15 Mar 2021

MIDTERM

- multiple docs OK (Python + R)
- still need source and PDF for everything.
- POLYGLOT. R markdown: Python + R chunks
  polyglot NOTEBOOKS.

HOPFIELD (1982)

- dynamics of individual neuron.
  - increasing current → higher propensity to spike.
    ($I = 0$ → 'reactive')
    $I >$ thresh → bursting/spiking/chaotic solutions)

N COUPLED neurons: (Hindmarsh - Rose) 3N coupled equations

(want to consider 10^3 neurons)

- how do neurons 'think' or '<u>remember</u>'?

NEURAL NETWORKS and physical systems
with emergent collective computational abilities.

- each neuron is in state $\{0,1\}$ (firing or not firing) V_i
- lots of neurons.
- neurons are connected (<u>complete</u> network) : T_{ij} weights $\in \mathbb{R}$

(1) RULE. $\begin{cases} V_i \to 1 & \text{if } \sum_j T_{ij} V_j > U_i \\ V_i \to 0 & \text{if } \sum T_{ij} V_j < U_i \end{cases}$ (typically 0)

(local) POSITIVE feedback loop (if $T_{ij} > 0$)

- encoding MANY stable states in the system (coexisting attractors)

- ASYNCHRONOUS $\Rightarrow$ biological systems +

- store a set of states V^s: $s = 1 \cdots n$?

'memories' or 'items' : <u>attractors</u>

state $V_1 = \{ \underbrace{0, 1, 1, 0, 0, 1, 0, 1 \ldots}_{N} \}$ $\quad \begin{pmatrix} 1,1 \\ 0,0 \\ 0,1 \end{pmatrix} \to (-1, 1)$

$\vdots$

n

encode by setting weights T_{ij} to ② $\overbrace{\sum_{s'} \underbrace{(2V_i^{s'} - 1)}_{(0,1) \to (-1,1)} \underbrace{(2V_j^{s'} - 1)}_{*}}$

If both neurons are on or off in state s' then add 1 to weight otherwise subtract 1

$T_{ii} = 0$

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① & ② $\sum_j T_{ij} V_j^{s'} = \sum_s (2V_i^s - 1) \left[\underbrace{\sum_j V_j^{s'} (2V_j^s - 1)} \right] = H_j^{s'}$

mean $= 0$ unless $s = s'$ ($N/2$)

$\to \quad (2V_i^{s'} - 1) N/2$

(7)

$$E = -\frac{1}{2} \sum_{i \neq j}\sum T_{ij} \underline{V_i V_j}$$

monotonically decreasing function

$$\Delta E = - \underbrace{\Delta V_i \sum_{j \neq i} T_{ij} V_j}$$

$\approx$ LYAPUNOV function

(global asymptotic stability)

LOCAL minimum

→ Stored V_s states:

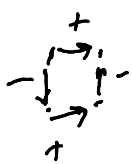
SPINGLASS / Ising model

PHYSICS: magnetism, many-body systems

('spins') +1/-1

SPIN GLASS: spin system with random connections

FRUSTRATED: many local attractors



- "Stochastic mean processing time" = $1/w$

- computer experiments.

  $N=30$ } proof requires symmetry
  $N=100$ } $T_{ij} = T_{ji}$
  relaxed in simulations

$N=30$ 'never displayed an ergodic wandering'

$4/w$ ($4N$ transitions)

50 trials ~ 2 or 3 end states

cycles?

'<u>chaotic</u> wandering' (but always in a small region)

<u>HAMMING</u> <u>distance</u>

number of bit differences

101001 H distance = 2
101111

2^N discrete states:

- 1982: $N=100$