Introduction to Image Processing

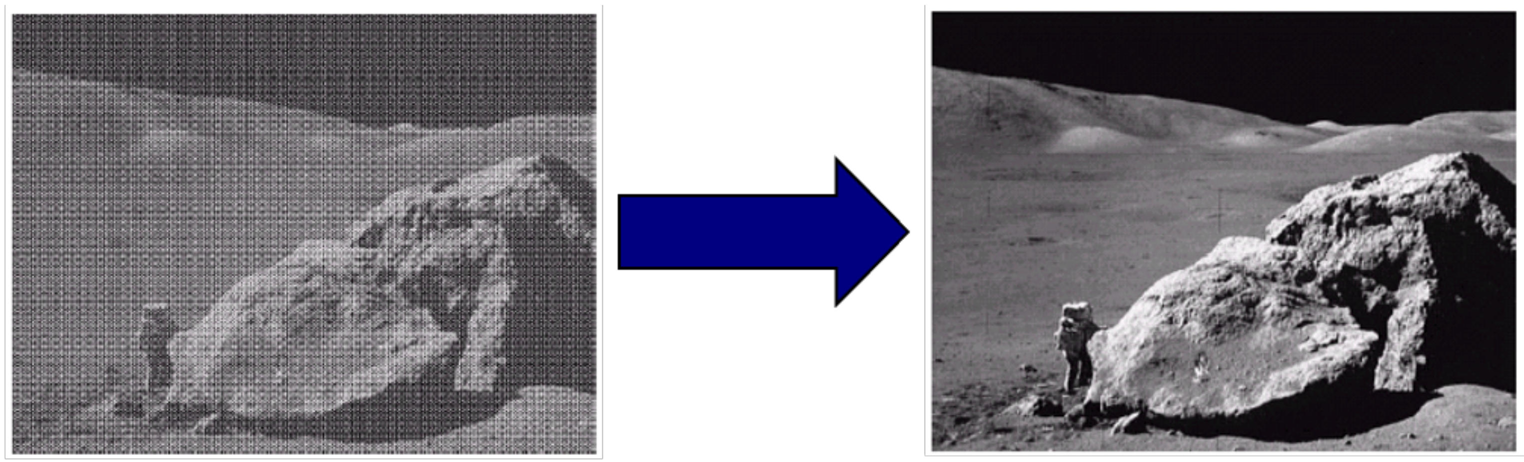
Ch 5. Image Restoration and Reconstruction

Kuan-Wen Chen

Ch 5. Image Restoration and Reconstruction

5.1 A Model of the Image Degradation/Restoration process

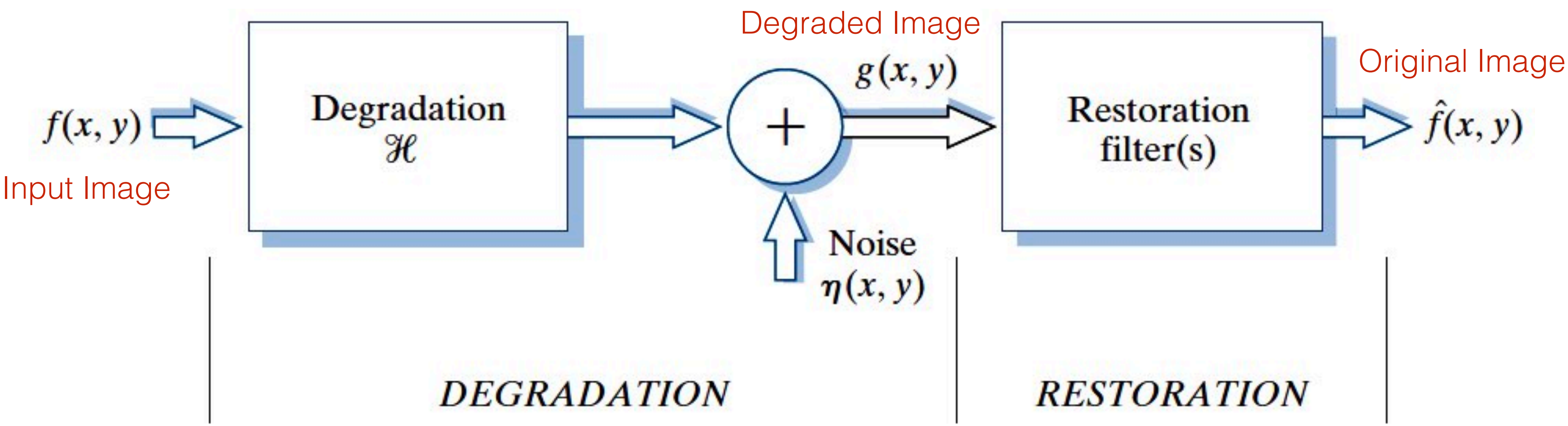
- **What is Image Restoration?**
  - Image restoration attempts to recover an image that has been degraded by using a priori knowledge of the degradation phenomenon, i.e. modeling the degradation and applying the inverse process
  - Similar to image enhancement, but more objective. (i.e. noise removal)



credit of this slide: C. Nikou

5.1 A Model of the Image Degradation/Restoration process

- The principal sources of noise in digital images arise during image acquisition (digitization) and/or transmission.



$$g(x,y) = \underline{(h \star f)}(x,y) + \underline{\eta(x,y)}$$

$$G(u,v) = \underline{H(u,v)}F(u,v) + \underline{N(u,v)}$$

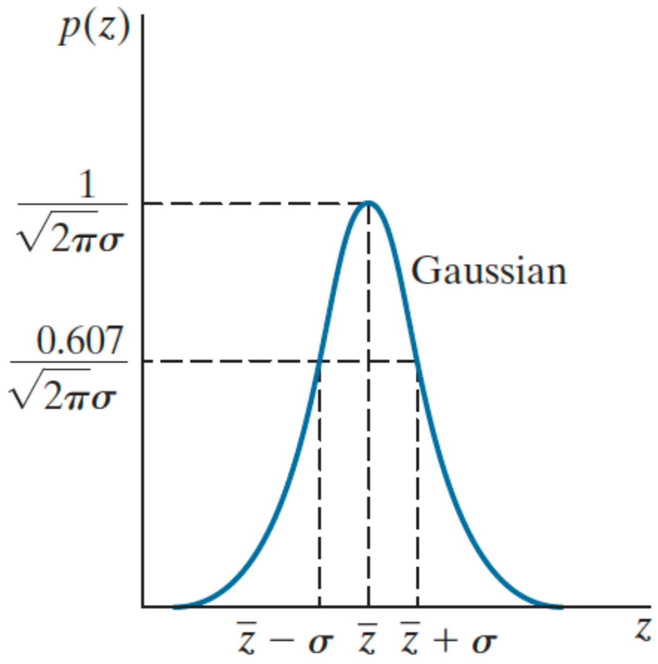
If we have some priori knowledge, we can restore the original image

5.2 Noise Models

- Noise discussed in this chapter
  - Gaussian Noise
  - Rayleigh Noise
  - Erlang (Gamma) Noise
  - Exponential Noise
  - Uniform Noise
  - Salt-and-Pepper Noise
  - Periodic Noise

- Noise property (in this book, partially invalid in some applications)
  - Spatially-independent (except periodic noise)
  - Signal-uncorrelated

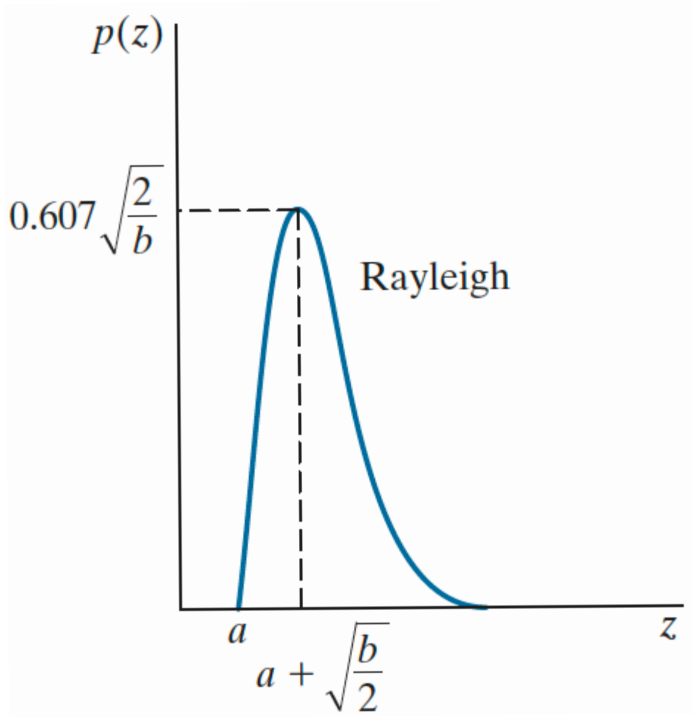
- Gaussian Noise
  - Most common model



$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z - \bar{z})^2}{2\sigma^2}} \qquad -\infty < z < \infty$$

electronic circuit noise and sensor noise caused by poor illumination and/or high temperature

- Rayleigh Noise

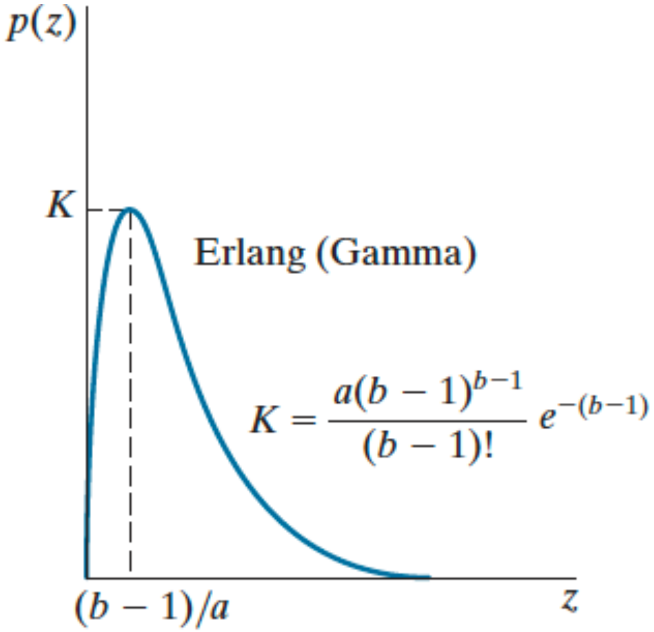


$$p(z) = \begin{cases} \frac{2}{b}(z - a)e^{-(z - a)^2/b} & z \geq a \\ 0 & z < a \end{cases}$$

$$\bar{z} = a + \sqrt{\pi b/4} \qquad \sigma^2 = \frac{b(4 - \pi)}{4}$$

characterizing noise phenomena in range imaging

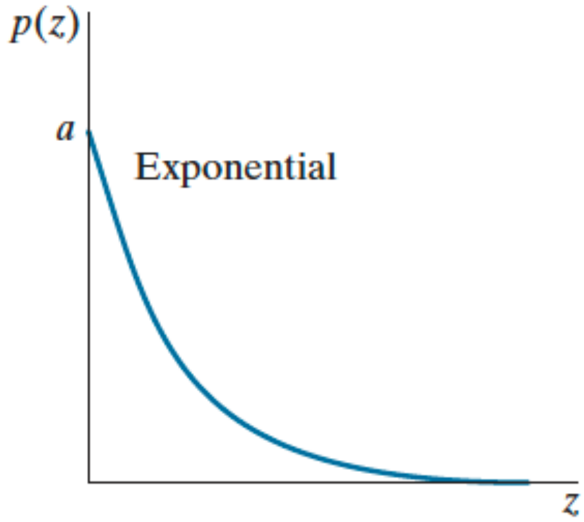
- Erlang (Gamma) Noise



$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

$$\bar{z} = \frac{b}{a} \qquad \sigma^2 = \frac{b}{a^2}$$

- Exponential Noise
  - A special case of Erlang with $b = 1$

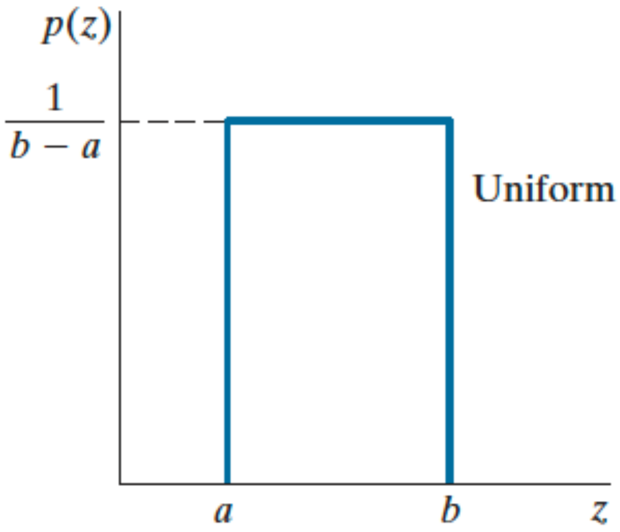


$$p(z) = \begin{cases} ae^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

$$\bar{z} = \frac{1}{a} \qquad \sigma^2 = \frac{1}{a^2}$$

laser imaging

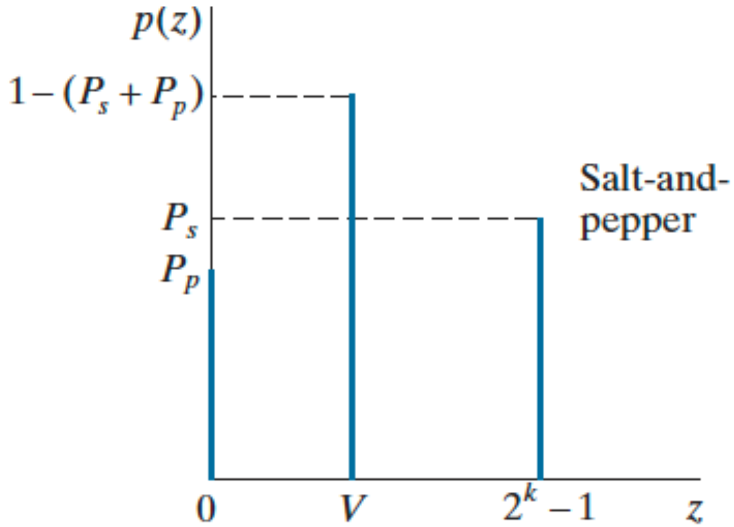
- Uniform Noise



$$p(z) = \begin{cases} \dfrac{1}{b-a} & a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

- the least descriptive of practical situations
- used extensively in simulations

- Salt-and-Pepper Noise
  - *bipolar impulse noise*
  - *data-drop-out noise*
  - *spike noise*



$$p(z) = \begin{cases} P_s & \text{for } z = 2^k - 1 \\ P_p & \text{for } z = 0 \\ 1-(P_s+P_p) & \text{for } z = V \end{cases}$$

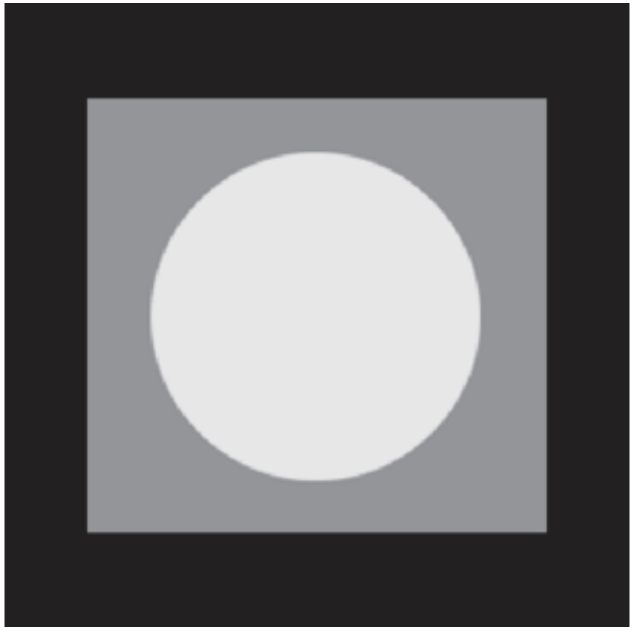
$$\bar{z} = (0)P_p + K(1 - P_s - P_p) + (2^k - 1)P_s$$

$$\sigma^2 = (0-\bar{z})^2 P_p + (K-\bar{z})^2(1 - P_s - P_p) + (2^k - 1)^2 P_s$$

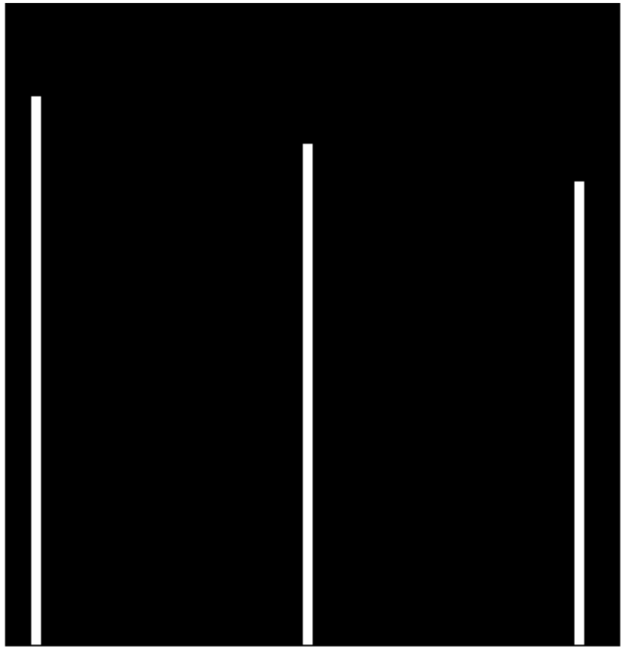
in situations where quick transients, such as faulty switching, take place during imaging

