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# Dynamic Model for a simplified spar floater supporting the DTU 10 MW wind turbine

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## **AUTHORS**

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## Part 1: Model Formulation

In this section, the analytical formulation for a floating wind turbine is performed. The main parts are simplified into spar buoy, a tower, a rotor and a nacelle with rigid connections. The mooring is also abridged into a linear spring which provides the necessary restoring force.

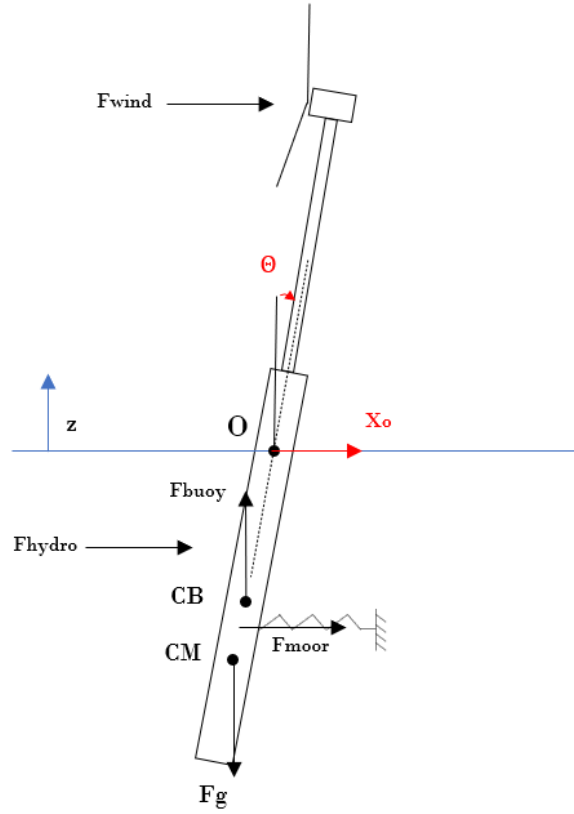


Figure 1: Model formulation diagram.

### Question 1: Calculation of Inertial parameters and center of mass

The total mass  $M_{tot}$  is calculated as the sum of mass of the turbine structure components. It can be expressed as follows:

$$M_{tot} = \sum M_{comp} = M_{floater} + M_{tower} + M_{nacelle} + M_{rotor} \quad (1)$$

The total mass of the floating wind turbine is  $1.2118 \cdot 10^7$  kg.

The center of mass of the system is the superposition of masses and center of masses of its components. This is computed using weighted relative position of the masses of each component given

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as:

$$z_{CM,tot} = \frac{1}{M_{tot}} \sum M_{comp} \cdot z_{CM,comp} \quad (2)$$

$$= \frac{M_{floater} \cdot z_{CM,floater} + M_{tower} \cdot z_{CM,tower} + M_{nacelle} \cdot z_{CM,nacelle} + M_{rotor} \cdot z_{CM,rotor}}{M_{tot}}$$

The center of mass of the system is  $-86.1109$  m, meaning that mass of the structure is concentrated underwater due to heavy spar buoy which helps stabilize the structure.

The moment of inertia around point O (see Figure 1) is calculated using the parallel axis theorem which is the transposition of inertia about the center of mass to another axis parallel to it.

$$I_O = I_{CM,floater} + M_{floater} \cdot z_{CM,floater}^2 + I_{CM,tower} + M_{tower} \cdot z_{CM,tower}^2 + I_{CM,nacelle} + M_{nacelle} \cdot z_{CM,nacelle}^2 + I_{CM,rotor} + M_{rotor} \cdot z_{CM,rotor}^2 \quad (3)$$

The moment of inertia at the waterline is  $1.4566 \cdot 10^{11}$  kg·m<sup>2</sup>.

## Question 2: Local surge displacement

From Figure 2 representing the translation motion of the turbine-sparbuoy system, the local surge displacement,  $x(z)$ , can be represented as a function of  $x_0$  and  $\theta$ . Assuming small angles where  $\sin \theta \approx \theta$ , the kinematics of each  $z$ -position along the turbine/sparbuoy can be transformed into the  $(x_0, \theta)$  model using the transformation:

$$x(z) = x_0 + z\theta \quad (4)$$

$$\dot{x}(z) = \dot{x}_0 + z\dot{\theta} \quad (5)$$

$$\ddot{x}(z) = \ddot{x}_0 + z\ddot{\theta} \quad (6)$$

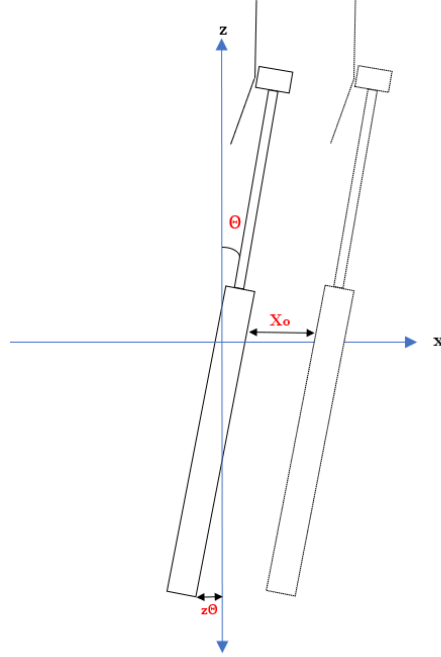


Figure 2: Coordinate transformation diagram.

**Question 3: Express  $F_{moor}$  and  $\tau_{moor}$  as a function of  $(x_0, \theta)$**

The mooring system is considered to be a linear spring system, so the restoring force is proportional to the surge displacement. The surge displacement at the mooring connection height can be found from the transformation given by Equation 4. Since this restoring force acts opposite to the displacement it also is negative.

$$\begin{aligned} F_{moor} &= -K_{moor} \cdot x(z_{moor}) \\ &= -K_{moor} \cdot (z_{moor} \cdot \theta + x_0) \end{aligned} \quad (7)$$

$$\begin{aligned} \tau_{moor} &= F_{moor} \cdot z_{moor} \\ &= -K_{moor} \cdot (z_{moor} \cdot \theta + x_0) \cdot z_{moor} \end{aligned} \quad (8)$$

**Question 4: Restoring moment from gravity and hydrostatic forces**

The restoring moment from gravity and hydrostatic forces,  $\tau_{Buoy}$ , is calculated in the model using the case where the system is freely floating with no mooring and no wind or wave forcing. The restoring moment is therefore derived from the balance of the hydrostatic and gravity forces experienced by the system, as well as the system's second moment of area around the waterplane area. In the full

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restoring matrix for the system, this is represented by term  $C_5$ :

$$C_5 = \tau_5/x_5 = mg(z_b = z_g) + \rho g I_{11}^A \quad (9)$$

Here,  $x_5$  is the term that represents the pitch,  $\theta$ . Rearranging Equation 9 and substituting  $\theta$  for  $x_5$ , the restoring moment can be represented the following, where  $I_{11}^A = \int x^2 dA$ :

$$\tau_{\text{Buoy}} = \theta (mg(z_b - z_g) + \rho g I_{11}^A) \quad (10)$$

A positive restoring moment from the buoy assists the system stability. When designing spar-buoy systems, it is standard practice to design the center of gravity below the center of buoyancy. It is possible for the center of gravity to be above the center of buoyancy and for the system to still be stable if the difference is small enough. However, this would likely lead to increased dynamic response from the system.

#### Question 5: Integrated hydrodynamic forces and moments

The horizontal hydrodynamic force,  $df$ , for a section of spar buoy,  $dz$ , is given by Equation 11. Here, the first term is the inertia load per section due to added mass and the second term is the Froude-Krylov force both of these together makeup the non-viscous forcing experienced by a submerged body under wave loading. The drag force is the third term. This equation was derived from Morison equation [1].

$$d\tilde{F}_{hydro} = \rho_w \left( C_m A (\dot{u}(z) - \ddot{x}(z)) dz + A \dot{u}(z) dz + \frac{1}{2} C_D D (u(z) - \dot{x}(z)) | u(z) - \dot{x}(z) | dz \right) \quad (11)$$

Here,  $A$  is the cross sectional area of the spar-buoy and  $D$  is its diameter in a vertical position. Substituting transformations for surge velocity and accelerations from Equations 5 & 6, the sectional hydrodynamic forcing can be written as follows:

$$d\tilde{F}_{hydro} = \rho_w (C_m A (\dot{u}(z) - (\ddot{x}_0 + z\ddot{\theta})) dz + A \dot{u}(z) dz + \frac{1}{2} C_D D (u(z) - (\dot{x}_0 + z\dot{\theta})) | u(z) - (\dot{x}_0 + z\dot{\theta}) | dz) \quad (12)$$

The equation is simplified by combining two like terms. Also, in the linear model used in this report, the  $C_m A (\ddot{x}_0 + z\ddot{\theta})$  term is moved to the left hand side and is accounted for in the added mass matrix. However, that is not shown in the analytical representation here.

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$$d\tilde{F}_{hydro} = \rho_w((C_m + 1)A\dot{u}(z) - C_m A(\ddot{x}_0 + z\ddot{\theta}) + \frac{1}{2}C_D D(u(z) - \dot{x}_0 - z\dot{\theta}) | u(z) - (\dot{x}_0 + z\dot{\theta}) dz \quad (13)$$

Now, the above equation can be integrated from the limits  $z_{bot}$ , the bottom point of spar-buoy to  $z = 0$ , the mean sea level. Thus, the total hydrodynamic forcing on the submerged structure is given as:

$$\tilde{F}_{hydro} = \int_{z_{bot}}^0 d\tilde{F}_{hydro} dz \quad (14)$$

By multiplying Equation 11 by the sectional coordinate  $z$  we can estimate the hydrodynamic moment to be:

$$\tilde{\tau}_{hydro} = \int_{z_{bot}}^0 d\tilde{F}_{hydro} \cdot z dz \quad (15)$$

### Question 6: Simplified, linear model for surge and pitch

The surge and pitch of the system can be modeled in a simplified linear system (Equation 16) where the mooring forces are linearized and considered in the restoring matrix,  $C$ . Additionally, the linearized force and moment equations for wind and hydrodynamic forcing are used as well.

$$\left( \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \right) \begin{bmatrix} \ddot{x}_0 \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} x_0 \\ \theta \end{bmatrix} = \begin{bmatrix} F_{Hydrodynamic} + F_{Wind} \\ \tau_{Hydrodynamic} + \tau_{Wind} \end{bmatrix} \quad (16)$$

Each of the terms in Equation 16 can be calculated for motion mode: surge and pitch. These are summarized below.

