Generalized linear mixed models

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- Data: Josh Banta, Adrian Stier, Sea McKeon, David Julian, Jada-Simone White

- NSERC (Discovery)
- SHARCnet

Outline

1. Examples and definitions
2. Estimation
   - Overview
   - Methods
3. Inference
4. Challenges & open questions

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(Generalized) linear mixed models

(G)LMMs: a statistical modeling framework incorporating:

- combinations of categorical and continuous predictors, and interactions
- (some) non-Normal responses (e.g. binomial, Poisson, and extensions)
- (some) nonlinearity (e.g. logistic, exponential, hyperbolic)
- non-independent (grouped) data

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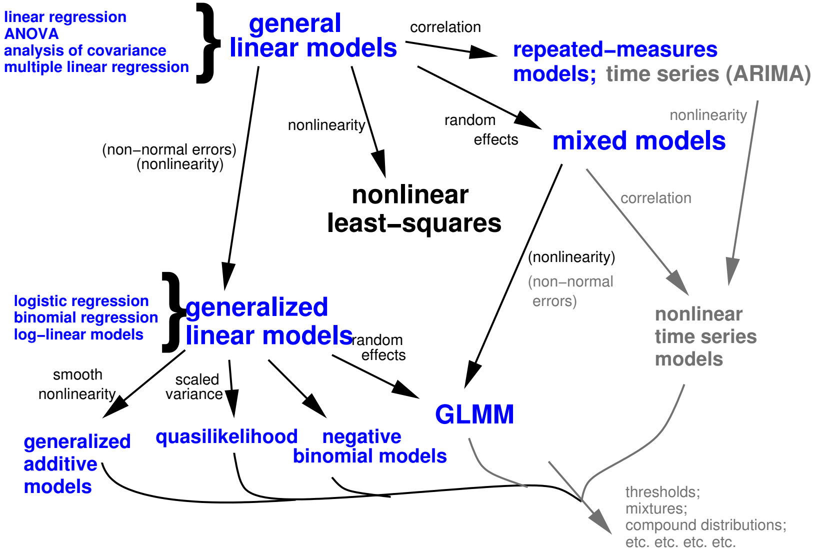
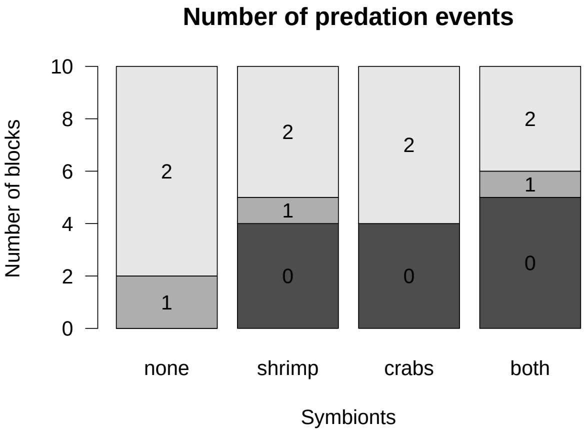
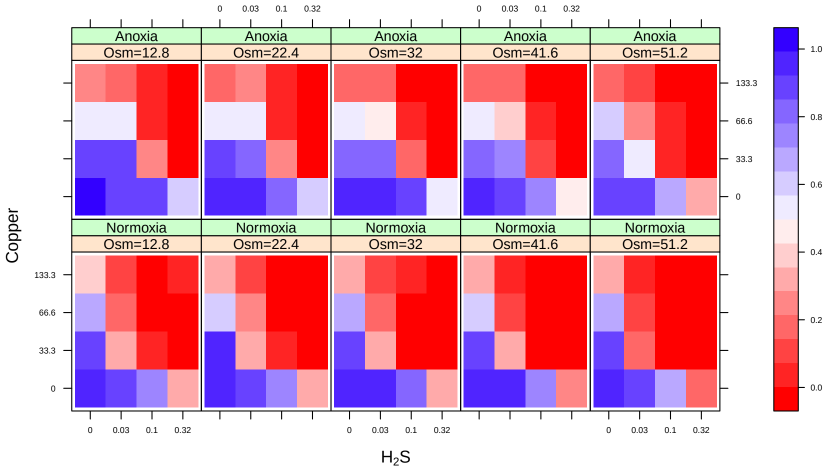

