2102 509 – Introduction to Optimization Techniques
Homework # 1: Due Date 1st October 2025.
Submit the homework in the class before lecture.
LATE HOMEWORK WILL NOT BE ACCEPTED. DUPLICATION OF HOMEWORK IS STRICTLY FORBIDDEN.

Name:..... Collaborator:.....

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1. (20 points) Let f be a real-valued function of n variables.

1.1 Assume that f is continuously differentiable. Then prove that f is convex on the convex set S if and only if

$$f(y) \geq f(x) + \nabla f(x)^T (y - x) \quad \text{for all } x, y \in S.$$

1.2 Assume that f is twice continuously differentiable. Then prove that f is convex on the convex set S if $\nabla^2 f(x)$ is positive semi-definite for all $x \in S$, by using the result obtained from Problem 1.1.

2. (15 points) Prove that if f is convex, then any stationary point is also a global minimizer.

3. (10 points) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = x^5 - 5x^3 - 20x + 5$$

Determine the minimizer x^* of f ($x^* \geq 0$) with the accuracy of at least 10 significant digits by using Newton-Raphson method.

4. (20 points) Write your MATLAB functions for solving the one-dimensional optimization problem using quadratic interpolation and golden section methods. Print out your programs. The format of the functions are as follows.

function [x_{min}, f_{min}, IFLAG, IFunc] = quadractic(a, b, epsilon, itmax)
function [x_{min}, f_{min}, IFLAG, IFunc] = golden(a, b, epsilon, itmax)

where x_{min} is the estimate of the minimizer of f, f_{min} is value of $f(x_{min})$, IFLAG is set to be 0 if the search is successful and -999 otherwise, IFunc is the number of function evaluation calls, $[a, b]$ is a given interval that brackets x_{min}, epsilon is the parameter used in the stopping criterion, itmax denotes the maximum number of the iterations allowed. Write comments in the codes so that they can be examined.

(20 points) Then use the quadratic and golden functions to compute the value of x that minimizes the function f in Problem 3. Locate the value of x with at least 4 significant digit accuracy. Print the results for every iteration. Compare the obtained results with the exact solution in Problem 3 and make a discussion regarding the performance of both methods.

Suchin Arunsawatwong, 17th September 2025