

Task 1

8.154 - 8.157 (Promotion)
 hypothesis: Normal distribution (the change in heart rate).
 95% CI: $\bar{x}_d \pm t_{n-2, 1-\frac{\alpha}{2}} \cdot \frac{s_d}{\sqrt{n}}$ = (13,67, 46,72)

①

$$\bar{x}_d = 25,2$$

$$S_d = 2,4$$

$$n = 29$$

$$1 - \alpha = 0,95$$

$$\alpha = 0,05$$

$$\frac{\alpha}{2} = 0,025$$

$$t_{n-2, \frac{\alpha}{2}} = 2,202$$

$$t_{28, 0,025} = 2,202$$

Paired + - test (2-sided)

→ Data from 2010 → just take the first 10:

(baseline heart rate)		$d = (2000 - 2010)$
2000	2010	
84	85	-1
87	77	10
90	81	9
94	81	13
98	74,5	23,5
86	83,75	2,25
88	76,5	12,50
84	77,75	6,25
86	79,25	6,75
98	84,25	13,75

$$H_0: \Delta = 0 \text{ vs } H_1: \Delta \neq 0$$

→ no diff. between mean baseline (2000) heart rate and 2010's one.

After 10 years of doing treadmill, the baseline heart rate was reduced!

Good heart indicator.

$$\bar{d} = 9,5$$

$$\bar{d} = 9,5$$

$$S_d = 6,764$$

$$n = 10$$

so, the baseline heart rate significantly changed over 10 years period.

$$\text{Test Statistic } t = \frac{\bar{d} - 0}{S_d / \sqrt{n}} = \frac{9,5}{6,764 / \sqrt{10}} = 4,44 \geq 0 \Rightarrow p\text{-value} = 2 \cdot P(t_{n-2} \geq t) =$$

$$\{ P(t_9 \leq 3,85) = 0,995$$

$$\} P(t_9 \leq 4,781) = 0,9995$$

since: $3,85 < 4,44 < 4,781$,

then: $0,995 < P(t_9 \leq 4,44) < 0,9995$

$$= 2 \cdot P(t_9 \geq 4,44) =$$

$$= 2 \cdot (1 - P(t_9 \leq 4,44))$$

$$0,005 > 1 - P(t_9 \leq 4,44) > 0,0005$$

$$0,01 > 2 \cdot [1 - P(t_9 \leq 4,44)] > 0,001$$

H₀ rejected!

$$\text{• } \boxed{\text{95% CI}} = \bar{d} \pm t_{9,0,975} \cdot \frac{s_d'}{\sqrt{10}} = 9,3 \pm 3,868 \cdot \frac{6,764}{\sqrt{10}}$$

$t_{9,0,975} = 2,262$

$$\bar{d} = 9,3$$

$$s_d' = 6,764$$

$$t_{9,0,975} = 2,262$$

$$\boxed{(9,66, 12,304)}$$

Task 2: 8.110 - 8.114 (Microb.)

$$\text{• } n=8 \quad \begin{array}{l} \text{inoculated: } I \rightarrow \bar{I} \pm t_{7,0,975} \cdot \frac{s_I}{\sqrt{8}} = 1,634 \pm 2,365 \cdot \frac{0,98}{\sqrt{8}} = (1,28, 2,98) \\ \text{uninoculated: } U \rightarrow \bar{U} \pm t_{7,0,975} \cdot \frac{s_U}{\sqrt{8}} = 1,084 \pm 2,365 \cdot \frac{0,51}{\sqrt{8}} = (0,65, 1,51) \end{array}$$

$$\bar{I} = 1,634$$

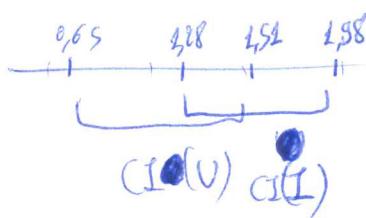
$$\bar{U} = 1,084$$

$$s_I = 0,980$$

$$s_U = 0,510$$

$$t_{7,0,975} = 2,365$$

overlap



• Only if $CI(U) = CI(I)$ then there would be no signif. diff between the mean pod weights for the 2 groups. Since $CI(U) \neq CI(I)$ even though they may overlap, this means that we expect their corresponding mean pod weights to be (significantly) different.

