

live coding (lecture 2)

18 November 2025

```
library(ggplot2); theme_set(theme_bw())  
## requires ggplot2 version >= 4.0.0  
library(lme4)  
library(glmmTMB)  
library(lmerTest)  
library(ggplot2)  
library(tidyverse)  
library(plotly)  
library(buildmer)  
library(broom.mixed)  
library(bbmle)  
library(DHARMA)  
library(effects)
```

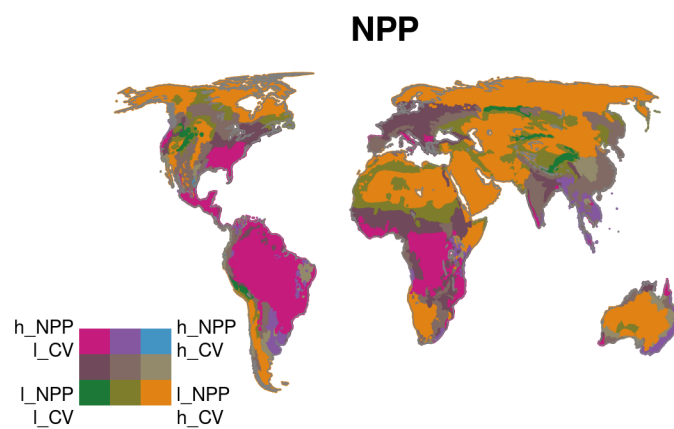
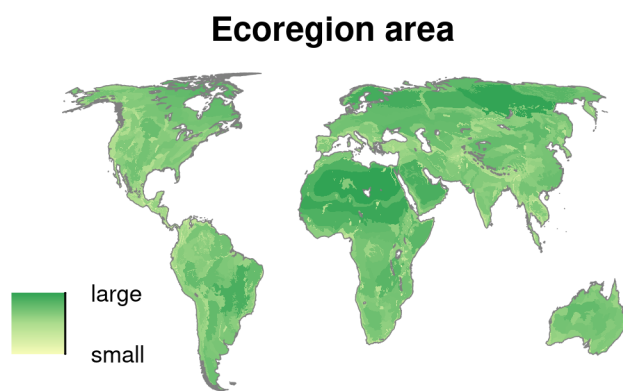
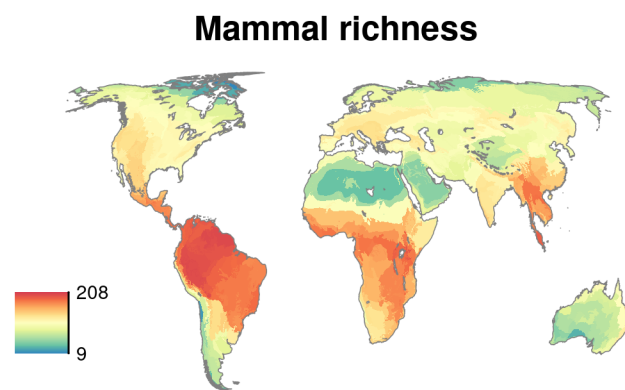
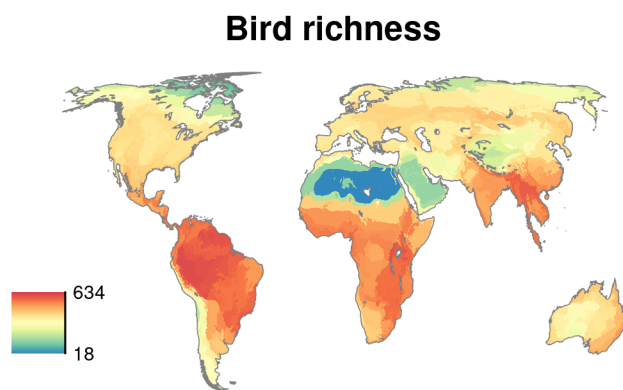
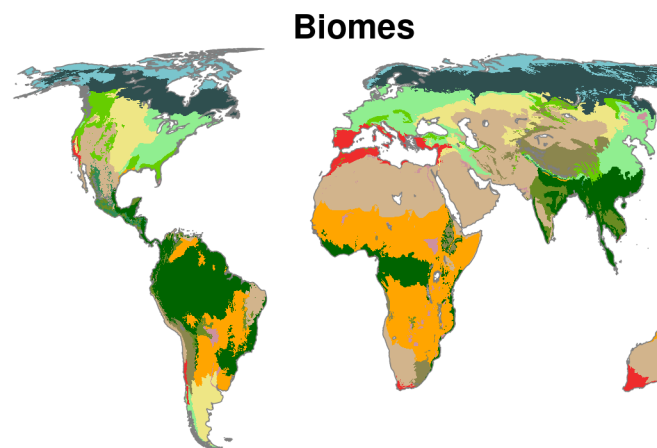
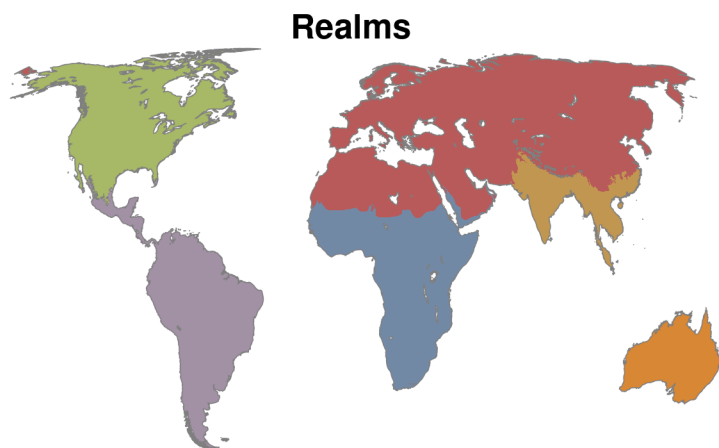
You may want to get previously fitted models (this uses the much-maligned `attach()` function)

```
dd <- readRDS("data/ecoreg.rds")  
mod_list <- readRDS("outputs/mod_list.rds")  
attach(mod_list)  
buildmer_list <- readRDS("outputs/buildmer.rds")  
attach(buildmer_list)  
confint_list <- readRDS("outputs/confint.rds")  
attach(confint_list)  
confint_tidy <- readRDS("outputs/confint_tidy.rds")
```

(These time-consuming fits etc. are actually done using `lecture2_batch.R`)

