Continuation methods

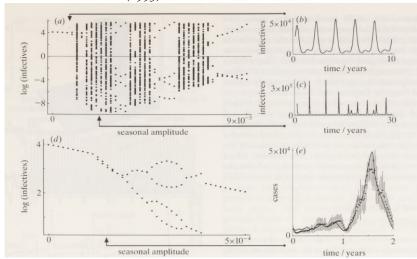
Ben Bolker

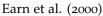
2021-03-08 ©BMB (except textbook material/other images)

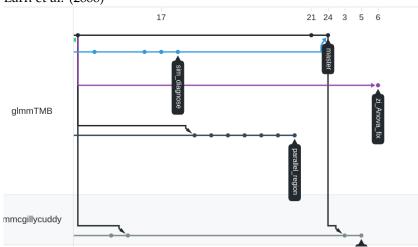
Numerical bifurcation analysis

- brute-force
- run model over a (1D or 2D?) grid of values
 - discard transient
 - figure out periodicity (Poincaré map/stroboscopic map)
 - figure out Lyapunov exponents?

Bolker and Grenfell (1993)







- Blyth, Renson, and Marucci (2020)
- Starting from a known attractor/equilibrium point of a system, move in parameter space (typically 1D) to detect *nearby* points/cycles.
- Tricks: backward-time solution may find unstable points/orbits

grind

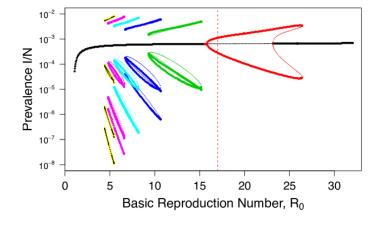
- tutorial
- code
- MATLAB version
- Uses Newton-Raphson to find steady-state solutions (not necessarily stable)
- evaluate Jacobian numerically

remotes::install_github("hansschepers/grindr")
library(Grind)

- disadvantages: not very powerful
- advantages: pretty basic, can see what it's doing

XPPAUT

- Old-school
- Handy for graphical exploration of 2D systems
- interface to AUTO (Doedel et al. 2007)
- Used by Krylova and Earn (2013)
 - supplementary material from Krylova et al.



- disadvantages: old-school, need to integrate with Python/R for nice plots
- advantages: fast, easy to use, graphical, powerful

PyDSTool/PyCont

Python based, interface to AUTO (Clewley et al. 2007)

- PyDSTool documentation
- PyCont documentation
- Hindmarsh-Rose example on GitHub
- disadvantages: inscrutable Python objects
- advantages: fast, powerful

References

- Blyth, Mark, Ludovic Renson, and Lucia Marucci. 2020. "Tutorial of Numerical Continuation and Bifurcation Theory for Systems and Synthetic Biology." arXiv:2008.05226 [Q-Bio], August. http: //arxiv.org/abs/2008.05226.
- Bolker, B. M., and Bryan Thomas Grenfell. 1993. "Chaos and Biological Complexity in Measles Dynamics." Proceedings of the Royal Society of London. Series B: Biological Sciences 251 (1330): 75–81. https://doi.org/10.1098/rspb.1993.0011.
- Clewley, Robert H., W. E. Sherwood, M. D. LaMar, and J. M. Guckenheimer. 2007. "PyDSTool, a Software Environment for Dynamical Systems Modeling." URL Http://Pydstool. Sourceforge. Net. https://pydstool.github.io/PyDSTool/FrontPage.html.
- Doedel, Eusebius J., Thomas F. Fairgrieve, Björn Sandstede, Alan R. Champneys, Yuri A. Kuznetsov, and Xianjun Wang. 2007. "AUTO-07P: Continuation and Bifurcation Software for Ordinary Differential Equations."
- Earn, David J. D., Pejman Rohani, Benjamin M. Bolker, and Bryan T. Grenfell. 2000. "A Simple Model for Complex Dynamical Transitions in Epidemics." Science 287 (5453): 667-70. https: //doi.org/10.1126/science.287.5453.667.
- Krylova, Olga, and David J. D. Earn. 2013. "Effects of the Infectious Period Distribution on Predicted Transitions in Childhood Disease Dynamics." Journal of the Royal Society Interface 10 (84): 20130098. https://doi.org/10.1098/rsif.2013.0098.