Election 2020: Live Results And Analysis

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Beware The Hot Pumpkin

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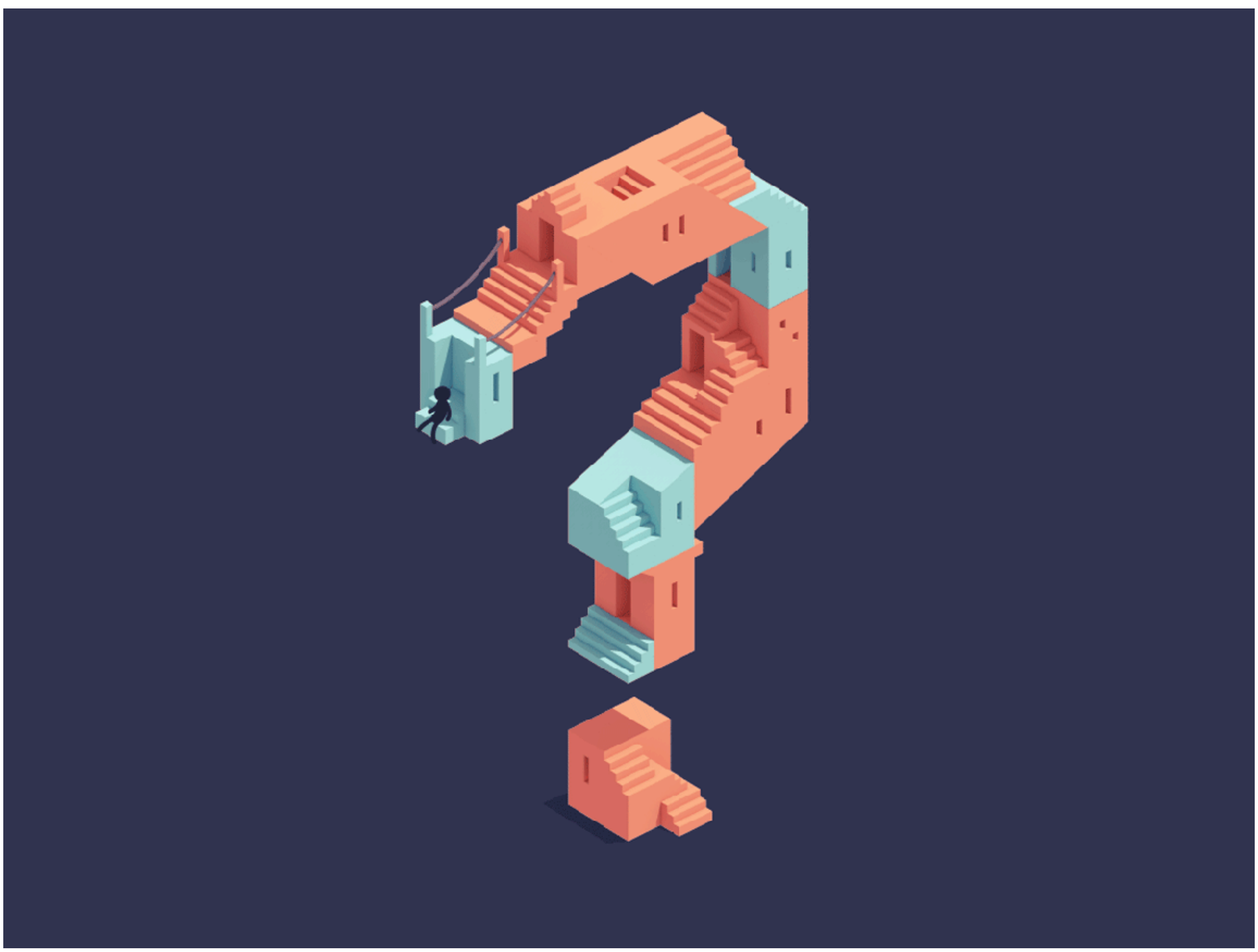

ILLUSTRATION BY GUILLAUME KURKDJIAN

Welcome to The Riddler. Every week, I offer up problems related to the things we hold dear around here: math, logic and probability. Two puzzles are presented each week: the Riddler Express for those of you who want something bite-size and the Riddler Classic for those of you in the slow-puzzle movement. Submit a correct answer for either,[1] and you may get a shoutout in next week's column. Please wait until Monday to publicly share your answers! If you need a hint or have a favorite puzzle collecting dust in your attic, find me on Twitter.

