogenous income densities $h_T(y(\theta))$ and elasticity of taxable income $\zeta(y(\theta);\theta)$. The first-order condition from individual i's utility maximization problem is $(1-T'(\theta^i))-\left(\frac{y(\theta^i)}{w(\theta^i)}\right)^{1/\sigma}\cdot\frac{1}{w(\theta^i)}=0$. Implicitly differentiating with respect to the marginal "keep rate" $1-T'(\theta^i)$, we can solve for the local labor supply response:

$$\zeta(y(\theta);\theta)=\frac{\partial y(\theta)}{\partial(1-T'(\theta))}\cdot\frac{1-T'(\theta)}{y(\theta)}=\frac{\sigma}{1+\frac{T''(\theta)}{1-T'(\theta)}\sigma y(\theta)}. \tag{4}$$

To update the marginal tax rate schedule in the simulations, we use the fact that $\zeta(y(\theta);\theta)=\frac{\partial y(\theta)}{\partial(1-T'(\theta))}\cdot\frac{1-T'(\theta)}{y(\theta)}$ and that $\int_{s=0}^{\infty}\left(1-g(s)\right)h_T(z(s))dz(s)=0$ to rewrite the condition in equation (3) as

[5] We assume there are no income effects, simplifying the computation of the marginal value of public funds and marginal social welfare weights. We verify that $\int_{\Theta}g(\theta)dF(\theta)\approx 1$.

[6] All integrals are computed numerically by the trapezoidal method, using the *trapz* function in MATLAB.

$$T^{*\prime}(\theta) = \frac{\int_{s=0}^{\theta} \left(g(s) - 1\right) h_T(y(s)) dy(s)}{\frac{\partial y(\theta)}{\partial(1 - T^{*\prime}(\theta))} h_T(y(\theta))}. \tag{5}$$

We compute the alternative tax schedule from the right-hand side of equation (5). We use the current tax schedule in place of $T^{*\prime}(\theta)$ on the left-hand side. We update the marginal tax rate schedule as the weighted average of the alternative tax schedule and the current tax schedule. We apply a weight of .001 to the alternative tax schedule to aid in a gradual progression towards convergence to the fixed-point tax schedule. We also impose an upper bound of 1.0 and a lower bound of −0.1 on the alternative tax schedule to facilitate convergence; these limits do not bind at the optimum in any of our specifications.

1.3 Step 3: Update the labor supply, consumption, and lump-sum grant for each type

We rearrange the first-order condition from the individual's utility maximization problem, yielding

$$y(\theta) = w(\theta)^{(1+\sigma)} \cdot (1 - T'(\theta))^{\sigma} \tag{6}$$

and update the labor supply of each type from the right-hand side. Next, we simply update the lump-sum grant to ensure that the budget constraint is satisifed (i.e., $\int_{\Theta} T(y(\theta);\theta) h_T(y(\theta)) dy(\theta) = R$). We then update consumption in each state as $c(\theta) = y(\theta) - T(y(\theta);\theta)$ using the updated income distribution, lump-sum grant, and marginal tax rate schedule.

1.4 Step 4: Converge to the fixed-point tax schedule

We repeat steps 2 and 3 until (i) the updating of the marginal tax rate schedule is trivially small and (ii) the difference between the proposed alternative tax schedule and the current tax schedule is trivially small. For criterion (i), we define a trivially small update as occurring when the magnitude of the vector of differences between the current and updated tax schedules is less than 10^{-5}. For criterion (ii), we define a trivially small difference as present when the percent difference in marginal tax rates at all income levels is less than .01 percent.[7] Once these criteria are satisfied, the procedure is complete and the resulting tax schedule is the fixed-point tax schedule.

[7] We exclude the tax rate for the top income earners—which is expected to be zero—when checking this criterion to avoid division by zero. See Seade (1977) for the "zero at the top" result that arises when using a bounded ability distribution.

1.5 Step 5: Check for optimality of the fixed-point tax schedule

Our final step is to confirm that the fixed-point tax schedule is the optimal nonlinear tax schedule. We check the second-order condition that the incomes corresponding to the fixed-point tax schedule are non-decreasing in income-earning ability type.

2 Implementation in MATLAB

We use the five MATLAB files detailed below to implement the fixed-point algorithm:

- "run_simulations.m" executes the simulations.
- "economy.m" defines an `economy`-class object that implements the fixed-point algorithm when the policymaker has the baseline redistributive preferences.
- "economy_weakRedist.m" defines a subclass that modifies the `economy` class to implement the fixed-point algorithm when the policymaker has weak redistributive preferences relative to the baseline.
- "economy_strongRedist.m" defines a subclass that modifies the `economy` class to implement the fixed-point algorithm when the policymaker has strong redistributive preferences relative to the baseline.
- "economy_invOpt.m" defines a subclass that modifies the `economy` class to implement the fixed-point algorithm when the policymaker has redistributive preferences that rationalize the status quo U.S. tax system.

All of the files above can be found in the "code" directory of the replication files.[8] Run the script "run_simulations.m" in MATLAB to output a PDF figure of the optimal marginal tax rate schedules to the "Figures" subfolder of the "output" folder.

[8] The following files with additional functions used in the simulations can be found in the "lib" directory of the replication files: "consolidator.m" (called by "interpcon.m"), "ksr.m" (kernel smoothing regression), "ksr_vw.m" (kernel smoothing regression with variable window width), and "interpcon.m" (linear interpolation). The first three files are from https://www.mathworks.com/matlabcentral/fileexchange/, while the last file is a custom function.

References

Chetty, Raj. 2012. "Bounds on Elasticities with Optimization Frictions: A Synthesis of Micro and Macro Evidence on Labor Supply." *Econometrica* 80 (3):969–1018.

Piketty, Thomas, Emmanuel Saez, and Gabriel Zucman. 2018. "Distributional National Accounts: Methods and Estimates for the United States." *Quarterly Journal of Economics* 133 (2):553–609.

Saez, Emmanuel. 2001. "Using Elasticities to Derive Optimal Income Tax Rates." *The Review of Economic Studies* 68 (1):205–229.

Seade, Jesus K. 1977. "On the Shape of Optimal Tax Schedules." *Journal of Public Economics* 7 (2):203–235.