28 Jan 2021

What does the SIR model TELL US about control, SIZE of epidemic, etc?

$R_0 = \frac{\beta}{\gamma}$ expected cases/case in a susceptible population

$r = \beta - \gamma$: time^{-1}

exp growth rate in a susc pop'n

$\boxed{\frac{\log(2)}{r}}$: DOUBLING TIME / HALF LIFE

r 0.23/day ?

e^{rt} $\quad 1 + x + \frac{x^2}{2} + \dots$

$e^{0.23} \approx 1.2586 \dots$

$x(t) = 2x(0)$

$= x_0 e^{rt} \rightarrow \frac{\log 2}{r}$

SCALES rather than RATES: $\sim \frac{0.7}{0.23} \approx 3$ days

SIGN r growing or shrinking

$R_0 \gtrless 1$: equivalent

$\beta - \gamma$ and $\frac{\beta}{\gamma}$

How much control do we need to suppress an epidemic?

$R_{eff} = R_0 \cdot S$ S is a proportion $= \frac{\beta S}{\gamma}$

contacts will be wasted

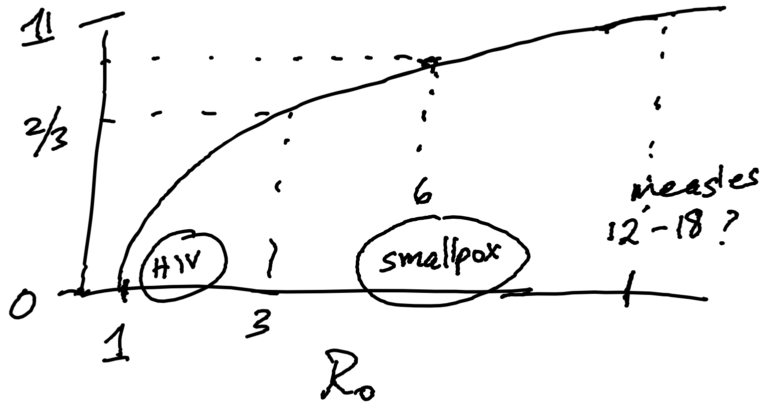
FORCE OF INFECTION : rate of new infections PER susceptible

$+ \beta S I \cdot \boxed{\frac{1}{\gamma}}$

vaccinate /protect prop p of population

how big does p have to be so that $R_{eff} < 1$?

$R_0 \cdot \underbrace{\boxed{(1-p)}}_{\text{frac unprotected}} \le 1 \rightarrow p = 1 - \frac{1}{R_0}$ HYPERBOLIC



COVID $\simeq 3$

2-6

STRENGTH-BASED : control before infection

SPEED-BASED : control after infection.

~ TESTING + TRACE for COVID;

HIV:

---

how good does this have to be?

$$\frac{dI}{dt} = \beta S I - \underbrace{\gamma I}_{\text{recovery}} - \underbrace{\alpha I}_{\text{control}}$$

$$\beta SI - \gamma I - \alpha I < 0$$

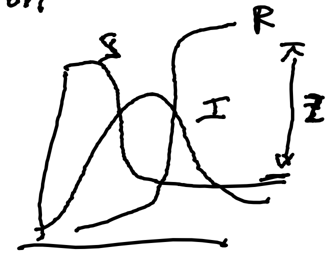
Suppose $S = 1$ (DFE)

$$\beta I - \gamma I - \alpha I < 0$$

$$((\beta - \gamma) - \alpha) I < 0$$

$$\alpha > r$$

speed-based intervention



$\frac{dI}{dt}$ near DFE $:= \beta I - \gamma I$ $(S \approx 1)$

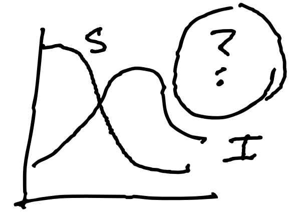
$$I(t) = I(0)\, e^{(\beta - \gamma) t}$$

exp rate of growth $\equiv r$

Jan 28
p 5

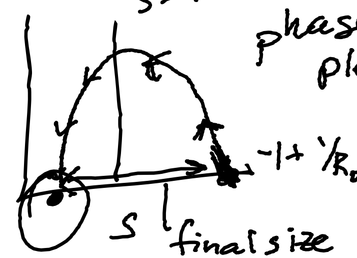
$$\frac{dS}{dt} = -\beta S I$$

$$\frac{dI}{dt} = \beta S I - \gamma I$$



CAN'T solve for $S(t), I(t)$

we CAN solve for $I(S)$



$$\left(\frac{dI}{dS}\right) = \frac{\beta S \cancel{I} - \gamma \cancel{I}}{-\beta S \cancel{I}}$$

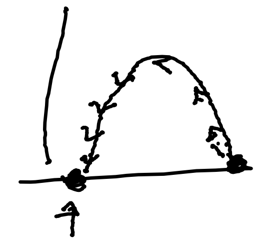
$$= \frac{\beta S - \gamma}{-\beta S} = -1 + \left(\frac{\gamma}{\beta}\right)\cdot\frac{1}{S} = -1 + \frac{1}{R_0 S} \quad ???$$

FINAL SIZE: integrate : $\int dI = \int \left(-1 + \frac{1}{R_0 S}\right) dS$

$$I - I(0) = -(S - S(0)) + \frac{1}{R_0}\left(\log\left(S/S(0)\right)\right)$$

$$I - I(0) = -(S - S(0)) + \frac{1}{R_0}\log\left(S/S(0)\right)$$

$t \to \infty$



$$I(\infty) - I(0) = -(S(\infty) - S(0)) + \frac{1}{R_0}\log\left(\frac{S(\infty)}{S(0)}\right)$$

$I(\infty) \to 0$

$I(0) \approx 0$ (novel epidemic

$S(0) \approx 1$

$$0 = -(S(\infty) - 1) + \frac{1}{R_0}\log(S(\infty))$$

$$S(\infty) = 1 - Z \Big\} \boxed{\text{FINAL SIZE}}$$

$$-Z = \frac{1}{R_0}\log(1 - Z)$$

HARD TO SOLVE! $\Rightarrow$ Newton's method: Lambert W