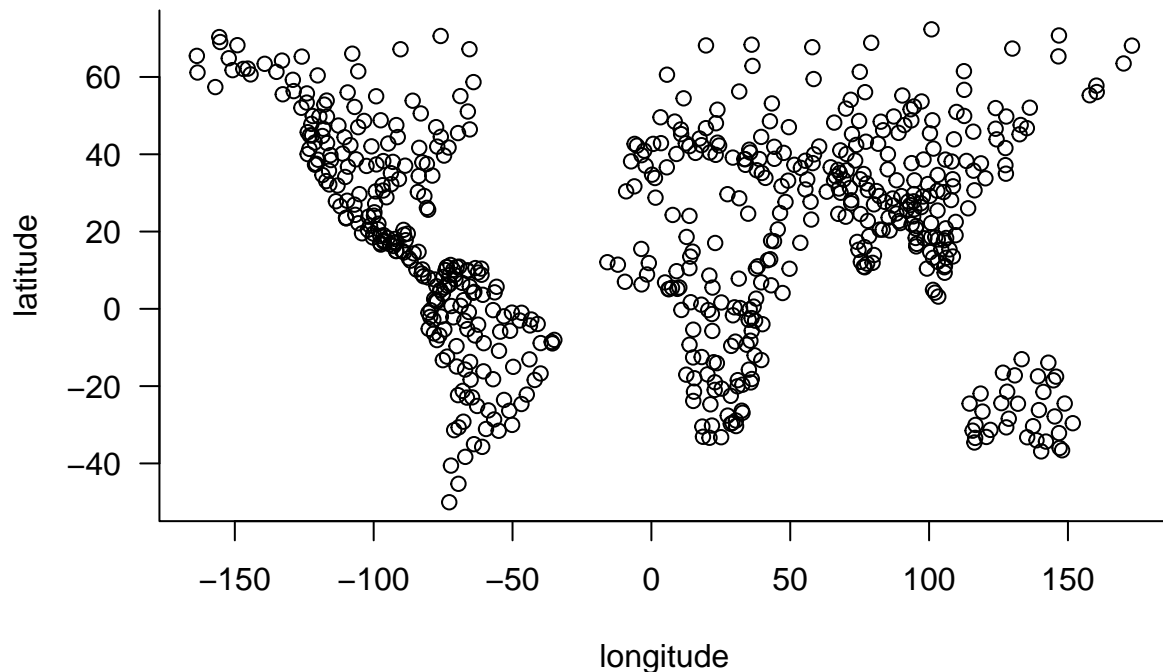
introduction to the example

From Moritz et al. (2023): 620 global observations



Sampling locations/ecoregions:

```
par(las=1, bty = "l")
plot(y ~ x, data = dd, xlab = "longitude", ylab = "latitude")
```



(not bothering to do a proper projection, land boundaries etc.)

exploration

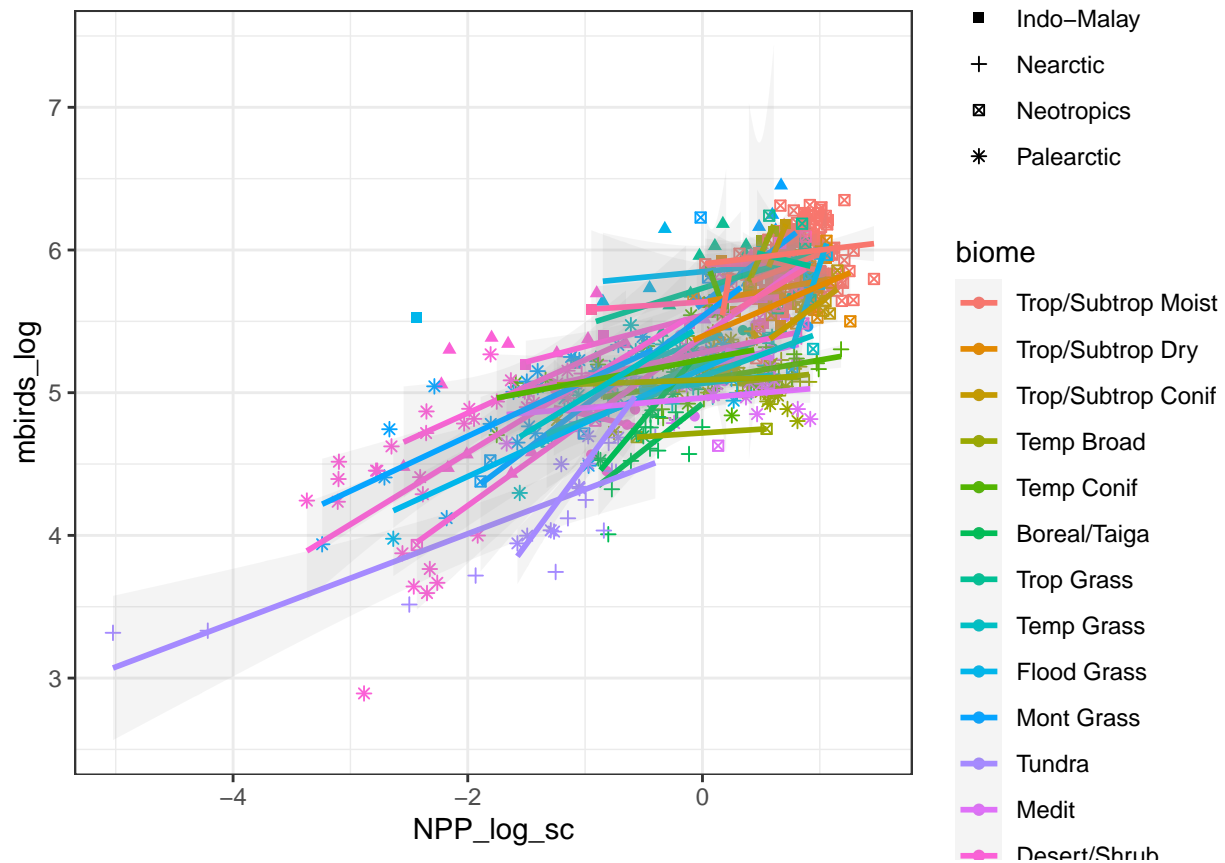
```
dd <- readRDS("data/ecoreg.rds")
```

Draw figures: `ggplot` is particular good for its flexibility — you can easily switch the way you incorporate different predictors (as facets, colours, etc.).

- plot high-dimensional data using all possible mappings - x/y position, colour, shape, line type, plus facets (“small multiples à la Tufte)
- flexibility to switch among different views of the data on the fly
- overlay some kinds of summary statistics/summaries on the fly (means, regression lines etc.)
- tools for grouping (by facet, with `geom_ellipse` or `ggalt::geom_encircle()` ...)

Competitors with similar capabilities include `tinyploth` (R): `plotnine`, `seaborn` (Python); `gadfly`, `AlgebraOfGraphic` (Julia), but `ggplot` is mature (e.g. see `ggplot` extensions gallery)

```
gg0 <- ggplot(dd, aes(NPP_log_sc, mbirds_log, colour = biome, shape = flor_realms)) +
  geom_point() +
  geom_smooth(method = "lm", aes(group = biome_FR), alpha = 0.1)
print(gg0)
```



Organize plots by biome or floristic realm:

```
gg1 <- gg0 + facet_wrap(~biome)
gg2 <- gg0 + facet_wrap(~flor_realms)
```

With pop-up tooltips (nice for identifying points interactively):

```
ggplotly(gg2)
```

(could also try out `ggiraph` for this?)

model fitting

full model

This is the full model used in Moritz et al. (2023). The fixed effects include all two-way interactions between the covariates of interest (NPP, Fire, annual coefficients of variation of each, all appropriately scaled and/or log-transformed). Random effects include main effects only, varying at the three levels of realms (e.g. “Afrotropics”, biomes (e.g. “Tropical grassland”), and their interaction (e.g. “Tropical grasslands in the Afrotropics”). We didn’t even try to fit interactions; these covariance matrices are already 5x5 (15 parameters each ...)