Mass Matrix:

$$\begin{aligned} M_{11} &= M_{Tot} = 1.2118 \cdot 10^7 \text{ kg} \\ M_{12} &= M_{21} = M_{Tot} \cdot z_g = -1.0435 \cdot 10^9 \text{ kg}\cdot\text{m} \\ M_{22} &= I_{33}^b + I_{11}^b = I_O = 1.4566 \cdot 10^{11} \text{ kg}\cdot\text{m}^2 \end{aligned}$$

Added Mass Matrix:

$$A_{11} = \int_{z_{bot}}^0 \rho \frac{\pi}{4} D^2 C_m dz = -\frac{\pi}{4} \rho D^2 C_m z_{bot} = 1.2118 \cdot 10^7 \text{ kg}$$

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$$A_{22} = \int_{z_{\text{bot}}}^0 \rho \frac{\pi}{4} D^2 C_m z^2 dz = -\frac{\pi}{12} \rho D^2 C_m z_{\text{bot}}^3 = 5.8166 \cdot 10^{10} \text{ kg}\cdot\text{m}^2$$

$$A_{12} = A_{21} = \int_{z_{\text{bot}}}^0 \rho \frac{\pi}{4} D^2 C_m z dz = -\frac{\pi}{8} \rho D^2 C_m z_{\text{bot}}^2 = -7.2708 \cdot 10^8 \text{ kg}\cdot\text{m}$$

Restoring Matrix:

$$C_{11} = -F_{\text{moor}}/x_O = K_{\text{moor}} = 6.6700 \cdot 10^4 \text{ N/m}$$

$$C_{12} = -\tau_{\text{moor}}/x_O = K_{\text{moor}} \cdot z_{\text{moor}} = -4.0020 \cdot 10^6 \text{ N}$$

$$C_{21} = -F_{\text{moor}}/\theta = K_{\text{moor}} \cdot z_{\text{moor}} = -4.0020 \cdot 10^6 \text{ N/rad}$$

$$C_{22} = C_5 - \tau_{\text{moor}}/\theta = (\rho g I_{11}^A + m g (z_b - z_g)) + K_{\text{moor}} \cdot z_{\text{moor}}^2 = 3.3519 \cdot 10^9 \text{ N}\cdot\text{m/rad}$$

### Question 7: Natural Frequencies for surge and pitch

Since we are computing the natural frequency of the structure the damping matrix is zero and the external forcing matrix is also zero. Thus the equation of motion can be represented as:

$$(\mathbf{M} + \mathbf{A})\ddot{x} + \mathbf{C}x = 0 \quad (17)$$

Now considering a sinusoidal nature for displacement:

$$x(t) = \phi e^{j\omega t}$$

Substituting, the equation of motion becomes:

$$(-\omega^2(\mathbf{M} + \mathbf{A}) + \mathbf{C}) \phi e^{j\omega t} = 0$$

Therefore, the equation can be further simplified into a quadratic expression whose solution is given by eigenvalue analysis.

$$(-\omega^2(\mathbf{M} + \mathbf{A}) + \mathbf{C}) = 0 \quad (18)$$

The two non trivial solutions are  $\omega_1^2$  &  $\omega_2^2$  which are differentiated by the  $\phi_1$  &  $\phi_2$  eigenvectors. Hence, the natural frequency for surge and pitch can be found by:

$$f_n = \frac{\sqrt{\omega_n^2}}{2\pi}$$

Thus, the natural frequency of surge is estimated to be  $f_1 = 0.0083 \text{ Hz}$  and for the pitch  $f_2 = 0.0326 \text{ Hz}$ .

## Part 2: Dynamic Analysis

In this section, the linear model is implemented for dynamic response to wind and wave forcing. The total time simulation studied for all responses 1600s and the first 1000 seconds are considered to be transient and next 600s is stable.

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### Question 8 : First order system with a state vector

The linear model for dynamic response given in Equation 16 can be re-written as a first-order system by using the state vector:

$$\underline{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} x_0 \\ \theta \\ \dot{x}_0 \\ \dot{\theta} \end{bmatrix} \quad (19)$$

First, the state variables can be substituted into Equation 16:

$$\left( \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \right) \begin{bmatrix} \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} F_{\text{Hydrodynamic}} + F_{\text{Wind}} \\ \tau_{\text{Hydrodynamic}} + \tau_{\text{Wind}} \end{bmatrix} \quad (20)$$

Then, rearranging and solving for  $\dot{q}_3$  and  $\dot{q}_4$ :

$$\begin{bmatrix} \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \left( \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \right)^{-1} \left( \begin{bmatrix} F_{\text{Hydrodynamic}} + F_{\text{Wind}} \\ \tau_{\text{Hydrodynamic}} + \tau_{\text{Wind}} \end{bmatrix} - \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \right) \quad (21)$$

This means that for every known pitch and surge, the second time derivative of pitch and surge can be determined by the model.

### Question 9: Introducing linear damping

A linear damping term,  $B_{11}\dot{x}_0$  can be added to the model for surge where  $B_{11} = 2 \cdot 10^5 \text{ N/(m/s)}$ . Temporarily neglecting the hydrodynamic and wind forcing, this adjusts the model to become:

$$\begin{bmatrix} \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \left( \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \right)^{-1} \left( - \begin{bmatrix} B_{11} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} q_3 \\ q_4 \end{bmatrix} - \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \right) \quad (22)$$

### Question 10: Decay tests for surge and pitch with no hydrodynamic or wind forcing

(a) **Initial surge of 1 m:**  $\underline{q} = [1 \ 0 \ 0 \ 0]^T$

The first decay test introduces a unit surge displacement as an initial condition. The system is then released and the time-series response as well as the frequency response is analyzed in Figures 3 and 4. Only one response frequency is seen for surge. The same frequency is the primary response for pitch with a secondary response at a higher frequency. The two response frequencies are very similar to the estimated frequencies from the eigenvalue analysis. The decreasing amplitude of the time response of the structure shows the effect of the damping term on the model.



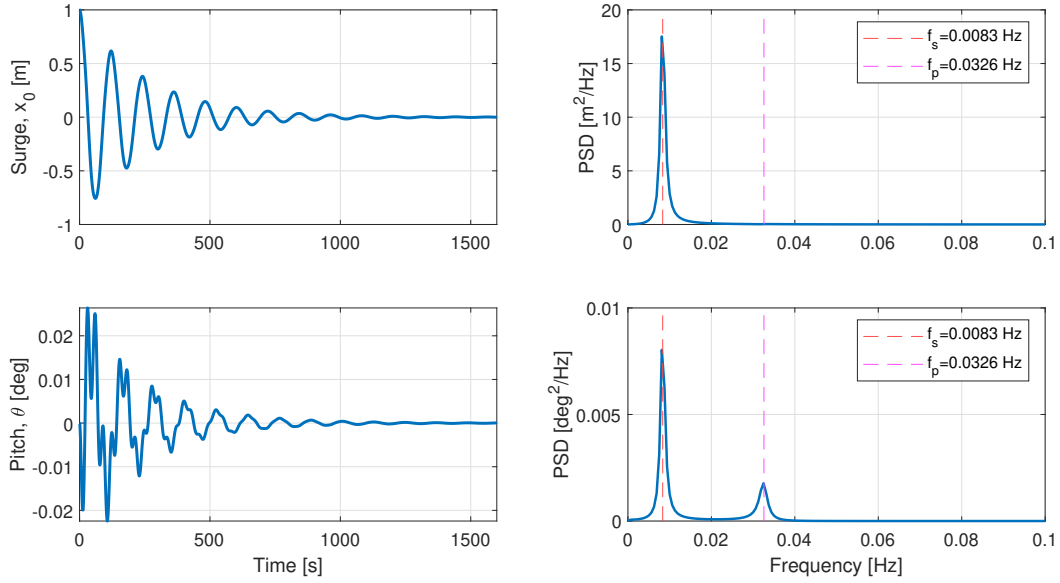


Figure 3: Plots showing the time-series and frequency response to an initial surge of 1 m. The left-hand plots show the time series decay responses for the turbine-floater-spar-buoy system surge (top) and pitch (bottom). The right-hand side plots show the frequency of response.

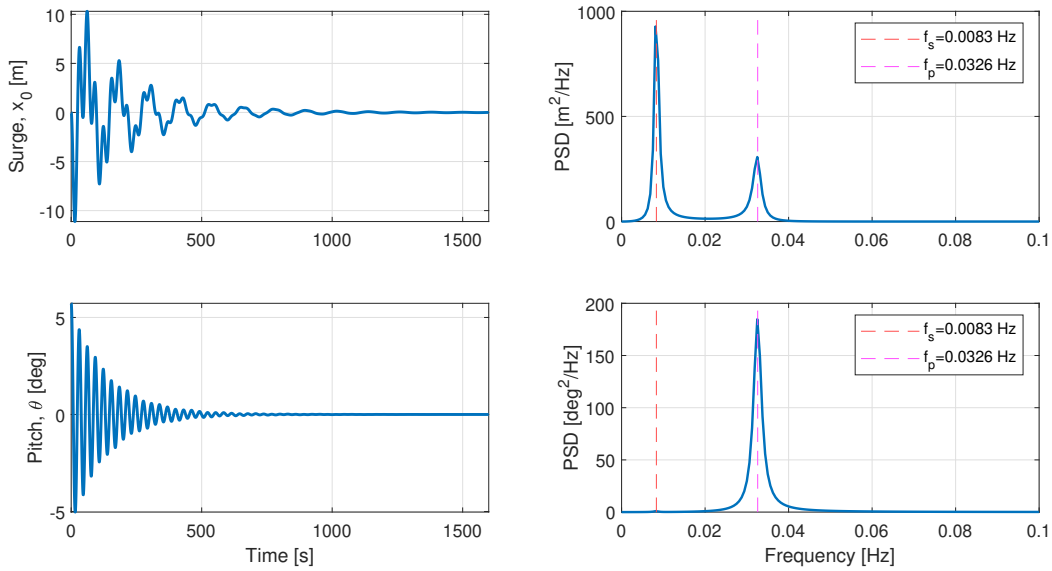


Figure 4: Plots showing the time-series and frequency response to an initial pitch of 0.1 radians. The left-hand plots show the time series decay responses for the turbine-floater-spar-buoy system surge (top) and pitch (bottom). The right-hand side plots show the frequency of response.

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(b) Initial pitch of 0.1 radians:  $\underline{q} = [0 \ 0.1 \ 0 \ 0]^T$

When the initial condition is a 0.1-radian-pitch displacement, the system's dynamic response mirrors the surge response. However, in this case, the surge response has a single response frequency and the pitch response has two frequencies: a primary frequency response at the same frequency as the surge displacement case, and a secondary frequency response that matches the frequency of the pitch. Comparing Figures 3 and 4, the lower of the two observed frequencies is the surge frequency and the higher of the two frequencies is the pitch frequency. Given that each motion mode experiences both its frequency and the frequency of the other motion mode when the other mode is perturbed, it can be concluded the pitch and surge motions are coupled.

### Question 11: Decay tests for surge and pitch with hydrodynamic forcing

By including the hydrodynamic forcing expressed in Equation 11 we can re-simulate the decay tests done in question 10 and compare the results. These hydrodynamic forcing terms add onto the damping matrix as they are essentially hindering the motion of the structure under water by dissipating energy.