Riddler Express

While waiting in line to vote early last week, I overheard a discussion between election officials. Apparently, there may have been a political sign that was within 100 feet of the polling place, against New York State law.

This got me thinking. Suppose a polling site is a square building whose sides are 100 feet in length. An election official takes a string that is also 100 feet long and ties one end to a door located in the middle of one of the building's sides. She then holds the other end of the string in her hand.

What's the area of the region outside of the building she can reach while holding the string?

Submit your answer

Riddler Classic

From Ricky Jacobson comes a party game that's just in time for Halloween:

Instead of playing hot potato, you and 60 of your closest friends decide to play a socially distanced game of hot pumpkin.

Before the game starts, you all sit in a circle and agree on a positive integer *N*. Once the number has been chosen, you (the leader of the group) start the game by counting "1" and passing the pumpkin to the person sitting directly to your left. She then declares "2" and passes the pumpkin one space to *her* left. This continues with each player saying the next number in the sequence, wrapping around the circle as many times as necessary, until the group has collectively counted up to *N*. At that point, the player who counted "*N*" is eliminated, and the player directly to his or her left starts the next round, again proceeding to the same value of *N*. The game continues until just one player remains, who is declared the victor.

In the game's first round, the player 18 spaces to your left is the first to be eliminated. Ricky, the next player in the sequence, begins the next round. The second round sees the elimination of the player 31 spaces to Ricky's left. Zach begins the third round, only to find himself eliminated in a cruel twist of fate. (Woe is Zach.)

What was the smallest value of *N* the group could have used for this game?

Extra credit: Suppose the players were numbered from 1 to 61, with you as Player No. 1, the player to your left as Player No. 2 and so on. Which player won the game?

Extra extra credit: What's the smallest *N* that would have made *you* the winner?

Submit your answer

Solution to last week's Riddler Express

Congratulations to 👏 Eric Mentzell 👏 of Takoma Park, Maryland, winner of last week's Riddler Express.

Last week, four-time WNBA champion Sue Bird and Seattle Storm teammate Breanna Stewart were interested in testing whether Bird had a "hot hand" — that is, if her chances of making a basket depended on whether or not her previous shot went in. Bird happened to know that her chances of making any given shot was *always* 50 percent, independent of her shooting history, but she agreed to perform an experiment.

In each trial of the experiment, Bird took three shots, while Stewart recorded which shots Bird made or missed. Stewart then looked at all the trials with at least one shot that was immediately preceded by a made shot. She randomly picked one of these trials, and then randomly picked a shot that was immediately preceded by a made shot. (If there was only one such shot to pick from, she chose that shot.)

What was the probability that Bird made the shot that Stewart picked?

At first glance, you might have thought that the answer was 50 percent. After all, Bird acknowledged that she had a 50 percent chance of making any given shot. Right?

Wrong. And Riddler Nation was not fooled. To see why the answer wasn't 50 percent, many solvers, like Deborah Abel, listed out all eight possible shot sequences. If we let "X" represent a made shot and "O" a missed shot, then the eight possible shot sequences were: OOO, OOX, OXO, XOO, OXX, XOX, XXO, and XXX. Because Bird had an equal chance of making or missing any shot, all eight of these shot sequences were equally likely.

During her analysis, Stewart first looked at shots that were “immediately preceded by a made shot.” This didn’t occur for sequences OOO and OOX, so we know that Stewart was limiting her analysis exclusively to the remaining six sequences, and that each had a one-in-six chance of being selected.

Next, Stewart “randomly picked a shot that was immediately preceded by a made shot.” Here’s what happened if you looked at the six possible sequences one at a time:

- OXO: Stewart looked at the last shot. For this sequence, Bird made zero percent of the shots Stewart could have picked.
- XOO: Stewart looked at the second shot. For this sequence, Bird made zero percent of the shots Stewart could have picked.
- OXX: Stewart looked at the last shot. For this sequence, Bird made 100 percent of the shots Stewart could have picked, which gives a 1/6 chance of selecting a made shot in this sequence.
- XOX: Stewart looked at the second shot. For this sequence, Bird made zero percent of the shots Stewart could have picked.
- XXO: Stewart could have looked at the second or third shots. For this sequence, Bird made 50 percent of the shots Stewart could have picked, which gives a 1/12 chance of selecting a made shot in this sequence.
- XXX: Stewart could have looked at the second or third shots. For this sequence, Bird made 100 percent of the shots Stewart could have picked, which gives a 1/6 chance of selecting a made shot in this sequence.

Putting this all together, the probability Bird had made the shot that Stewart picked was 1/6 + 1/12 + 1/6, or **5/12** — the solution!