• 2 independent samples: I, U plants. (2 sample t-test).

→ We have to test if their variances are equal or not (of the populations that these 2 samples come from): $H_0: \sigma_I^2 = \sigma_U^2$ vs $H_1: \sigma_I^2 \neq \sigma_U^2$.

$$F = \left(\frac{\sigma_I^2}{\sigma_U^2} \right)^{-1} = \left(\frac{(0,98)^2}{(0,51)^2} \right)^{-1} = \left(\frac{0,1764}{0,2601} \right)^{-1} = (0,678)^{-1,975} > 1 \Rightarrow p = \alpha \cdot Pr(F_{7,7} \geq 0,678) =$$

$$= 2 \cdot [1 - Pr(F_{7,7} \leq 0,678)]$$

$$= 2,379, H_0$$

accept (equal variances)

So, we'll use the two-sample t-test for independent samples with equal variances.

Task 2

(2)

$$H_0: \mu_1 = \mu_0 \text{ vs } H_1: \mu_1 \neq \mu_0$$

$$(\sigma_I^2 = \sigma_U^2 = \sigma^2).$$

$$\text{Test Statistic: } t = \frac{\bar{I} - \bar{U}}{S \cdot \sqrt{\frac{1}{8} + \frac{1}{8}}} = \frac{1,634 - 1,084}{0,967 \cdot 0,5} = \frac{0,55}{0,48335} = 1,14$$

$$n_I = n_U = n = 8, \quad u_I + u_U = 14$$

$$\bar{I} = 1,634$$

$$\bar{U} = 1,084$$

$$S_I = 0,98$$

$$S_U = 0,51$$

$$S = \sqrt{\frac{(8-1)S_I^2 + (8-1)S_U^2}{(8+8-2)}} = \sqrt{\frac{7 \cdot (0,98)^2 + 7 \cdot (0,51)^2}{14}} = 0,967$$

$$[\text{p-value}] = 2 \cdot P(t_{14} > t) = 2 \cdot [1 - P(t_{14} \leq 1,14)] = 2 \cdot [1 - 0,9831] = 0,033$$

- A 95% CI for the diff in mean food weights between the 2 groups, is given by:

$$[CI] = (\bar{I} - \bar{U}) \pm t_{14, 0,975} \cdot S \cdot \sqrt{\frac{1}{8} + \frac{1}{8}} =$$

$$= 0,55 \pm 2,145 \cdot 0,8335 =$$

$$= 0,55 \pm 0,5 = [(0,05, 1,05)]$$

0 is not included, which confirms the findings of the previous

1-sample t -test result. (and that the inoculated plants show statist. more growth).

Task 3

: 9.7 - 9.8

- A t -test will not be useful because the data don't seem to be normally distributed \Rightarrow we can see that if we plot the distribution of length of stays for each hospital.



So, the 2 samples are non-normal and independent.
 $n_1 = 14, n_2 = 13$ (or otherwise: the 2 samples from the same distribution)

- Wilcoxon rank-sum test: $\begin{cases} H_0: \text{The median length of stays are the same for the 2 hospitals} \\ H_1: \text{the lengths are different.} \end{cases}$

Posto: combined first hospital, 2nd → second hospital
hos (1st) rank

5 (1st)	1	
8 (1st)	2	2
10 (1st)	3	3,5
10	4	3,5
13 (1st)	5	5
12 (1st)	6	6
86 (1st)	7	7
97	8	8
89 (1st)	9	9
32 (1st)	10	10
33 (1st)	11	11
35	12	12
94 (1st)	13	13,5
94	14	13,5
60 (1st)	15	15,5
60	16	15,5
68	17	17
73	18	18
76	19	19
86	20	20
87	21	21
96	22	22
125	23	23
238	24	24

$n_1 = 12, n_2 = 13, \min(n_i) = n_2 > 10$, so we'll use the normal approximation method.