5.2 Noise Models

- Noise examples

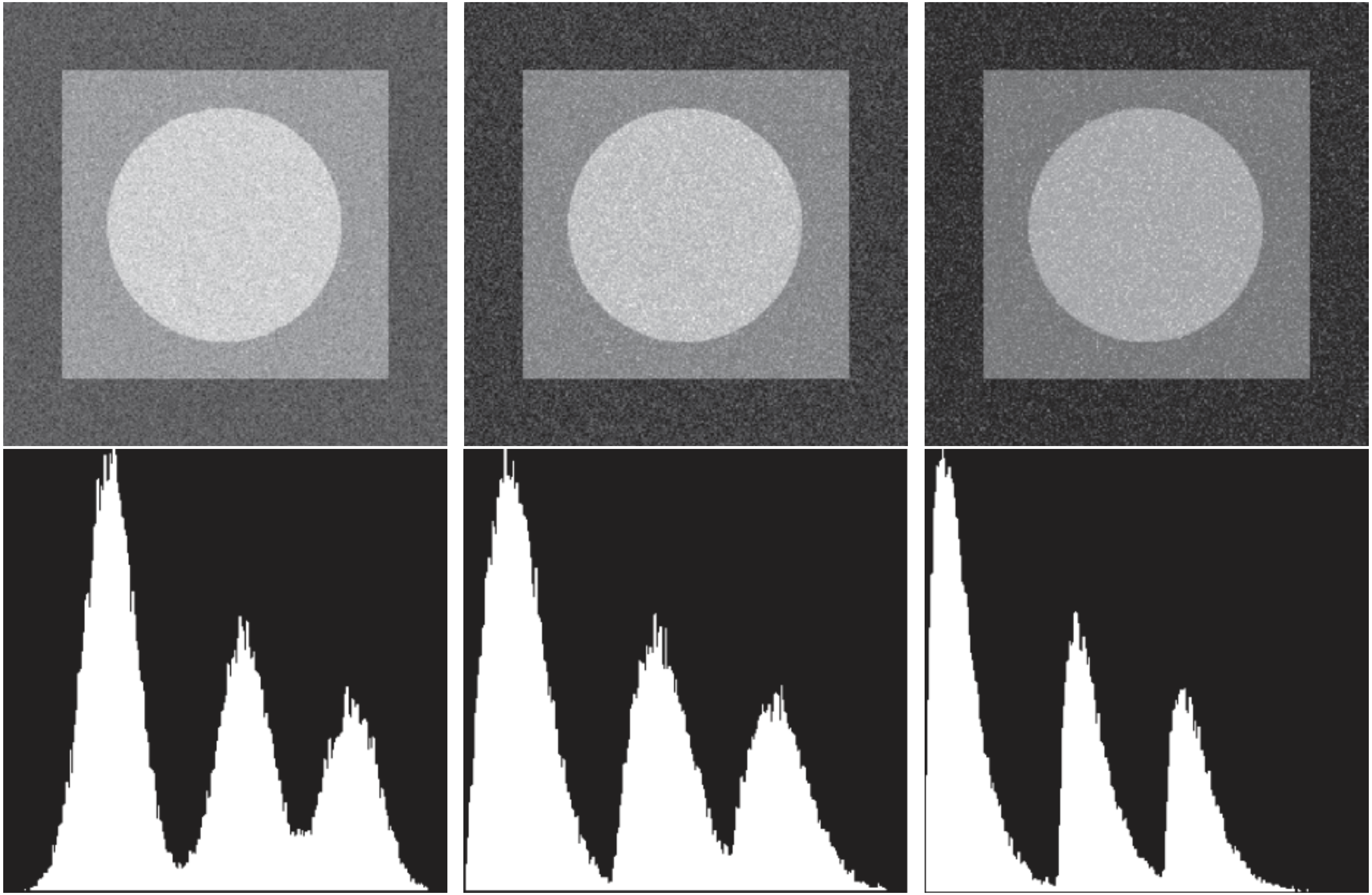


Original image



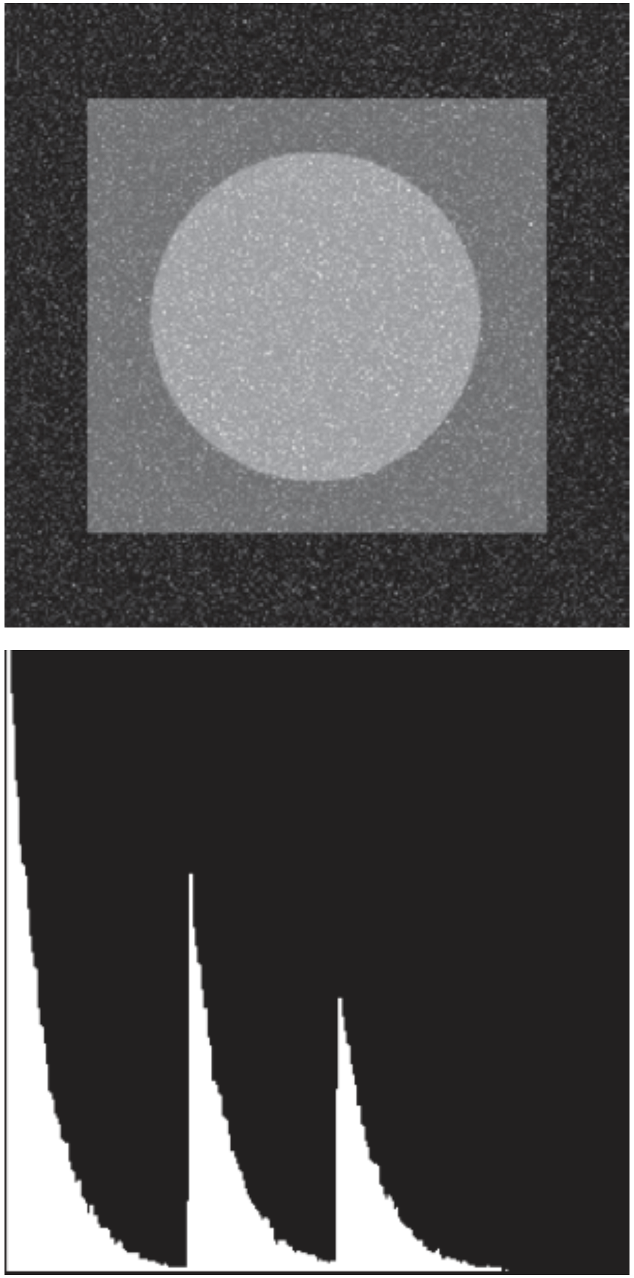
Histogram

- Noise examples

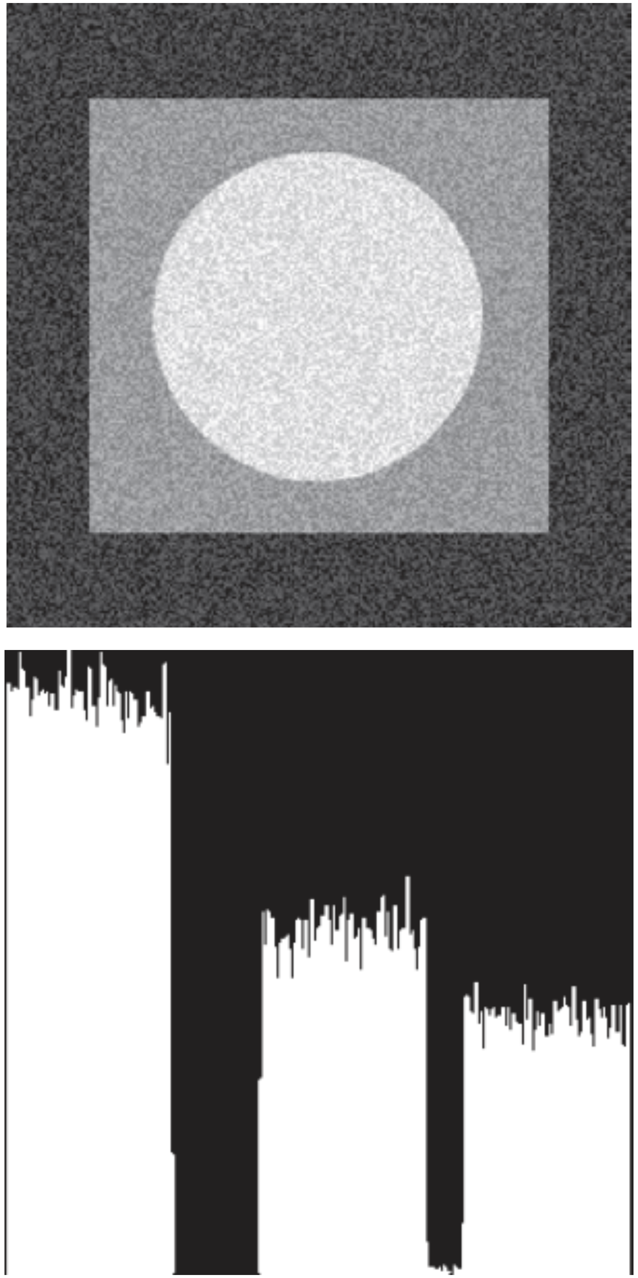


Gaussian Noise Rayleigh Noise Erlanga Noise

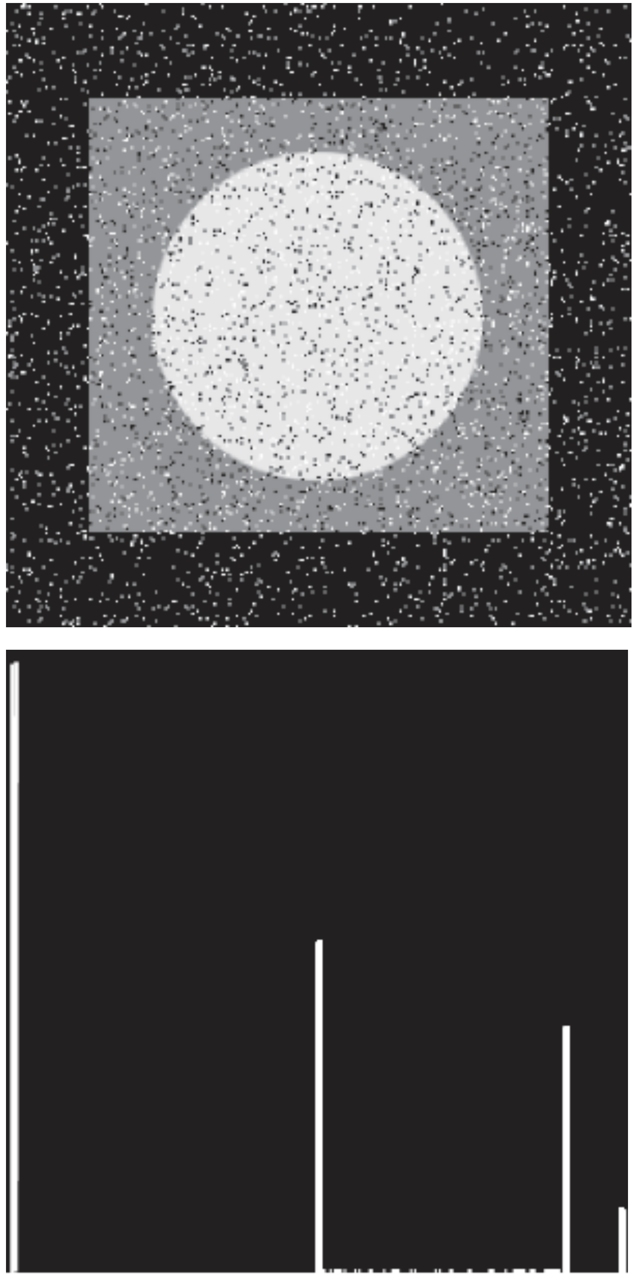
- Noise examples



Exponential Noise



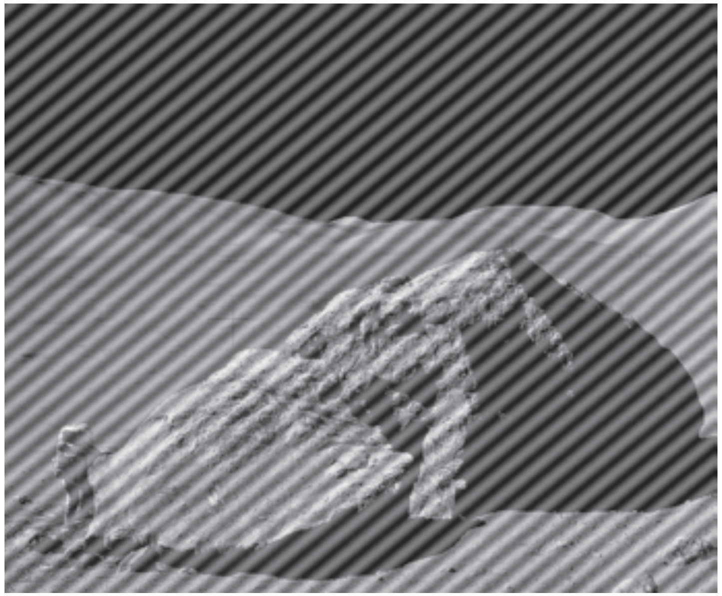
Uniform Noise



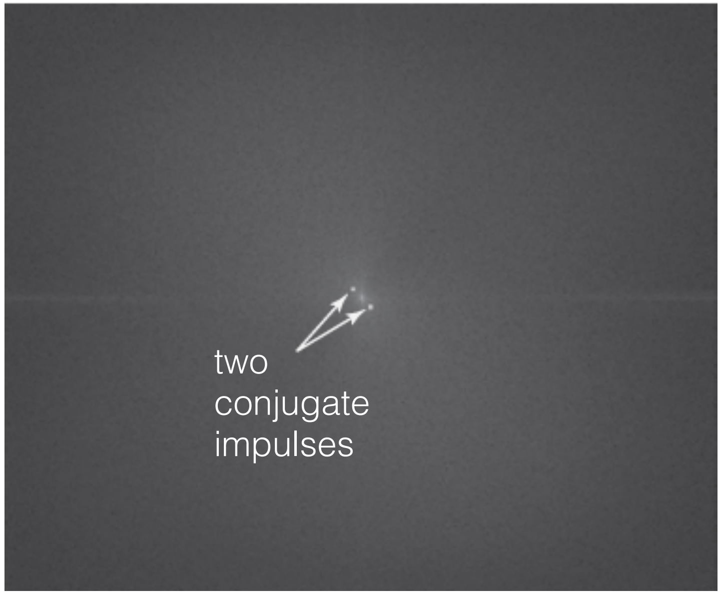
Salt-and-pepper Noise

5.2 Noise Models

- Periodic Noise
  - Arises from electrical or electromechanical interference during image acquisition
  - The only type of spatially dependent noise in this chapter
  - Periodic noise can be reduced significantly via frequency domain filtering (Sec. 5.4)



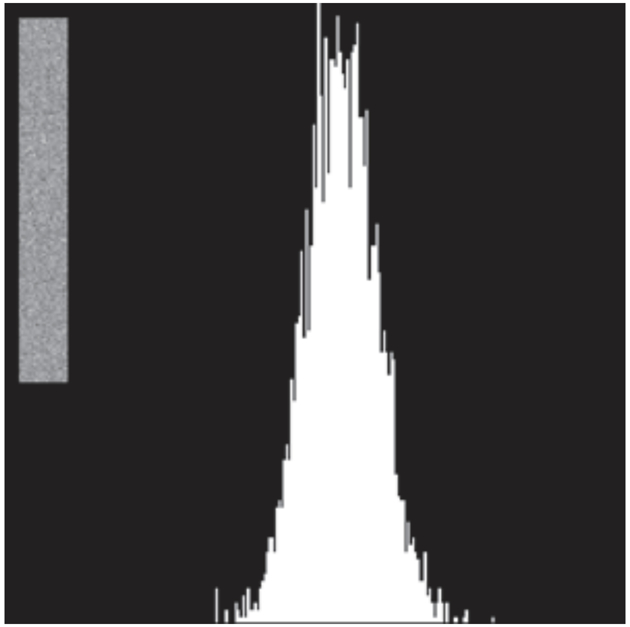
Image



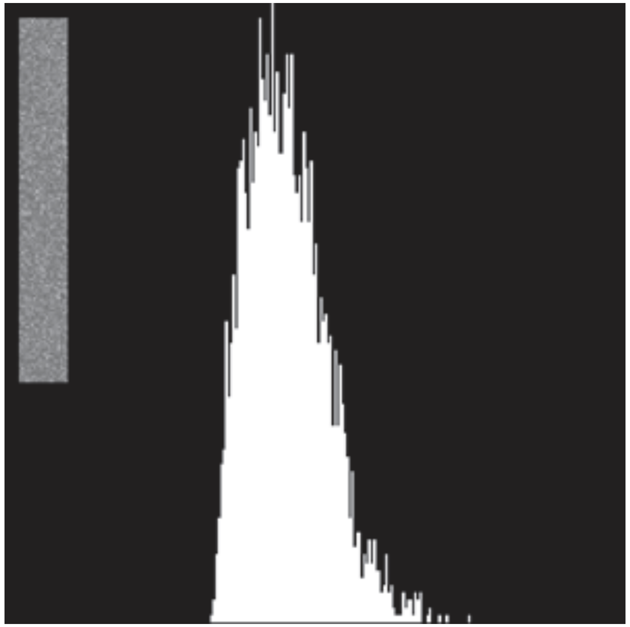
Fourier Spectrum

5.2 Noise Models

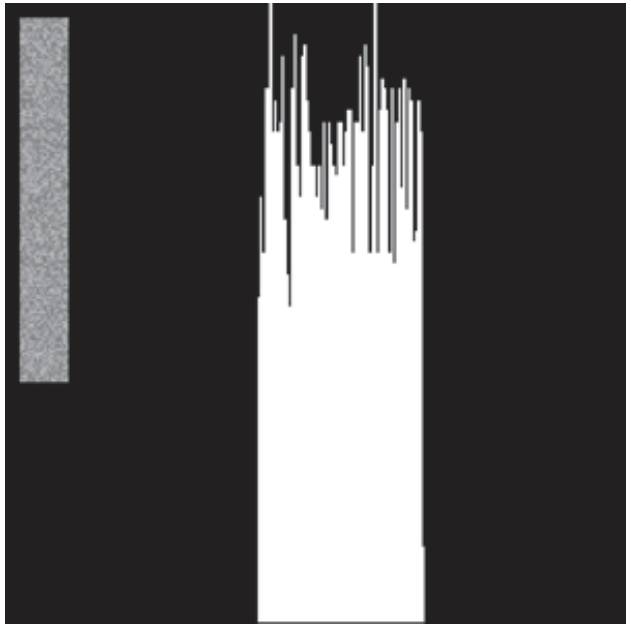
- Estimation of Noise Parameters
  - If the imaging system is available, one simple way to study the characteristics of system noise is to capture a set of “flat” images.



