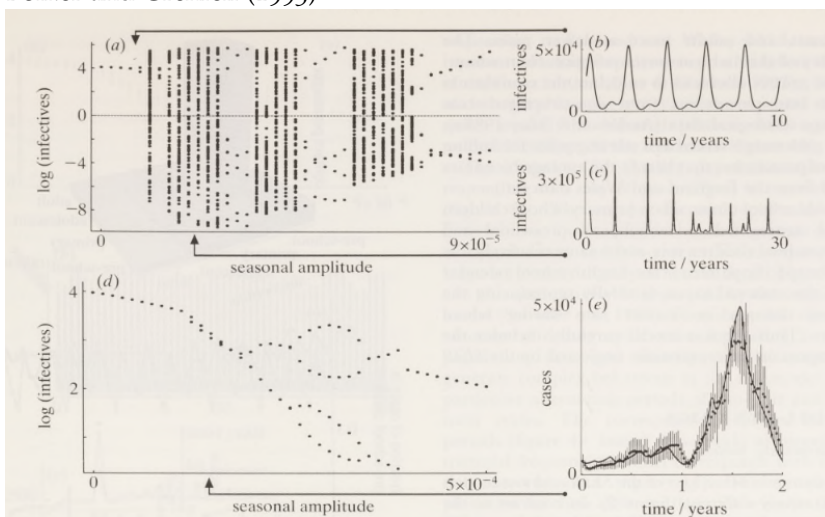# Continuation methods

Ben Bolker

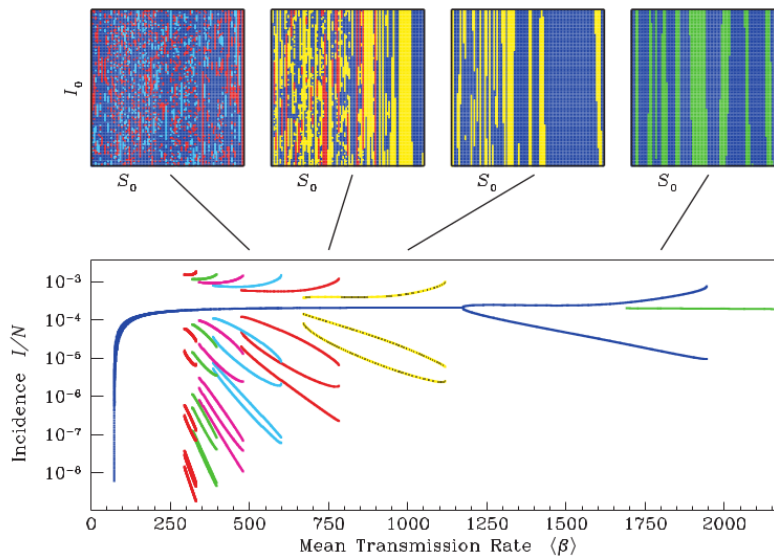*2021-03-10 ©BMB (except textbook material/other images)*

Numerical bifurcation analysis

- brute-force
- run model over a (1D or 2D?) grid of values

  - discard transient
  - figure out periodicity (Poincaré map/stroboscopic map)
  - figure out Lyapunov exponents?

Bolker and Grenfell (1993)

Earn et al. (2000)

Continuation methods

- Doedel, Keller, and Kernevez (1991)
- Blyth, Renson, and Marucci (2020)
- Starting from a known attractor/equilibrium point of a system, move in parameter space (typically 1D) to detect *nearby* points/cycles.
- Tricks: *backward-time* solution may find unstable points/orbits

grind

- tutorial

- code

- MATLAB version

- Uses Newton-Raphson to find steady-state solutions (not necessarily stable)
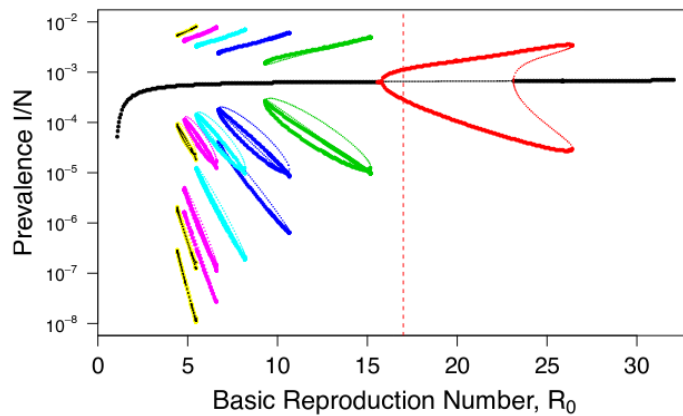
- evaluate Jacobian numerically

```r
remotes::install_github("hansschepers/grindr")
library(Grind)
```

- **disadvantages**: not very powerful
- **advantages**: pretty basic, can see what it's doing

XPPAUT

- Old-school
- Handy for graphical exploration of 2D systems
- interface to AUTO (Doedel et al. 2007)
- Used by Krylova and Earn (2013)

  - supplementary material from Krylova et al.

- **disadvantages**: old-school, need to integrate with Python/R for nice plots
- **advantages**: fast, easy to use, graphical, powerful

PyDSTool/PyCont

Python based, interface to AUTO (Clewley et al. 2007)

- PyDSTool documentation

- PyCont documentation

- Hindmarsh-Rose example on GitHub

- **disadvantages**: inscrutable Python objects

- **advantages**: fast, powerful

References

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