Random effects term for factor analysis: factAnal

Steve Walker

$$(\texttt{linForm}|\texttt{varFac}) \tag{1}$$

Let B be an l by d matrix of loadings over l variables (often species) characterized by d latent axes. All elements above the diagonal of this loadings matrix are constrained to be zero, all other elements are free parameters.

Let J_{var} and J_{obs} be an l The random effects model matrix for the factor analysis term is then given by the following expression.

$$Z^\top = (B^\top J_{\text{var}}^\top) * J_{\text{obs}}^\top \tag{2}$$

$$\text{link}\left(\mathcal{M}\right) = XB \tag{3}$$

$$x_{ij} \sim \mathcal{N}(0,1) \tag{4}$$

where $\mathcal{M}$ is an n by m matrix giving the mean of the response matrix, Y; X is an n by d matrix of latent factors, B is a d by m matrix of loadings. This equation can be vectorized as,

$$\text{link}\left(\text{vec}(\mathcal{M})\right) = \left(B^\top \otimes I_n\right) \text{vec}(X) \tag{5}$$

1 Random effects model matrix and relative covariance factor

Let X be an l by d matrix of loadings over l objects (often species) characterized by d latent dimensions. Let J be an l The random effects model matrix for the factor analysis term is then given by the following expression.

$$Z^\top = \left(X^\top J_{\text{obj}}^\top\right) * \left(J_{\text{var}}^\top\right) \tag{6}$$

The relative covariance factor is simply an m by m identity matrix, where m is the number of multivariate observations.

The parameters of $Z^\top$ are in the matrix X.