Continuation methods

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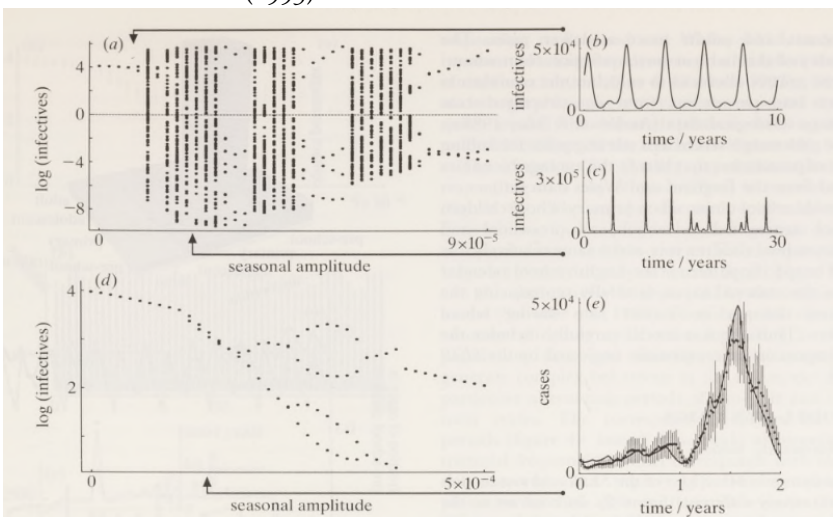
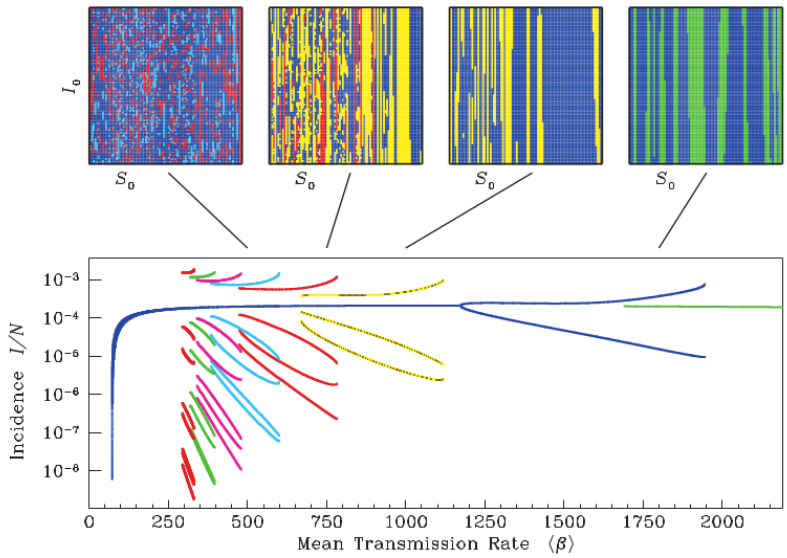
Numerical bifurcation analysis

- brute-force
- run model over a (1D or 2D?) grid of values
  - discard transient
  - figure out periodicity (Poincaré map/stroboscopic map)
  - figure out Lyapunov exponents?

Bolker and Grenfell (1993)



Earn et al. (2000)



Continuation methods

- Blyth, Renson, and Marucci (2020)
- Starting from a known attractor/equilibrium point of a system, move in parameter space (typically 1D) to detect *nearby* points/cycles.
- Tricks: *backward-time* solution may find unstable points/orbits

grind

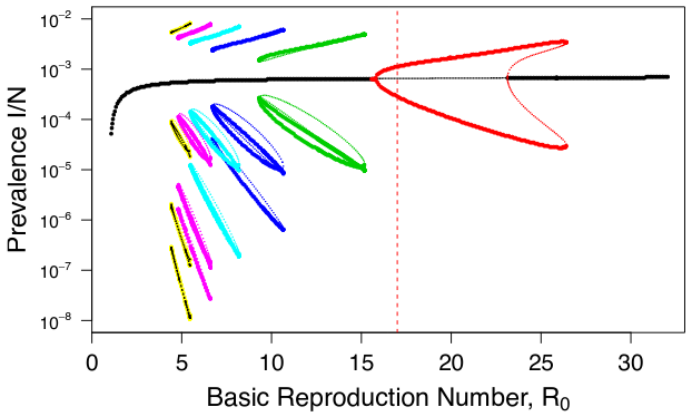
- tutorial
- code
- MATLAB version
- Uses Newton-Raphson to find steady-state solutions (not necessarily stable)
- evaluate Jacobian numerically

```
remotes::install_github("hansschepers/grindr")
library(Grind)
```

- **disadvantages**: not very powerful
- **advantages**: pretty basic, can see what it's doing

XPPAUT

- Old-school
- Handy for graphical exploration of 2D systems
- interface to AUTO (Doedel et al. 2007)
- Used by Krylova and Earn (2013)
  - supplementary material from Krylova et al.



- **disadvantages**: old-school, need to integrate with Python/R for nice plots
- **advantages**: fast, easy to use, graphical, powerful

PyDSTool/PyCont

Python based, interface to AUTO (Clewley et al. 2007)

- PyDSTool documentation
- PyCont documentation
- Hindmarsh-Rose example on GitHub
- **disadvantages**: inscrutable Python objects
- **advantages**: fast, powerful

References

Blyth, Mark, Ludovic Renson, and Lucia Marucci. 2020. "Tutorial of Numerical Continuation and Bifurcation Theory for Systems and Synthetic Biology." *arXiv:2008.05226 [Q-Bio]*, August. http://arxiv.org/abs/2008.05226.

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Clewley, Robert H., W. E. Sherwood, M. D. LaMar, and J. M. Guckenheimer. 2007. "PyDSTool, a Software Environment for Dynamical Systems Modeling." *URL Http://Pydstool. Sourceforge. Net*. https://pydstool.github.io/PyDSTool/FrontPage.html.

Doedel, Eusebius J., Thomas F. Fairgrieve, Björn Sandstede, Alan R. Champneys, Yuri A. Kuznetsov, and Xianjun Wang. 2007. "AUTO-07P: Continuation and Bifurcation Software for Ordinary Differential Equations."

Earn, David J. D., Pejman Rohani, Benjamin M. Bolker, and Bryan T. Grenfell. 2000. "A Simple Model for Complex Dynamical Transitions in Epidemics." *Science* 287 (5453): 667–70. https://doi.org/10.1126/science.287.5453.667.

Krylova, Olga, and David J. D. Earn. 2013. "Effects of the Infectious Period Distribution on Predicted Transitions in Childhood Disease Dynamics." *Journal of the Royal Society Interface* 10 (84): 20130098. https://doi.org/10.1098/rsif.2013.0098.