for the first hospital (shown as 1st)
 $\boxed{R_1} = 1+8+3,5+5+6+7+9+10+12+13,5+$

$$LS,5 = \boxed{83,5}$$

$$E(R_2) = \frac{n_2(n_1+n_2+1)}{12} = \frac{12(12+13+1)}{12} = 137,5$$

- We notice that there exist ties between the 2 datasets, so we'll use $\boxed{(3,6)}$ from eq (9.8) with 2 common obs. each.
- We have $\boxed{g=3}$ tied groups, so:

$$\boxed{\text{Var}(R_2)} = \frac{n_1 n_2}{12} \cdot \left[n_1 + n_2 + 1 - \frac{\sum_{i=1}^g t_i(t_i-1)}{(n_1 + n_2)(n_1 + n_2 - 1)} \right] =$$

$$2 \cdot \frac{12 \cdot 13}{12} \cdot \left[12 + 13 + 1 - \frac{3 \cdot [2(2^2 - 1)]}{(12+13)(12+13-1)} \right] = \\ = 11,91667 \cdot \left(25 - \frac{18}{24 \cdot 23} \right) =$$

$$= \boxed{297,5282}.$$

So, the test statistic is:

$$\boxed{T} = \frac{|R_2 - E(R_2)| - 0,5}{\sqrt{\text{Var}(R_2)}} =$$

$$= \frac{|83,5 - 137,5| - 0,5}{\sqrt{297,5282}} = \frac{53,5}{17,299} = \boxed{3,10}$$

$$\boxed{p\text{-value}} = 2(1 - \phi(T)) =$$

$$= 2(1 - \phi(3,10)) \approx \boxed{0,002}$$

0,9930

⇒ H_0 rejected, results show that there is a significantly statistical difference between the length of stay of the 2 hospitals.

Task 4

9.13-9.26.

[Otolaryngology, Pediatrics]

(3)

- H₀: The duration of effusion in breast-fed babies and the duration of effusion in bottle-fed babies come from the same distribution.

rs

H₁: → - they come from diff. distributions.

- The distribution of effusion in both samples is very skewed and seems to be far from normal. ⇒ (so a parametric paired-sample t-test ~~shouldn't~~ be used in this case).

- The 2-samples are paired (common age, sex, socioeconomic status) ⇒ we'll use the Wilcoxon signed-rank test. non-zero diff.: n = 23 → use the normal approximation method (eq. 9.6)

# pair	breast-fed (br) (days)	bottle-fed (bot) (days)	d = br - bot	sign	Range at ranks	Avg. ranks
1	20	18	2	+	2 ✓	5-7
2	21	35	-24	-	24 ✓	19
3	3	7	-4	-	4 ✓	3
4	24	182	-158	-	158 ✓	21
5	7	6	1	+	1 ✓	2,5
6	28	33	-5	-	1-4	2,5
7	58	223	-165	-	10-11	10,5
8	7	7	0		22	22
9	33	57	-24			
10	17	76	-59	-	18 ✓	18
11	17	186	-169	-	53 ✓	20
12	19	29	-17	-	169 ✓	23
13	52	39	13	+	17 ✓	17
14	14	15	-1	+	13 ✓	16
15	12	21	-9	-	1 ✓	2,5
16	30	28	2	+	9 ✓	13
17	7	8	-1	+	5-7	6
18	15	27	-12	-	1 ✓	1-4
19	65	77	-12	-	19 ✓	19,5
20	10	19	-9	-	19 ✓	14,5
21	7	8	-1	-	5-7	6
22	19	16	3	+	1 ✓	2-4
23	34	28	6	+	3 ✓	8
24	25	20	5	+	6 ✓	12
					5 ✓	10-11

N=23

$R_1 = \text{rank-sum of positive diff.} = 6 + 21,5 + 16 + 6 + 8 + 18 + 10,5 \Rightarrow R_1 = 61$

$$E(R_1) = \frac{n(n+1)}{4} = \frac{23 \cdot 24}{4} = 138$$

$$\Rightarrow R_1 = 61$$

Ties exist ($g=4$ tied groups), so for the variance of R_1 , we'll use the

(3b) formula (eq 9.6)

$$\text{Var}(R_1) = \frac{n(n+1)(2n+1)}{24} - \sum_{i=1}^{g=4} \frac{(r_i - t_i)^2}{98} = \frac{93 \cdot 94 \cdot 197}{24} - \frac{(6-3)^2 + (24-12)^2 + (25-15)^2 + (18-8)^2}{98} = 1081 - 2 = 1079$$