I actually used text manipulation to set up the formula, so I could build lots of variations on the model. This makes the code more compact and reproducible (less cutting and pasting), at the expense of transparency.

Here I’m analyzing just the bird data (`mbirds_log`); for the paper we analyzed diversity of birds, mammals and amphibians separately.

```
form <- mbirds_log ~ (NPP_log_sc + Feat_log_sc + NPP_cv_sc + Feat_cv_sc)^2 +
  (1 + NPP_log_sc + Feat_log_sc + NPP_cv_sc + Feat_cv_sc | biome) +
  (1 + NPP_log_sc + Feat_log_sc + NPP_cv_sc + Feat_cv_sc | flor_realms) +
  (1 + NPP_log_sc + Feat_log_sc + NPP_cv_sc + Feat_cv_sc | biome_FR)
```

Fit the model and take a first glance at the RE covariance matrix:

```
m_full <- lmer(form, data = dd)
```

(about 10 seconds)

```
VarCorr(m_full)
```

```
## Groups      Name          Std.Dev. Corr
## biome_FR    (Intercept) 0.107644
##              NPP_log_sc 0.122015 0.055
##              Feat_log_sc 0.024896 -0.975 -0.276
##              NPP_cv_sc 0.034689 -0.999 -0.015 0.965
##              Feat_cv_sc 0.061427 0.981 0.247 -1.000 -0.972
## biome       (Intercept) 0.125732
##              NPP_log_sc 0.158137 -0.470
##              Feat_log_sc 0.013087 -0.996 0.389
##              NPP_cv_sc 0.091421 -0.512 0.999 0.434
##              Feat_cv_sc 0.044459 0.901 -0.806 -0.859 -0.834
## flor_realms (Intercept) 0.236549
##              NPP_log_sc 0.130949 -0.346
##              Feat_log_sc 0.120903 0.608 -0.955
##              NPP_cv_sc 0.032681 0.012 0.908 -0.764
##              Feat_cv_sc 0.077954 0.357 -0.293 0.360 0.045
## Residual                    0.196065
```

An example of text manipulation, used to illustrate *more* maximal models (4-way interactions in the fixed effects, two-way interactions in the random effects terms)

```
vars <- c("NPP_log_sc", "Feat_log_sc", "NPP_cv_sc", "Feat_cv_sc")
all_vars <- paste(vars, collapse = "+")

max_form <- sprintf("(%s)^4 + ((%s)^2 | biome) + ((%s)^2 | flor_realms) + ((%s)^2 | biome_FR)",
  all_vars, all_vars, all_vars, all_vars)
max_form2 <- reformulate(max_form, response = "mbirds_log")
```

Fit the ridiculously full model (use `calc.derivs=FALSE` to skip gradient/Hessian checking, speeding things up)

```
m_fullmax <- lmer(max_form2, data = dd, control = lmerControl(calc.derivs = FALSE))
```

(this model takes about 100 minutes to fit ...)

buildmer

Trying out the `buildmer` package to automatically reduce the model:

```
m_buildmer <- buildmer(form, data = dd)
```

Decides on intercepts only at each stage. Unfortunately it also does model selection on the fixed effects by default, which I **strongly** disagree with (for philosophy about model selection/averaging see Bolker (2024))

Use `include` to force `buildmer` to keep all of the fixed-effect terms:

```
m_buildmer_fix <- buildmer(form, data = dd,
                           buildmerControl =
                             buildmerControl(
                               include= ~(NPP_log_sc + Feat_log_sc + NPP_cv_sc + Feat_cv_sc)^2))
```

For birds and amphibians, the selected model included independent effects of the predictors at the biome/realms interaction level and intercept-only (mean diversity) variation at the biome and realms levels; for mammals, the (independent) predictor effects were included at the realms level, with mean-diversity effects at the biome and biome/realms levels.

In Moritz et al. (2023), we fitted all combinations of [intercept-only, diagonal, full] at each level [biome, realms, biome \times realms]: 3 choices at each of three hierarchical levels \rightarrow 81 sub-models to fit. We chose the *minimum-AIC, non-singular* model, which turned out to be a diagonal covariance matrix at the (biome \times realms) level and intercepts only at the other two levels:

```
form2 <- mbirds_log ~ (NPP_log_sc + Feat_log_sc + NPP_cv_sc + Feat_cv_sc)^2 +
  (1 + NPP_log_sc + Feat_log_sc + NPP_cv_sc + Feat_cv_sc || biome_FR) +
  (1 | biome) +
  (1 | flor_realms)
```

```
m_moritz <- lmer(form2, data = dd)
```

For comparison, let's try this in glmmTMB as well:

```
m_glmmTMB <- glmmTMB(form2, data = dd, REML = TRUE)
```

Kliegl method

Check the individual VarCorr terms:

```
sapply(VarCorr(m_full), det)
```

```
##      biome_FR      biome    flor_realms
## -1.613694e-41  3.538280e-43 -6.052761e-33
```

All three terms are singular (determinant $\ll 1$), so let's replace them all with diagonal versions:

```
## string-processing a formula is usually a no-no but I'm in a hurry ...
form_diag <- as.formula(gsub("|", "||", deparse1(form), fixed = TRUE))
m_kliegl1 <- lmer(form_diag, data = dd)
```

```
## boundary (singular) fit: see help('isSingular')
```

```
## use internal machinery for brevity
lme4:::prt.VC(VarCorr(m_kliegl1), digits = 3)
```

```
## Random effects:
## Groups      Name      Std.Dev.
## biome_FR    Feat_cv_sc  0.0725
## biome_FR.1  NPP_cv_sc   0.0746
## biome_FR.2  Feat_log_sc  0.0316
## biome_FR.3  NPP_log_sc   0.0883
## biome_FR.4  (Intercept)  0.1285
## biome       Feat_cv_sc  0.0308
## biome.1     NPP_cv_sc   0.0000
## biome.2     Feat_log_sc  0.0000
## biome.3     NPP_log_sc   0.1321
## biome.4     (Intercept)  0.1219
```

```
## flor_realms Feat_cv_sc 0.0670
## flor_realms.1 NPP_cv_sc 0.0000
## flor_realms.2 Feat_log_sc 0.0959
## flor_realms.3 NPP_log_sc 0.0918
## flor_realms.4 (Intercept) 0.2201
## Residual 0.1955
```

This suggests we should drop NPP_cv_sc from the flor_realms term; NPP_cv_sc and Feat_log_sc from biome; and keep all terms for biome_FR

```
form_kliegl2 <- mbirds_log ~
  (NPP_log_sc + Feat_log_sc + NPP_cv_sc + Feat_cv_sc)^2 +
  (1 + NPP_log_sc + Feat_cv_sc || biome) +
  (1 + NPP_log_sc + Feat_log_sc + Feat_cv_sc || flor_realms) +
  (1 + NPP_log_sc + Feat_log_sc + NPP_cv_sc + Feat_cv_sc || biome_FR)
m_kliegl2 <- lmer(form_kliegl2, data = dd)
isSingular(m_kliegl2)
```

```
## [1] FALSE
```