(a) Comparing initial surge of 1 m

By giving an initial surge displacement of 1 m and releasing the motion of the floater is analysed. Simulating with this initial condition the equation of motion is solved by *ode4.m*.

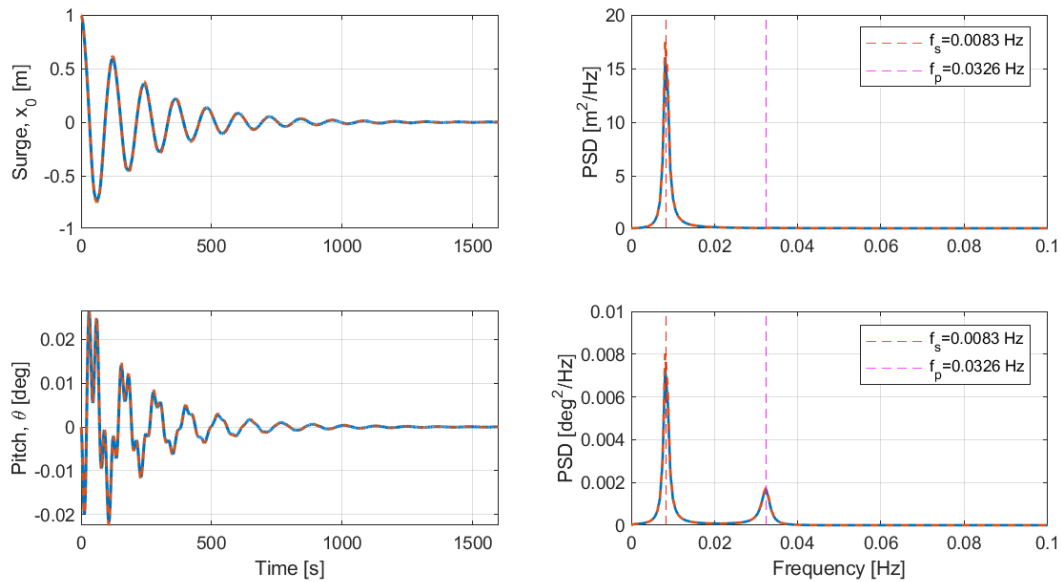


Figure 5: Surge condition with(blue) and without hydrodynamic forcing(dotted orange).

The response obtained with hydrodynamic damping is plotted over the result from question 10. It can be observed that both conditions respond similarly for an initial surge displacement. We see the

same coupled response in pitch when surge is excited. Meaning that the coupling effect comes from the diagonal term of the damping matrix as expected. The similarity in the two cases suggests that the introduction of hydrodynamic forcing as a damping effect is not that significant when compared to the restoring forces by mooring systems. There is only a small change in the PSD peak for surge  $<10\%$  from  $17.525\text{ m}^2/\text{Hz}$  to  $16.06\text{ m}^2/\text{Hz}$ .

### (b) Comparing initial pitch of 0.1 radians

By giving an initial pitch displacement of 0.1 rad and releasing the motion of the floater is analysed. Simulating with this initial condition the equation of motion is solved by *ode4.m*.

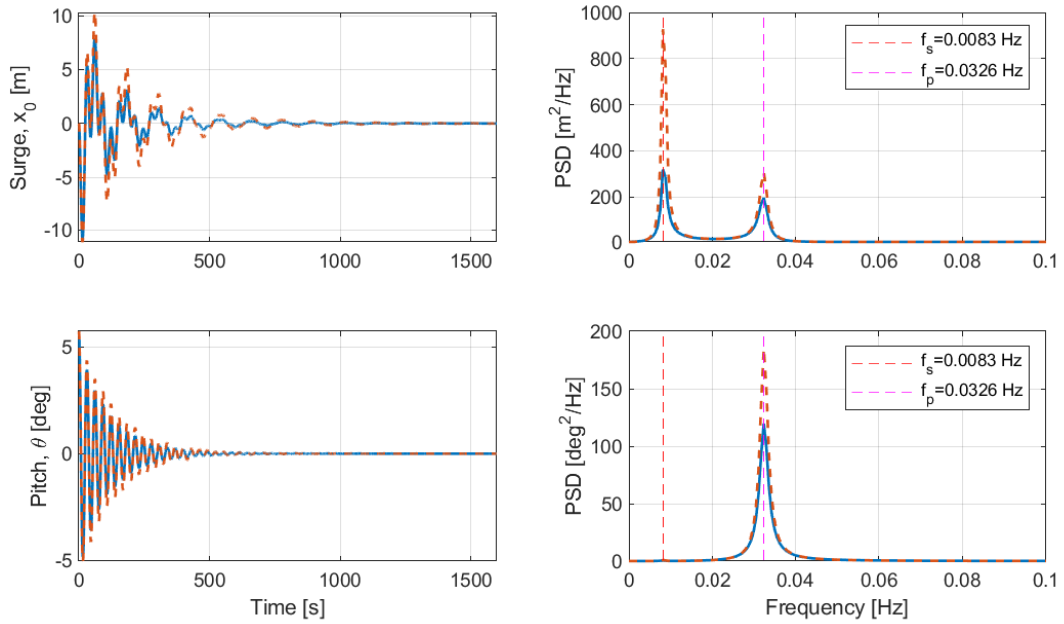


Figure 6: Pitch condition with(blue) and without hydrodynamic forcing(dotted orange).

Now by instigating a pitch of approximately 5 degrees and releasing the motion of the structure is simulated and plotted over the results without forcing from earlier question. The responses are similar again but the magnitudes are reduced for the case including hydrodynamic forcing. An all-around 30 % reduction is noticed. Thus, the inference is that the hydrodynamic forcing is able to provide additional damping effect when the structure submerged is in a pitch-dominant motion. This is not the case for surge initial condition, and likely part of the small difference seen is due to the motion coupling.

### (c) Instigating an initial pitch of 1 radian

A pitch decay test is conducted now for an initial condition which gives a large pitch of 1 radian. Here, the hydrodynamic forcing is disabled once again to purely study the damping effect of mooring

system and compared with it enabled.

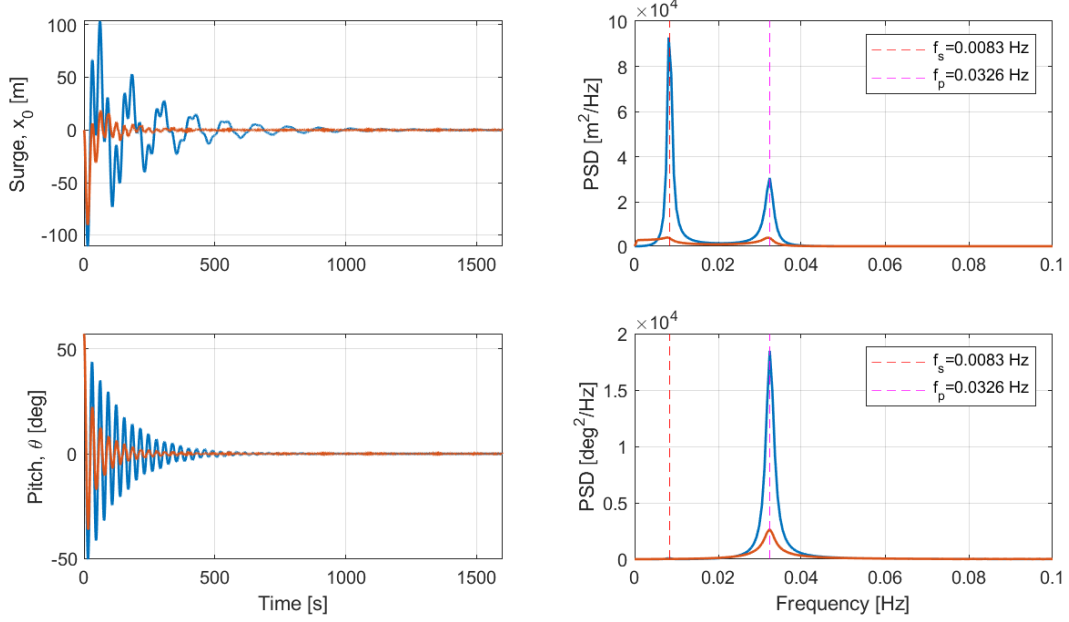


Figure 7: Pitch condition with(orange) and without hydrodynamic forcing(blue).

It is evident now that since the pitch condition is 1 radian a purely hydro static analysis of the response yields in highly unrealistic outputs of surge in excess of 50m. Since our model assumed small angles for  $\theta$  the surge displacement given by Eq.4 no longer holds valid for a 50+ degree initial pitch. This would require a more extensive modelling for surge displacement one without such a small angle assumption. Including the hydro forcing it is observed that the damping effect due to it is still realistic in terms of pitch response. But the surge response is still off as it reports a surge of almost 80m.

#### Question 12: Forcing tests for surge and pitch with only regular wave forcing

In this case, the dynamic response from regular wave forcing is considered. The regular waves have the properties  $H = 6$  m and  $T = 10$  s. For each of the calculations, the wave kinematics are computed assuming an upright buoy position. In Figure 8, the time-response shows that in the steady-state condition, there is a constant oscillation of both surge and pitch and the damping is only able to reduce the amplitude of the oscillations to 1.5 m for surge and approximately 0.65 degrees for pitch. These are very small oscillations, however. Additionally, the primary frequency response is 0.1 Hz, which is the forced response frequency from the waves. There is no observable response in either the surge or pitch's natural frequencies.

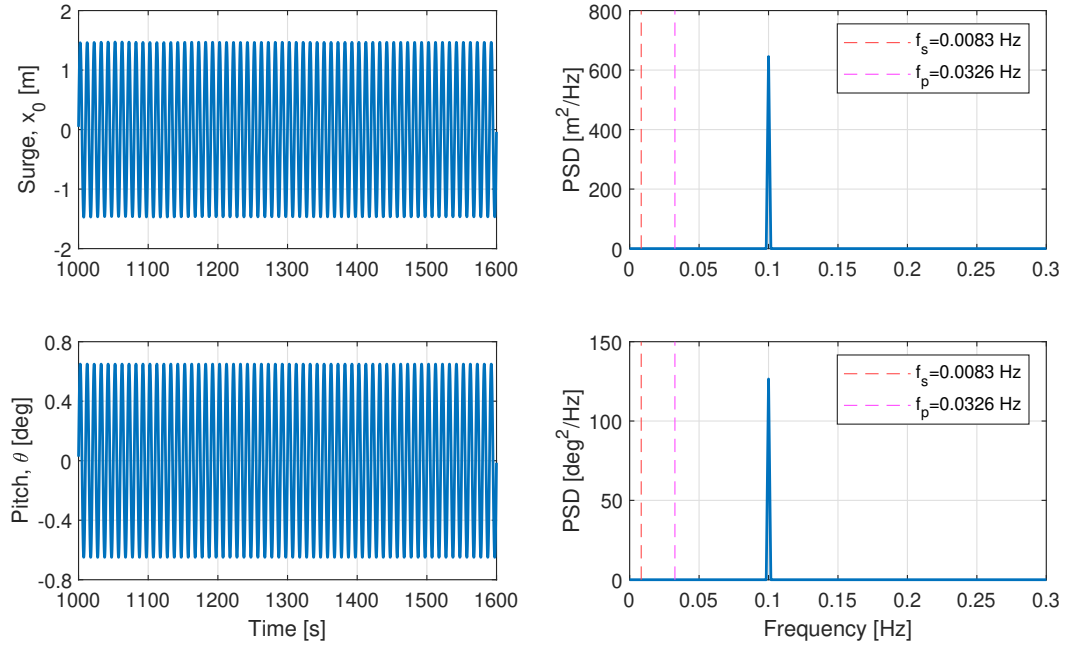


Figure 8: With regular wave forcing, the structure experiences regular oscillations of both surge and pitch. The frequency of both responses is 0.1 Hz, identical to the forcing frequency.

