Epidemic models 1

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motivation

- P & I data from Philadelphia 1918 flu:

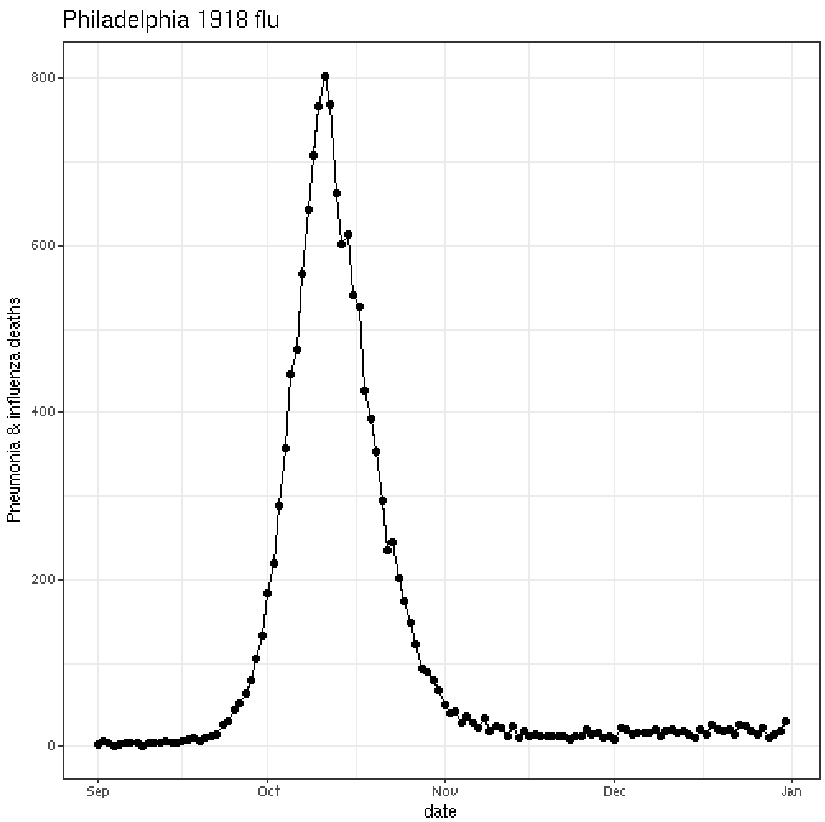

Figure 1: plot of chunk philadata

some questions

exponential growth

- one variable (=1D model)
- how does disease spread? → equation

what do we want to figure out?

what shall we assume?

- classify individuals as S, I (**compartmental** model; **microparasite** or **intensity-independent**)
- disease is transmitted from S to I
- $S \to I$ instantaneously (zero latent period, no E)
- population is **homogeneous** (no heterogeneity in susceptibility, infectiousness, contact)
- fixed population size (birth = migration = 'natural' death = 0)
- transmission rate is time-invariant

- assumption 2 is OK (Pasteur, Koch's postulates ...)
- all the rest are approximations

start simple!

- parsimony
- robustness?
- applicability/estimation?

Levins (1966) (also Orzack et al. (1993), Levins (1993), Weisberg (2007))

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what variables should we use?

- time (t)
- state variable: incidence, prevalence, death rate, death toll (= cumulative death?)
- deaths loosely connected to transmission

but deaths are observed!

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when are deaths a good **proxy** for incidence?

- infection -> death time is fixed

- homogeneity? (might not matters?)
- mortality curve is shifted epidemic

(COVID context ... we observe case reports, number of tests, hospitalizations, and deaths)

- **incidence**: number of infections per unit time (rate or flow)
- **prevalence**: number of currently infected people (quantity or stock)

prevalence is closer to the **mechanism**

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model components:

- $I(t)$ (state variable: prevalence)
- $I(0)$ (initial conditions)
- β (parameter) = avg contacts **per susceptible per infective per unit time**

$$I(t + \Delta t) \approx I(t) + \beta I(t) \Delta t$$

Take $\lim \Delta t \to 0$ (and solve):

$$\frac{dI}{dt} = \beta I \to I(t) = I(0) exp(\beta t)$$

model criticism

- Ignored discrete nature of individuals
- Ignored time-varying β (e.g. **diurnal** fluctuations)
- Ignored finite infectious periods (recovery/death)

Next: What if we make infectious periods finite? (i.e., including recovery (**clearance**) or death

$$dI/dt = \beta I - \gamma I$$

mean infectious period

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\begin{aligned}
I(t) &= I(0)\exp(-\gamma t) \\
\text{proportion uninfected} &= \exp(-\gamma t) \\
\text{proportion infected} &= 1-\exp(-\gamma t) (= \text{CDF} := C(t)) \\
\text{PDF} : &= C'(t) = \gamma \exp(-\gamma t) \\
\text{substitute } x &= \gamma t, d\,x = \gamma\, dt \\
\text{mean} = E[t] &= \int t \exp(-\gamma t)\, dt = \int x \exp(-x)\, dx/\gamma = 1/\gamma
\end{aligned}
$$

dimensional analysis

rates and characteristic times/scales

- is I a proportion or a density or a number ... ?
- what are the units of β\$, \$$gamma$?

nondimensionalization

- standardize any values that can be eliminated **without loss of (mathematical) generality**
- what can we do here?
- $gamma=$1
- I ? (depends on how we have defined it initially) →

references

Levins, R. 1966.. *American Scientist* 54: 421–431. https://www.jstor.org/stable/27836590.

Levins, R. 1993.. *Quarterly Review of Biology* 68 (4): 547–555.

Orzack, SH et al. 1993.. *Quarterly Review of Biology* 68 (4): 533–546.

Weisberg, M. 2007.. *Biology & Philosophy* 21 (5) (January): 623–645. doi:10.1007/s10539-006-9051-9. http://www.springerlink.com/content/v141t07655057614/.