linear regression
ANOVA
analysis of covariance
multiple linear regression
general linear models
correlation
repeated−measures models; time series (ARIMA)
nonlinearity
random effects
mixed models
(non−normal errors) (nonlinearity)
nonlinear least−squares
correlation
(nonlinearity)
(non−normal errors)
logistic regression
binomial regression
log−linear models
generalized linear models
random effects
nonlinear time series models
smooth nonlinearity
scaled variance
GLMM
generalized additive models
quasilikelihood
negative binomial models
thresholds;
mixtures;
compound distributions;
etc. etc. etc. etc.

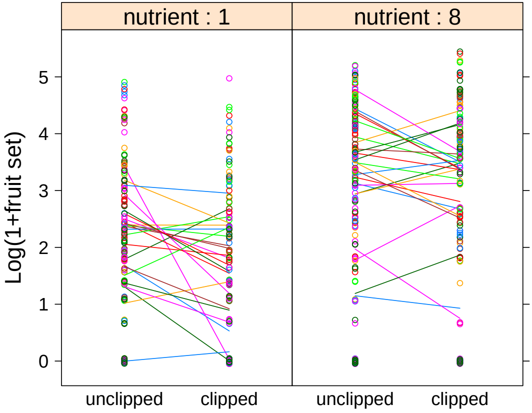
Coral protection from seastars (*Culcita*) by symbionts (McKeon et al., 2012)



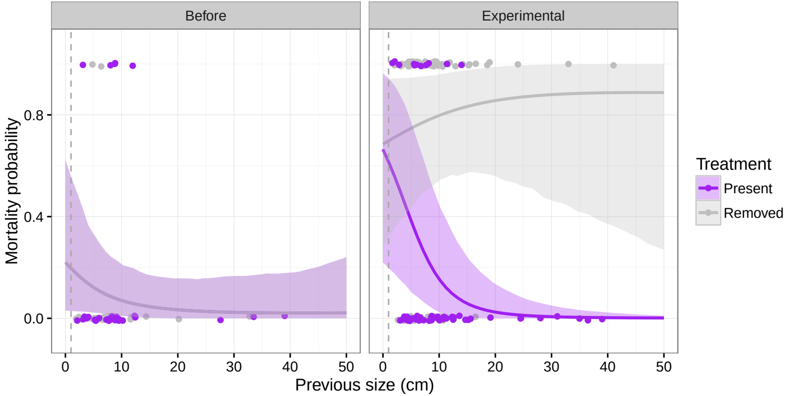
Environmental stress: *Glycera* cell survival (D. Julian unpubl.)



Arabidopsis response to fertilization & herbivory (Banta et al., 2010)



Coral demography (J.-S. White unpubl.)



Technical definition

$$\underbrace{Y_i}_{\text{response}} \sim \overbrace{\text{Distr}}^{\substack{\text{conditional}\\\text{distribution}}} \left(\underbrace{g^{-1}(\eta_i)}_{\substack{\text{inverse}\\\text{link}\\\text{function}}}, \underbrace{\phi}_{\substack{\text{scale}\\\text{parameter}}}\right)$$

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$$\underbrace{\boldsymbol{b}}_{\substack{\text{conditional}\\ \text{modes}}} \sim \text{MVN}(\boldsymbol{0}, \underbrace{\Sigma(\boldsymbol{\theta})}_{\substack{\text{variance-}\\ \text{covariance}\\ \text{matrix}}})$$

What are random effects?

