4.24 With reference to the discussion on linearity in Section 2.6, demonstrate that

(a)* The 2-D continuous Fourier transform is a linear operator.

(b) The 2-D DFT is a linear operator also.

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M + vy/N)}$$

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(ux/M + vy/N)}$$

4.28 Show the validity of the following 2-D *discrete* Fourier transform pairs from Table 4.4:

(a)* $\delta(x,y) \Leftrightarrow 1$

(b)* $1 \Leftrightarrow MN\delta(u,v)$

(c) $\delta(x - x_0, y - y_0) \Leftrightarrow e^{-j2\pi(ux_0/M + vy_0/N)}$

(d)* $e^{j2\pi(u_0x/M + v_0y/N)} \Leftrightarrow MN\delta(u - u_0, v - v_0)$

(e) $\cos(2\pi\mu_0 x/M + 2\pi\nu_0 y/N) \Leftrightarrow$
$(MN/2)\left[\delta(u + \mu_0, v + v_0) + \delta(u - u_0, v - v_0)\right]$

(f)* $\sin(2\pi\mu_0 x/M + 2\pi\nu_0 y/N) \Leftrightarrow$
$(jMN/2)\left[\delta(u + \mu_0, v + v_0) - \delta(u - u_0, v - v_0)\right]$

4.32 We mentioned in Example 4.10 that embedding a 2-D array of even (odd) dimensions into a larger array of zeros of even (odd) dimensions keeps the symmetry of the original array, provided that the centers coincide. Show that this is true also for the following 1-D arrays (i.e., show that the larger arrays have the same symmetry as the smaller arrays). For arrays of even length, use arrays of 0's ten elements long. For arrays of odd lengths, use arrays of 0's nine elements long.

(a)* $\{a, b, c, c, b\}$

(b) $\{0, -b, -c, 0, c, b\}$

(c) $\{a, b, c, d, c, b\}$

(d) $\{0, -b, -c, c, b\}$

$$F(\mu,\nu) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,z)e^{-j2\pi(\mu t + \nu z)}\,dt\,dz$$

$$f(t,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(\mu,\nu)e^{j2\pi(\mu t + \nu z)}\,d\mu\,d\nu$$

4.48* A continuous Gaussian lowpass filter in the continuous frequency domain has the transfer function

$$H(\mu,\nu) = Ae^{-(\mu^2 + \nu^2)/2\sigma^2}$$

Show that the corresponding filter kernel in the continuous spatial domain is

$$h(t,z) = A2\pi\sigma^2 e^{-2\pi^2\sigma^2(t^2 + z^2)}$$

4.52 Do the following:

(a) Show that the Laplacian of a continuous function $f(t,z)$ of two continuous variables, t and z, satisfies the following Fourier transform pair:

$$\nabla^2 f(t,z) \Leftrightarrow -4\pi^2(\mu^2+\nu^2)F(\mu,\nu)$$

(*Hint:* See Eq. (3-50) and study entry 12 in Table 4.4.)

(b)* The result in (a) is valid only for continuous variables. How would you implement the continuous frequency domain transfer function $H(\mu,\nu) = -4\pi^2(\mu^2+\nu^2)$ for discrete variables?

(c) As you saw in Example 4.21, the Laplacian result in the frequency domain was similar to the result in Fig. 3.46(d), which was obtained using a spatial kernel with a center coefficient equal to −8. Explain why the frequency domain result was not similar instead to the

5.18 An industrial plant manager has been promoted to a new position. His first responsibility is to characterize an image filtering system left by his predecessor. In reading the documentation, the manager discovers that his predecessor established that the system is linear and position invariant. Furthermore, he learns that experiments conducted under negligible-noise conditions resulted in an impulse response that could be expressed analytically in the frequency domain as

$$H(u,v) = e^{-[u^2/150 + v^2/150]} + 1 - e^{-[(u-50)^2/150 + (v-50)^2/150]}$$

The manager is not a technical person, so he employs you as a consultant to determine what, if anything, he needs to do to complete the characterization of the system. He also wants to know the function that the system performs. What (if anything) does the manager need to do to complete the characterization of his system? What filtering function does the system perform?

5.26 During acquisition, an image undergoes uniform linear motion in the vertical direction for a time T. The direction of motion then switches 180° in the opposite direction for a time T. Assume that the time it takes the image to change directions is negligible, and that shutter opening and closing times are negligible also. Is the final image blurred, or did the reversal in direction "undo" the first blur? Obtain the overall blurring function $H(u,v)$ first, and then use it as the basis for your answer.

