Brief stuff on deterministic chaos

Ben Bolker

Chaos

- sensitive dependence on initial conditions
- bounded

Lyapunov exponent/spectrum

- Long-term average dynamics
- Expansion/contraction of phase space
-

Fractal dimension

- box-counting
- Kaplan-Yorke
- Correlation dimension (Grassberger-Procaccia)
  - $C(\epsilon) = \lim_{N\to\infty} g/N^2$
  - $C(\epsilon) \propto \epsilon^{\nu}$
  - log-log graph

Embedding

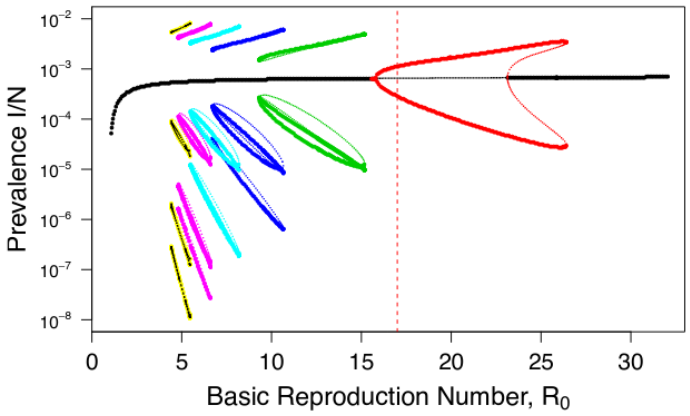
- Takens theorem

Poincaré map

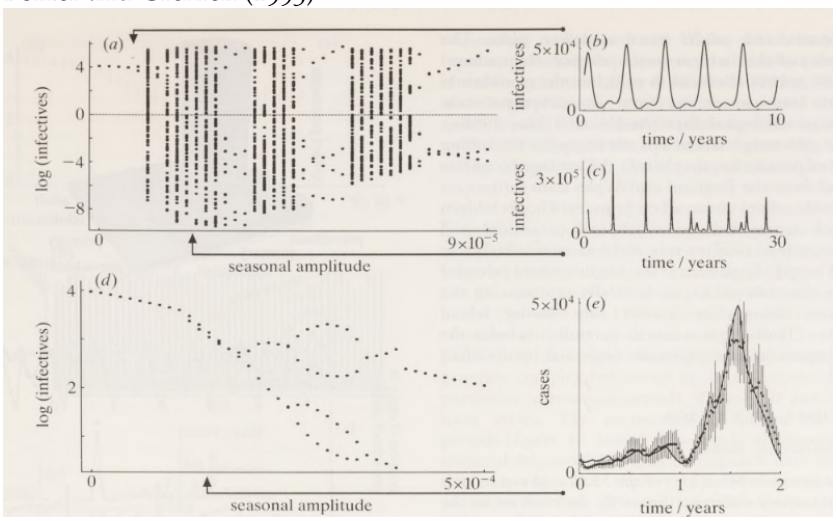
continuation methods

- Blyth, Renson, and Marucci (2020); PyDSTool; XPPAUT

  Krylova and Earn (2013)



Bolker and Grenfell (1993)



References

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Krylova, Olga, and David J. D. Earn. 2013. "Effects of the Infectious Period Distribution on Predicted Transitions in Childhood Disease Dynamics." *Journal of the Royal Society Interface* 10 (84): 20130098. https://doi.org/10.1098/rsif.2013.0098.