

# rmaxima

test page

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# library(rmaxima)
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integrate(x^3, x);  
L: sqrt(1 - 1/(R^2));  
assume(R>=0)  
integrate(x, x, 0, L);
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$$\frac{x^4}{4} \sqrt{1 - \frac{1}{R^2}} [R \geq 0] \frac{R^2 - 1}{2 R^2}$$

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inv_gamma(x, alpha, beta) := beta^alpha / gamma(alpha) * (1/x)^(alpha + 1) * %e^(-beta / x);  
assume(s>0)$  
assume(nu>0)$  
inv_scaled_chisq(x, nu, s) := inv_gamma(x, nu/2, nu/2 * s^2);  
norm(x, mu, sigma) := 1 / (sigma * sqrt(2 * %pi)) * %e^(-1/2 * ((x - mu)/sigma)^2);  
t(x, nu) := gamma((nu + 1)/2) / (sqrt(nu * %pi) * gamma(nu / 2)) * (1 + x^2/nu)^(-(nu + 1)/2);
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$$\text{inv\_gamma}(x, \alpha, \beta) := \frac{\beta^\alpha}{\Gamma(\alpha)} \left(\frac{1}{x}\right)^{\alpha+1} e^{-\frac{\beta}{x}}$$

$$\text{inv\_scaled\_chisq}(x, \nu, s) := \text{inv\_gamma}\left(x, \frac{\nu}{2}, \frac{\nu}{2} s^2\right)$$

$$\text{norm}(x, \mu, \sigma) := \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2}$$

$$t(x, \nu) := \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi} \Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

The predictive distribution of the mean of the missing data is

$$\bar{y}^{\text{miss}} \mid \mu, \sigma^2, \bar{y}^{\text{obs}} \sim N\left(\mu, \frac{\sigma^2}{(N-n)}\right)$$

Averaging over  $\mu$ :

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assume(sigma > 0, n > 0, N > 0)$
ratsubstflag: true$
integrate(norm(ymiss, mu, sigma/sqrt(N - n)) * norm(mu, yobs, sigma/sqrt(n)), mu, minf, inf);

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$$\frac{\sqrt{N-n} \sqrt{n} e^{\frac{(n^2 - N n) y_{obs}^2 + (2 N n - 2 n^2) y_{miss} y_{obs} + (n^2 - N n) y_{miss}^2}{2 N \sigma^2}}}{\sqrt{2} \sqrt{\pi} \sqrt{N} \sigma}$$

and averaging over  $\sigma^2$ :

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% * inv_scaled_chisq(sigma^2, n-1, s^2);
ratsubst(sqrt(t), sigma, %);
integrate(%, t, 0, inf);

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$$\frac{\sqrt{N-n} (n-1)^{\frac{n-1}{2}} \sqrt{n} 2^{-\frac{n-1}{2} - \frac{1}{2}} s^{2(n-1)} \sigma^{-2 \left(\frac{n-1}{2} + 1\right) - 1} e^{\frac{(n^2 - N n) y_{obs}^2 + (2 N n - 2 n^2) y_{miss} y_{obs} + (n^2 - N n) y_{miss}^2}{2 N \sigma^2}} - \frac{(n-1) s^4}{2 \sigma^2}}{\sqrt{\pi} \sqrt{N} \Gamma\left(\frac{n-1}{2}\right)}$$

$$\frac{\sqrt{N-n} (n-1)^{\frac{n-1}{2}} \sqrt{n} 2^{-\frac{n-1}{2} - \frac{1}{2}} s^{2n-2} t^{-\frac{n}{2} - 1} e^{\frac{(n^2 - N n) y_{obs}^2 + (2 N n - 2 n^2) y_{miss} y_{obs} + (n^2 - N n) y_{miss}^2}{2 N t}} - \frac{(n-1) s^4}{2 t}}{\sqrt{\pi} \sqrt{N} \Gamma\left(\frac{n-1}{2}\right)}$$

$$\frac{\sqrt{N-n} (n-1)^{\frac{n-1}{2}} \sqrt{n} 2^{-\frac{n-1}{2} - \frac{1}{2}} s^{2n-2} \int_0^\infty t^{-\frac{n}{2} - 1} e^{\frac{(n^2 - N n) y_{obs}^2 + (2 N n - 2 n^2) y_{miss} y_{obs} + (n^2 - N n) y_{miss}^2}{2 N t}} - \frac{(n-1) s^4}{2 t} dt}{\sqrt{\pi} \sqrt{N} \Gamma\left(\frac{n-1}{2}\right)}$$