Gaussian noise



Rayleigh noise



Uniform noise

5.3 Restoration in the Presence of Noise Only –Spatial Filtering

When an image is degraded only by additive noise, i.e. ***H*** = ***I***

$$g(x,y) = f(x,y) + \eta(x,y)$$

$$G(u,v) = F(u,v) + N(u,v)$$

We can use spatial filters of different kinds to remove different kinds of noise

- **Mean Filters**
- **Order-Statistics Filters**
- **Adaptive Filters**

5.3 Restoration in the Presence of Noise Only –Spatial Filtering

- **Mean Filters**
  - Arithmetic Mean Filter

    - the same as the box filter in Ch. 3

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(r,c)\in S_{xy}} g(r,c)$$

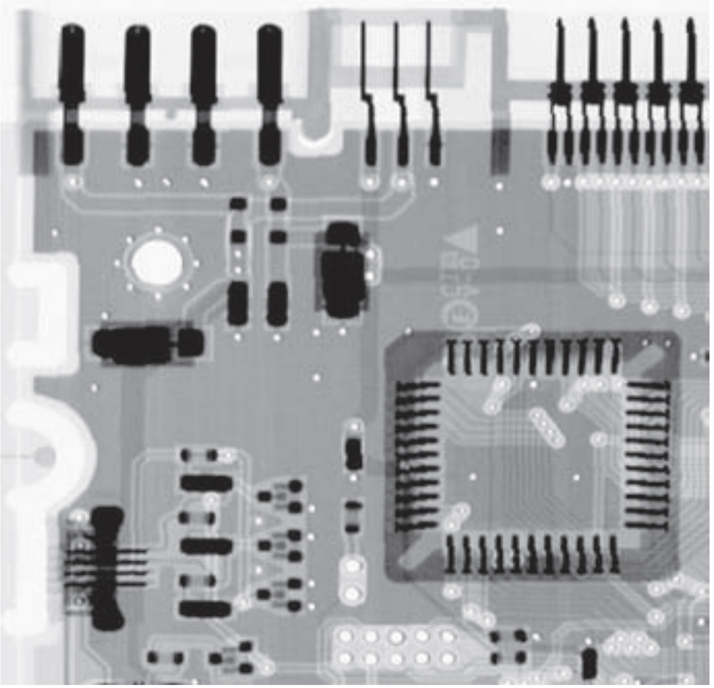
  - Geometric Mean Filter

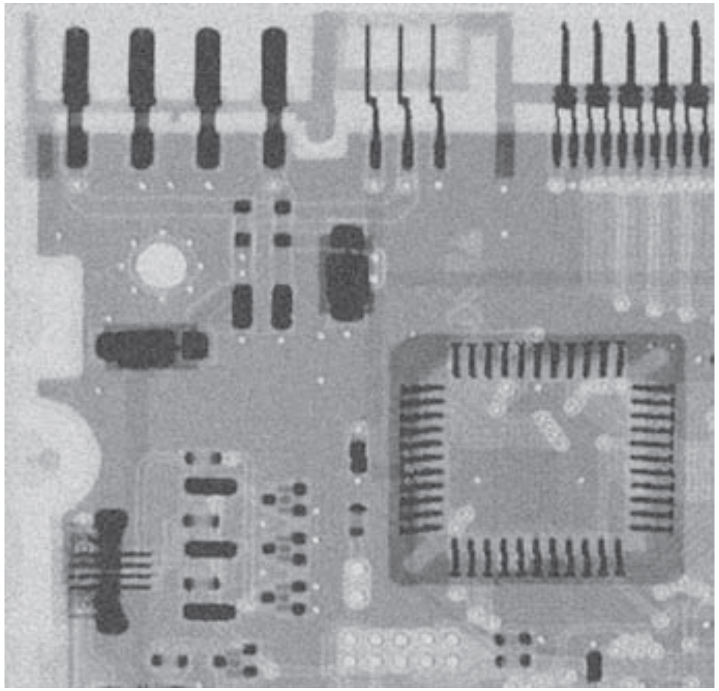
    - achieves similar smoothing to the arithmetic mean, but tends to lose less image detail.

$$\hat{f}(x,y) = \left[\prod_{(r,c)\in S_{xy}} g(r,c) \right]^{\frac{1}{mn}}$$

- Noise removal examples

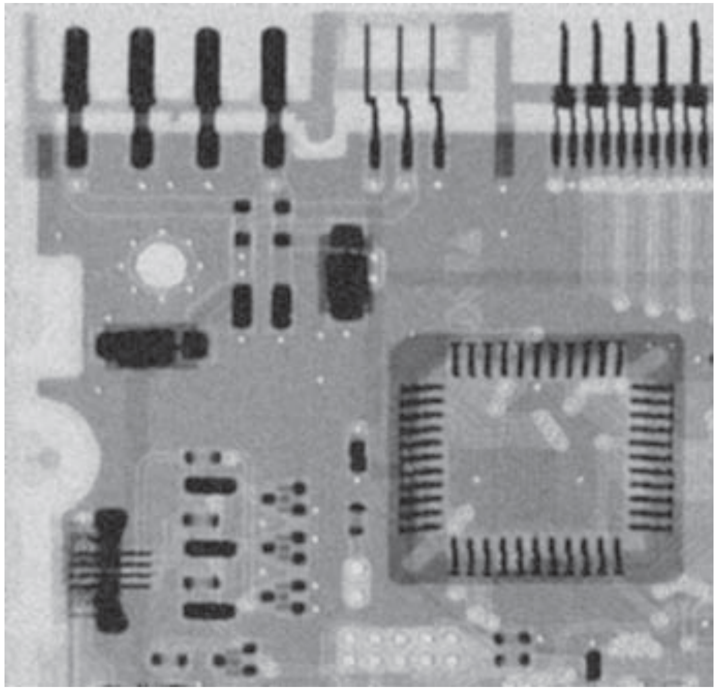
Original Image

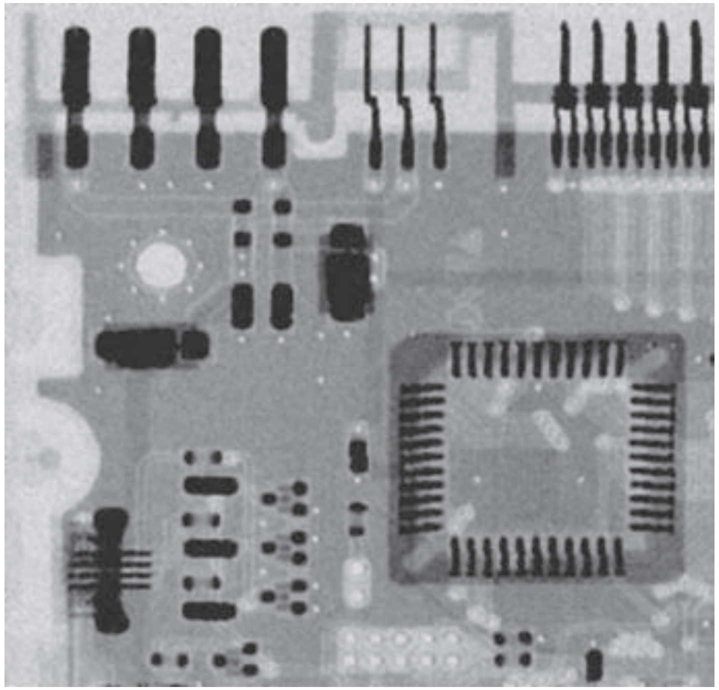




Corrupted with additive Gaussian noise

Arithmetic mean filter





Geometric mean filter

sharper

5.3 Restoration in the Presence of Noise Only –Spatial Filtering

- **Mean Filters**
  - Harmonic Mean Filter
    - works well for salt noise, but fails for pepper noise
    - also does well for other types of noise like Gaussian noise

$$\hat{f}(x,y) = \frac{mn}{\sum_{(r,c)\in S_{xy}} \frac{1}{g(r,c)}}$$

  - Contraharmonic Mean Filter

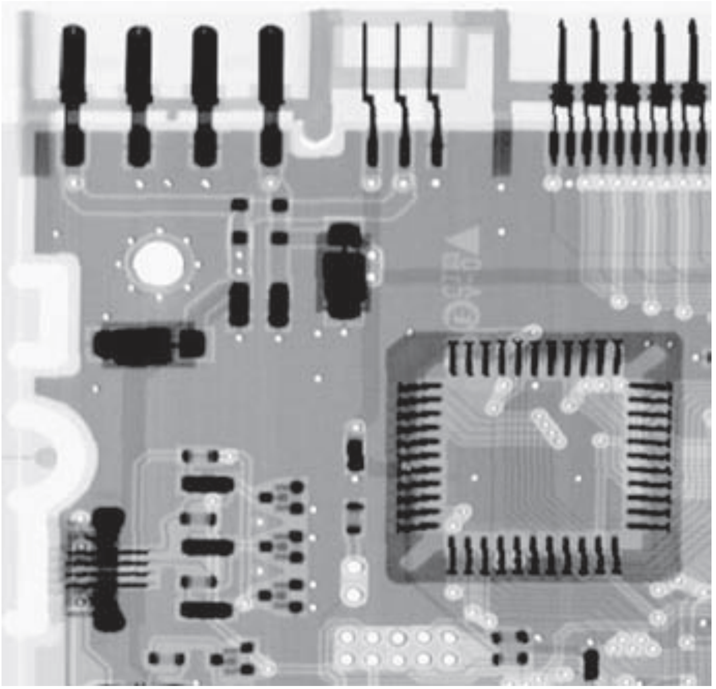
$$\hat{f}(x,y) = \frac{\sum_{(r,c)\in S_{xy}} g(r,c)^{Q+1}}{\sum_{(r,c)\in S_{xy}} g(r,c)^{Q}}$$

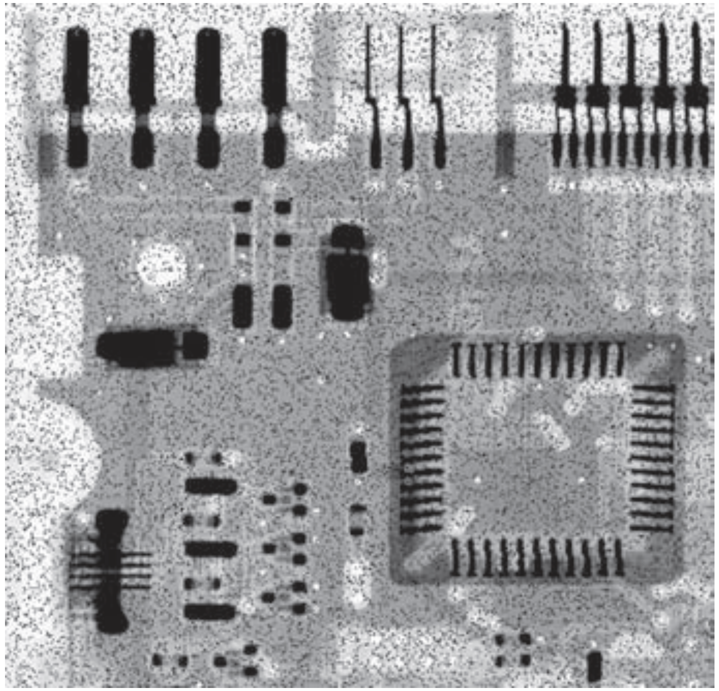
- Q is the *order* of the filter.
- Positive values of Q eliminate pepper noise.
- Negative values of Q eliminate salt noise.
- It cannot eliminate both simultaneously.

- Q = 0, arithmetic mean filter
- Q = -1, harmonic mean filter

- Noise removal examples

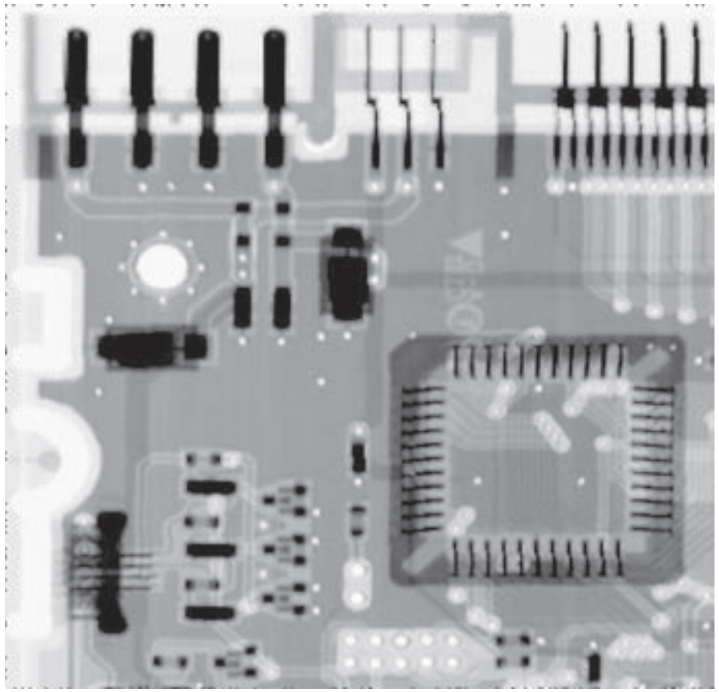
Original Image





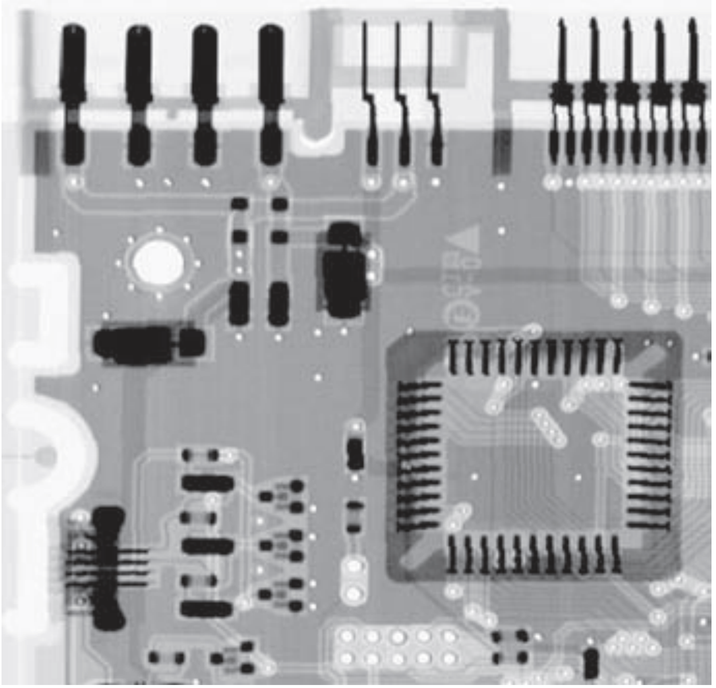
Corrupted by pepper noise at 0.1

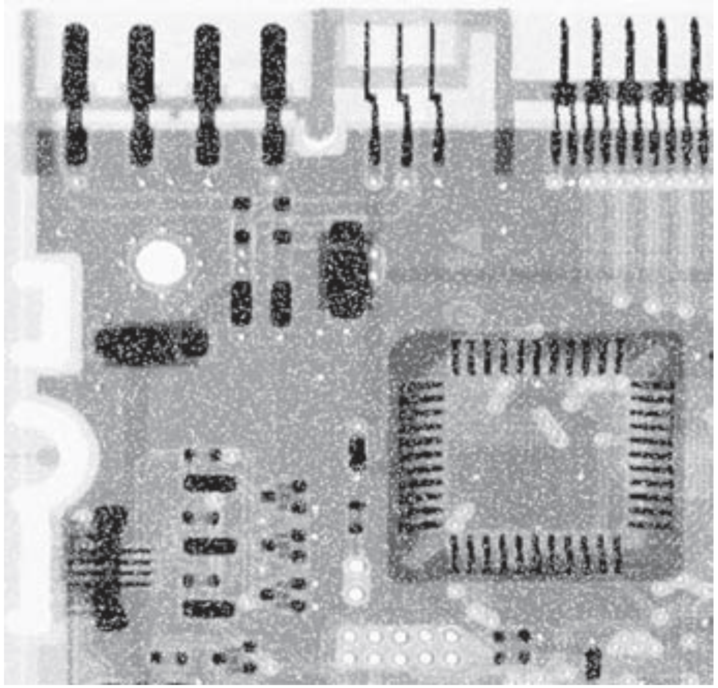
Filtering with a 3x3 Contraharmonic Filter with Q = 1.5