Can we get away with full covariances once we drop these terms?

```
form_kliegl3 <- as.formula(gsub("||", "|", deparse1(form_kliegl2), fixed = TRUE))
m_kliegl3 <- lmer(form_kliegl3, data = dd)
```

```
## boundary (singular) fit: see help('isSingular')
```

```
## Warning: Model failed to converge with 1 negative eigenvalue: -7.1e-01
```

```
sapply(VarCorr(m_kliegl3), det)
```

```
##      biome_FR      biome flor_realms
## 2.477975e-27 7.076794e-16 0.000000e+00
```

reduced-rank model

```
form_kliegl4 <- mbirds_log ~
  (NPP_log_sc + Feat_log_sc + NPP_cv_sc + Feat_cv_sc)^2 +
  rr(1 + NPP_log_sc + Feat_cv_sc | biome, d=2) +
  rr(1 + NPP_log_sc + Feat_log_sc + Feat_cv_sc | flor_realms, d=3) +
  rr(1 + NPP_log_sc + Feat_log_sc + NPP_cv_sc + Feat_cv_sc | biome_FR, d=4)
m_kliegl4 <- glmmTMB(form_kliegl4, dd, REML=TRUE)
```

```
mod_list <- tibble::lst(m_moritz, m_full, m_fullmax,
  m_kliegl4,
  m_buildmer_mod = m_buildmer@model,
  m_buildmer_fix_mod = m_buildmer_fix@model,
  m_glmmTMB)
```

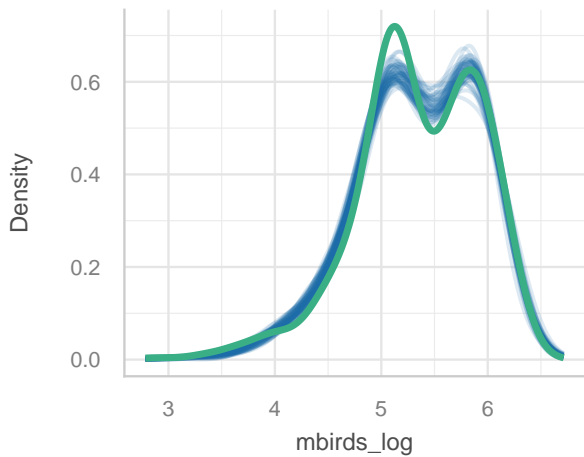
```
odir <- "outputs"
if (!dir.exists(odir)) dir.create(odir)
saveRDS(mod_list, file = file.path(odir, "mod_list.rds"))
saveRDS(tibble::lst(m_buildmer, m_buildmer_fix),
  file = file.path(odir, "buildmer.rds"))
```

diagnostics

check_model

```
performance::check_model(m_moritz)
```

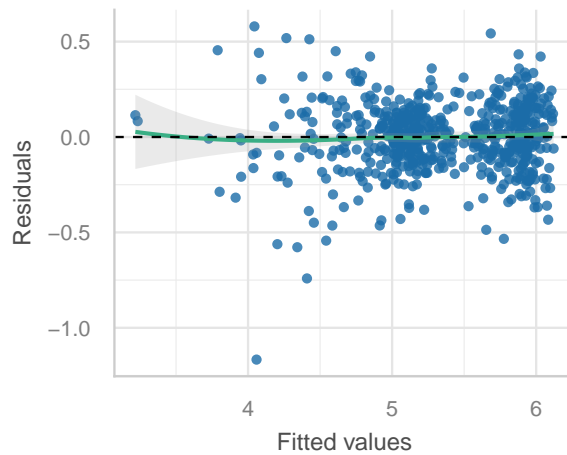

Model-predicted lines should resemble observed data line. Reference line should be flat and horizontal.



— Observed data — Model-predicted data

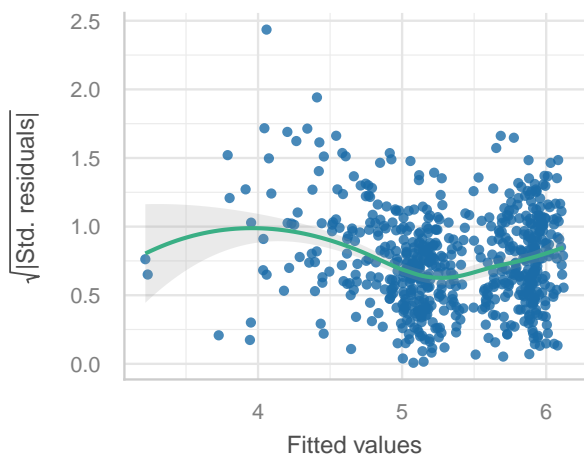
Linearity

Reference line should be flat and horizontal



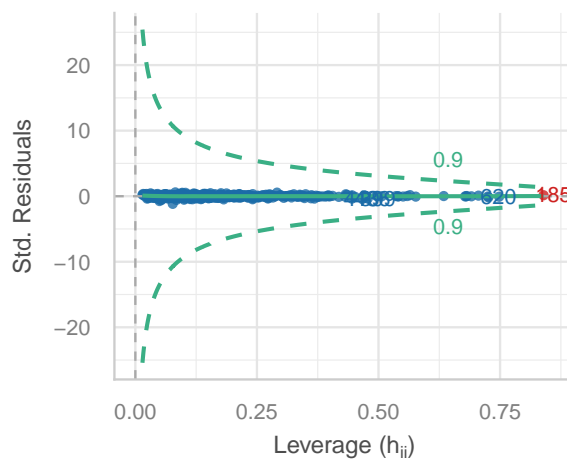
Homogeneity of Variance

Reference line should be flat and horizontal



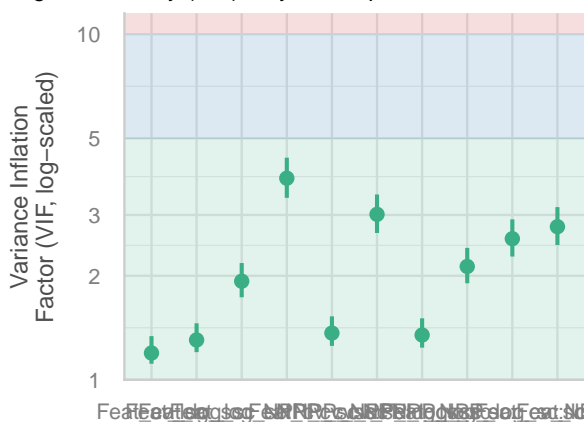
Influential Observations

Points should be inside the contour lines



Collinearity

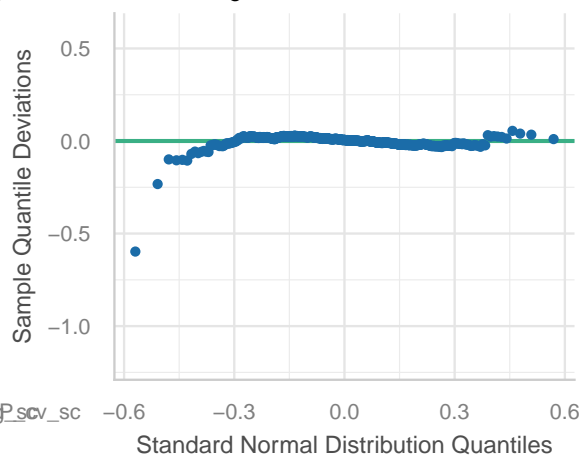
High collinearity (VIF) may inflate parameter uncertainty Dots should fall along the line



