Hw3 es 2 point 1

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We could use a proof similar to the one done in the lecture or doing it a little bit differently

2) $\| A_k \|_F^2 = \| \sum_{i=1}^{k} \sigma_i u_i v_i^T \|_F^2 = \| U \Sigma V^T \|_F^2 \Rightarrow$

where $U = \begin{pmatrix} | & & | \\ u_i & \cdots & u_m \\ | & & | \end{pmatrix}, V^T = \begin{pmatrix} - v_i - \\ \vdots \\ - v_m - \end{pmatrix}, \Sigma = \begin{pmatrix} \sigma_1 & \cdots & & 0 \\ \vdots & \sigma_k & 0 & \vdots \\ 0 & \cdots & \ddots & 0 \end{pmatrix}$

now $\|A\|_F^2 = Tr(A^T A) \Rightarrow Tr\left((U \Sigma V^T)^T (U \Sigma V^T) \right) = Tr(V \Sigma^T U^T U \Sigma V^T) \Rightarrow$

Since U and V are orthonormal basis then $UU^T = U^T U = V^T V = V V^T = I$

$\Rightarrow Tr(V \Sigma^T \Sigma V^T) \Rightarrow$

Now the trace of the product is cyclic so $\Rightarrow Tr(V^T V \Sigma^T \Sigma) = Tr(\Sigma^T \Sigma) = \| \Sigma \|_F^2$

Since Σ is a matrix done in this way $\Sigma = \begin{pmatrix} \sigma_1 & & & \\ & \ddots & & \\ & & \sigma_k & \\ & & & 0 & \\ & & & & \ddots \\ & & & & & 0 \end{pmatrix}$ Then the frobineaus norm is $\| \Sigma \|_F^2 = \sum_{i=1}^{k} \sigma_i^2$

so $\| A_k \|_F^2 = \| \Sigma \|_F^2 = \sum_{i=1}^{k} \sigma_i^2$