So, the test statistic is: $T = \frac{|R_1 - E(R_1)| - \frac{1}{2}}{\sqrt{\text{Var}(R_1)}} = \frac{76,5}{\sqrt{1079}} = 2,288 \text{ N}_{(8,33)}$

p-value = $2 \cdot [1 - \Phi(T)] = 2 \cdot [1 - \Phi(2,33)] = 0,0204$, \Rightarrow results stat. significant.

H_0 reject \Rightarrow the duration of effusion is diff. distributed between the 2 cases {breast-fed babies, bottle-fed}

Looking at the signs of the differences in duration, we observe a shorter duration of effusion in breast-fed babies compared to the bottle-fed ones.

Task 5

10.13

Had Urethritis before

We'll use the Chi-Square test →
(independent samples) → 2×2 contingency table →

	related gonorrhoea	non-gonorrhoea related	
YES	160	50	210
NO	40	55	95
	200	1005	305

⑨

$$\hat{p}_1 = \frac{160}{200} = 80\%$$

$$\hat{p}_2 = \frac{50}{105} \approx 48\%$$

→ p_1 : probability of a patient having gonorrhoea while he had prior episode of urethritis

→ p_2 : probability of a patient having nongonococcal urethritis (while he had prior episode of urethritis)

$H_0: p_1 = p_2 = p$

$H_1: p_1 \neq p_2$

no relationship between present diagnosis and prior episode of urethritis.

- Expected Table values: $E_{11} = \frac{210 \cdot 200}{305} = 137,7$ $E_{12} = \frac{95 \cdot 200}{305} = 62,3$

$$E_{21} = \frac{210 \cdot 105}{305} = 72,3$$

$$E_{22} = \frac{95 \cdot 105}{305} = 32,7$$

• $E_{ij} > 5 \forall i, j$ so the normal approximation method can be used.

- Test Statistic: $\chi^2 = \frac{(160 - 137,7)^2}{137,7} + \frac{(50 - 62,3)^2}{62,3} + \frac{(40 - 72,3)^2}{72,3} + \frac{(55 - 32,7)^2}{32,7} =$

$\approx 38,18$

- p-value = $P(\chi^2 > \chi^2) = P(\chi^2 > 38,18) = 1 - P(\chi^2 \leq 38,18)$.

$$\{ P(\chi^2 \leq 10,83) = 0,999 \}$$

$$\{ P(\chi^2 \leq 38,18) > P(\chi^2 \leq 10,83) \rightarrow P(\chi^2 \leq 38,18) > 0,999 \}$$

$$\{ P(\chi^2 \leq 38,18) > 1 - P(\chi^2 \leq 10,83) \rightarrow 1 - P(\chi^2 \leq 38,18) < 0,001 \}$$

p-value < 0,001

Results are stat. significantly.

{ We reject H_0 , so there is an association between present diagnosis and prior episodes of urethritis }

Also, By comparing \hat{p}_1 and \hat{p}_2 estimations it seems that you have a larger probability of being diagnosed with gonorrhoea ~~in~~ if you had prior episodes of urethritis.

10.6-10.7. "Receiving an antibiotic" and "Receiving a bacterial culture" are both categorical variables, each one having only two possible outcomes (yes/no).

We'll use the Chi-Square test on the observed data:

		Received bacterial culture		
		YES	NO	
Received Antibiotic	YES	2	5	7
	NO	4	14	18
		6	19	25

For example: $O_{22} = 2$ because in the table 2.13 (with the hospital data), only 2 patients both received an antibiotic and a bacterial culture.

$$\cdot p_1 = \frac{2}{6} = 0.33 = 33\%$$

p_1 : prob. to receive bact. culture when also received antibiotic

$$\cdot p_2 = \frac{5}{19} = 0.26 = 26\% \quad p_2: \text{prob. to receive bact. culture} \rightarrow$$

→ We want to test the hypothesis $H_0: p_1 = p_2 = p$ vs $H_1: p_1 \neq p_2$

↳ no relationship between the 2 categorical variables.