So while Bird had a 50 percent chance of making any given shot, Stewart’s methodology for selecting a shot was somehow biased toward shots that Bird missed. What was going on here?

Solver Madeline Argent of Launceston, Australia offered an explanation. Among the six shot sequences listed above, there were eight cases in which Bird made one of her first two shots. Of these eight, Bird made four of the next shots and missed four of them. So if Stewart had selected among all *shots* that were immediately preceded by a made shot, Bird would have made half the selected shots. But because Stewart first randomly selected a *trial*, the last two shot sequences — XXO and XXX, in which Bird made most of her shots — were unfairly weighted equally alongside the other four sequences, even though they had twice as many made shots for Stewart to choose from.

That was what happened when Bird took three shots per trial. Meanwhile, solver Dean Ballard found similar results when Bird took more shots. The probability she had made a selected shot approached 50 percent as the number of shots increased, but it never quite reached 50 percent.

Clearly, this methodology for determining whether a basketball player had a "hot hand" was flawed. It may surprise some readers that this was precisely the methodology used in an attempt to debunk the "hot hand" back in 1985 — a debunking that itself was later debunked. If you'd like to read more on this, check out this 2019 article that connects all of this to the Monty Hall problem. (Kudos to submitter Drew Mathieson for the link!)

So if Stewart performed the experiment as outlined, and found that Bird had made *half* the selected shots (rather than 5/12 of them), she would have rightfully concluded that Bird *did* have a "hot hand."

Solution to last week's Riddler Classic

Congratulations to 👏 Lucas Robinson 👏 of Oakwood, Ohio, winner of last week's Riddler Classic.

Last week, four-time NBA champion LeBron James was playing a game of sudden-death, one-on-one basketball with Los Angeles Lakers teammate Anthony Davis. They flipped a coin to see which of them had first possession, and whoever made the first basket won the game.

Both players had a 50 percent chance of making any shot they took. However, Davis was the superior rebounder and would always rebound any shot that either of them missed. Every time Davis rebounded the ball, he dribbled back to the three-point line before attempting another shot.

Before each of Davis's shot attempts, James had a probability p of stealing the ball and regaining possession before Davis could get the shot off. What value of p made this an evenly matched game of one-on-one, in which both players had an equal chance of winning *before* the coin was flipped?

Suppose James's probability of winning when he had possession was J, while James's probability of winning when *Davis* had possession was D. We can write an equation for J: For James to win when he had possession, he either had to score (with probability 1/2) or, upon missing (also with probability 1/2), he'd have to somehow win after Davis got the rebound and gained possession. In other words, $J = 1/2 + 1/2 \cdot D$. We can similarly write an equation for D: For James to win when Davis had possession, he either had to steal the ball (with probability p) and regain possession, or, upon not getting the steal (with probability $1-p$), he needed Davis to miss (with probability 1/2) so James could have another chance at victory. In other words, $D = pJ + (1-p)D/2$.

That gave you two equations with three unknowns. What was missing? Based on the coin flip, James started with possession half the time, and Davis started with possession the other half the time. James's probability of winning was therefore $J/2 + D/2$, which the problem stated was to equal 1/2, so the third equation was $J/2 + D/2 = 1/2$, or $J + D = 1$.

Solving this system of three equations gave you the result that $J = 2/3$ (James had a two-thirds chance of winning when he had possession), $D = 1/3$ (James had a one-third chance of winning when Davis had possession) and $p = 1/3$. In other words, for the game to be fair, James had to have a **one-third** chance of stealing the ball.

Solver Quoc Tran extended the puzzle, simulating how James's fortunes changed in the more realistic scenario where Davis didn't nab *every* rebound. For the game to be fair, we already saw that James needed to steal one-third of the time when Davis rebounded every shot. Meanwhile, if James and Davis had had equal chances of getting a rebound, then to maintain parity, James couldn't steal at all. But when Davis had a rebounding edge, interesting mathematics was happening, as shown below:

Probability of a James victory as a function of James'

How fitting that this game was fair where purple met gold.

Want more riddles?

Well, aren't you lucky? There's a whole book full of the best puzzles from this column and some never-before-seen head-scratchers. It's called "The Riddler," and it's in stores now!

Want to submit a riddle?

Email Zach Wissner-Gross at riddlercolumn@gmail.com

Footnotes

1. Important small print: In order to 👏 win 👏, I need to receive your correct answer before 11:59 p.m. Eastern time on Monday. Have a great weekend!