Random effects term for factor analysis: factAnal

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To fit a (possibly unbalanced) factor analysis random effects term using `lme4ord`, one uses the following `R` `formula`.

$$\texttt{respVar} \sim \texttt{factAnal}(0 + \texttt{obsFac} \,|\, \texttt{varFac}) \tag{1}$$

where `respVar` is the response vector and `obsFac` and `varFac` are `R` factors indicating the multivariate observation and variable associated with each row in the data set. For example, in ecology `codeFac` would normally indicate the site and `varFac` the species associated with each abundance observation in `respVar`.

Let $\boldsymbol{B}$ be an l by d matrix of loadings over l variables (often species) characterized by d latent axes. All elements above the diagonal of this loadings matrix are constrained to be zero, all other elements are free parameters.

Let $\boldsymbol{J}_{\mathrm{var}}$ and $\boldsymbol{J}_{\mathrm{obs}}$ be an l The random effects model matrix for the factor analysis term is then given by the following expression.

$$\boldsymbol{Z}^\top = (\boldsymbol{B}^\top \boldsymbol{J}_{\mathrm{var}}^\top) * J_{\mathrm{obs}}^\top \tag{2}$$

$$\text{link}\left(\mathcal{M}\right) = \boldsymbol{X}\boldsymbol{B} \tag{3}$$

$$x_{ij} \sim \mathcal{N}(0,1) \tag{4}$$

where $\mathcal{M}$ is an n by m matrix giving the mean of the response matrix, $\boldsymbol{Y}$; $\boldsymbol{X}$ is an n by d matrix of latent factors, $\boldsymbol{B}$ is a d by m matrix of loadings. This equation can be vectorized as,

$$\text{link}\left(\text{vec}(\mathcal{M})\right) = \left(\boldsymbol{B}^\top \otimes \boldsymbol{I}_n\right) \text{vec}(\boldsymbol{X}) \tag{5}$$

1 Random effects model matrix and relative covariance factor

Let $\boldsymbol{X}$ be an l by d matrix of loadings over l objects (often species) characterized by d latent dimensions. Let $\boldsymbol{J}$ be an l The random effects model matrix for the factor analysis term is then given by the following expression.

$$\boldsymbol{Z}^\top = \left(\boldsymbol{X}^\top \boldsymbol{J}_{\text{obj}}^\top\right) * \left(\boldsymbol{J}_{\text{var}}^\top\right) \tag{6}$$

The relative covariance factor is simply an m by m identity matrix, where m is the number of multivariate observations.

The parameters of $\boldsymbol{Z}^\top$ are in the matrix $\boldsymbol{X}$.