● Low (< 5)

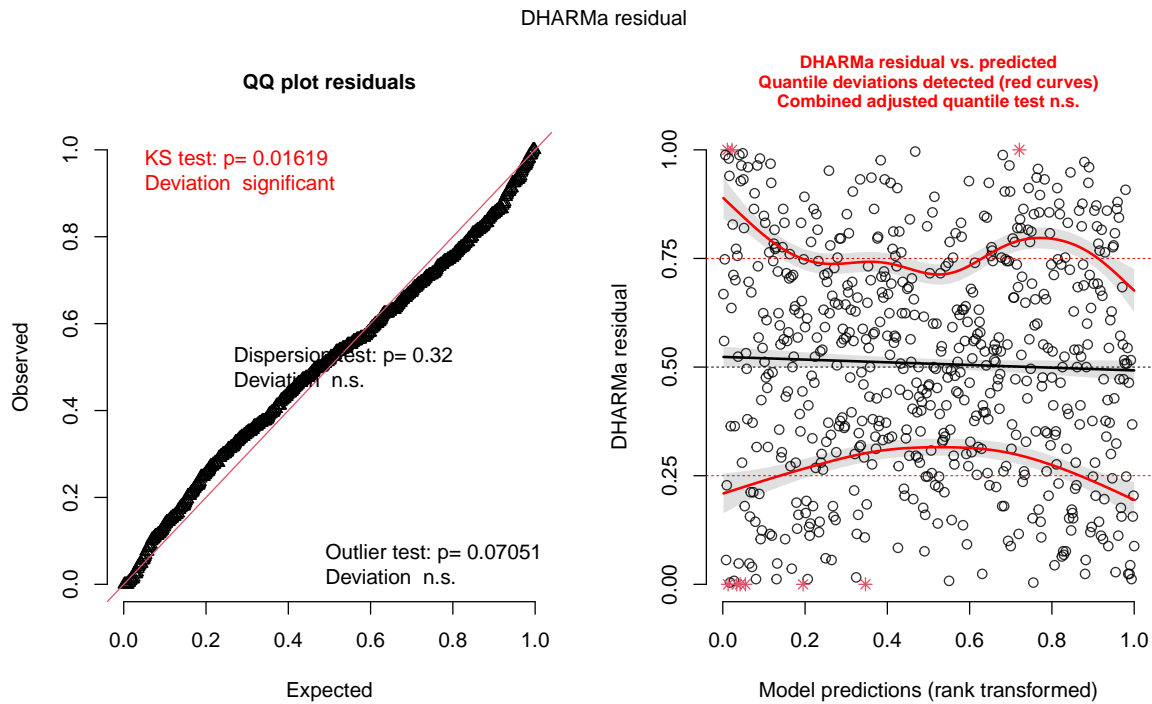
Normality of Residuals

/ Dots should fall along the line



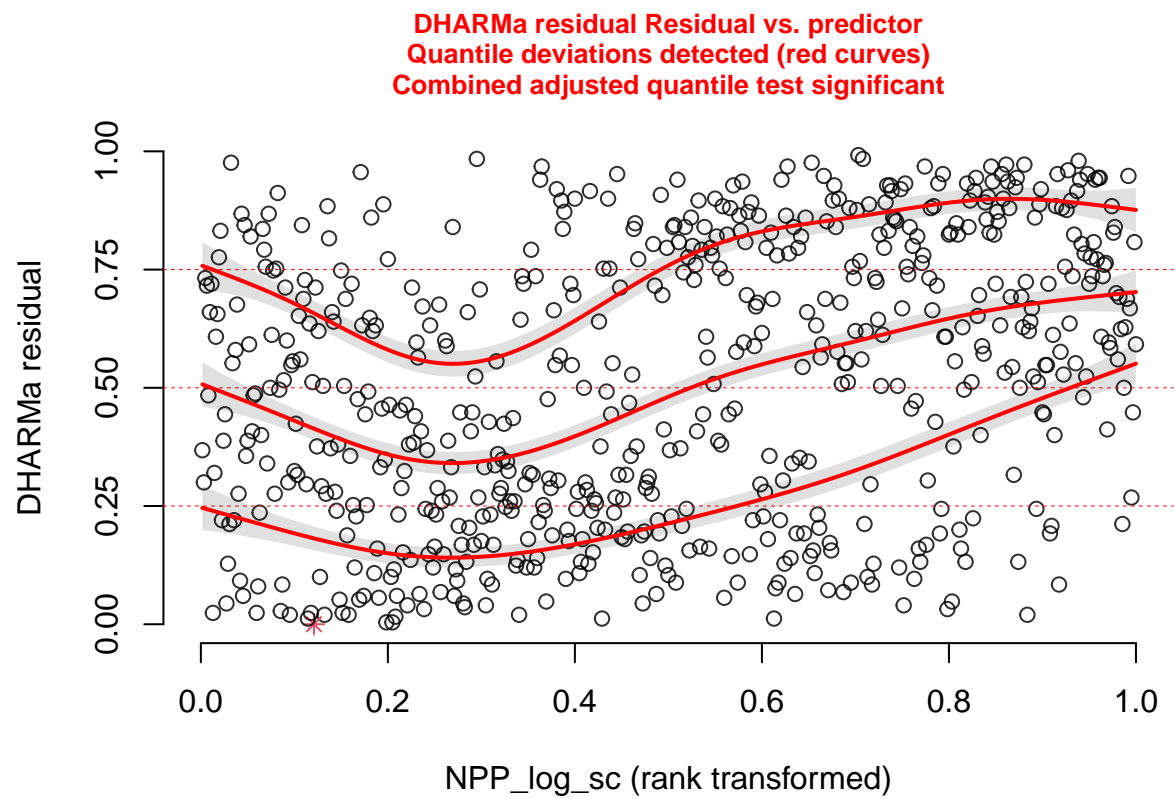
DHARMa

```
pp1 <- DHARMa::simulateResiduals(m_moritz)
## plot(pp1)
pp2 <- DHARMa::simulateResiduals(m_moritz, use.u = TRUE)
plot(pp2)
```

