Brief stuff on deterministic chaos

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Chaos

- sensitive dependence on initial conditions (sdic); nearby trajectories diverge (exponentially)
- bounded

  (exponential growth is an example of **unbounded** sdic)

Lyapunov exponent/spectrum

- Long-term average dynamics
- Expansion/contraction of phase space

```r
par(las=1,bty="l")
library(ellipse)
ee <- ellipse(x=0,t=1)
M <- matrix(c(1.2,0.5,0.5,1),nrow=2)
MASS::eqscplot(ee,type="l",xlim=c(-5,5))
for (i in 2:5) {
    ee <- t(M %*% t(ee))
    lines(ee[,1],ee[,2],col=i)
}
```
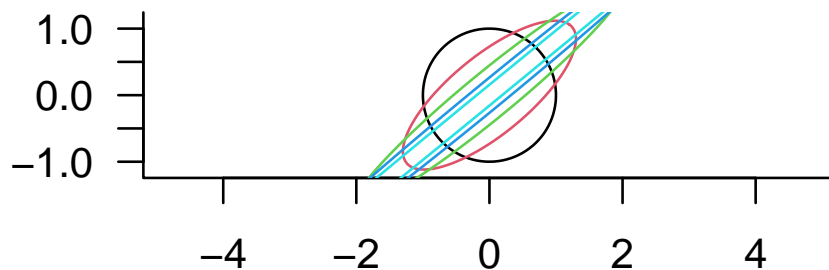

Fractal dimension

- Kaplan-Yorke:
- Correlation dimension (Grassberger-Procaccia)

  - $C(\epsilon) = \lim_{N \to \infty} g/N^2$
  - $C(\epsilon) \propto \epsilon^\nu$
  - log-log graph

- box-counting

Embedding

- Takens theorem

Poincaré map

References