A method for …

- accounting for among-individual, within-block correlation
- compromising between
  complete pooling (no among-block variance)
  and fixed effects (large among-block variance)
- handling levels selected at random from a larger population
- sharing information among levels (*shrinkage estimation*)
- estimating variability among levels
- allowing predictions for unmeasured levels

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Random-effect myths

- levels of random effects must always be sampled at random
- a complete sample cannot be treated as a random effect
- random effects are always a **nuisance variable**
- nothing can be said about the predictions of a random effect
- you should always use a random effect no matter how few levels you have

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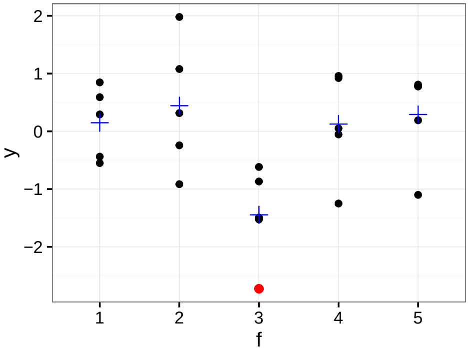
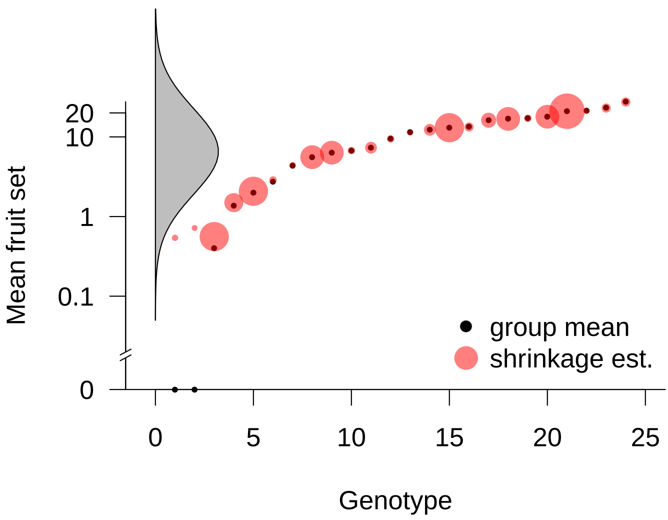
Maximum likelihood estimation

- Best fit is a compromise between two components (consistency of data with fixed effects and conditional modes; consistency of random effect with RE distribution)
- Goodness-of-fit *integrates* over conditional modes



Shrinkage: *Arabidopsis* conditional modes



Estimation methods

deterministic : various approximate integrals (Breslow, 2004)

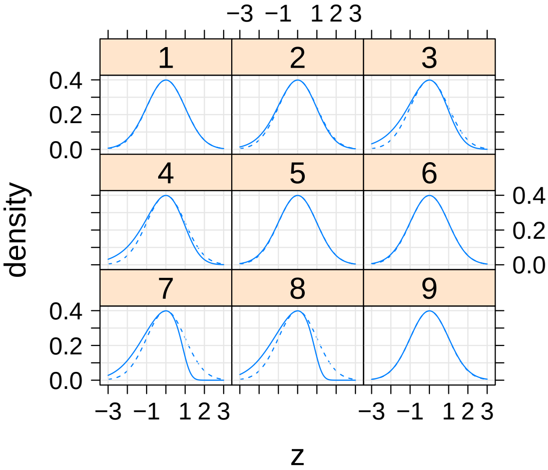
- Penalized quasi-likelihood, Laplace, Gauss-Hermite quadrature, ... (Biswas, 2015); best methods needed for large variance, small clusters
- flexibility and speed vs. accuracy

...

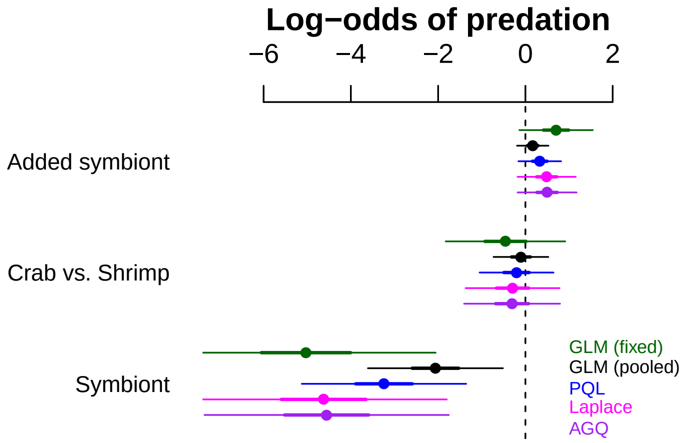
stochastic (Monte Carlo): frequentist and Bayesian (Booth and Hobert, 1999; Ponciano et al., 2009; Sung and Geyer, 2007)