- Noise removal examples

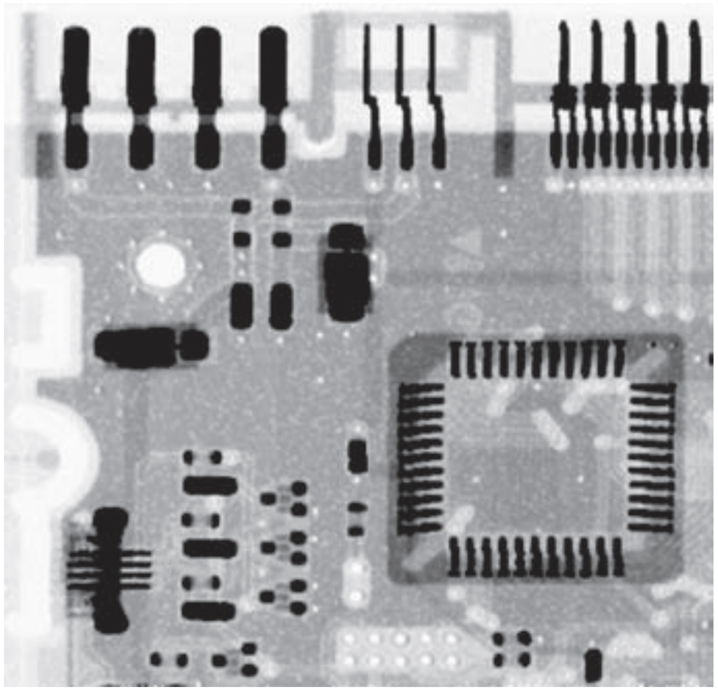
Original Image





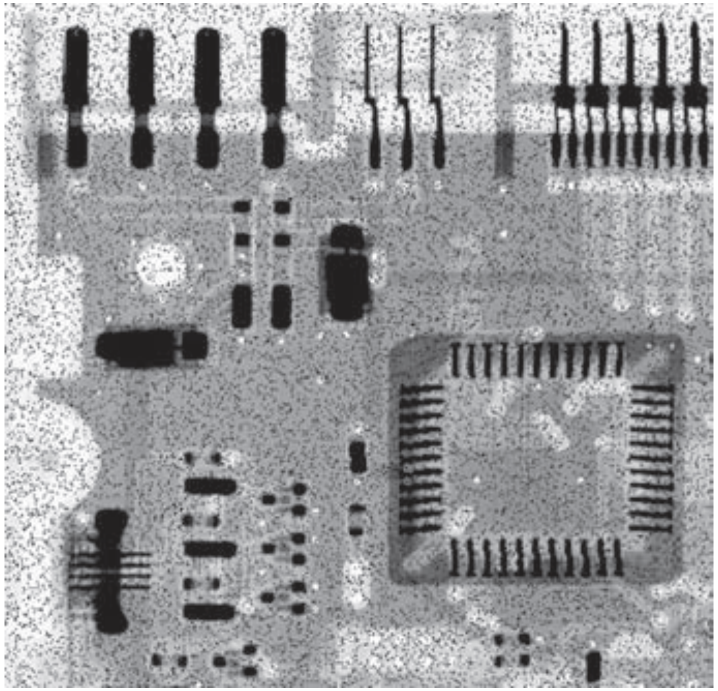
Corrupted by
salt noise at 0.1

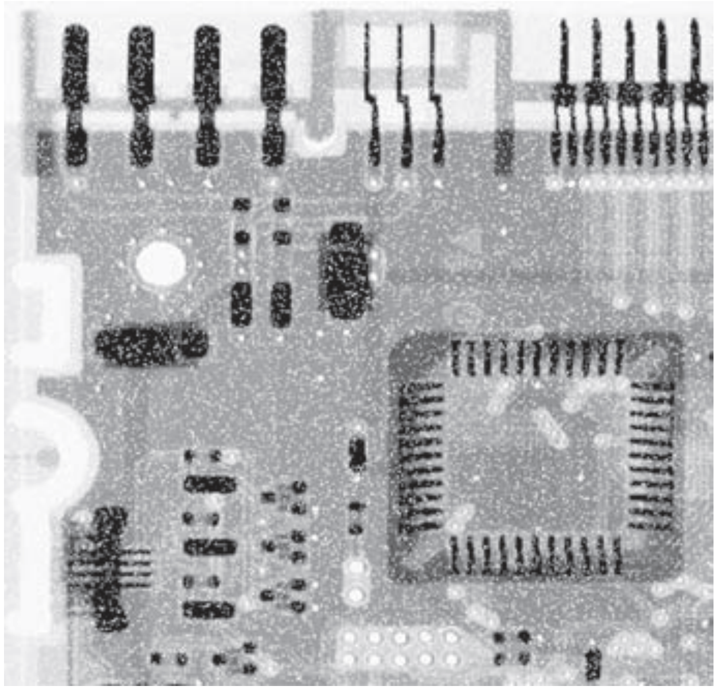
Filtering with a 3x3
Contraharmonic Filter with Q = -1.5



- Noise removal examples

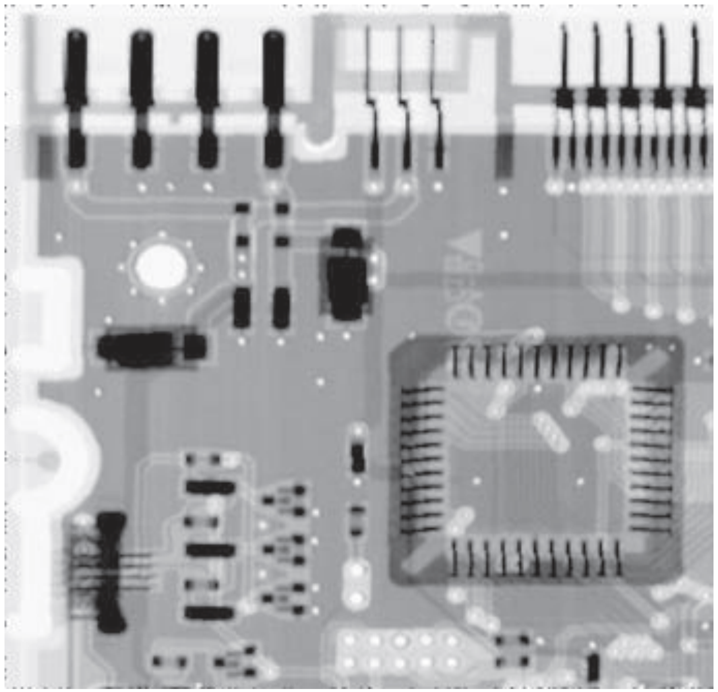
Corrupted by pepper noise at 0.1

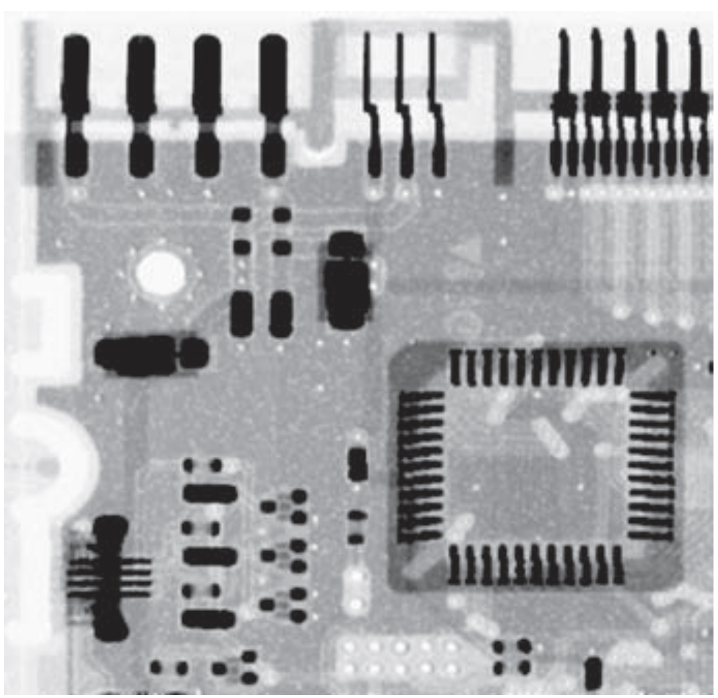




Corrupted by salt noise at 0.1

Contraharmonic with $Q = 1.5$

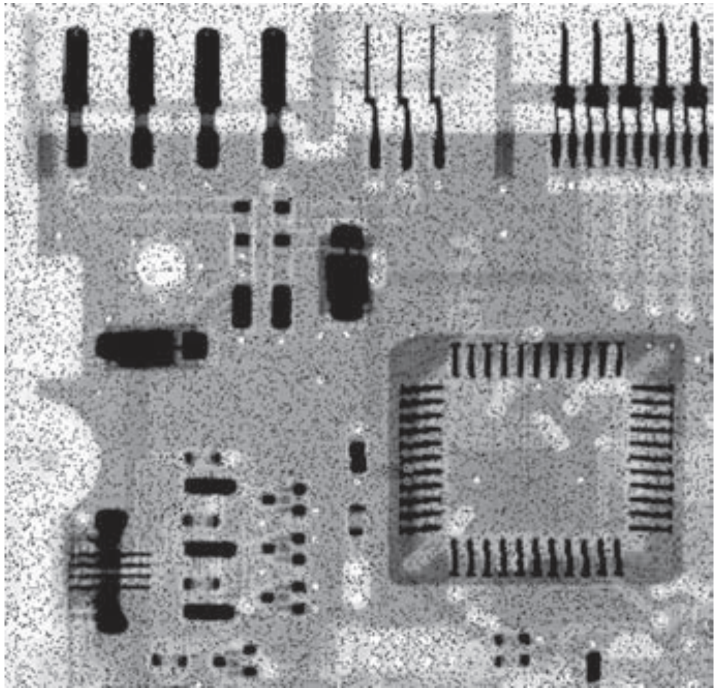


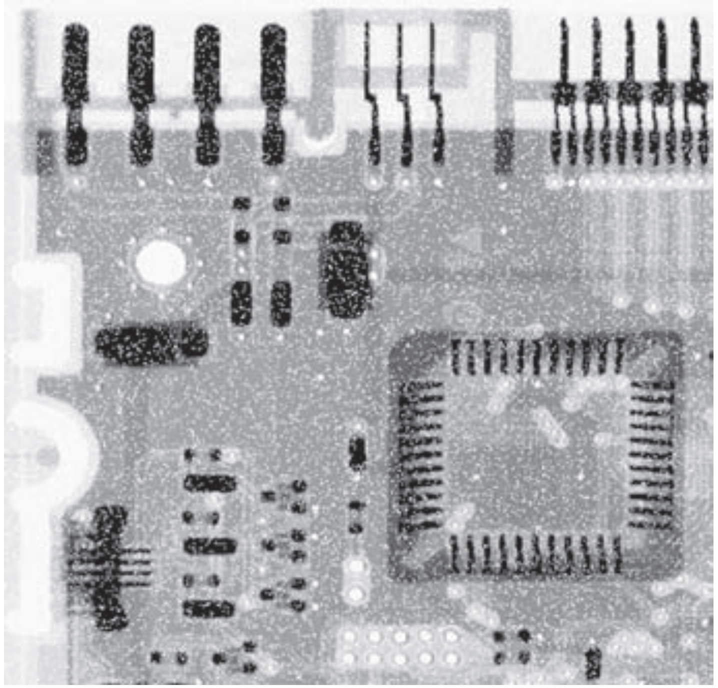


Contraharmonic with $Q = -1.5$

- Noise removal examples

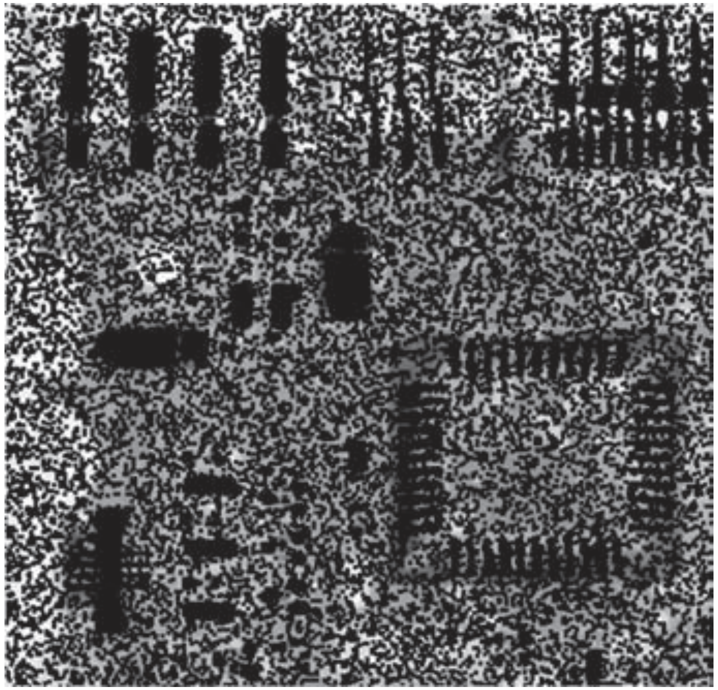
Corrupted by pepper noise at 0.1

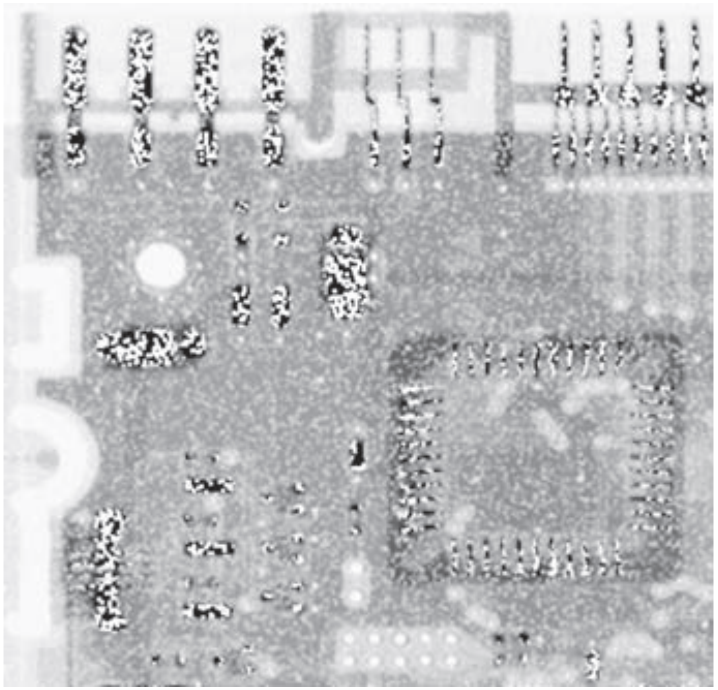




Corrupted by salt noise at 0.1

Contraharmonic with Q = -1.5





Contraharmonic with Q = 1.5

5.3 Restoration in the Presence of Noise Only –Spatial Filtering

- **Order-Statistics Filters**
  - Median Filter
    - excellent noise-reduction capabilities for random noise
    - less blurring
    - particularly good for both salt and pepper noise

$$\hat{f}(x,y) = \underset{(r,c)\in S_{xy}}{\text{median}}\{g(r,c)\}$$

  - Max and Min Filter

    - Max filter is good for pepper noise and Min filter is good for salt noise

Max Filter

$$\hat{f}(x,y) = \max_{(r,c)\in S_{xy}}\{g(r,c)\}$$

Min Filter

$$\hat{f}(x,y) = \min_{(r,c)\in S_{xy}}\{g(r,c)\}$$

5.3 Restoration in the Presence of Noise Only –Spatial Filtering

- **Order-Statistics Filters**
  - Midpoint Filter
    - Good for randomly distributed noise, like Gaussian or uniform noise

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(r,c)\in S_{xy}}\{g(r,c)\} + \min_{(r,c)\in S_{xy}}\{g(r,c)\}\right]$$

  - Alpha-Trimmed Mean Filter
    - delete the *d*/2 lowest and the *d*/2 highest intensity values, then averaging these remaining pixels
    - useful for situations involving multiple types of noise, such as a combination of salt-and-pepper and Gaussian noise.

