

$$c_x = x_1 - x_c \quad (1)$$

$$c_y = y_1 - y_c \quad (2)$$

$$c = \begin{bmatrix} c_x \\ c_y \end{bmatrix} \quad (3)$$

$$D(q) = jacobian(c) = \frac{\partial c}{\partial q} \quad (4)$$

$$\dot{c} = \frac{\partial c}{\partial q} \dot{q} = D(q) \dot{q} \quad (5)$$

$$\ddot{c} = \frac{d}{dt}(D(q) \dot{q}) = D(q) \ddot{q} + \frac{\partial}{\partial q}(D(q) \dot{q}) \dot{q} = 0 \quad (6)$$

$$d = \frac{\partial}{\partial q}(D(q) \dot{q}) \dot{q} \quad (7)$$

$$e = D(q) * (M(q) \setminus (D(q)^\top * (\alpha * c + \beta * \dot{c}))) \quad (8)$$

$$\begin{bmatrix} M(q)_{4x4} & -D(q)_{4x2}^\top \\ D(q)_{2x4} & 0_{2x2} \end{bmatrix} \begin{bmatrix} \ddot{q}_{4x1} \\ \lambda_{2x1} \end{bmatrix} = \begin{bmatrix} (-C(q, \dot{q}) - G(q))_{4x1} \\ -(d + e)_{2x1} \end{bmatrix} \quad (9)$$