- usually slower but flexible and accurate

Laplace-approximation diagnostics



Estimation: *Culcita* (McKeon et al., 2012)



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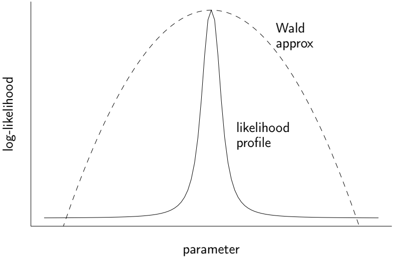
Wald tests

- typical results of `summary`
- exact for ANOVA, regression: approximation for GLM(M)s
- fast
- approximation is sometimes awful (Hauck-Donner effect)



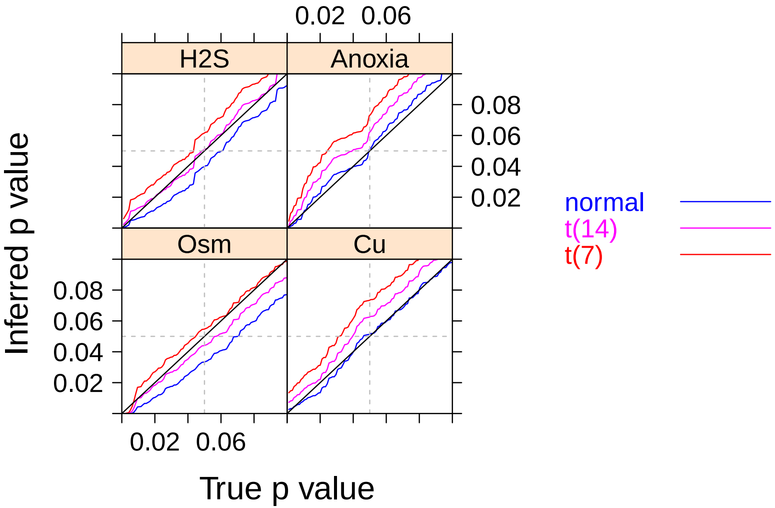
Likelihood ratio tests

- better than Wald, but still have to two problems:
  - "denominator degrees of freedom" (when estimating scale)
  - for GLMMs, distributions are approximate anyway (Bartlett corrections)
  - Kenward-Roger correction? (Stroup, 2014)
- Profile confidence intervals: expensive/fragile

Parametric bootstrapping

- fit null model to data
- simulate "data" from null model
- fit null and working model, compute likelihood difference
- repeat to estimate null distribution
- should be OK but ??? not well tested
  (assumes estimated parameters are "sufficiently" good)

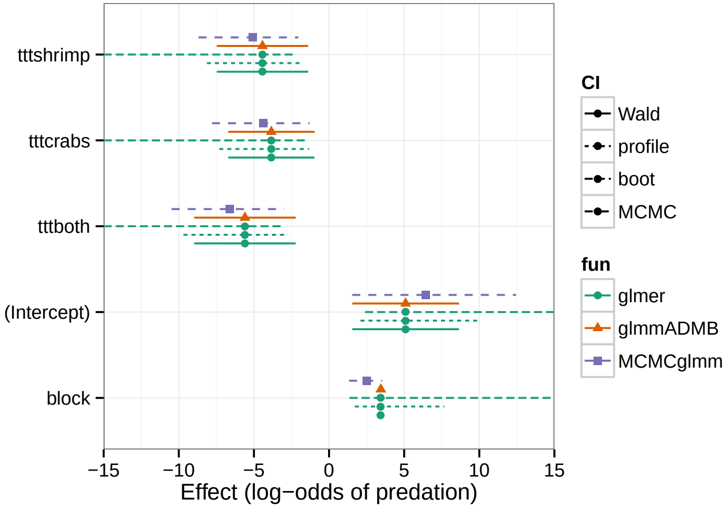
Parametric bootstrap results (*Glycera*)



