• DISCONTINUOUS. Bifurcations.

Hopf bifurcation of H-H model

stable
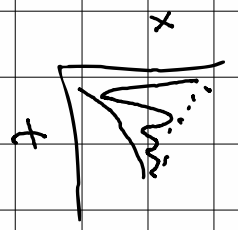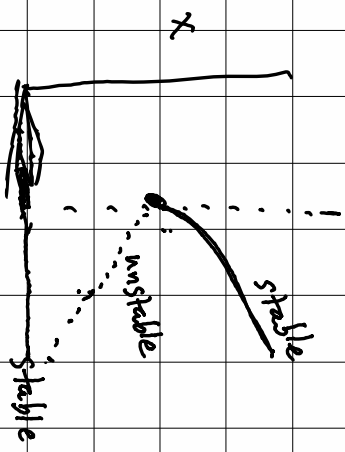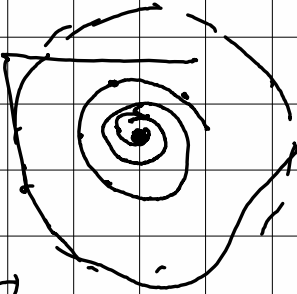
voltage →

limit cycle

unstable

stable

stable

x

bif parameter

stable

HYSTERESIS.

x

t

x

t
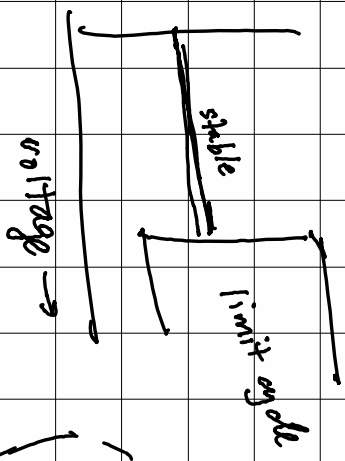
λ_1, λ_2 — negative real part
positive real part

Re → 0

$-0.00001 \pm 5i$

Damping time

$-(\text{re part})$, period
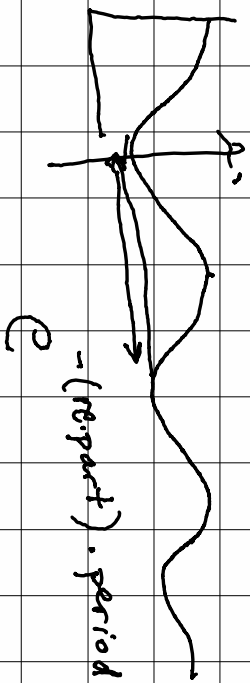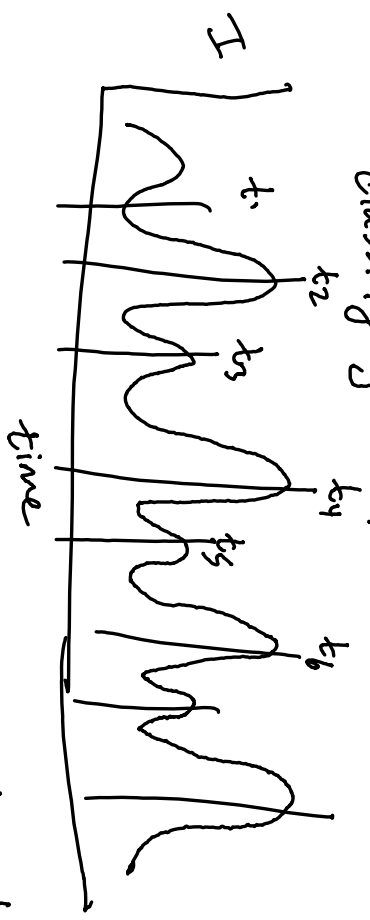
COEXISTING ATTRACTORS

classifying the periodicity of the attractor.

I

t_1 t_2 t_3 t_4 t_5 t_6

time

↓ Poincaré map (take annual points)

→ $\{1.00, 2.01, \boxed{1.001}, \boxed{2.0101}, 1.00, 2.01, ... \}$

t_1 — t_2 — t_3 — t_4 — t_5 — t_6

2.0100070808o9
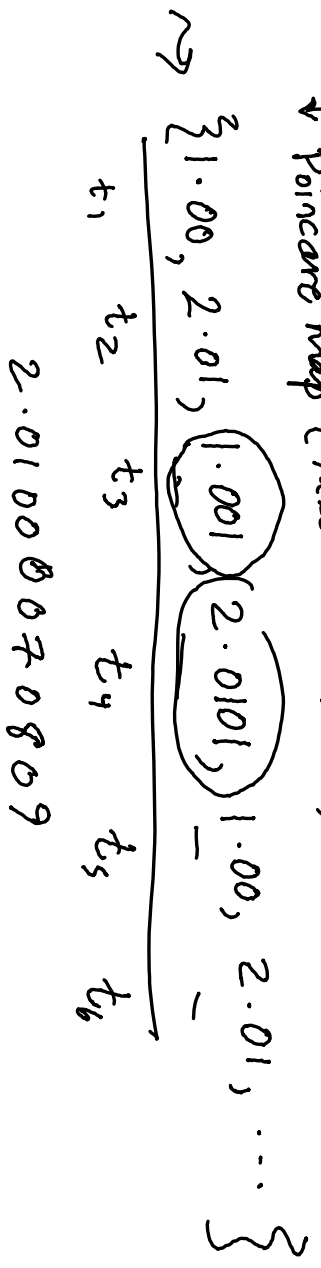
NUMERICAL continuation methods

- suppose I have parameters $\vec{p}$ for which I've identified an attractor (steady state, limit cycle, ...)

- analytically? run eq'ns numerically & to find the exact value.
  $\llcorner$ grad $(\vec{x}) = 0$
  $\vec{x}^* $

- CONTINUATION method.
  $\vec{p} + \vec{\delta p}$: start from $\vec{x}^*$.
  Use Newton-Raphson. to find $\vec{x}^*(\vec{p} + \vec{\delta p})$

  $\llcorner$ UNSTABLE equilibria
  identify bifurcation points