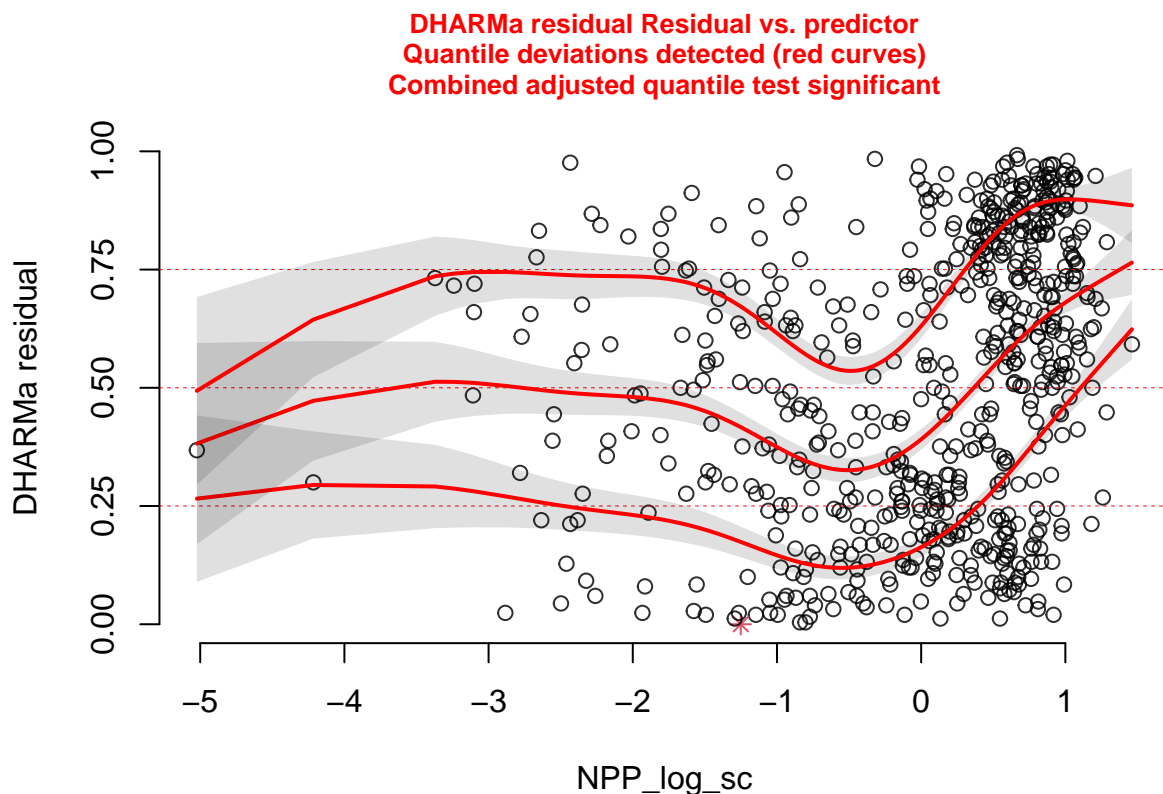


It can be useful to plot against particular covariates, and to turn off the automatic scaling/ranking:

```
plotResiduals(pp1, form = dd$NPP_log_sc)
```



```
plotResiduals(pp1, form = dd$NPP_log_sc, rank = FALSE)
```



model comparison

```
bbmle::AICtab(mod_list)
```

##		dAIC	df
##	m_buildmer_mod	0.0	11
##	m_glmmTMB	31.5	19
##	m_moritz	31.5	19
##	m_kliegl4	43.5	40
##	m_buildmer_fix_mod	44.6	16
##	m_full	72.0	57
##	m_fullmax	334.6	215

plotting (coefficient, effects plots)

Downstream processing (contrasts, marginal effects, plotting, etc.): `effects`, `broom.mixed`, `sjPlot`, `marginalEffects`, `ggeffects`, `emmeans` (see Mixed Models Task View)

coefficient plots (comparing models)

You **must** (??) decide on a model before comparing parameter estimates — this step is for understanding sensitivity, not for choosing which model to use.

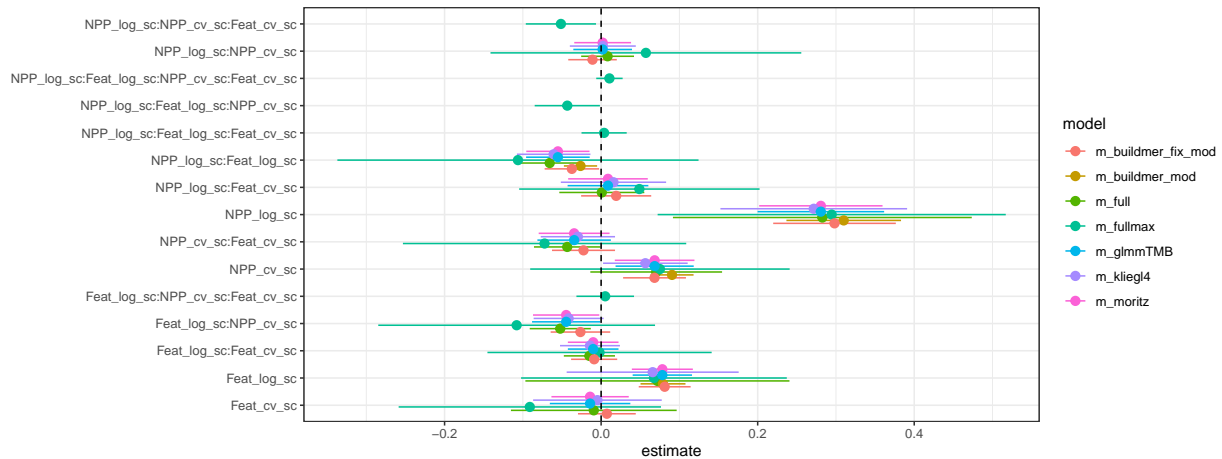
```
tt <- mod_list |>
  purrr::map_dfr(tidy, effects = "fixed", .id = "model", conf.int = TRUE) |>
```

```

filter(term != "(Intercept)")

gg_coef <- ggplot(tt, aes(estimate, term, colour = model)) +
  geom_pointrange(aes(xmin = conf.low, xmax = conf.high),
    position = position_dodge(width = 0.75)) +
  geom_vline(xintercept = 0, lty = 2) +
  labs(y = "")
print(gg_coef)