→ Expected values table:

$$E_{11} = \frac{7 \cdot 6}{25} = 1.68, \quad E_{12} = \frac{7 \cdot 19}{25} = 5.32$$

$$E_{21} = \frac{18 \cdot 6}{25} = 4.32, \quad E_{22} = \frac{18 \cdot 19}{25} = 13.68$$

$E_{11} < 5$, so we cannot use the contingency table method.

Instead, we'll use Fisher's

Exact Test

(1)

First, we enumerate all possible tables with the same row and column margins as the observed table:

a	b
c	d

0	7
6	19

1	6
5	19

2	5
4	14

3	4
3	15

4	3
2	16

a=5

5	2
1	17

a=6

6	1
0	18

$$N_1 = 7, N_2 = 18 \\ M_1 = 6, M_2 = 19. \quad N = 25$$

or (10.7)

→ Now, we compute the exact probabilities of each table (using eq. (10.6)):

a	b
c	d

$$P(a, b, c, d) = \frac{(a+b)! (c+d)! (a+c)! (b+d)!}{n! a! b! c! d!}$$

$$= \frac{N_1! N_2! M_1! M_2!}{n! a! b! c! d!}$$

(5)

Task 5 | 10.6-10.7 (cont.)

13-24-25-26-27-28

$$\text{azj.} \cdot P(X=0, 7, 6, 12) = \frac{7! \cdot 18! \cdot 6! \cdot 18!}{20! \cdot 21! \cdot 22! \cdot 23! \cdot 24! \cdot 25!} = 0,105. \quad [=P(X=0), \text{ from eq. (10.7)}]$$

$$\cdot P(X=1) = \frac{7! \cdot 18! \cdot 6! \cdot 19!}{25! \cdot 21! \cdot 20! \cdot 23! \cdot 24!} = 0,339$$

$$\cdot P(X=2) = 0,363, \quad P(X=3) = 0,161, \quad P(X=4) = 0,03, \quad P(X=5) = 0,0021$$

↑ observed table →

$$P(X=6) = 0,00004$$

$$(3) P_a = P(0) + P(1) + P(2) = 0,806$$

$$P_b = P(3) + P(4) + P(5) + P(6) = 0,557$$

p-value

$$2 \times \min(P_a, P_b, 0.5) = 2 \times 0,9 = 1 \rightarrow H_0 \text{ accept, so there}$$

is no association between receiving bacterial culture and receiving antibiotics while in the hospital.

Task 6 | 10.19-10.22

- Otolaryngology

group	Num	Num of children with otorrhea at 2 weeks
Antibiotic ear drops	76	4
Oral antibiotics	77	34
Observation (no treatment)	75	41
Total:	228	79

• Observation Group: After 2 weeks of no treatment, the child can either have or not the otorrhea disease. So the number of children from the obs. group that still have otorrhea after 2 weeks, follows the binomial distribution where "success" (p) is considered to have the disease after 2 weeks! So, $\hat{p} = \frac{41}{75} = 0,55$

is a point estimate for the prevalence of otorrhea. Since $n\hat{p}\hat{q} > 5$, we can use the normal-theory method to calculate a 95% CI for the prevalence of otorrhea (eq. 6.19).

The CI is: $\hat{p} \pm Z_{1-\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$ $\Rightarrow 1-\alpha=0,95 \Rightarrow Z_{0,975}=1,96$

$$\Rightarrow 0,55 \pm 1,96 \cdot \sqrt{\frac{0,55 \cdot 0,45}{75}} = 0,55 \pm 1,96 \cdot 0,0574 = 0,55 \pm 0,113 = (0,44, 0,66)$$

• Same logic as before now for the ear drop group:

$$n=76 \\ x=4$$

$$\hat{p} = \frac{x}{n} = \frac{4}{76} = 0,053, \quad \hat{q} = 1 - \hat{p} = 0,947, \quad n\hat{p}\hat{q} = 76 \cdot 0,053 \cdot 0,947 = 3,81 < 5$$

► Instead, we'll use the exact method (eq. 6.20)

But, here we
cannot use
the normal
approximation
method.