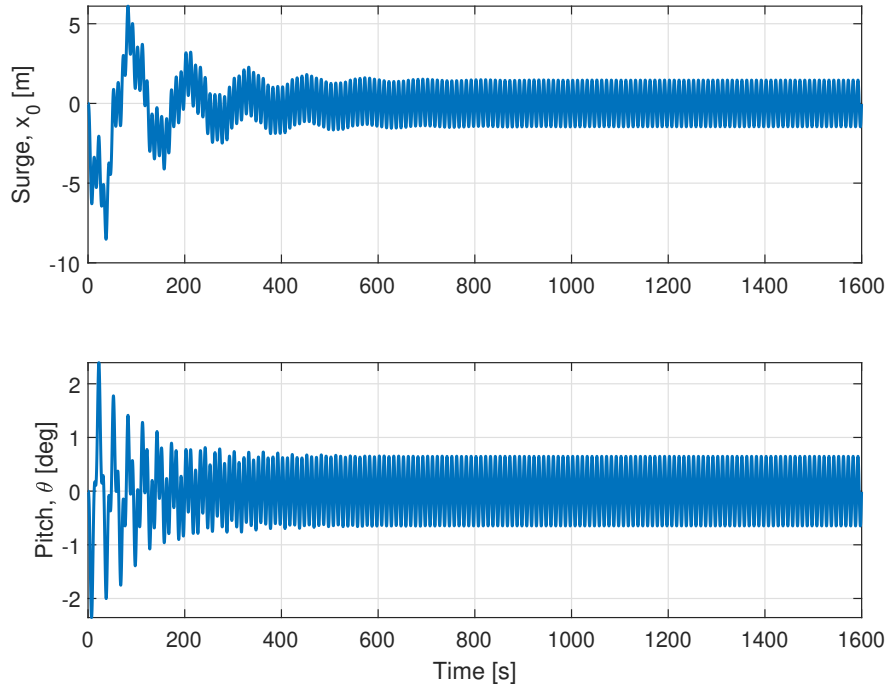


Figure 9: Full time series response for regular wave forcing.

Statistic	Surge [m]	Pitch [deg]
Mean	0.1042e-4	-0.0275e-4
Standard Deviation	1.0372	0.4592
Maximum	1.4714	0.6494
Minimum	-1.4723	-0.6495

Table 1: Summary of statistics for Surge and Pitch with only regular wave forcing.

The statistics presented above it can be seen that pitch and surge have zero mean value with almost the same max and min values indicating steady response. This can be interpreted as the loading due to regular waves is minimal and hence the oscillations reflect the same.

### Question 13: Forcing tests for surge and pitch with regular wave and steady wind forcing

In addition to regular waves a steady wind of  $8\text{ m/s}$  is introduced as wind forcing. The response plots and PSD plots are shown below. with the exclusion of first 1000s as transient time from both the frequency domain and the time series. The statistics for steady state time domain also also tabulated below.

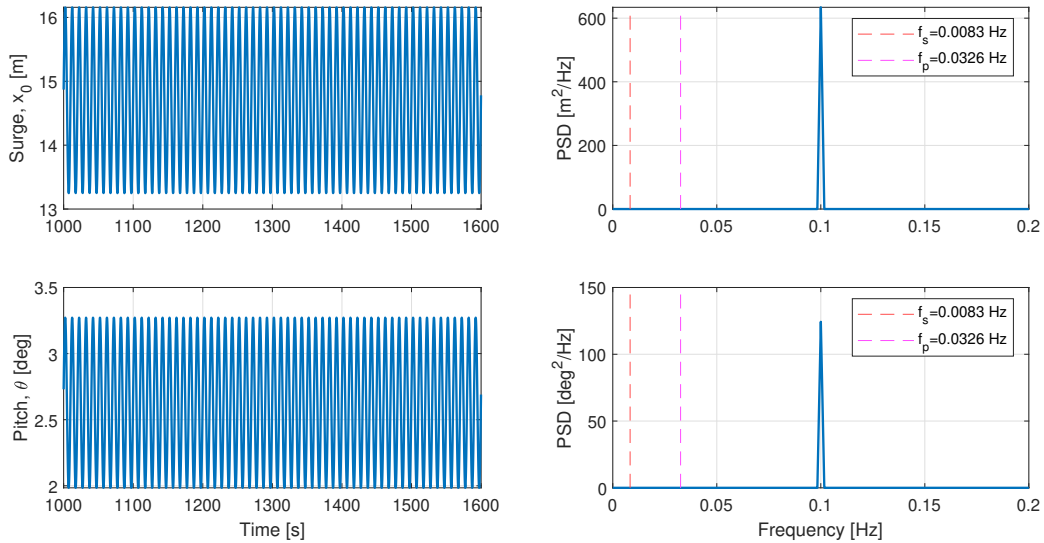


Figure 10: Time series and PSD for regular waves with steady wind of  $8\text{ m/s}$ .

With the mean steady wind of  $8\text{ m/s}$  comes the rotor thrust forcing. This forcing calculation is done using the global thrust coefficient  $C_T$ . First a mean thrust force is calculated and then a time varying thrust time series is computed and the resultant time series for forcing is determined via an adhoc reduction of dynamic part to compensate for turbulence as directed in the report hints [2].

Here, it is observed that the although the standard deviation is same as that of question 12 there

is noticeable increase to mean, maxima and minima for surge and pitch both. This is attributed to the fact that the mean loads for wind are comparatively more significant and hence influence the response of the structure more than the wave loading alone and also that the wind forcing unlike wave is constant and not sinusoidal in nature blowing in the same direction. As for the PSD plot the excitation frequency is at 0.1 Hz as this is still the forcing frequency due to wave conditions.

Statistic	Surge [m]	Pitch [deg]
Mean	14.7020	2.6272
Standard Deviation	1.0282	0.4552
Maximum	16.1587	3.2718
Minimum	13.2487	1.9846

Table 2: Summary of statistics for Surge and Pitch with regular wave and steady wind forcing.

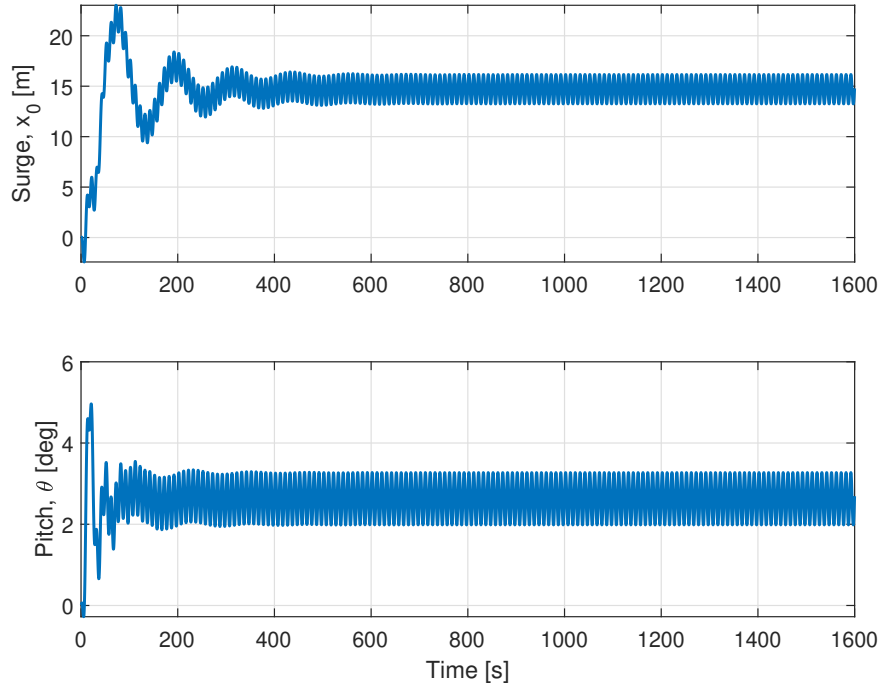


Figure 11: Full time series response for regular wave forcing with steady wind of 8 m/s.

When comparing the above figure with Figure 9 it can be observed that the pitch transient signal is much more different in that the steady state is reached much faster when a constant wind forcing is introduced this is due to the aerodynamic damping that helps restore stable response more quickly.

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**Question 14: Forcing tests for surge and pitch with irregular wave and steady wind forcing**

This analysis considers the structure's dynamic response to forcing from irregular waves and steady wind. A JONSWAP spectrum is used to simulate a time-series case of irregular wave kinematics. The parameters used for the JONSWAP spectrum are:  $H_S = 6$  m,  $T_P = 10$  s, and  $\gamma = 3.3$ . The same 8 m/s steady wind speed is used.

The steady-state time-series and frequency response is presented in Figure 12. The average surge is approximately 14.5 m and the average pitch is approximately 2.5 degrees. From Figure 8, it can be seen that the response to waves is oscillatory and centered around 0. Additionally, Figure 10 shows the effect of steady wind forcing on surge and pitch. Therefore, the values of the average surge and pitch are due to wind forcing, and the oscillations are from the irregular wave forcing. The frequency response shows a peak frequency of 0.1, which is consistent with the regular wave forcing due to the same  $H$  and  $T$  parameters being used. The relative magnitude of the 0.1 Hz frequency peak is higher for the pitch response than the surge response. This shows the effect of the wind damping on the surge of the system.

However, frequencies are also consistently observed from 0.05 Hz to 0.15 Hz for both pitch and surge. This is due to the irregular waves within the sea state having similar but not regular frequencies. There is a small frequency response at the surge frequency, but this is likely due to surge-pitch coupling.