```



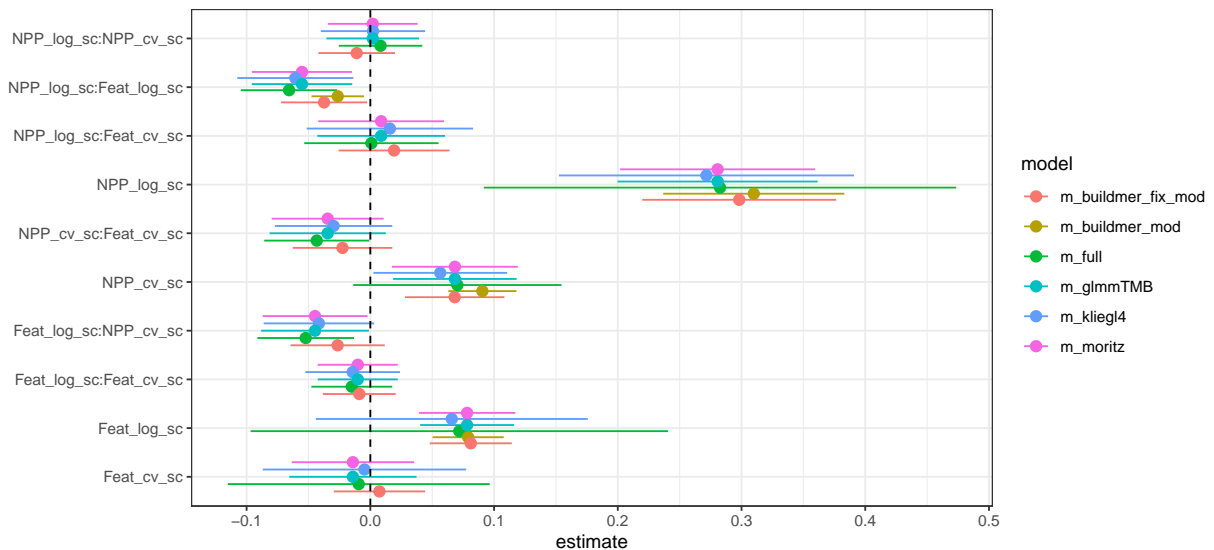
Drop “fullmax” model:

```

mod_list2 <- mod_list[names(mod_list) != "m_fullmax"]
tt2 <- mod_list2 |>
  purrr::map_dfr(tidy, effects = "fixed", .id = "model", conf.int = TRUE) |>
  filter(term != "(Intercept)")

print(gg_coef + tt2)

```

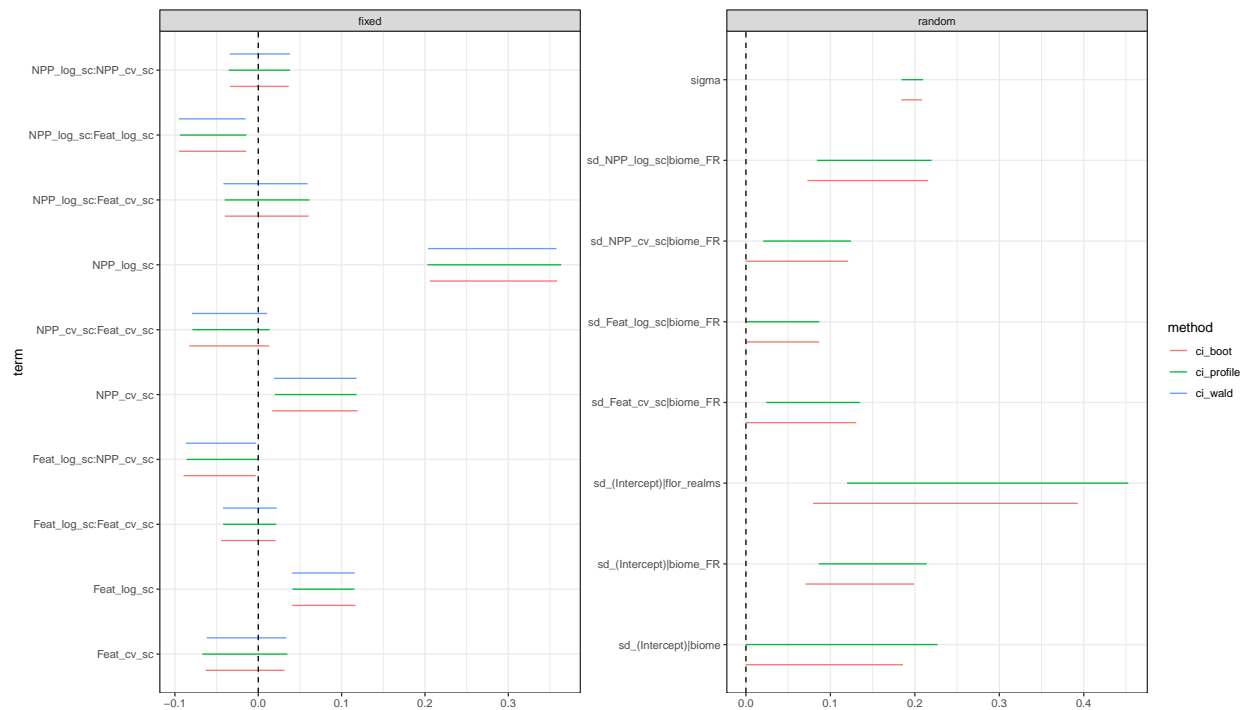


coefficient plots (comparing CI approximations)

```
ci_wald <- confint(m_moritz, method = "Wald", signames = FALSE)
ci_profile <- confint(m_moritz, method = "profile", signames = FALSE)
ci_boot <- confint(m_moritz, method = "boot", nsim = 1000, parallel = "multicore", ncpus = 6, signames = FALSE)
confint_list <- tibble::lst(ci_wald, ci_profile, ci_boot)
```

```
confint_tidy <- confint_list |>
  map(as.data.frame) |>
  map(~rownames_to_column(., var = "term")) |>
  bind_rows(.id = "method") |>
  mutate(effect = ifelse(grepl("(sd|cor|sigma)", term), "random", "fixed")) |>
  rename(lwr = "2.5 %", upr = "97.5 %") |>
  filter(term != "(Intercept)")
```