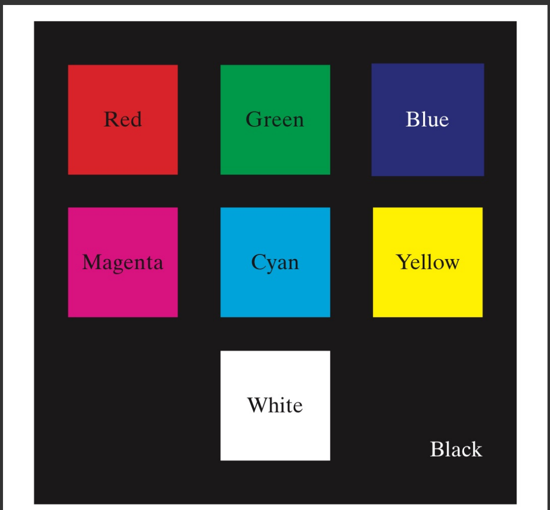
5.27* Consider image blurring caused by uniform acceleration in the x-direction. If the image is at rest at time $t = 0$ and accelerates with a uniform acceleration $x_0(t) = at^2/2$ for a time T, find the blurring function $H(u,v)$. You may assume that shutter opening and closing times are negligible.

6.13 Consider the following image composed of solid color squares. For discussing your answer, choose a gray scale consisting of eight shades of gray, 0 through 7, where 0 is black and 7 is white. Suppose that the image is converted to HSI color space. In answering the following questions, use specific numbers for the gray shades if using numbers makes sense. Otherwise, the relationships "same as," "lighter than," or "darker than" are sufficient. If you cannot assign a specific gray level or one of these relationships to the image you are discussing, give the reason.

(a)* Sketch the hue image.

(b) Sketch the saturation image.

(c) Sketch the intensity image.

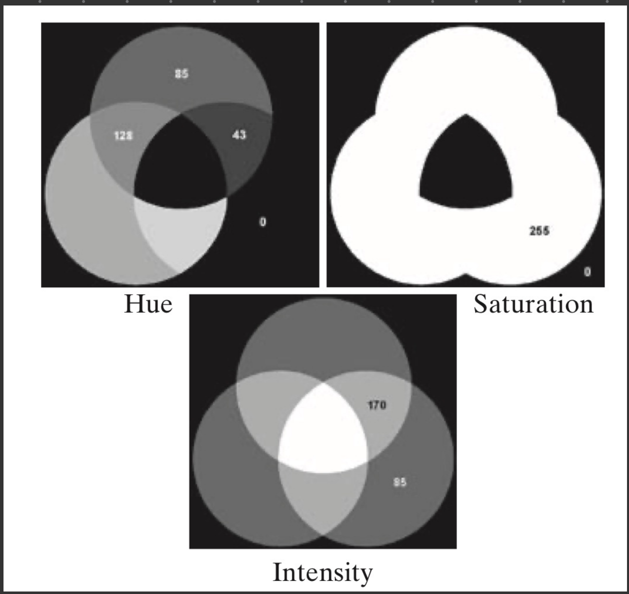


6.14 The following 8-bit images are the H, S, and I component images from Fig. 6.14. The numbers indicate gray-level values. Answer the following questions, explaining the basis for your answer in each. If it is not possible to answer a question based on the given information, state why you cannot do so.

(a)* Give the gray-level values of all regions in the hue image.

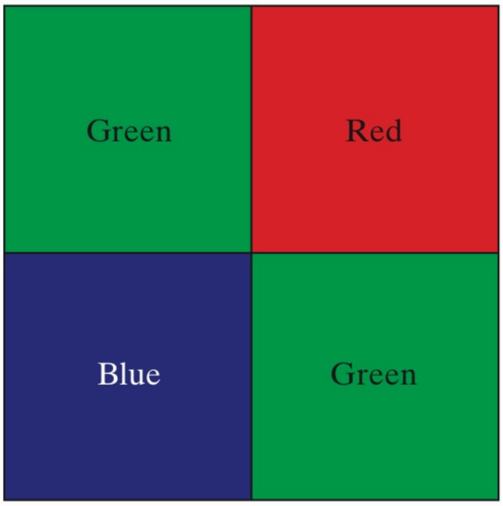
(b) Give the gray-level value of all regions in the saturation image.

(c) Give the gray-level values of all regions in the intensity image.



6.20 Explain the shape of the hue transformation function for the image complement approximation in Fig. 6.31(b) using the HSI color model.

6.25 Consider the following 500×500 RGB image, in which the squares are fully saturated red, green, and blue, and each of the colors is at maximum intensity. An HSI image is generated from this image. Answer the following questions.



(a) Describe the appearance of each HSI component image.

(b)* The saturation component of the HSI image is smoothed using an averaging kernel of size 125×125. Describe the appearance of the result. (You may ignore image border effects in the filtering operation.)

(c) Repeat (b) for the hue image.