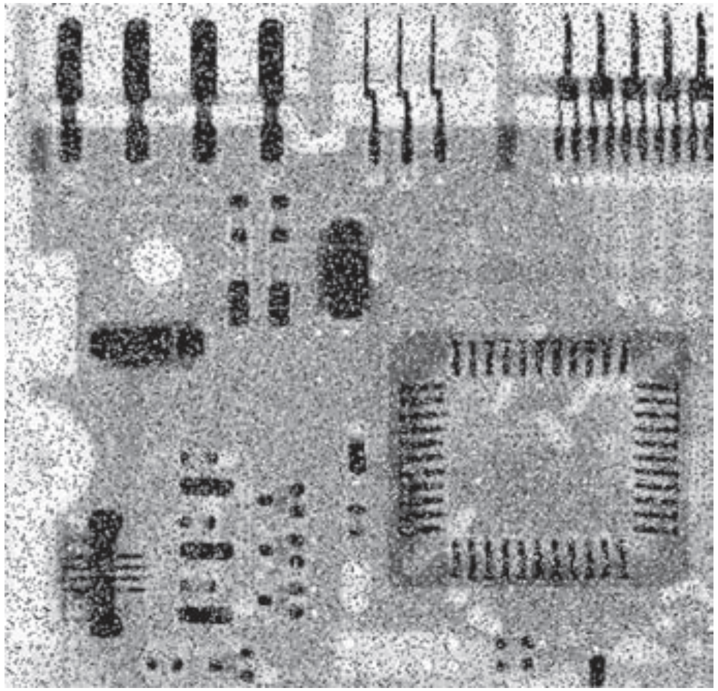
$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(r,c)\in S_{xy}} g_R(r,c)$$

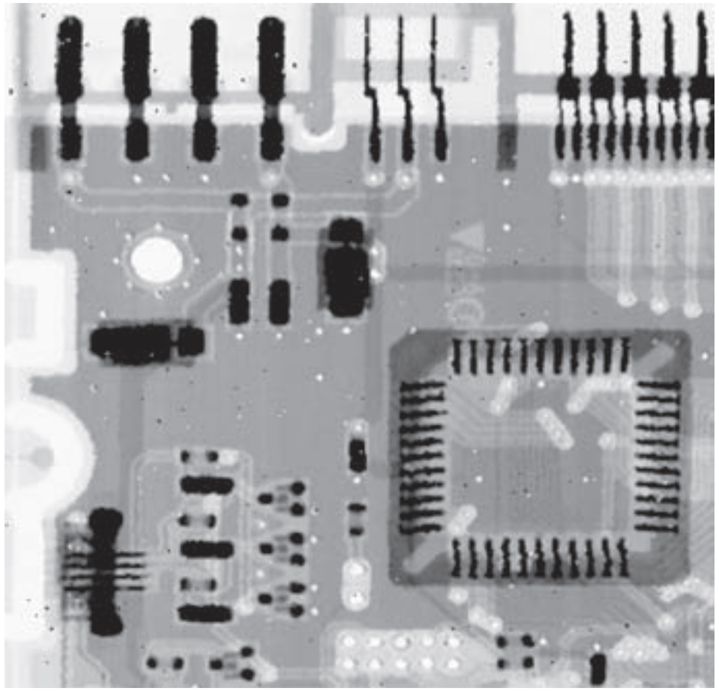
- $d = 0$, arithmetic mean filter
- $d = mn - 1$, median filter

- Noise removal examples

Repeated passes will reduce noise but blur the image

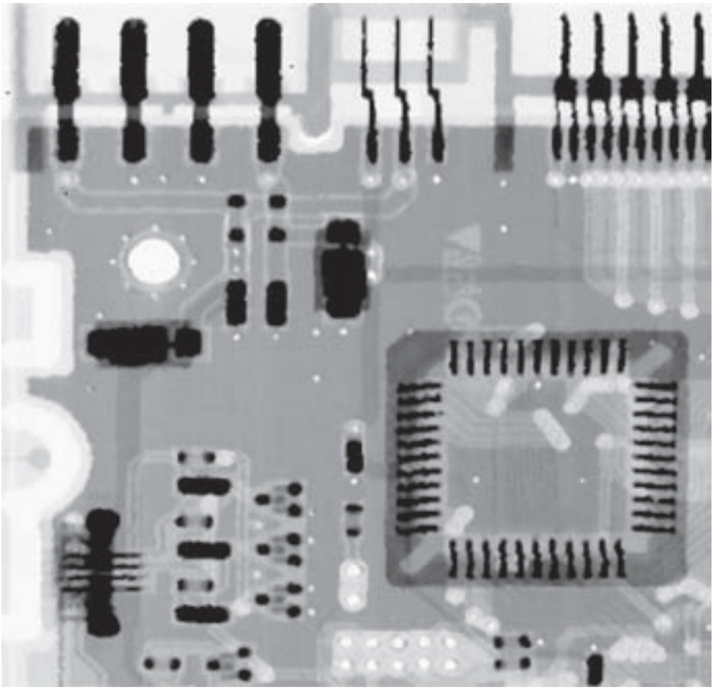
Corrupted by salt and pepper noise at 0.1

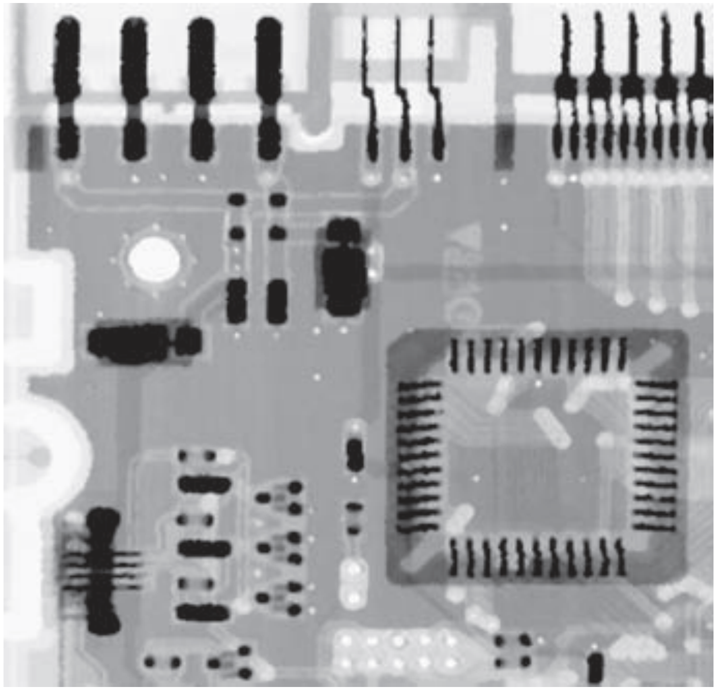




Median filter x1

Median filter x2

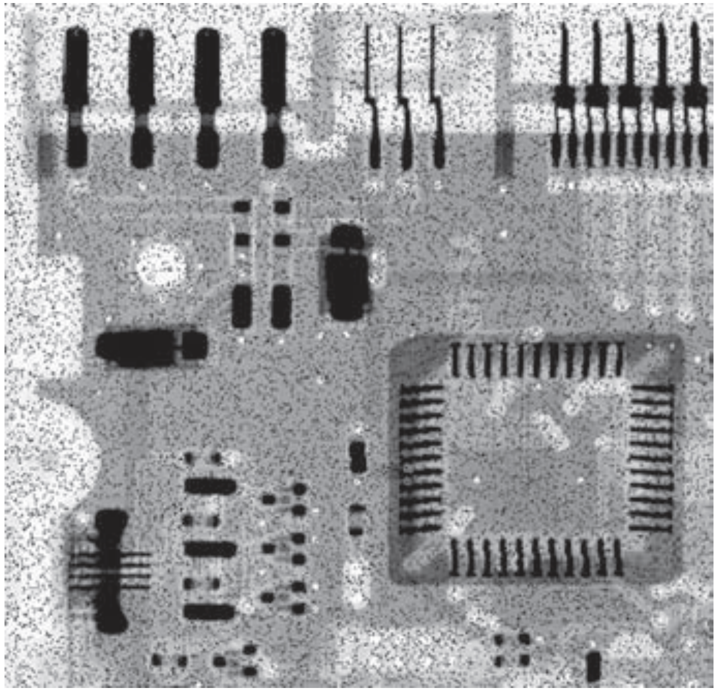


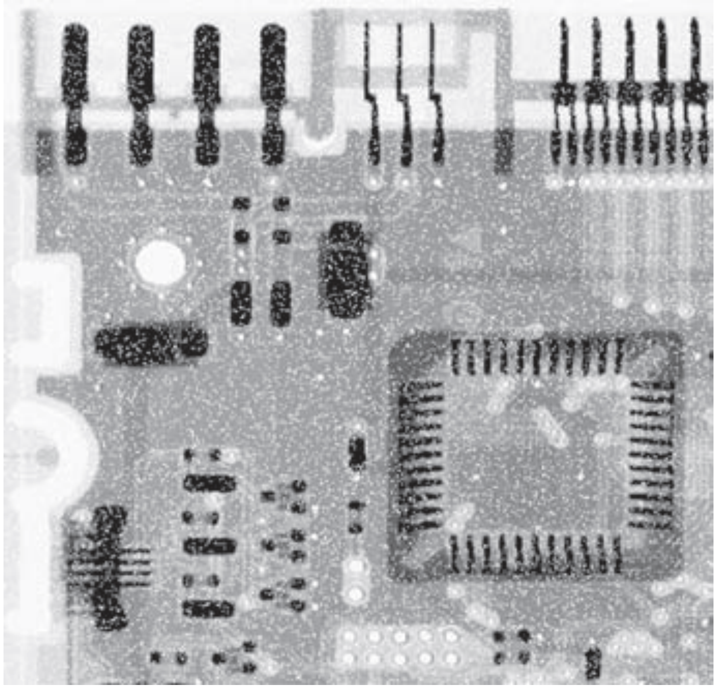


Median filter x3

- Noise removal examples

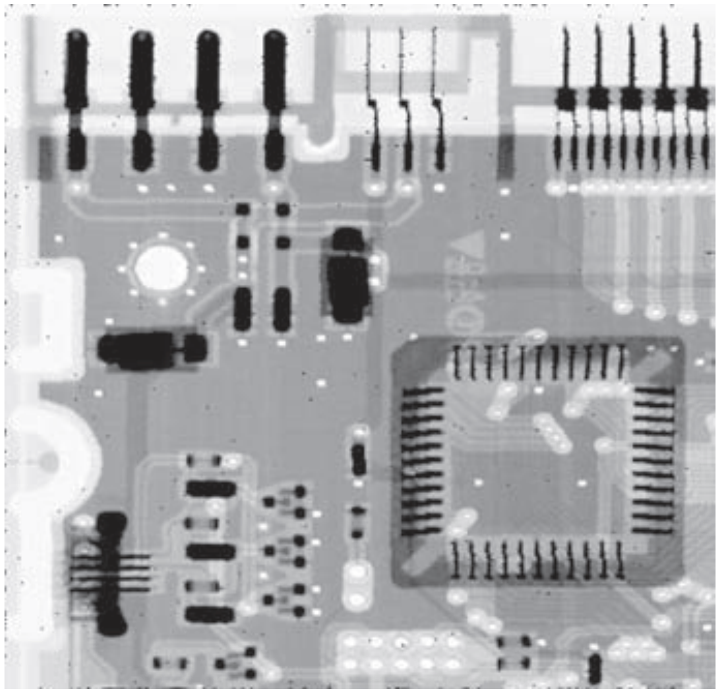
Corrupted by pepper noise at 0.1

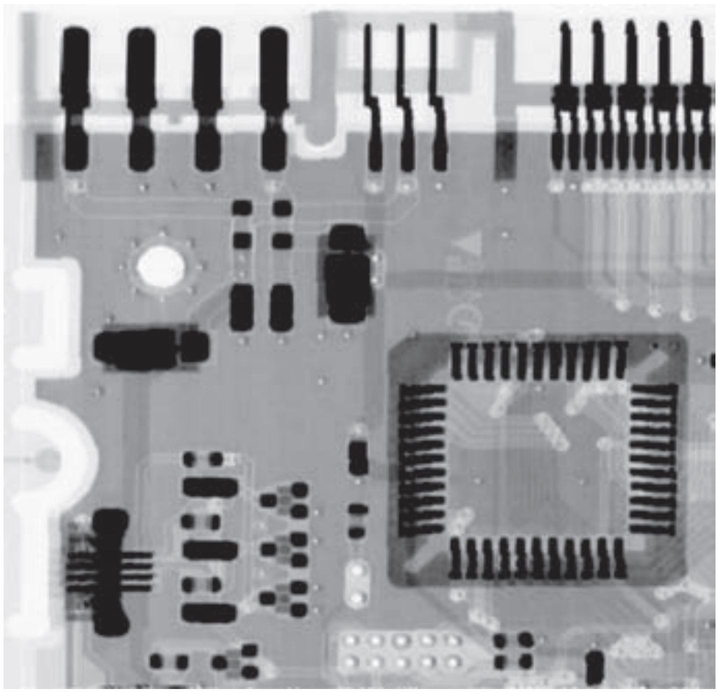




Corrupted by salt noise at 0.1

Max filter

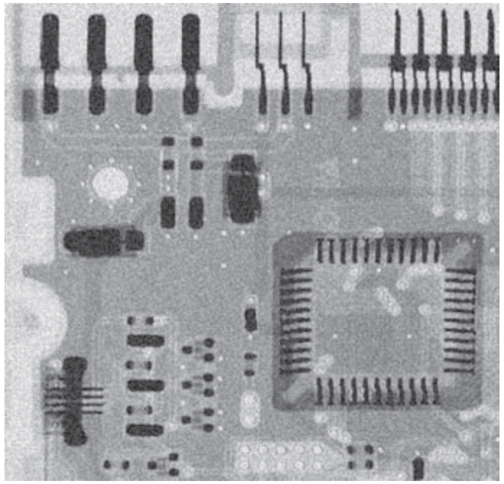


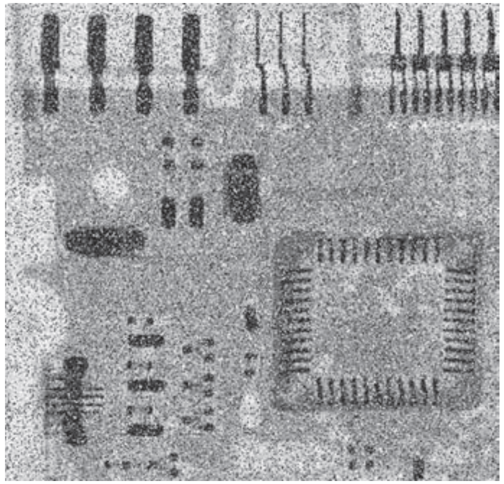


Min filter

- Noise removal examples

Corrupted by additive uniform noise



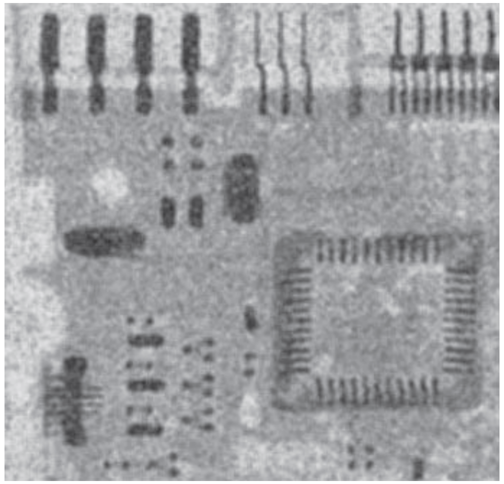


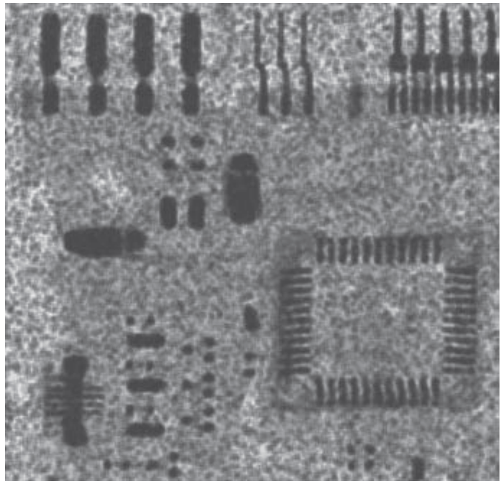
New Input Image

Corrupted by salt-and-pepper noise

5 x 5 for all filters

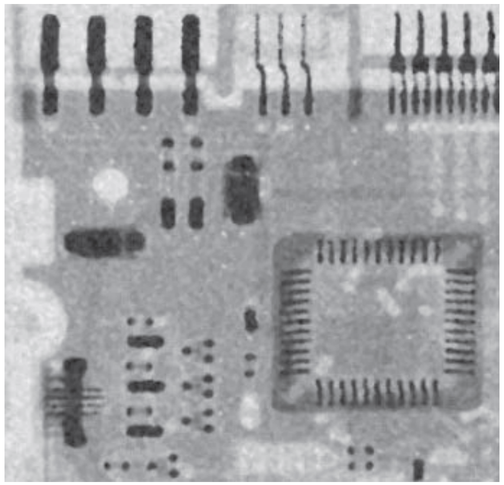
Arithmetic mean filter

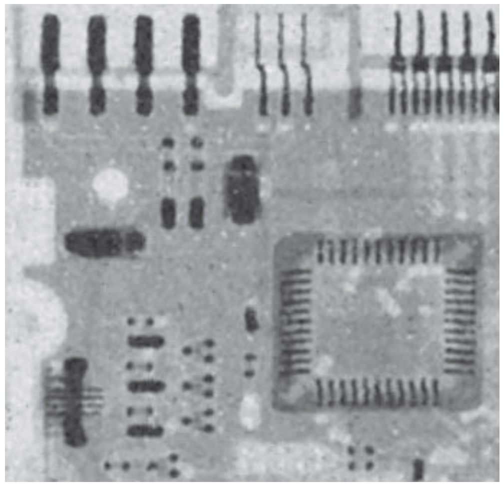




Geometric mean filter

Median filter





Alpha-trimmed median filter (d = 5)

slightly smoother

5.3 Restoration in the Presence of Noise Only –Spatial Filtering

- **Adaptive Filters**
  - The filters discussed so far are applied to an entire image without any regard for how image characteristics vary from one point to another.
  - The behaviour of **adaptive filters** changes depending on the characteristics of the image inside the filter region.
    - Adaptive, Local Noise Reduction Filter (Adaptive Mean Filter)
    - Adaptive Median Filter

credit of this slide: C. Nikou

5.3 Restoration in the Presence of Noise Only –Spatial Filtering

- **Adaptive Filters**
  - Adaptive, Local Noise Reduction Filter

noise variance: can be estimated from sample noisy images

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_{S_{xy}}^2}\left[g(x,y) - \bar{z}_{S_{xy}}\right]$$