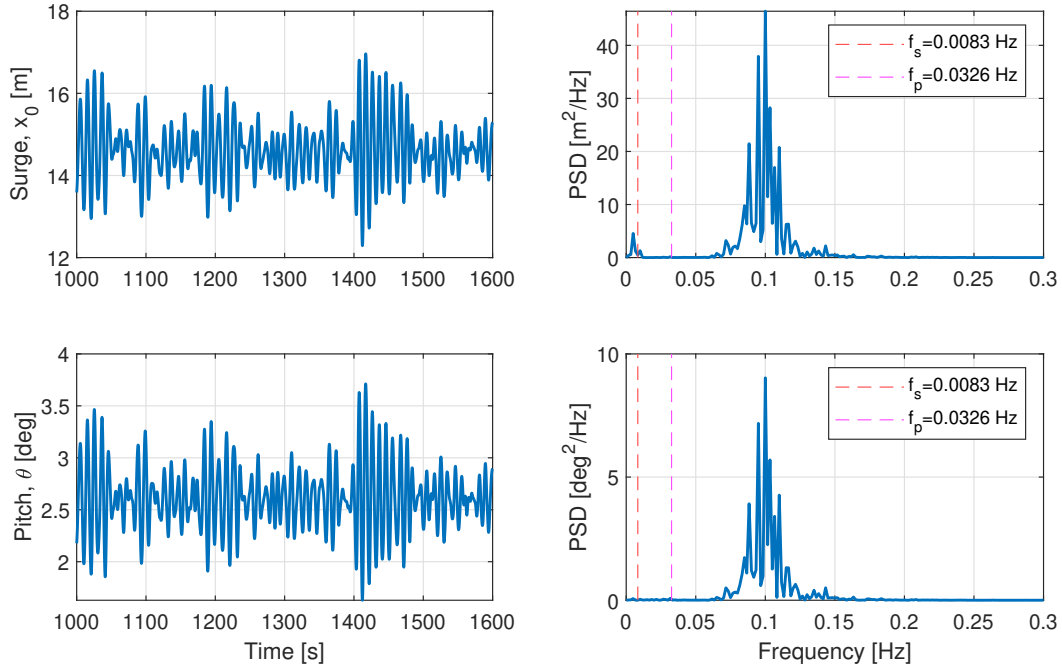


Figure 12: Steady-state time-series and frequency response of surge and pitch with irregular wave and steady wind forcing.



Statistic	Surge [m]	Pitch [deg]
Mean	14.5715	2.6044
Standard Deviation	0.6006	0.2595
Maximum	16.4920	3.4431
Minimum	12.9976	1.8799

Table 3: Summary of statistics for Surge and Pitch with irregular wave and steady wind forcing.

### Question 15: Forcing tests for surge and pitch with irregular wave and unsteady wind forcing

In this question wind forcing is altered from being steady to unsteady with the introduction of Kaimal wind time-series based on  $V_{10min} = 8$  m/s ,  $I = 0.14$  and  $l = 340.2$  m. The time and frequency response are shown in below figure excluding the transient signal.

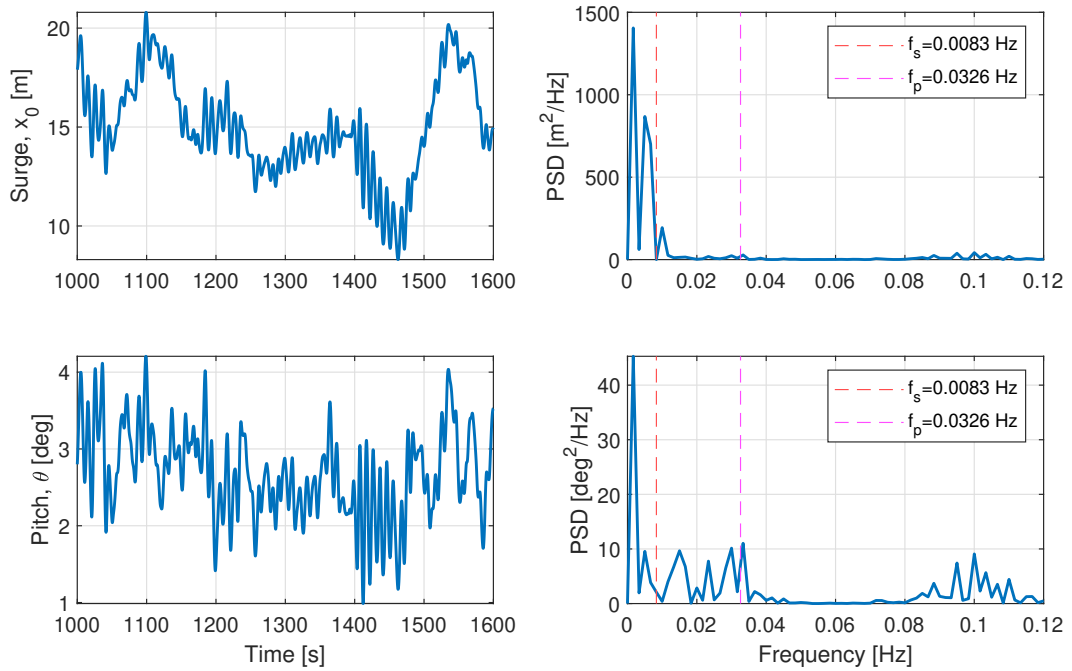


Figure 13: Dynamic response to irregular wave and unsteady wind forcing. The surge response is primarily driven by wind forcing where the pitch response is driven by wind forcing, surge coupling, and some wave forcing.

It can be observed from the above figure in conjunction with the statistics for forcing with irregular wave and unsteady wind that the oscillations for both times series have further increased owing to the turbulent nature of the wind forcing thus reflecting a significant change in standard deviation, maxima and minima when compared to the 3 previous forcing tests from question 12 through 14. The PSD plots also reflect a similar behaviour in that the surge peaks now observed at the very low end of the frequencies due to the Kaimal spectrum of wind(0.0016 Hz) which also falls in this zone

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and natural frequency of the surge is also instigated. For the pitch PSD it is noticeable that it is piqued by both wind and wave forcing once at the Kaimal spectrum region(0.0016 Hz) and again at the natural frequency of pitch as well. There is also some small peaks at the 0.1 Hz due to wave forcing frequency ( $1/T_P$ ).

Statistic	Surge [m]	Pitch [deg]
Mean	14.9844	2.6791
Standard Deviation	2.5013	0.5767
Maximum	20.7973	4.2064
Minimum	8.3008	0.9868

Table 4: Summary of statistics for Surge and Pitch with irregular wave and unsteady wind forcing.

### Part 3: Blade Pitch Control for Dynamic Stability

#### Question 16: Instantaneous controller for steady wind and no wave forcing

In this analysis, there are two load cases. The first is a steady wind of 10 m/s and the second is a steady wind of 16 m/s. The first load case is less than the turbine's rated wind speed while the second is greater than the rated wind speed. There is no wave forcing. The steady-state response for each case is shown in Figures 14 and 15.

Figure 14 shows the response for the steady  $V_{10 \text{ min}} = 10$  m/s case. While there is a response oscillation for surge and pitch, the amplitudes are very small because for below rated wind speeds, the thrust increases with the wind speed. Therefore, when the wind speed increases, the thrust increases and the turbine hub moves backwards, lowering the relative velocity and thrust. The turbine then moves forward due to the decrease in thrust. This is a stable dynamic scenario and is depicted in the figure. The frequency response shows that the response is primarily in surge, since the lower frequency has a much higher occurrence.

By contrast, Figure 15 shows a dynamically unstable response for the steady  $V_{10 \text{ min}} = 16$  m/s case. The unstable response is in the pitch mode, because that is the dominating frequency response. The turbine is oscillating between -0.5 degrees and just under 6 degrees of pitch and is surging in response to the pitch changes. This rocking motion is due to the turbine's control scheme, which decreases the thrust with increasing wind speed when the wind speed is greater than the rated wind speed. Like in Figure 15, the turbine ends up rocking back and forth, because its motion and changing thrust are opposite signs of each other.

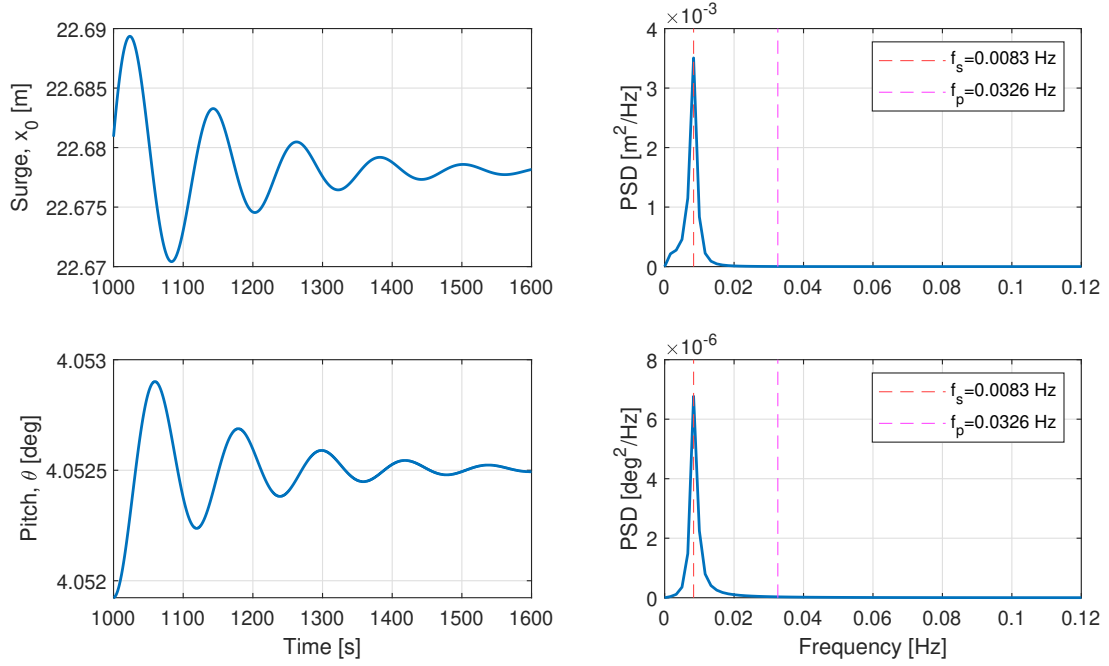


Figure 14: Steady-state response to steady wind forcing using an instantaneous controller for  $V_{10 \min} = 10$  m/s, less than rated wind speed.

