Model

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I think it makes sense that we use a model that's similar to vector-borne disease models because trematodes have two hosts: snails and ducks. Ignoring spatial features and sexuality, we're going to use the following model:

$$
\begin{aligned}
X'_S &= b - \beta_X X_S Y_I / N_X - \mu_X X_S - a X_S Y / N_X \\
X'_I &= \beta_X X_S Y_I / N_X - (\mu_X + \alpha) X_S - a X_I Y / N_X \\
Y'_S &= \mu_Y N_Y - \beta_Y Y_S X_I / N_Y - \mu_Y Y_S \\
Y'_I &= \beta_Y Y_S X_I / N_Y - \mu_Y Y_S
\end{aligned}
$$

Here, X and Y represent snail and duck population and subscripts represent the infection status. We keep the size of the duck population constant for simplicity. Snail population is hunted by the duck population and suffer from natural mortality.

Now, let's separate asexual and sexual reproduction. Sexual snails are diploids and asexual snails are triploid clones but we assume diploidy for both for simplicity. Following Ashby and Gupta (2014); Ashby and King (2015), we let birth rates for asexual and sexual population be

$$
\begin{aligned}
b^A_{ij} &= r(X^A_{ij,S} + f \sum_j X^A_{ij,I})(1 - h_b N) \\
b^S_{ij} &= c_b r \frac{(2 - \delta_{ij}) X^S_i X^S_j}{2 \sum_k X^S_k} (1 - h_b N),
\end{aligned}
$$

where X_i is the number of gamete i in sexual population following recombination. We write it as $X_i = u + fv$, following Ashby and Gupta (2014). As Gibson et al. (2017) showed that there the snail system is consistent with the two fold cost of sex, we let $c_b = 1$.

We want to model outcrossing and recombination of parasies. We can define P_{ij} be number of parasite genotypes in ducks after sexual recombination. Simply, we can write

$$
P_{ij} = \frac{(2 - \delta_{ij}) Y^S_i Y^S_j}{\sum_k Y^S_k}
$$

Then, our model becomes:

$$
\begin{aligned}
{X_{ij,S}}' &= b_{ij} - \beta_X X_{ij,S} Z_{ij}/N_X - \mu_X X_{ij,S} - a X_{ij,S} Y/N_X \\
{X_{ij,I}}' &= \beta_X X_{ij,S} Z_{ij}/N_X - (\mu_X + \alpha) X_{ij,S} - a X_{ij,S} Y/N_X \\
Y_S' &= \mu_Y N_Y - \beta_Y Y_S \left(\sum_{i,j} X_{ij,I} \right) - \mu_Y Y_S \\
Y_{ij,I}' &= \beta_Y Y_S X_{ij,I} - \mu_Y Y_{ij,I}
\end{aligned}
$$

References

Ashby, B. and S. Gupta (2014). Parasitic castration promotes coevolutionary cycling but also imposes a cost on sex. *Evolution 68*(8), 2234–2244.

Ashby, B. and K. C. King (2015). Diversity and the maintenance of sex by parasites. *Journal of evolutionary biology 28*(3), 511–520.

Gibson, A. K., L. F. Delph, and C. M. Lively (2017). The two-fold cost of sex: Experimental evidence from a natural system. *Evolution Letters 1*(1), 6–15.