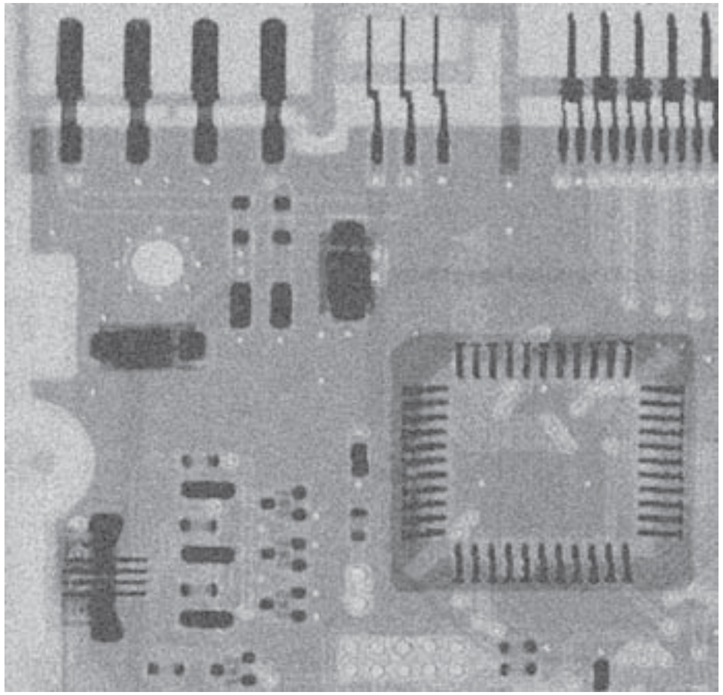
local mean

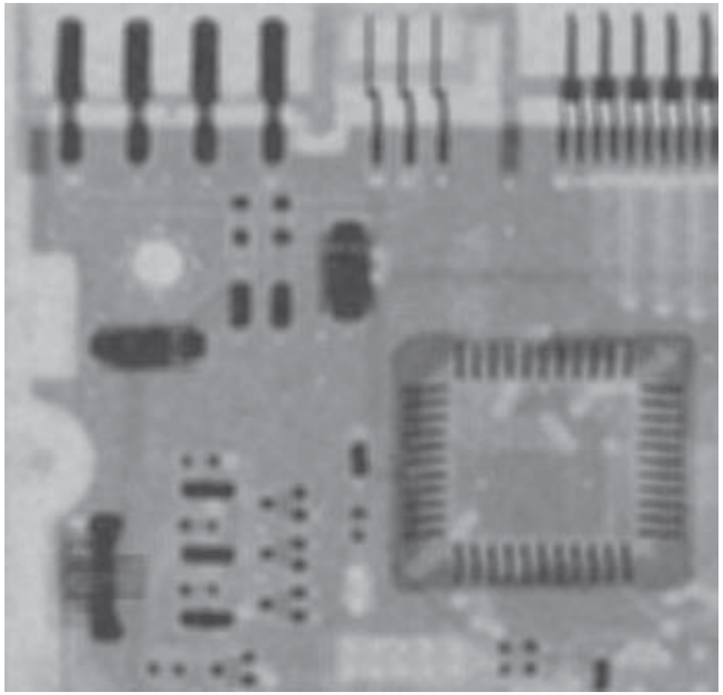
local variance

- If the variance of the noise = 0, return $g(x, y)$
- If local variance >> noise variance (e.g. edge), return a value close to $g(x, y)$
- If local variance ≈ noise variance, return local mean

- Noise removal examples

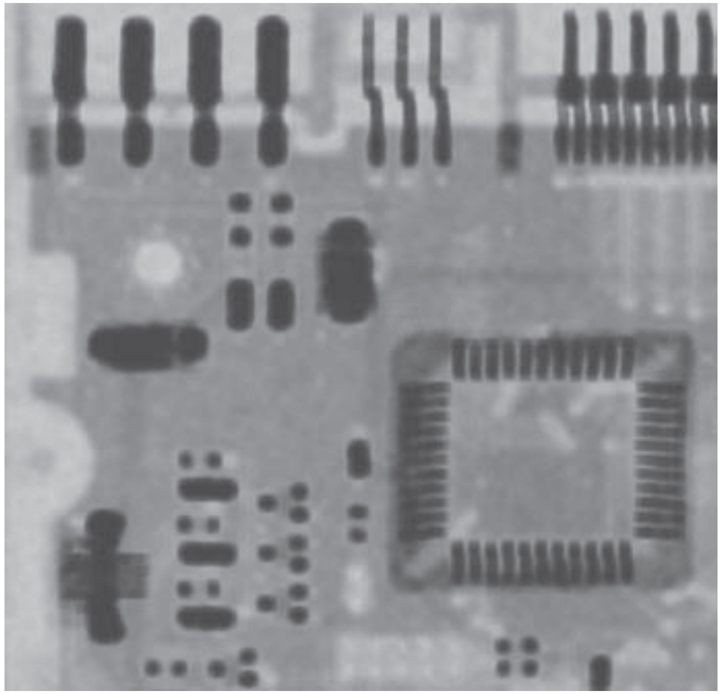
Corrupted with additive Gaussian noise

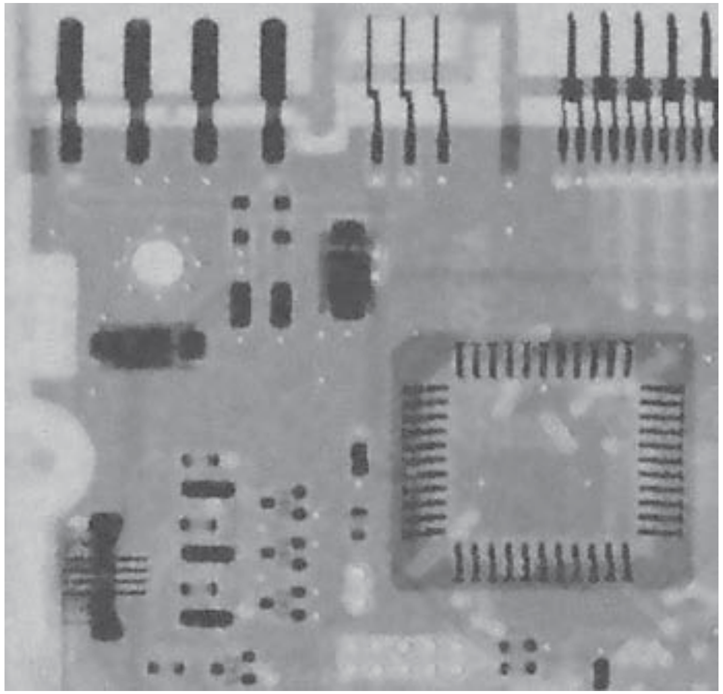




Arithmetic mean filter

Geometric mean filter





Adaptive mean filter

much sharper

5.3 Restoration in the Presence of Noise Only –Spatial Filtering

- **Adaptive Filters**
  - Adaptive Median Filter
    - The median filter performs relatively well on impulse noise as long as the spatial density of the impulse noise is not large.
    - The adaptive median filter can handle much more spatially dense impulse noise, and also performs some smoothing for non-impulse noise.

credit of this slide: C. Nikou

5.3 Restoration in the Presence of Noise Only –Spatial Filtering

- **Adaptive Filters**
  - Adaptive Median Filter
    - The key to understanding the algorithm is to remember that the adaptive median filter has three purposes:
      - Remove impulse noise
      - Provide smoothing of other noise
      - Reduce distortion (excessive thinning or thickenning of object boundaries)
    - In the adaptive median filter, the filter size changes depending on the characteristics of the image

credit of this slide: C. Nikou

5.3 Restoration in the Presence of Noise Only –Spatial Filtering

- **Adaptive Filters**
  - Adaptive Median Filter
    - Notation:

$$
\begin{aligned}
z_{\min} &= \text{minimum intensity value in } S_{xy} \\
z_{\max} &= \text{maximum intensity value in } S_{xy} \\
z_{\text{med}} &= \text{median of intensity values in } S_{xy} \\
z_{xy} &= \text{intensity at coordinates } (x, y) \\
S_{\max} &= \text{maximum allowed size of } S_{xy}
\end{aligned}
$$

5.3 Restoration in the Presence of Noise Only –Spatial Filtering

- **Adaptive Filters**
  - Adaptive Median Filter
    - **Algorithm**:

z_{min} = minimum intensity value in S_{xy}
z_{max} = maximum intensity value in S_{xy}
z_{med} = median of intensity values in S_{xy}
z_{xy} = intensity at coordinates (x, y)
S_{max} = maximum allowed size of S_{xy}

check z_{med} is an impulse or not

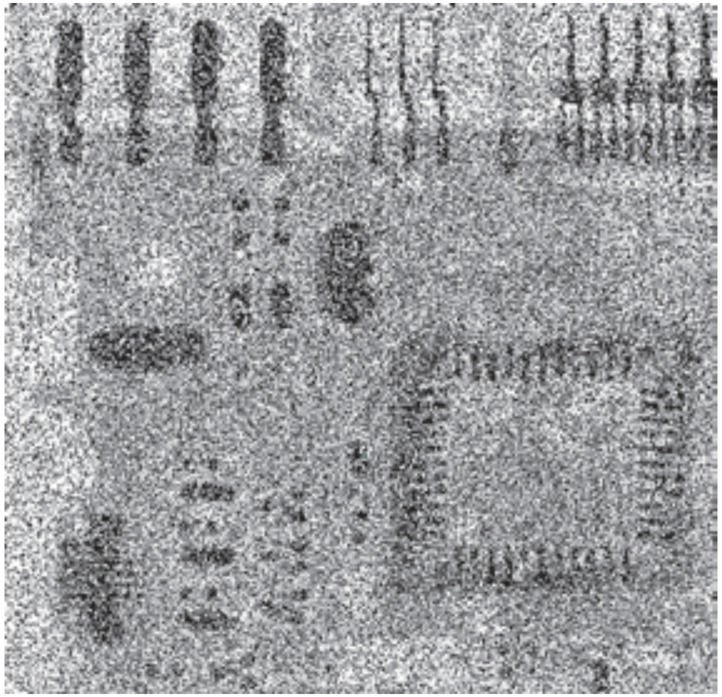
Level A: If $z_{\text{min}} < z_{\text{med}} < z_{\text{max}}$, go to Level B
Else, increase the size of S_{xy}
If $S_{xy} \leq S_{\text{max}}$, repeat level A
Else, output z_{med}.

check z_{xy} is an impulse or not

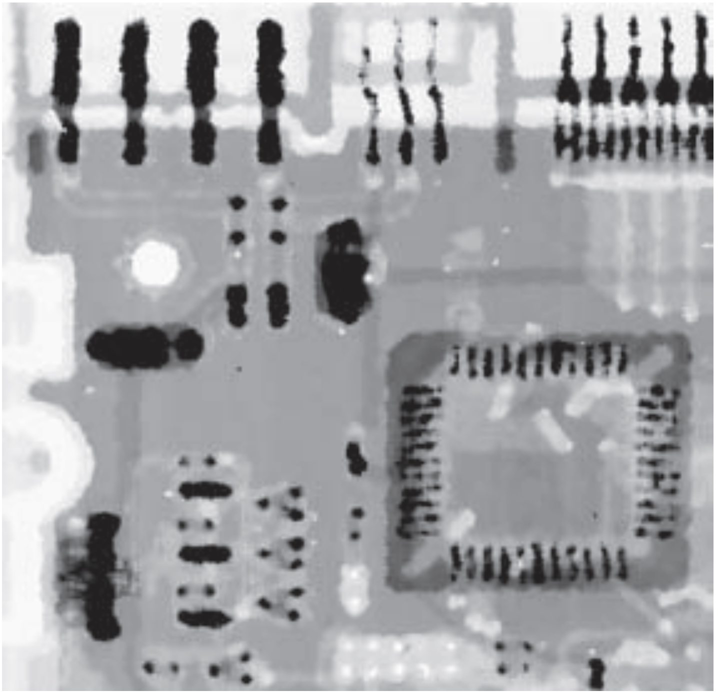
Level B: If $z_{\text{min}} < z_{xy} < z_{\text{max}}$, output z_{xy}
Else output z_{med}.

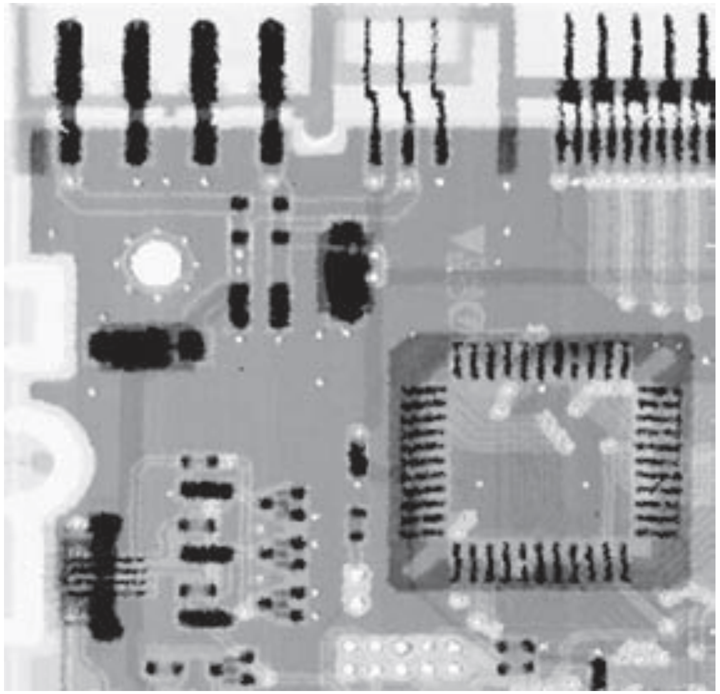
- Noise removal examples

Corrupted by salt-and-pepper noise at 0.2



median filter



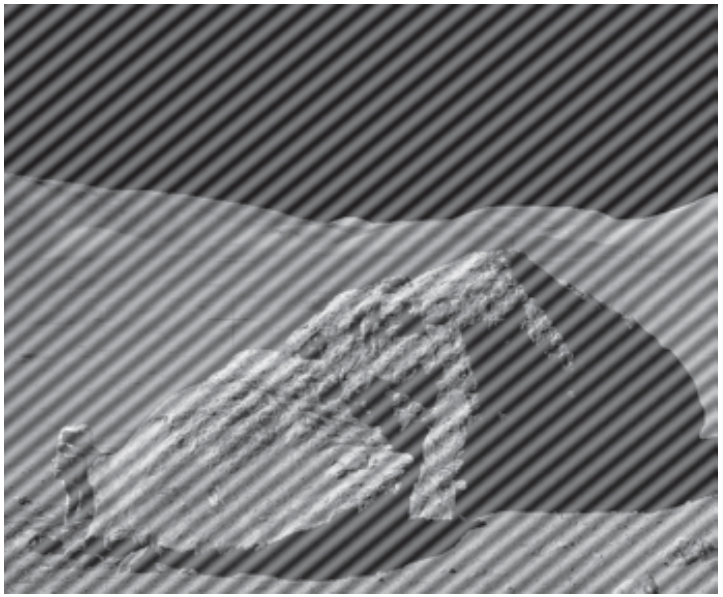


Adaptive median filter

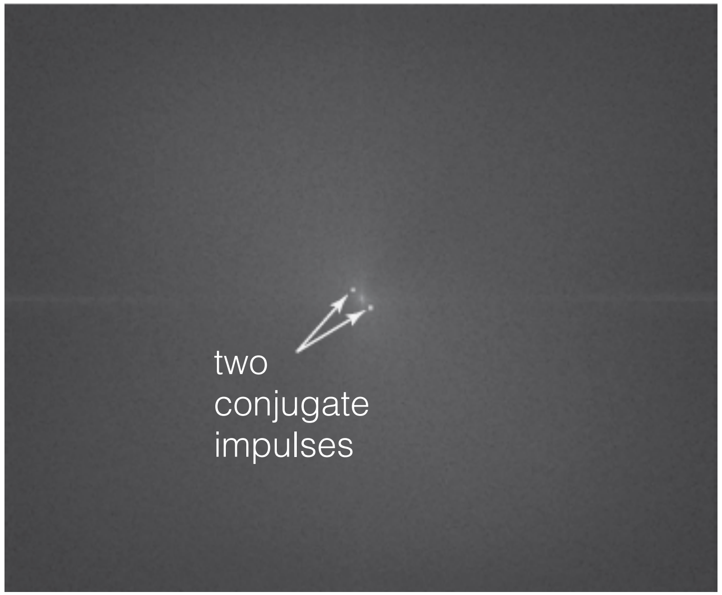
preserves sharpness and details

5.4 Periodic Noise Reduction by Frequency Domain Filtering

- Periodic Noise
  - Typically arises due to electrical or electromagnetic interference
  - Gives rise to regular noise patterns in an image.
  - Frequency domain techniques in the Fourier domain are most effective at removing periodic noise