Bayesian inference

- If we have a good sample from the posterior distribution (Markov chains have converged etc. etc.) we get most of the inferences we want for free by summarizing the marginal posteriors
- *post hoc* Bayesian methods: use deterministic/frequentist methods to find the maximum, then sample around it

Culcita confidence intervals



formula formats

- `fixed`: fixed-effect formula
- `random`: random-effect formula (in `lme4`, combined with fixed)
  - generally `x|g` (term|grouping variable)
  - simplest: `1|g`, single intercept term
  - nested: `1|g1/g2`
  - random-slopes: `r|g`
  - independent terms: `(1|g)+(x+0|g)` or `(x||g)`
- `lme`: `weights`, `correlation` for heteroscedasticity and residual correlation
- `MCMCglmm`: options for variance structure

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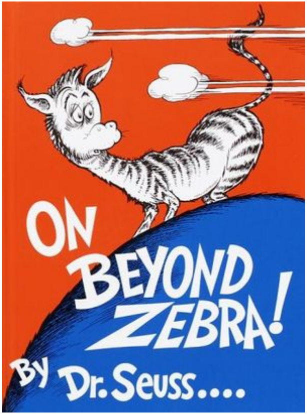
On beyond `lme4`

- `glmmADMB`, `glmmTMB`: zero-inflated and other distributions
- `brms`, `rstanarm`: interfaces to Stan
- `INLA`: spatial and temporal correlations

On beyond R

- Julia: MixedModels package
- SAS: PROC MIXED, NLMIXED
- AS-REML
- Stata (GLLAMM, xtmelogit)
- AD Model Builder; Template Model Builder
- HLM, MLWiN
- JAGS, Stan, rethinking package



Challenges

- Small clusters: need AGQ/MCMC
- Small numbers of clusters: need finite-size corrections (KR/PB/MCMC)
- Small data sets: issues with **singular** fits
  Barr et al. (2013) vs. Bates et al. (2015)
- Big data: speed!
- Model diagnosis
- Confidence intervals accounting for uncertainty in variances

See also: https://rawgit.com/bbolker/mixedmodels-misc/master/ecostats_chap.html https://groups.nceas.ucsb.edu/non-linear-modeling/projects

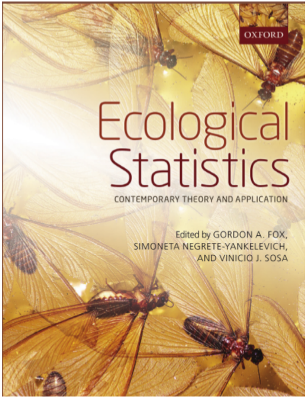
Spatial and temporal correlations

- Sometimes blocking takes care of non-independence ...
- but sometimes there is temporal or spatial correlation **within** blocks
- . . . also phylogenetic . . . (Ives and Zhu, 2006)
- "G-side" vs. "R-side" effects
- tricky to implement for GLMMs,
  but new possibilities on the horizon (Rousset and Ferdy, 2014; Rue et al., 2009);
  https://github.com/stevencarlislewalker/lme4ord

Next steps

- Complex random effects:
  regularization, model selection, penalized methods (lasso/fence)
- Flexible correlation and variance structures
- Flexible/nonparametric random effects distributions
- hybrid & improved MCMC methods
- Reliable assessment of out-of-sample performance

- http://ms.mcmaster.ca/~bolker/misc/private/14-Fox-Chap13.pdf
- https://rawgit.com/bbolker/mixedmodels-misc/master/ecostats_chap.html
- Bolker (2015)



(code ASPROMP8)

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