- To compare the prevalence of otorrhea for the ear drop group vs the observation group, we will use a two-sample test for binomial proportions: (we consider the two samples as independent)

p_1 : probability that after 2 weeks of treatment with antibiotic ear drops, you'll still have otorrhea.

p_2 : probability that after 2 week of no treatment, you'll still have otorrhea.

Hypothesis to test: $H_0: p_1 = p_2 = p$ vs $H_1: p_1 \neq p_2$.

$$n_1 = 76, \quad \hat{p}_1 = 0,053 \\ n_2 = 75, \quad \hat{p}_2 = 0,55$$

We'll try to use the Normal-Theory

Method (eq. 10.3). First we have to see if the normal approximation to the binomial distribution is valid for each of the 2 samples:

$$\hat{p} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2} = \frac{76 \cdot 0,053 + 75 \cdot 0,55}{76 + 75} \approx 0,3. \quad \hat{q} = 1 - \hat{p} = 0,7$$

$$\begin{aligned} \cdot n_1 \hat{p} \hat{q} &= 76 \cdot 0,3 \cdot 0,7 = 15,96 > 5. \\ \cdot n_2 \hat{p} \hat{q} &= 75 \cdot 0,3 \cdot 0,7 = 15,75 > 5. \end{aligned} \quad \left. \right\} \Rightarrow \text{So, YES we'll use the normal theory two-sample test for binomial proportions.}$$

The CI = (p_1, p_2) where p_1, p_2 satisfy:

$$\Pr(X \geq x | p = p_1) = \frac{\alpha}{2} = \sum_{k=x}^{76} \binom{76}{k} p_1^k$$

$$\Pr(X \leq x | p = p_2) = \frac{\alpha}{2} = \sum_{k=0}^x \binom{76}{k} p_2^k (1-p_2)^{76-k}$$

$$\left. \begin{array}{l} 1 - \alpha = 0,95 \\ \alpha = 0,05 \\ \frac{\alpha}{2} = 0,025 \end{array} \right\} \Pr(X \geq 4 | p = p_1) = 0,025 \rightarrow$$

$$\Pr(X \leq 4 | p = p_2) = 0,025$$

$$\rightarrow \left\{ \begin{array}{l} \sum_{k=4}^{76} \binom{76}{k} p_1^k (1-p_1)^{76-k} = 0,025 \\ \sum_{k=0}^4 \binom{76}{k} p_2^k (1-p_2)^{76-k} = 0,025 \end{array} \right. \quad \dots \text{Used}$$

the R function: `binom.test(4, 76, p = 0/76)` and got a 95% CI as:

$$CI = (p_1, p_2) = (0,0145, 0,1993)$$

Task 5 | 10.19 - 11.22 (Cont.)

(6)

The test statistic is: $Z = \frac{|\hat{p}_L - \hat{p}_R| - \left(\frac{1}{2n_L} + \frac{1}{2n_R}\right)}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_L} + \frac{1}{n_R}\right)}} = \frac{|0,053 - 0,055| - \left(\frac{1}{876} + \frac{1}{875}\right)}{\sqrt{0,3 \cdot 0,7 \cdot \left(\frac{1}{76} + \frac{1}{75}\right)}}$

$$= \frac{0,997 - 0,0132}{\sqrt{0,0055632}} = \frac{0,9838}{0,0746} = 13,985 \rightarrow \Phi(13,985) = 1.$$

* p-value $p = \min\{2(1 - \Phi(z)), 1\} = \min\{0,13, 1\} = 0,13$. So, since the p-value $< 0,001$ the results are highly significant and we reject the H_0 at the 95% significance level. So the prevalence of ~~otitis~~ between the 2 groups is different and judging the sample proportions ($\hat{p}_L = 0,053, \hat{p}_R = 0,055$) the antibiotic ear drops seem to be a better alternative to no treatment for the otitis disease.

End