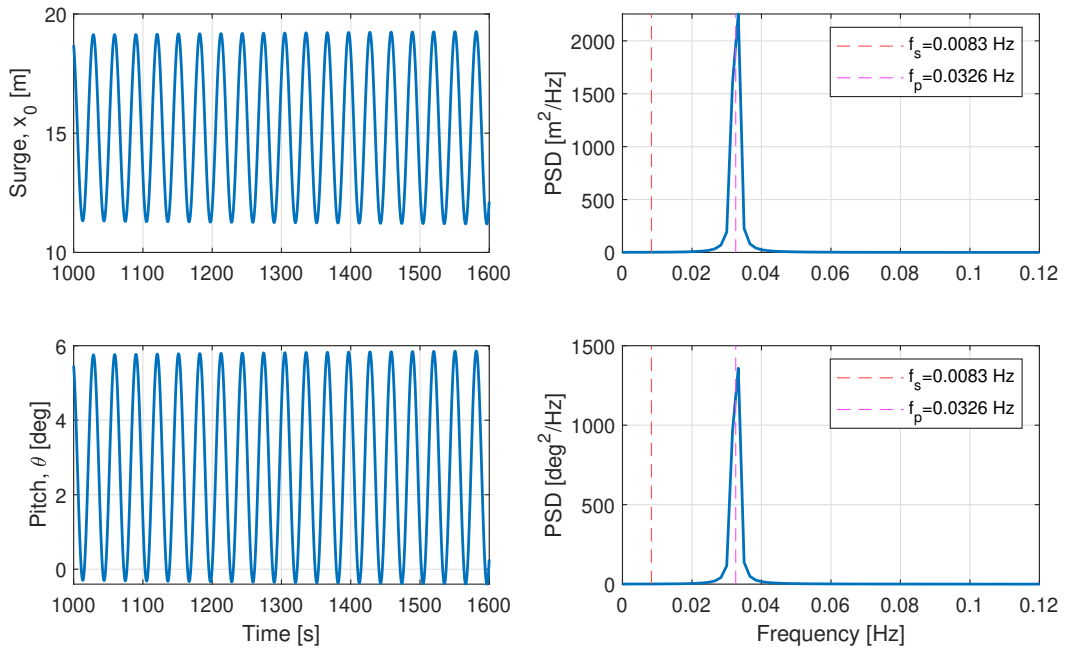


Figure 15: Steady-state response to steady wind forcing using an instantaneous controller for  $V_{10 \min} = 16$  m/s, greater than rated wind speed.

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### Question 17: Adapted floating controller for steady wind and no wave forcing

The previous case with a steady wind speed greater than the rated wind speed demonstrates the need for a time-delayed controller to prevent a dynamically unstable motion response. This can be achieved by changing the control scheme for the turbine above rated wind speeds. A simplified version of a common control model is presented in Equation 23.

$$\frac{dC_T}{dt} = -\gamma (C_T - C_T(V_{\text{rel}})) \quad (23)$$

In the equation,  $C_T$  is the "current"  $C_T$  where  $C_T(V_{\text{rel}})$  is what the  $C_T$  would be for an instantaneous  $C_T$  control scheme for the "current" relative velocity. The difference between these two terms is scaled by a constant factor,  $\gamma$ , to inform the update term for the next  $C_T$ ,  $\frac{dC_T}{dt}$ . In practice, the value for  $\gamma$  is dependent upon the magnitude of the time step,  $\Delta t$ .  $\gamma$  generally controls the speed of the controller feedback, and can be adjusted to dampen the oscillatory motion from the instability deriving from instantaneous changes in the  $C_T$ . However, as can be seen from the results of the control scheme in Figure 16,  $\gamma = 2$  is the wrong choice for a time step of 0.05 seconds. The response is still too fast, and the response of the turbine is very similar to the case in Figure 15 with the same steady wind speed and instantaneous control response.

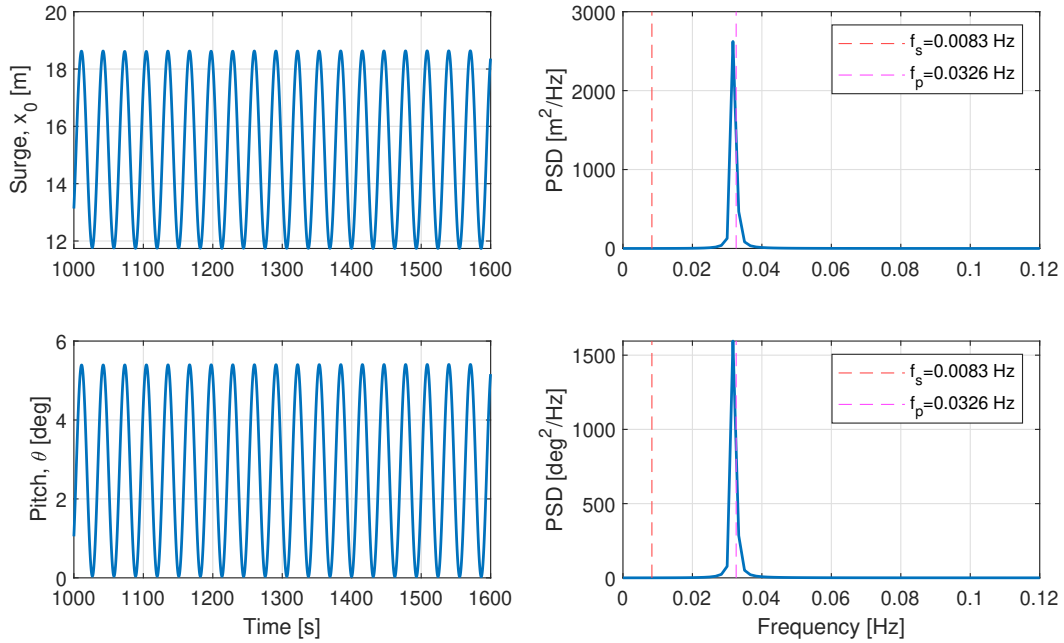


Figure 16: Dynamic response to  $V_{10 \text{ min}} = 16 \text{ m/s}$  with adapted thrust controller from Equation 23.

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### Question 18: Optimizing the adapted floating controller

Three values of  $\gamma$  are presented as a case study to compare control strategies. In order, these values are **0.1**, **0.5**, and **1.5**. The lower the gamma value, the lower the rate of change of  $C_T$  in time. As expected, the lowest value of  $\gamma$  produces the most "stable" response of the three (Figure 17). It appears that all three approach stability, but the case where  $\gamma = 1.5$  still shows a 2 meter surge amplitude and 2 degree pitch amplitude, so it is probably not time-delayed enough. However, the case where  $\gamma = 0.1$  is not necessarily the best case either. The original reason for the control scheme is to reduce the blade loading to extend the turbine's lifetime. Having a value of  $\gamma$  that is too low could reduce a turbine's ability to effectively reduce the loads experienced by the blades in time. Therefore, a value of  $\gamma$  which balances these two needs, possibly  $\gamma = 0.5$ , is the best case scenario. It allows the turbine to reduce the loads in high wind speeds effectively enough while also providing a slow enough response to dampen response oscillations.

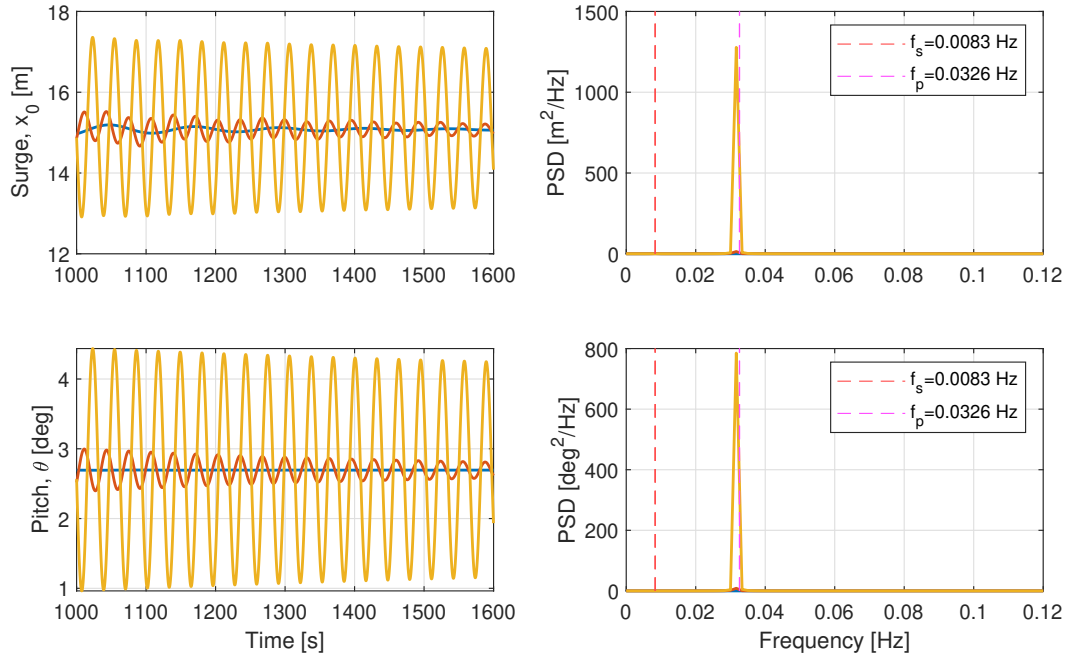


Figure 17: Steady-state response for the adapted pitch control where the wind is steady  $V_{10 \min} = 16$  m/s.  $\gamma = 0.1, 0.5, 1.5$  respectively for the blue, orange, and yellow traces.

---

## References

- [1] “Froude–krylov force - wikipedia.”
- [2] *46211 Offshore Wind Energy - Fall 2021 Assignment 5: Dynamic model for a simplified spar floater supporting the DTU 10MW wind turbine.* Technical University of Denmark.