Image



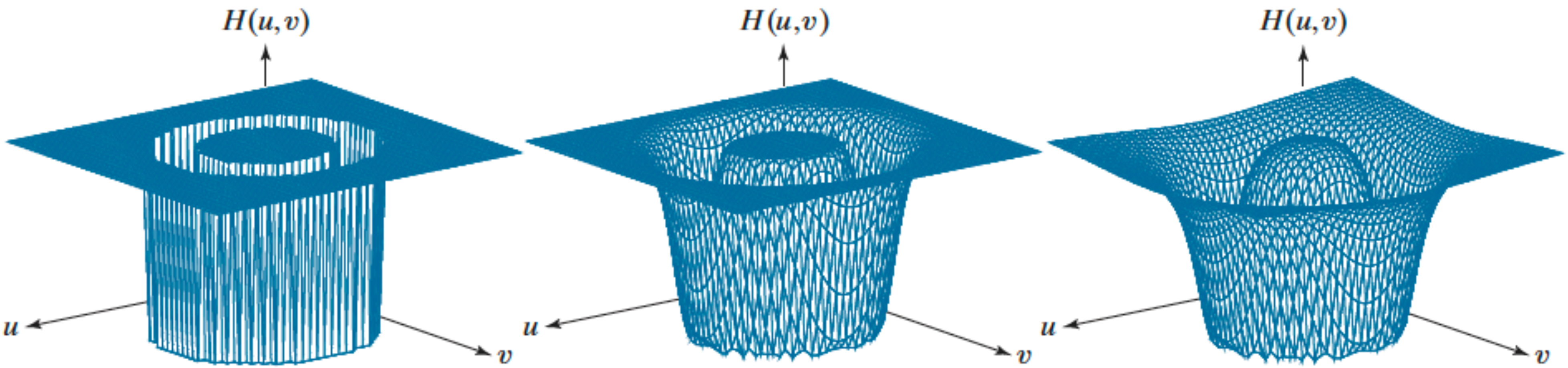
Fourier Spectrum

credit of this slide: C. Nikou

5.4 Periodic Noise Reduction by Frequency Domain Filtering

- Review - Bandreject Filter (notch reject filters) (Sec. 4.10)

Ideal (IBRF)	**Gaussian (GBRF)**	**Butterworth (BBRF)**
$H(u,v) = \begin{cases} 0 & \text{if } C_0 - \dfrac{W}{2} \le D(u,v) \le C_0 + \dfrac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = 1 - e^{-\left[\frac{D^2(u,v) - C_0^2}{D(u,v)W}\right]^2}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{D(u,v)W}{D^2(u,v) - C_0^2}\right]^{2n}}$



5.4 Periodic Noise Reduction by Frequency Domain Filtering

- Example

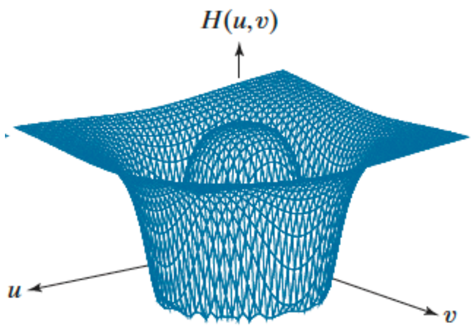
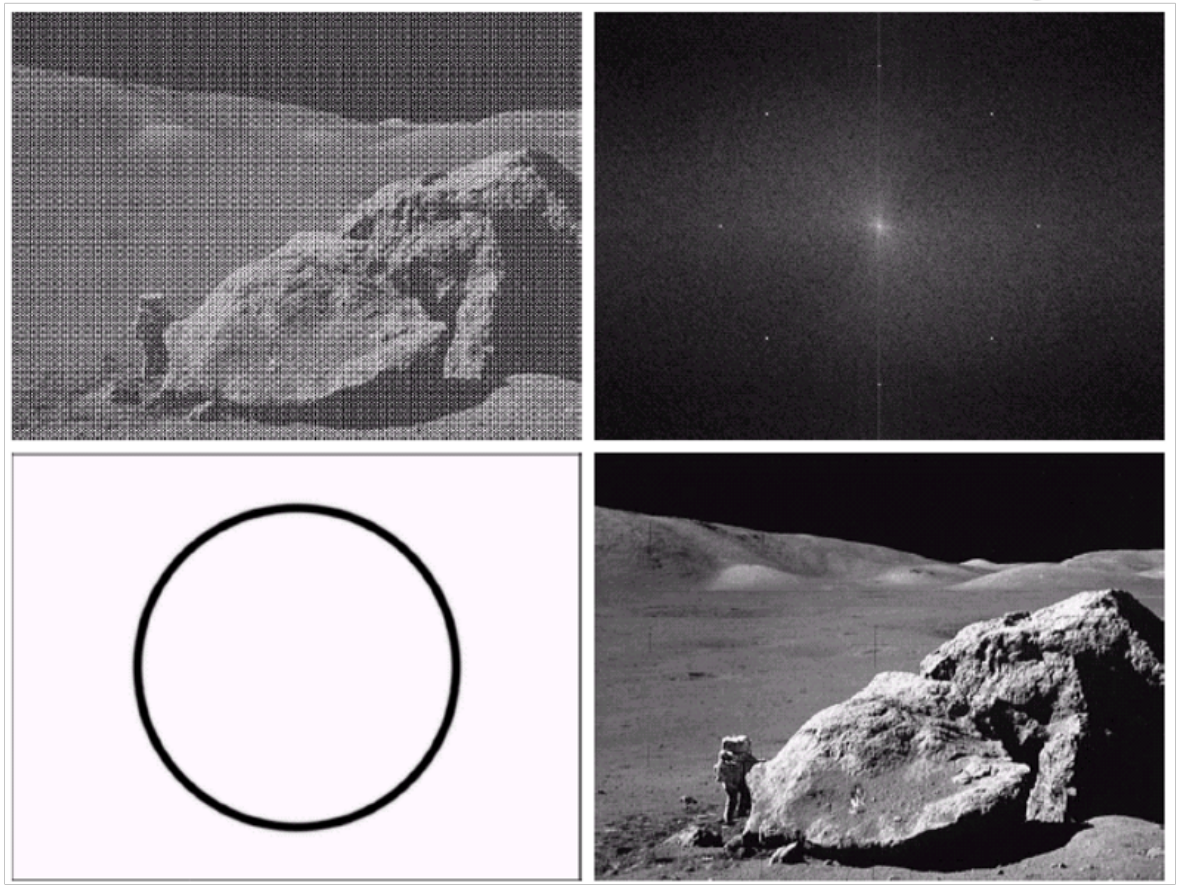


Image corrupted by sinusoidal noise

Fourier spectrum of corrupted image



Butterworth band reject filter

Filtered image

credit of this slide: C. Nikou

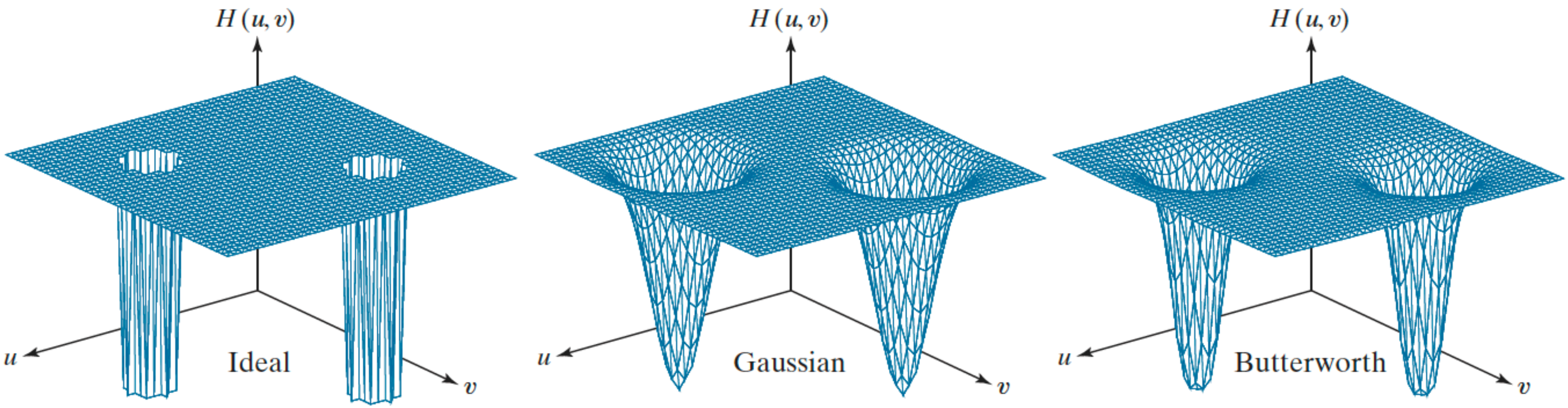
5.4 Periodic Noise Reduction by Frequency Domain Filtering

- Notch Filter
  - The general form of a notch filter transfer function:

$$H_{NR}(u,v) = \prod_{k=1}^{Q} H_k(u,v)H_{-k}(u,v)$$

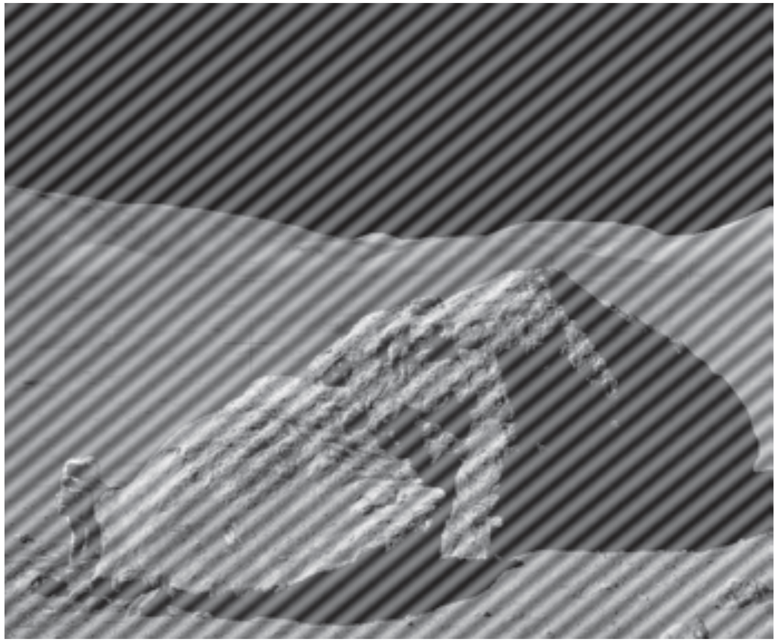
    - notches are specified as symmetric pairs

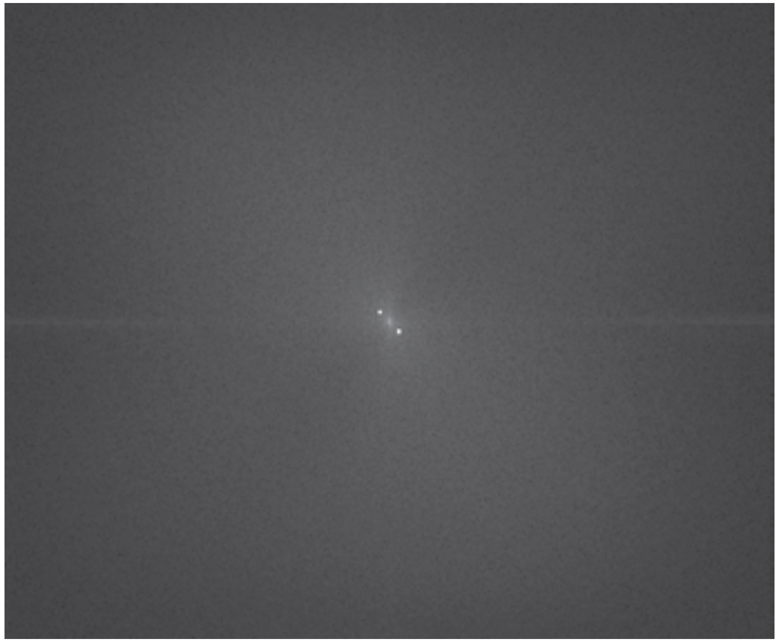
Example with one notch pair



- Noise removal examples

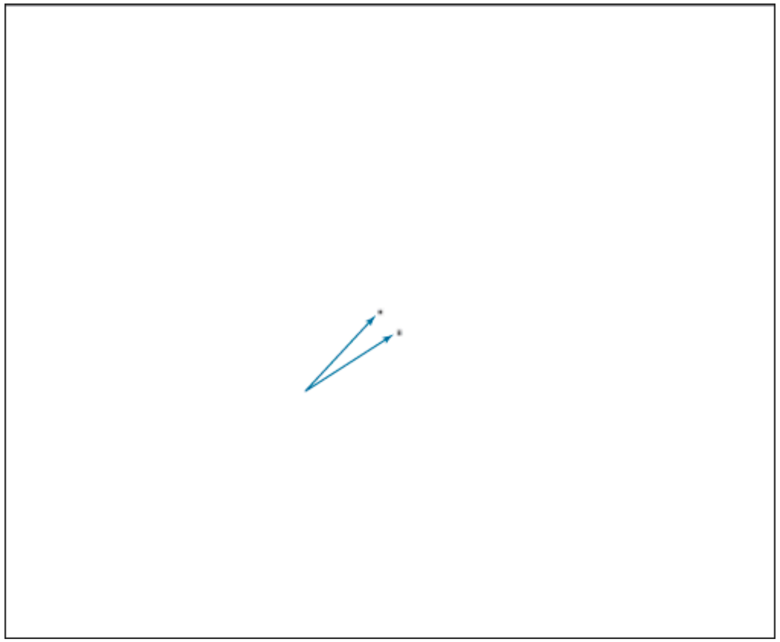
Image corrupted by sinusoidal interference.





Spectrum

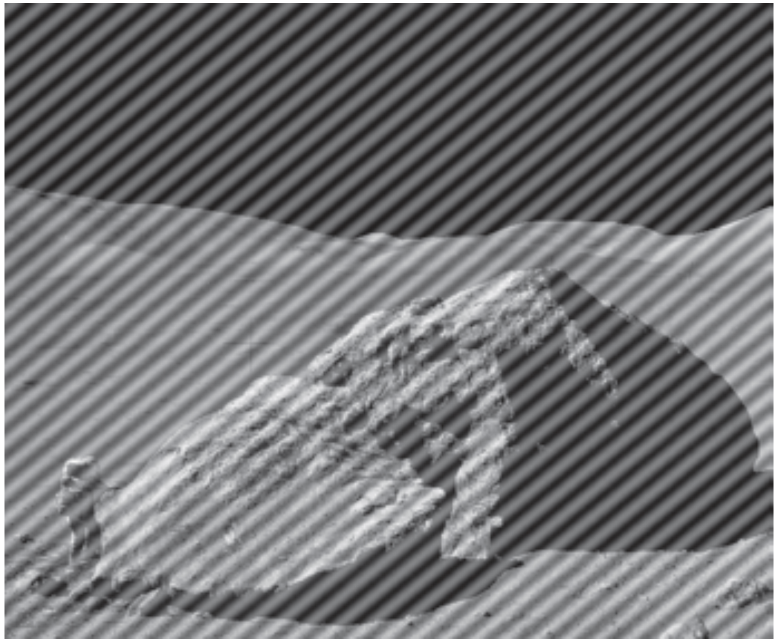
Notch filter



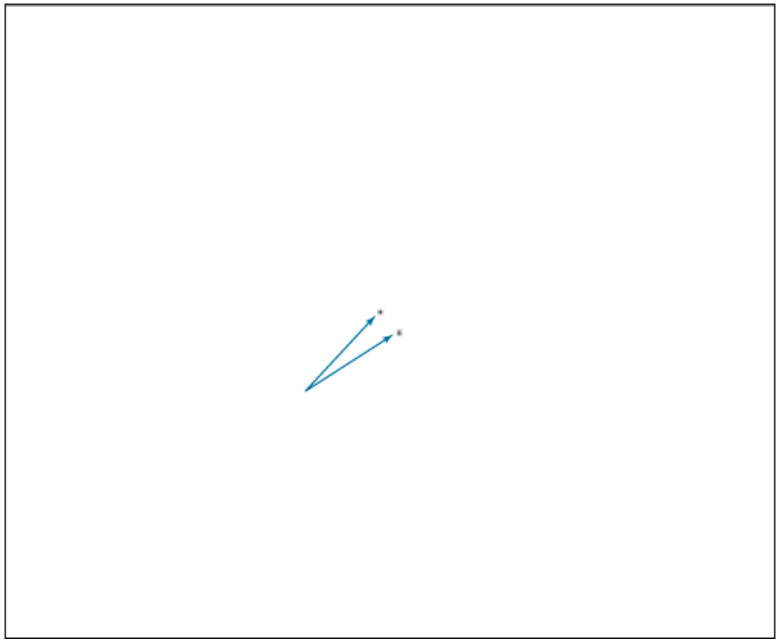
Result of notch reject filtering

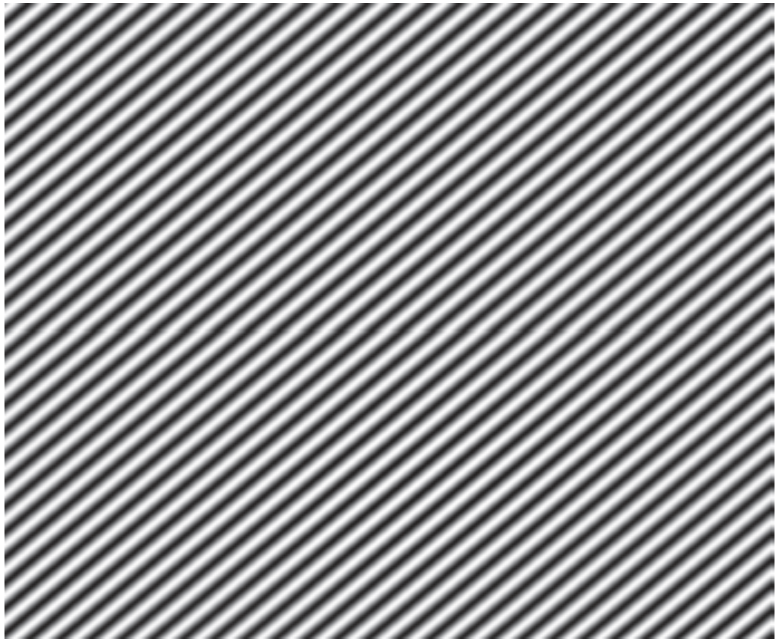
- Noise removal examples

Image corrupted by sinusoidal interference.