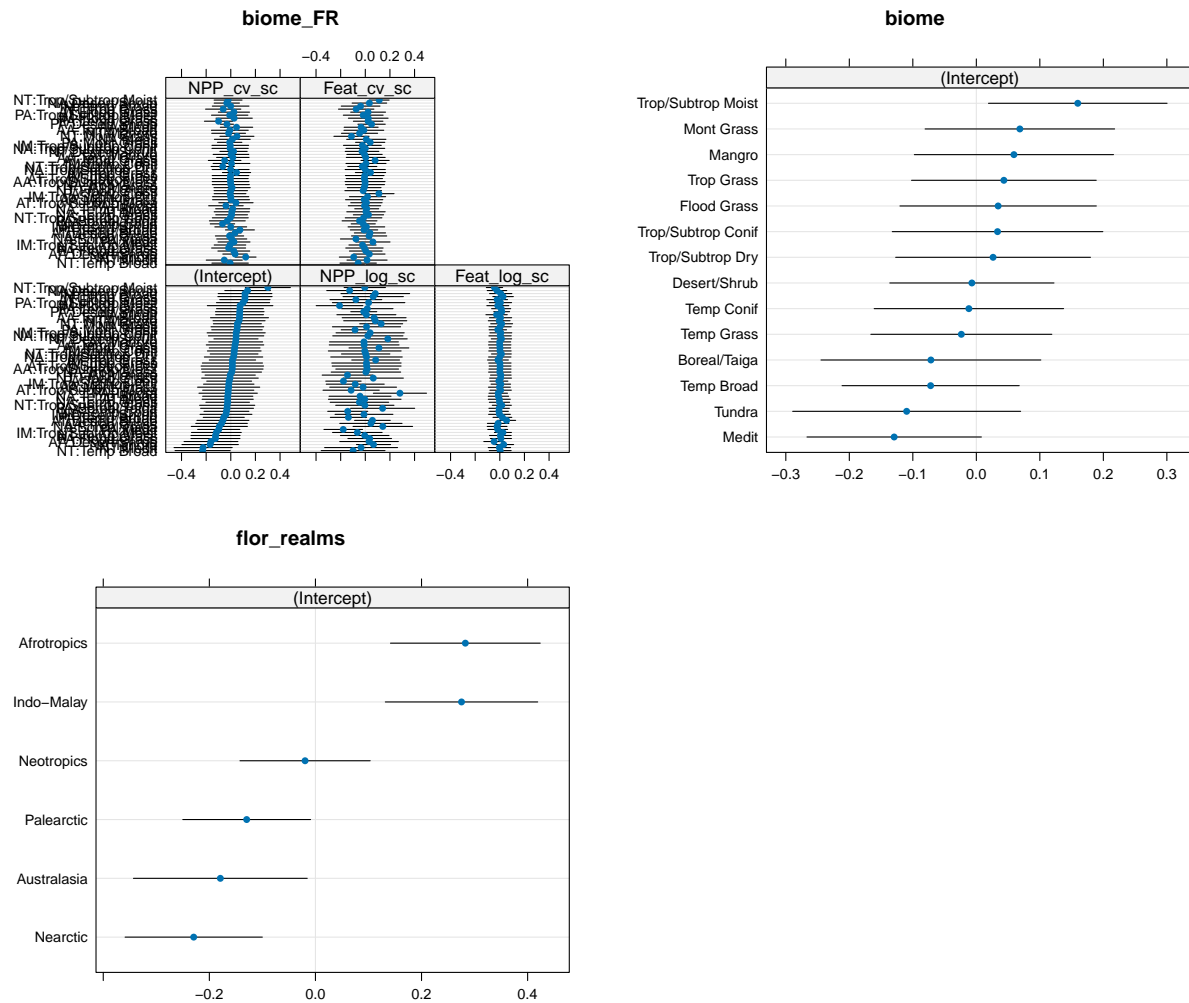
```
ggplot(confint_tidy, aes(xmin = lwr, xmax = upr, y = term, colour = method)) +
  geom_linerange(position = position_dodge(width = 0.75)) +
  geom_vline(lty = 2, xintercept = 0) +
  facet_wrap(~effect, scale = "free")
```



Plotting random effects

‘caterpillar plots’

```
cowplot::plot_grid(plotlist = lattice::dotplot(ranef(m_moritz)))
```



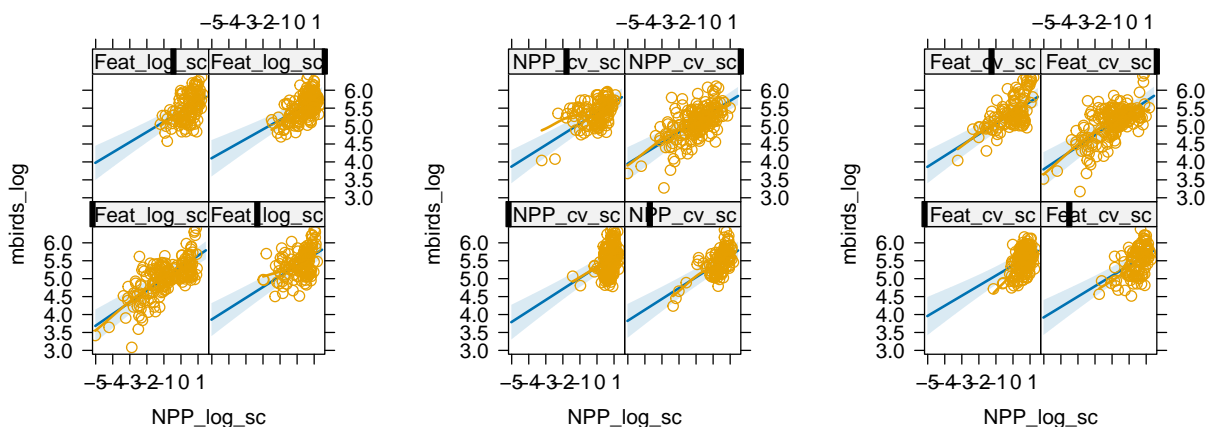
Effects plots

Effects plots, with *partial residuals* (also see the `remef` package, on GitHub).

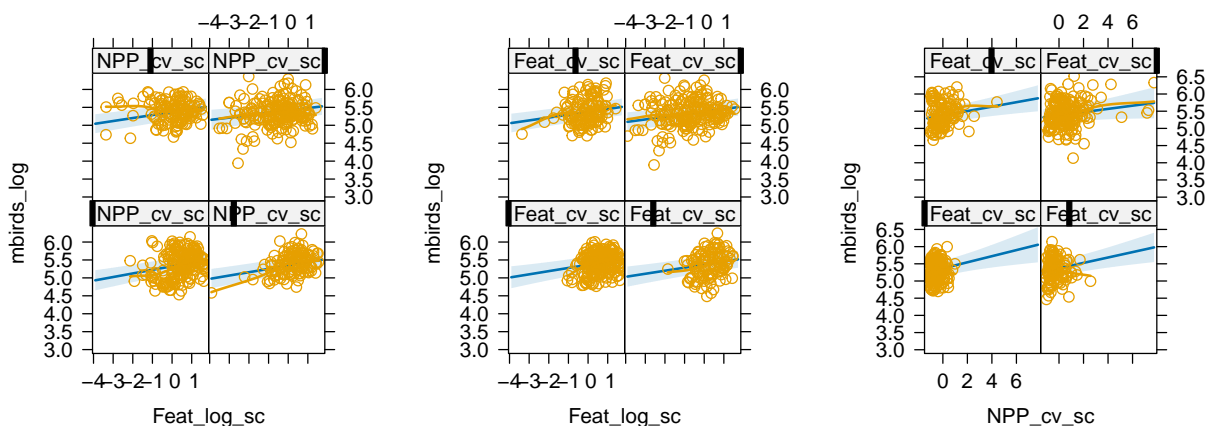
For conditional and marginal predictions/effects plots, see also `emmeans`/`sjPlot`/`ggeffects`/`marginaleffects` packages ...

```
plot(allEffects(m_buildmer_fix_mod, partial.residuals = TRUE))
```

NPP_log_sc*Feat_log_sc effect plot NPP_log_sc*NPP_cv_sc effect plot NPP_log_sc*Feat_cv_sc effect plot



Feat_log_sc*NPP_cv_sc effect plot Feat_log_sc*Feat_cv_sc effect plot NPP_cv_sc*Feat_cv_sc effect plot



Would like to try `marginaleffects` but struggling ...

References

- Bolker, B. M. (2024). Multimodel approaches are not the best way to understand multifactorial systems. *Entropy*, 26(6), 506. <https://doi.org/10.3390/e26060506>
- Moritz, M. A., Batllori, E., & Bolker, B. M. (2023). The role of fire in terrestrial vertebrate richness patterns. *Ecology Letters*, 26(4), 563–574. <https://doi.org/10.1111/ele.14177>