## MATLAB Code

### Main Code

```
1 % Assignment 5 Josh & Varun
2 clc; clear all
3 % Loading the given constants for the floater
4 global rho_w Cm CD D_spar z_bot g z_hub
5 load('model_constants.mat');
6 %% General Constants
7 global t Hs Tp rho_air A_r V_rated CT_0 aCT bCT gammaCT
8 for ig=1:1
9     fHighCut = 0.5;           % cut-off frequency
10    Tdur = 1600;               % total duration: 1000s transient + 600s
11                                response
12    dtode = 0.1;               % time-step for ode4 solver
13    dt = 0.05;                 % general time step
14    tode4 = [0:dtode:Tdur-dtode];
15    t = [0:dt:Tdur-dt];
16    z_buoy = z_bot/2;          % center of buoyancy force
17    Hs = 6;                    % linear wave amplitude and significant
18                                wave height [m]
19    Tp = 10;                   % linear wave period and significant wave
20                                period [s]
21    rho_air = 1.22;            % air density [kg/m^3]
22    A_r = 24885;               % 10 MW rotor area [m^2]
23    V_rated = 11.4;            % 10 MW rated wind speed [m/s]
24    CT_0 = 0.81;               % 10 MW nominal thrust coefficient
25    aCT = 0.5;                 % 10 MW "a" thrust parameter
26    bCT = 0.65;                % 10 MW "b" thrust parameter
27    gammaCT = 2;               % initial controller value
28    gammaJS = 3.3;             % JONSWAP peak enhancement factor
29    df = 1/Tdur;               % JONSWAP frequency spectra time step
30    TI = 0.14;                 % Kaimal wind spectra turbulence intensity
31    TL = 340.2;                % Kaimal wind spectra turbulence length
32                                scale [m]
33 end
34 %% Part 1, Model Formulation
35 % Question 1 : Mtot, zCMtot and IOtot calculation
36 for i1=1:1
```

---

```

33     M_tot = M_floater+ M_tower+M_nacelle+M_rotor;
34     zCM_tot=(M_floater*zCM_floater + M_tower*zCM_tower + (M_nacelle+
        M_rotor)*z_hub)/ M_tot;
35     IO_tot = (ICM_floater+ M_floater*zCM_floater^2)+(ICM_tower+M_tower*
        zCM_tower^2)+(M_nacelle+M_rotor)*z_hub^2;
36 end
37 % Question 2,3,4,5 : written in the report
38 % Question 6 : Building ODE system
39 for i6=1:1
40     I11A = pi/64*D_spar^4;
                                                    % second moment
        of area at the waterplane
41     M = [M_tot, M_tot*zCM_tot;
42          M_tot*zCM_tot, IO_tot];
                                                    % mass matrix
43     A = [-pi/4*rhow*D_spar^2*Cm*z_bot, -pi/8*rhow*D_spar^2*Cm*z_bot^2;
44          -pi/8*rhow*D_spar^2*Cm*z_bot^2, -pi/12*rhow*D_spar^2*Cm*z_bot
        ^3]; % added mass matrix
45     C = [K_moor, K_moor*z_moor;
46          K_moor*z_moor, rhow*g*I11A+M_tot*g*(z_buoy-zCM_tot)+K_moor*
        z_moor^2];
47 end
48 % Question 7: Estimating natural frequencies of surge and pitch
49 global w1 w5
50 for i7=1:1
51     [~,w15]=eig((M+A)^-1*C);
52     w1=w15(1,1)^0.5/(2*pi);
                                                    % Hz
53     w5=w15(2,2)^0.5/(2*pi);
                                                    % Hz
54 end
55 %% Part 2, Dynamic Analysis
56 % Question 8: written in the report
57 % Question 9: preparing for ode4 solver
58 for i9=1:1
59     B = [2*10^5, 0; 0, 0];
        matrix
                                                    % Damping
60 end
61 % Question 10: implementing ode4/dqdt algorithm
62 for i10=1:1
63     q01 = [1;0;0;0;0];
        condition
                                                    % first initial
64     q02 = [0;0.1;0;0;0];
        initial condition
                                                    % second
65     Y10_1 = ode4(@dqdtsparbuoy,tode4,q01,M+A,B,C,1);
        response 1
                                                    % unforced
66     Y10_2 = ode4(@dqdtsparbuoy,tode4,q02,M+A,B,C,1);
        response 2
                                                    % unforced
67     Y10_1(:,2)=Y10_1(:,2)*180/pi;
                                                    % convert pitch

```

---

```

        to degrees
68     Y10_2(:,2)=Y10_2(:,2)*180/pi;
69     % plotting PSD
70     labels10=["Surge , x_0 [m]","Pitch , \theta [deg]";"PSD [m^2/Hz]","
        PSD [deg^2/Hz]"];
71     figure
72     PSD(tode4,Y10_1(:,1:2),fHighCut,labels10)
73     figure
74     PSD(tode4,Y10_2(:,1:2),fHighCut,labels10)
75 end
76 % Question 11: with hyrdodynamic forcing , assuming no current u = 0
77 global u udot z
78 for i11=1:1
79     % part a
80     z = [0:-1:z_bot];
81     u=zeros(length(z),length(t));
82     udot=u;
83     Y11_1 = ode4(@dqdtsparbuoy,tode4,q01,M+A,B,C,2);%hydro forcing
        response 1
84     Y11_2 = ode4(@dqdtsparbuoy,tode4,q02,M+A,B,C,2);%hydro forcing
        response 2
85     Y11_1(:,2)=Y11_1(:,2)*180/pi;                % convert pitch to
        degrees
86     Y11_2(:,2)=Y11_2(:,2)*180/pi;
87     % plotting PSD
88     figure
89     PSD(tode4,Y11_1(:,1:2),fHighCut,labels10)
90     figure
91     PSD(tode4,Y11_2(:,1:2),fHighCut,labels10)
92     % part b
93     q03 = [0;1;0;0;0];
94     Y11_3 = ode4(@dqdtsparbuoy,tode4,q03,M+A,B,C,1);%no forcing
        response
95     Y11_4 = ode4(@dqdtsparbuoy,tode4,q03,M+A,B,C,2);%hydro forcing
        response
96     Y11_3(:,2)=Y11_3(:,2)*180/pi;                % convert pitch to
        degrees
97     Y11_4(:,2)=Y11_4(:,2)*180/pi;                % convert pitch to
        degrees
98     % plotting PSD
99     figure
100     PSD(tode4,Y11_3(:,1:2),fHighCut,labels10)
101     PSD(tode4,Y11_4(:,1:2),fHighCut,labels10)
102 end
103 % Question 12: Linear Wave Sea-State Hydrodynamic forcing
104 for i12=1:1

```



---

```

105     [u, udot]=LinearWaveKinematics();
106     q00=[0;0;0;0;0]; % zero initial
        condition
107     Y12 = ode4(@dqdtsparbuoy, tode4, q00, M+A, B, C, 2); % hydro forcing
        response 0
108     Y12(:,2)=Y12(:,2)*180/pi; % convert pitch to
        degrees
109     % plotting PSD
110     figure
111     PSD(tode4(10001:16000), Y12(10001:16000, 1:2), fHighCut, labels10)
        % make sure to eliminate transient
112 %     ylim([-1,1])
113 end
114 % Question 13: V10 Wind and Linear Wave Forcing
115 global V_10 V_hub
116 for i13=1:1
117     V_10 = 8;
118     V_hub = 8*ones(size(t));
119     Y13 = ode4(@dqdtsparbuoy, tode4, q00, M+A, B, C, 3); % hydro forcing
        response 0
120     Y13(:,2)=Y13(:,2)*180/pi; % convert pitch to
        degrees
121     % plotting PSD
122     figure
123     PSD(tode4(10001:16000), Y13(10001:16000, 1:2), fHighCut, labels10)
        % make sure to eliminate transient
124 end
125 % Question 14: V10 Wind and Jonswap Wave Forcing
126 for i14=1:1
127     [fvec, amp, S_JS] = jonswap(Hs, Tp, df, fHighCut, gammaJS);
128     [u, udot]=IrregularWaveKinematics(fvec, amp);
129     Y14 = ode4(@dqdtsparbuoy, tode4, q00, M+A, B, C, 3); % hydro forcing
        response 0
130     Y14(:,2) = Y14(:,2)*180/pi; % convert pitch to
        degrees
131     % plotting PSD
132     figure
133     PSD(tode4(10001:16000), Y14(10001:16000, 1:2), fHighCut, labels10)
        % make sure to eliminate transient
134 end
135 % Question 15: Kaimal Wind and Jonswap Wave Forcing
136 for i15=1:1
137     [~, V_hub, ~] = Kaimal_Timeseries(TI, TL, fHighCut);
138     Y15 = ode4(@dqdtsparbuoy, tode4, q00, M+A, B, C, 3); % hydro forcing
        response 0
139     Y15(:,2) = Y15(:,2)*180/pi; % convert pitch to

```

---

```

        degrees
140    % plotting PSD
141    figure
142    PSD(tode4(10001:16000),Y15(10001:16000,1:2),fHighCut,labels10)
        % make sure to eliminate transient
143 end
144 %% Part 3: Adaptation of pitch control for dynamic stability
145 % Adapting the CT pitch controller for a floating configuration
146 % Question 16: Steady wind and no waves
147 for i16=1:1
148     % 10 m/s case no waves
149     V_10 = 10;
150     V_hub = 10*ones(size(t));
151     u=zeros(length(z),length(t));
152     udot=u;
153     Y16_1 = ode4(@dqdtsparbuoy,tode4,q00,M+A,B,C,3); % hydro forcing
        response 0
154     Y16_1(:,2)=Y16_1(:,2)*180/pi; % convert pitch
        to degrees
155     % 16 m/s case no waves
156     V_10 = 16;
157     V_hub = 16*ones(size(t));
158     Y16_2 = ode4(@dqdtsparbuoy,tode4,q00,M+A,B,C,3); % hydro forcing
        response 0
159     Y16_2(:,2)=Y16_2(:,2)*180/pi; % convert pitch
        to degrees
160     % plotting PSD
161     figure
162     PSD(tode4(10001:16000),Y16_1(10001:16000,1:2),fHighCut,labels10)
        % make sure to eliminate transient
163     figure
164     PSD(tode4(10001:16000),Y16_2(10001:16000,1:2),fHighCut,labels10)
        % make sure to eliminate transient
165 end
166 % Question 17: CT adapted controller model
167 for i17=1:1
168     % 16 m/s case no waves
169     q00contr = [0;0;0;0;0];
170     Y17 = ode4(@dqdtsparbuoy,tode4,q00contr,M+A,B,C,4); % hydro
        forcing response 0
171     Y17(:,2) = Y17(:,2)*180/pi; % convert pitch
        to degrees
172     % plotting PSD
173     figure
174     PSD(tode4(10001:16000),Y17(10001:16000,1:2),fHighCut,labels10)
        % make sure to eliminate transient

```

---

```

175 end
176 % Question 18: Proper gamma for CT controller model
177 for i18=1:1
178     gammaCTrange = [0.1,0.5,1.5];
179     Y18 = zeros(length(Y17),length(gammaCTrange));
180     figure
181     for igam = 1:length(gammaCTrange)
182         gammaCT = gammaCTrange(igam);
183         Y18i = ode4(@dqdtsparbuoy,tode4,q00contr,M+A,B,C,4); % hydro
184         % Y18(:,igam) = Y18i(:,2); %
185         looking only at surge
186         Y18i(:,2) = Y18i(:,2)*180/pi; %
187         % convert pitch to degrees
188         PSD(tode4,Y18i(:,1:2),fHighCut,labels10)
189     hold on
190 end
191 end

```

### Functions, alphabetically listed

```

1 function dq = dqdtsparbuoy(tode4,q,M,B,C,forcing)
2 %% Description
3 % This function takes a mass matrix, M (add added mass before), damping
4 % matrix, B, restoring matrix, C, and initial
5 % conditions, q
6 %
7 % The function then calculates the relevant forcing vector, depending
8 % on
9 % the forcing input case, forcing
10 %
11 % The function then returns a generalized motion, dq
12 %
13 % This function is developed for solving the surge and pitch motions of
14 % a
15 % floating, moored spar-buoy wind turbine system
16 %
17 % Future development will include making this function robust for
18 % various
19 % degree-of-freedom considerations
20 %
21 % As it is currently implemented, M,B,C are all 2x2 matrices; F is a 2
22 % x1
23 % forcing vector with Force in 1 and torque in 2; q is a 4x1 vector
24 % that is
25 % split into two 2x1 vectors for solving the system. The return, dq, is
26 % 4x1