Notch filter

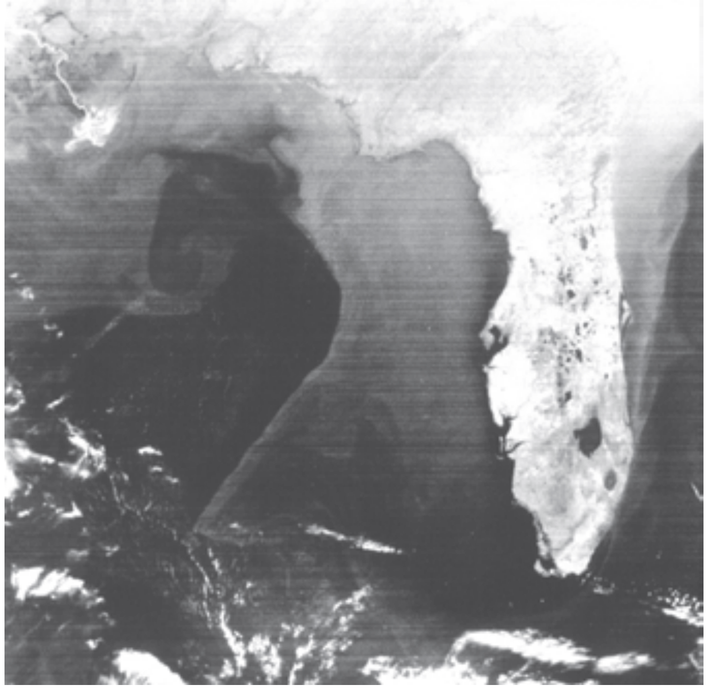


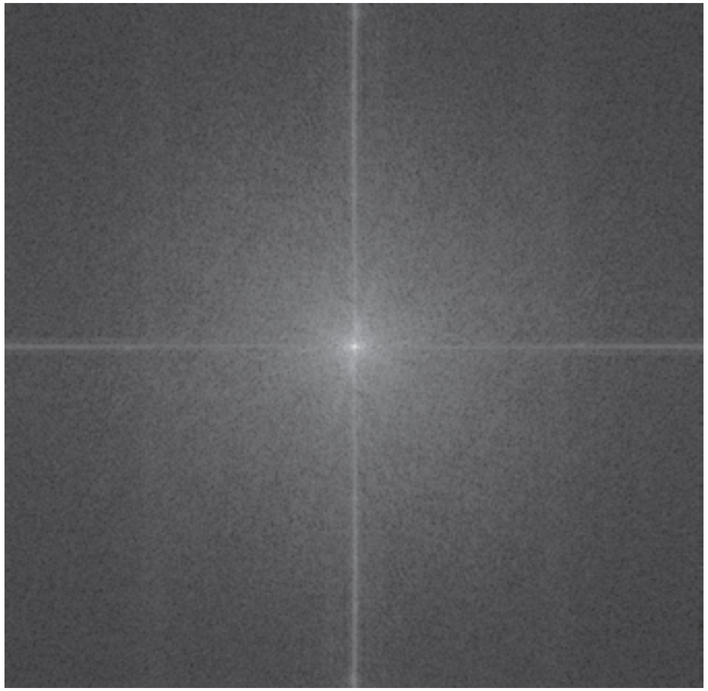


Corresponding notch pass filter

- Noise removal examples

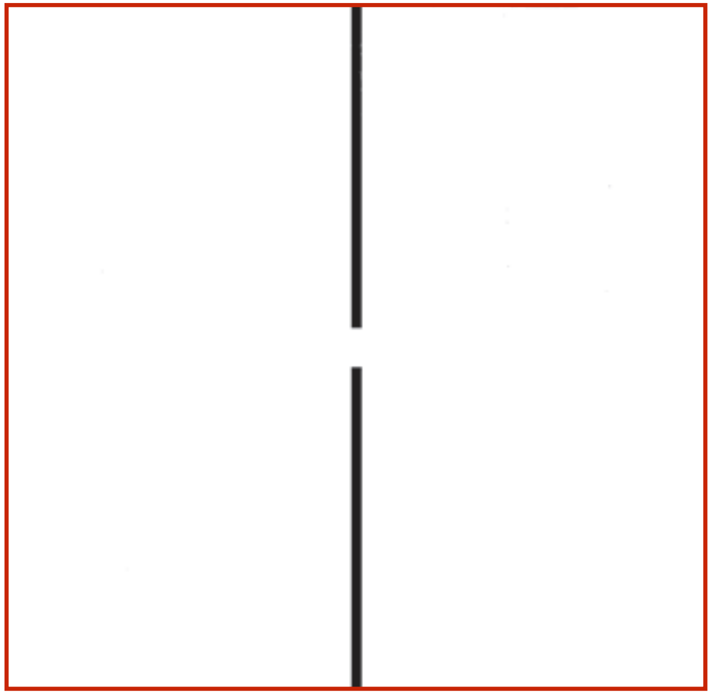
Satellite image

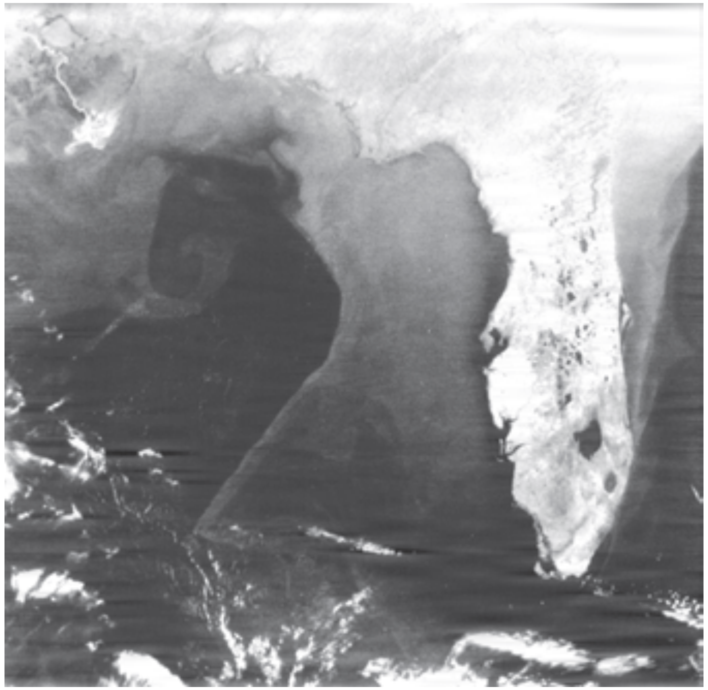




Spectrum

Notch reject filter

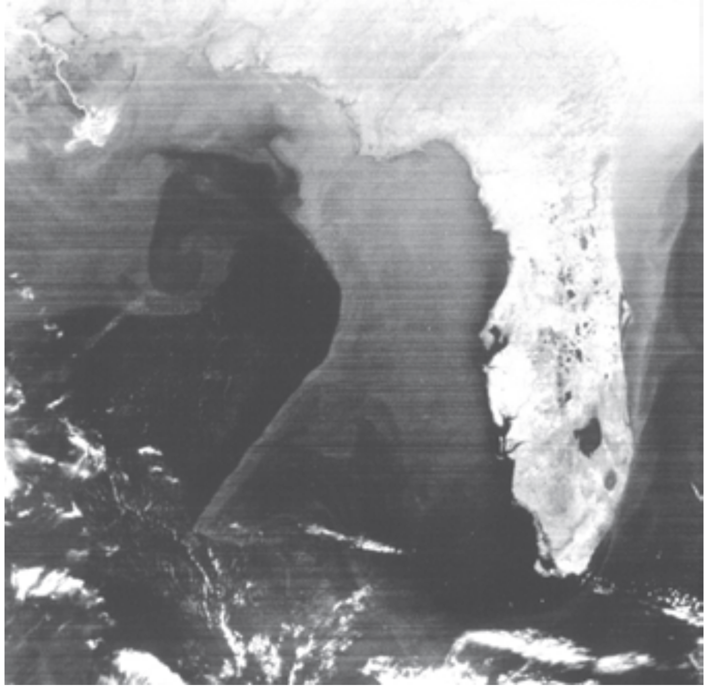




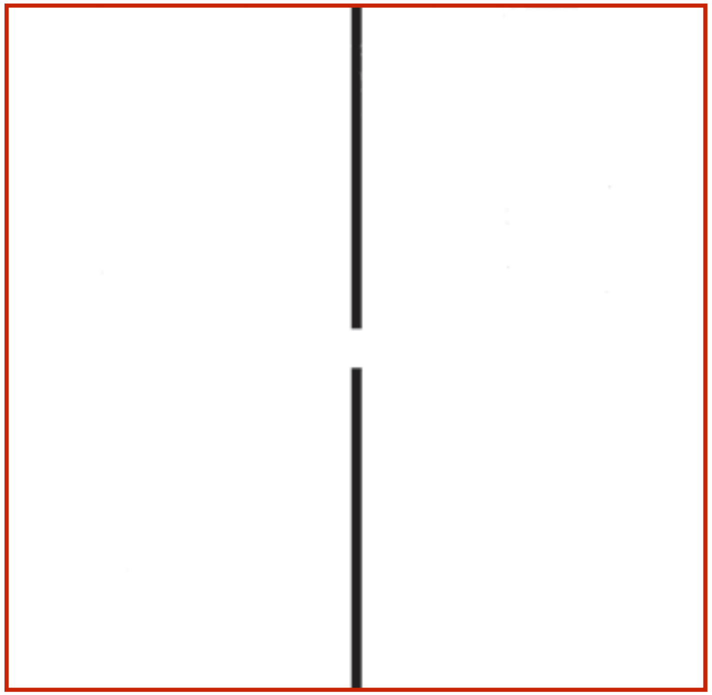
Result

- Noise removal examples

Satellite image



Notch reject filter

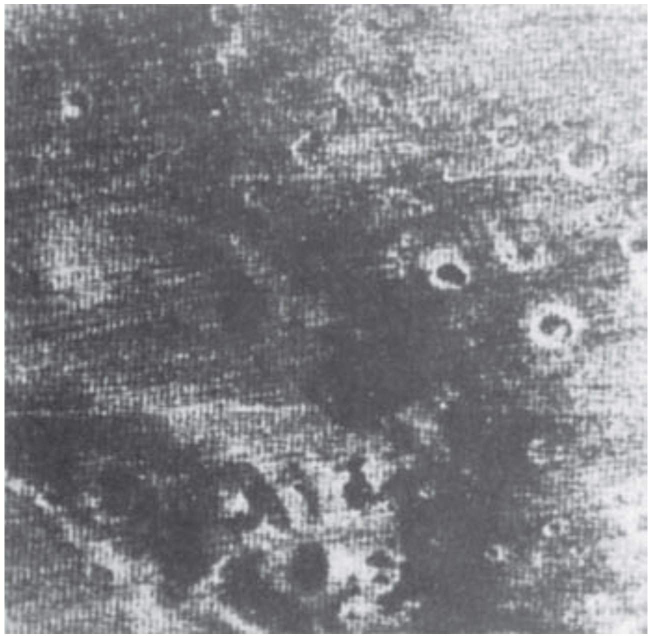


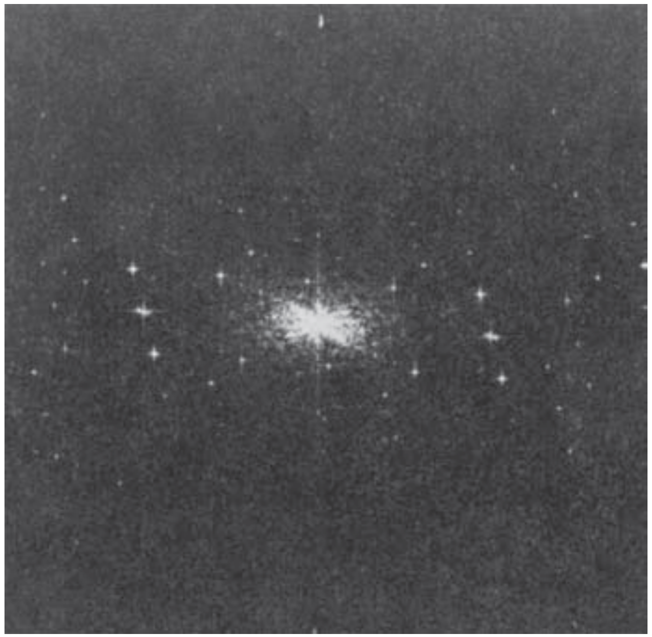
Corresponding notch pass filter

5.4 Periodic Noise Reduction by Frequency Domain Filtering

- **Optimum Notch Filter (Skip the details)**
  - Previous notch filtering discussed, the interference patterns have been simple to identify and characterize in the frequency domain
  - When several interference components are present, heuristic specifications of filter transfer functions are not always acceptable
  - Optimum, in the sense that it minimizes local variances

Image of the Martian terrain



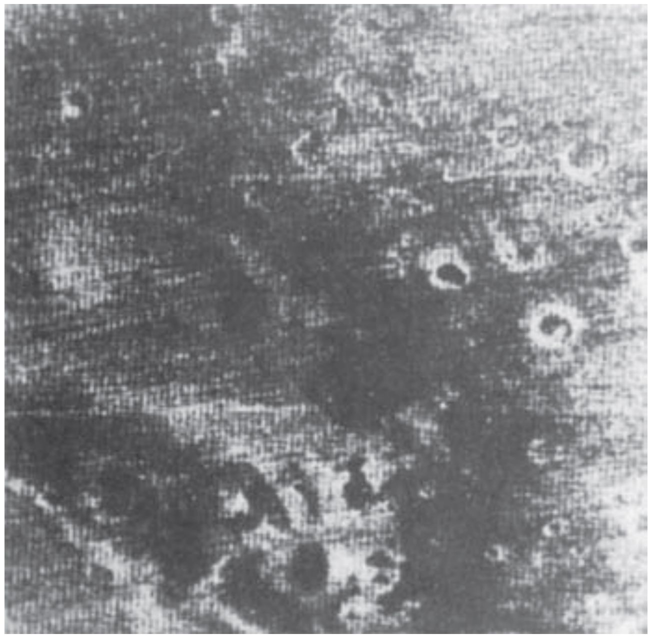


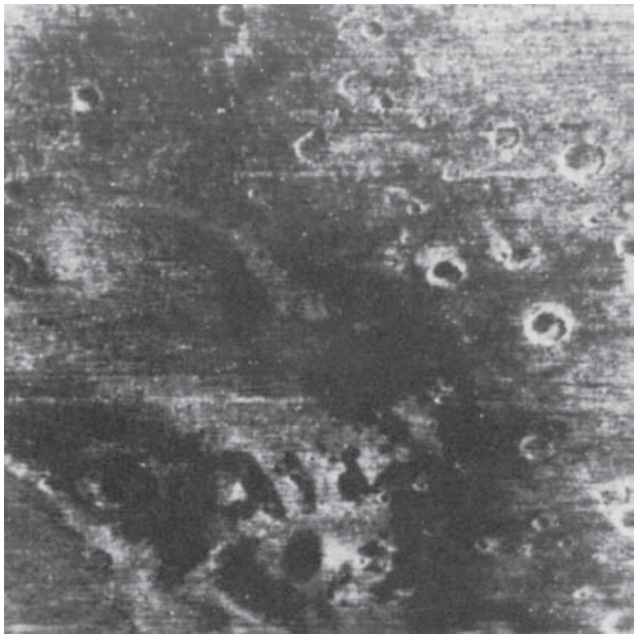
Spectrum

5.4 Periodic Noise Reduction by Frequency Domain Filtering

- **Optimum Notch Filter** (Skip the details)
  - Previous notch filtering discussed, the interference patterns have been simple to identify and characterize in the frequency domain
  - When several interference components are present, heuristic specifications of filter transfer functions are not always acceptable
  - Optimum, in the sense that it minimizes local variances

Image of the Martian terrain





Restored image