Extending the generalized linear mixed model for community ecologists

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1 Introduction

Data on ecological communities are very difficult to model. This difficulty arises Community data are multivariate, which does not in and of itself indicate data modelling challenges, It is multivariate and non-normal, phylogenetically, spatially, and temporally auto-correlated.

2 The model

2.1 The standard generalized linear mixed model

The generalized linear mixed model takes the form,

$$\boldsymbol{\eta} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{Z}\boldsymbol{b} \tag{1}$$

$$\boldsymbol{b} = \boldsymbol{\Lambda}_{\boldsymbol{\theta}}\boldsymbol{u} \tag{2}$$

$$\boldsymbol{u} \sim \mathcal{N}(0, \boldsymbol{I}) \tag{3}$$

$$\boldsymbol{y} \sim \mathcal{D}(\boldsymbol{\eta}, \boldsymbol{\phi}) \tag{4}$$

where ...

2.2 The extended model

GLMMs are suitable for modelling a wide variety of data. However, in community ecology the response variables are often species abundances or species presence-absence, and such data are often characterized by correlations between species even after the effects of environmental, phylogenetic, and space are accounted for. These correlations among species are typically due to either unmeasured site and species characteristics and species interactions. These correlations can be accounted for by allowing $\boldsymbol{Z}$ to depend on a vector of parameters, $\boldsymbol{\psi}$. Therefore the model that we will be using is given by,

$$\boldsymbol{\eta} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{Z}_{\boldsymbol{\psi}}\boldsymbol{\Lambda}_{\boldsymbol{\theta}}\boldsymbol{u} \tag{5}$$

2.3 Ecological structure in $\boldsymbol{Z}_{\boldsymbol{\psi}}$ and $\boldsymbol{\Lambda}_{\boldsymbol{\theta}}$

$$\boldsymbol{Z}_{\boldsymbol{\psi}} = \begin{bmatrix} \boldsymbol{Z}_{\text{FA}} & \boldsymbol{Z}_{\text{phylo}} & \boldsymbol{Z}_{\text{space}} \end{bmatrix} \tag{6}$$

2.4 Specifying mixed models with matrices

$$\boldsymbol{\eta} = \boldsymbol{X}\boldsymbol{\beta} + \begin{bmatrix} \boldsymbol{Z}_{\boldsymbol{\psi}_1} & \boldsymbol{Z}_{\boldsymbol{\psi}_2} & \dots & \boldsymbol{Z}_{\boldsymbol{\psi}_k} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Lambda}_{\boldsymbol{\theta}_1} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Lambda}_{\boldsymbol{\theta}_2} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{\Lambda}_{\boldsymbol{\theta}_k} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_2 \\ \dots \\ \boldsymbol{u}_k \end{bmatrix} \tag{7}$$