```

---

```

21 %% Implementation
22 % Find the proper time for forcing
23 global t z_hub gammaCT
24 [~,index]= min(abs(tode4-t));
25 % Dummy CT_vrel if not using controller
26 CT_vrel = 0;
27 % Calculate forcing
28 if forcing == 1
29     F = [0;0];
30                                     %
31                                     no external forcing
32 elseif forcing == 2
33     F = hydroforcing(index,q(3),q(4));
34                                     % hydro forcing only
35                                     % dxdt of the
36                                     hub
37                                     % dxdt of the
38                                     hub
39                                     %
40                                     time-delay controller
41                                     %
42                                     % hydro plus steady wind forcing
43                                     %
44                                     % dxdt of the
45                                     hub
46                                     %
47                                     time-delay controller
48                                     %
49                                     % hydro plus steady wind forcing
50                                     %
51                                     % dxdt of the
52                                     hub
53                                     %
54                                     time-delay controller
55                                     %
56                                     % hydro plus steady wind forcing
57                                     %
58                                     % dxdt of the
59                                     hub
60                                     %
61                                     time-delay controller
62                                     %
63                                     % hydro plus steady wind forcing
64                                     %
65                                     % dxdt of the
66                                     hub
67                                     %
68                                     time-delay controller
69                                     %
70                                     % hydro plus steady wind forcing
71                                     %
72                                     % dxdt of the
73                                     hub
74                                     %
75                                     time-delay controller
76                                     %
77                                     % hydro plus steady wind forcing
78                                     %
79                                     % dxdt of the
80                                     hub
81                                     %
82                                     time-delay controller
83                                     %
84                                     % hydro plus steady wind forcing
85                                     %
86                                     % dxdt of the
87                                     hub
88                                     %
89                                     time-delay controller
90                                     %
91                                     % hydro plus steady wind forcing
92                                     %
93                                     % dxdt of the
94                                     hub
95                                     %
96                                     time-delay controller
97                                     %
98                                     % hydro plus steady wind forcing
99                                     %
100                                    % dxdt of the
101                                    hub
102                                    %
103                                    time-delay controller
104                                    %
105                                    % hydro plus steady wind forcing
106                                    %
107                                    % dxdt of the
108                                    hub
109                                    %
110                                    time-delay controller
111                                    %
112                                    % hydro plus steady wind forcing
113                                    %
114                                    % dxdt of the
115                                    hub
116                                    %
117                                    time-delay controller
118                                    %
119                                    % hydro plus steady wind forcing
120                                    %
121                                    % dxdt of the
122                                    hub
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124                                    time-delay controller
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126                                    % hydro plus steady wind forcing
127                                    %
128                                    % dxdt of the
129                                    hub
130                                    %
131                                    time-delay controller
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133                                    % hydro plus steady wind forcing
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135                                    % dxdt of the
136                                    hub
137                                    %
138                                    time-delay controller
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140                                    % hydro plus steady wind forcing
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142                                    % dxdt of the
143                                    hub
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145                                    time-delay controller
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147                                    % hydro plus steady wind forcing
148                                    %
149                                    % dxdt of the
150                                    hub
151                                    %
152                                    time-delay controller
153                                    %
154                                    % hydro plus steady wind forcing
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156                                    % dxdt of the
157                                    hub
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159                                    time-delay controller
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161                                    % hydro plus steady wind forcing
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164                                    hub
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166                                    time-delay controller
167                                    %
168                                    % hydro plus steady wind forcing
169                                    %
170                                    % dxdt of the
171                                    hub
172                                    %
173                                    time-delay controller
174                                    %
175                                    % hydro plus steady wind forcing
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177                                    % dxdt of the
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180                                    time-delay controller
181                                    %
182                                    % hydro plus steady wind forcing
183                                    %
184                                    % dxdt of the
185                                    hub
186                                    %
187                                    time-delay controller
188                                    %
189                                    % hydro plus steady wind forcing
190                                    %
191                                    % dxdt of the
192                                    hub
193                                    %
194                                    time-delay controller
195                                    %
196                                    % hydro plus steady wind forcing
197                                    %
198                                    % dxdt of the
199                                    hub
200                                    %
201                                    time-delay controller
202                                    %
203                                    % hydro plus steady wind forcing
204                                    %
205                                    % dxdt of the
206                                    hub
207                                    %
208                                    time-delay controller
209                                    %
210                                    % hydro plus steady wind forcing
211                                    %
212                                    % dxdt of the
213                                    hub
214                                    %
215                                    time-delay controller
216                                    %
217                                    % hydro plus steady wind forcing
218                                    %
219                                    % dxdt of the
220                                    hub
221                                    %
222                                    time-delay controller
223                                    %
224                                    % hydro plus steady wind forcing
225                                    %
226                                    % dxdt of the
227                                    hub
228                                    %
229                                    time-delay controller
230                                    %
231                                    % hydro plus steady wind forcing
232                                    %
233                                    % dxdt of the
234                                    hub
235                                    %
236                                    time-delay controller
237                                    %
238                                    % hydro plus steady wind forcing
239                                    %
240                                    % dxdt of the
241                                    hub
242                                    %
243                                    time-delay controller
244                                    %
245                                    % hydro plus steady wind forcing
246                                    %
247                                    % dxdt of the
248                                    hub
249                                    %
250                                    time-delay controller
251                                    %
252                                    % hydro plus steady wind forcing
253                                    %
254                                    % dxdt of the
255                                    hub
256                                    %
257                                    time-delay controller
258                                    %
259                                    % hydro plus steady wind forcing
260                                    %
261                                    % dxdt of the
262                                    hub
263                                    %
264                                    time-delay controller
265                                    %
266                                    % hydro plus steady wind forcing
267                                    %
268                                    % dxdt of the
269                                    hub
270                                    %
271                                    time-delay controller
272                                    %
273                                    % hydro plus steady wind forcing
274                                    %
275                                    % dxdt of the
276                                    hub
277                                    %
278                                    time-delay controller
279                                    %
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1290                                   % dxdt of the
1291                                   hub
1292                                   %
1293                                   time-delay controller
1294                                   %
1295                                   % hydro plus steady wind forcing
1296                                   %
1297                                   % dxdt of the
1298                                   hub
1299                                   %
1300                                   time-delay controller
1301                                   %
1302                                   % hydro plus steady wind forcing
1303                                   %
1304                                   % dxdt of the
1305                                   hub
1306                                   %
1307                                   time-delay controller
1308                                   %
1309                                   % hydro plus steady wind forcing
1310                                   %
1311                                   % dxdt of the
1312                                   hub
1313                                   %
1314                                   time-delay controller
1315                                   %
1316                                   % hydro plus steady wind forcing
1317                                   %
1318                                   % dxdt of the
1319                                   hub
1320                                   %
1321                                   time-delay controller
1322                                   %
1323                                   % hydro plus steady wind forcing
1324                                   %
1325                                   % dxdt of the
1326                                   hub
1327                                   %
1328                                   time-delay controller
1329                                   %
1330                                   % hydro plus steady wind forcing
1331                                   %
1332                                   % dxdt of the
1333                                   hub
1334                                   %
1335                                   time-delay controller
1336                                   %
1337                                   % hydro plus steady wind forcing
1338                                   %
1339                                   % dxdt of the
1340                                   hub
1341                                   %
1342                                   time-delay controller
1343                                   %
1344                                   % hydro plus steady wind forcing
1345                                   %
1346                                   % dxdt of the
1347                                   hub
1348                                   %
1349                                   time-delay controller
1350                                   %
1351                                   % hydro plus steady wind forcing
1352                                   %
1353                                   % dxdt of the
1354                                   hub
1355                                   %
1356                                   time-delay controller
1357                                   %
1358                                   % hydro plus steady wind forcing
1359                                   %
1360                                   % dxdt of the
1361                                   hub
1362                                   %
1363                                   time-delay controller
1364                                   %
1365                                   % hydro plus steady wind forcing
1366                                   %
1367                                   % dxdt of the
1368                                   hub
1369                                   %
1370                                   time-delay controller
1371                                   %
1372                                   % hydro plus steady wind forcing
1373                                   %
1374                                   % dxdt of the
1375                                   hub
1376                                   %
1377                                   time-delay controller
1378                                   %
1379                                   % hydro plus steady wind forcing
1380                                   %
1381                                   % dxdt of the
1382                                   hub
1383                                   %
1384                                   time-delay controller
1385                                   %
1386                                   % hydro plus steady wind forcing
1387                                   %
1388                                   % dxdt of the
1389                                   hub
1390                                   %
1391                                   time-delay controller
1392                                   %
1393                                   % hydro plus steady wind forcing
1394                                   %
1395                                   % dxdt of the
1396                                   hub
1397                                   %
1398                                   time-delay controller
1399                                   %
1400                                   % hydro plus steady wind forcing
1401                                   %
1402                                   % dxdt of the
1403                                   hub
1404                                   %
1405                                   time-delay controller
1406                                   %
1407                                   % hydro plus steady wind forcing
1408                                   %
1409                                   % dxdt of the
1410                                   hub
1411                                   %
1412                                   time-delay controller
1413                                   %
1414                                   % hydro plus steady wind forcing
1415                                   %
1416                                   % dxdt of the
1417                                   hub
1418                                   %
1419                                   time-delay controller
1420                                   %
1421                                   % hydro plus steady wind forcing
1422                                   %
1423                                   % dxdt of the
1424                                   hub
1425                                   %
1426                                   time-delay controller
1427                                   %
1428                                   % hydro plus steady wind forcing
1429                                   %
1430                                   % dxdt of the
1431                                   hub
1432                                   %
1433                                   time-delay controller
1434                                   %
1435                                   % hydro plus steady wind forcing
1436                                   %
1437                                   % dxdt of the
1438                                   hub
1439                                   %
1440                                   time-delay controller
1441                                   %
1442                                   % hydro plus steady wind forcing
1443                                   %
1444                                   % dxdt of the
1445                                   hub
1446                                   %
1447                                   time-delay controller
1448                                   %
1449                                   % hydro plus steady wind forcing
1450                                   %
1451                                   % dxdt of the
1452                                   hub
1453                                   %
1454                                   time-delay controller
1455                                   %
1456                                   % hydro plus steady wind forcing
1457                                   %
1458                                   % dxdt of the
1459                                   hub
1460                                   %
1461                                   time-delay controller
1462                                   %
1463                                   % hydro plus steady wind forcing
1464                                   %
1465                                   % dxdt of the
1466                                   hub
1467                                   %
1468                                   time-delay controller
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1470                                   % hydro plus steady wind forcing
1471                                   %
1472                                   % dxdt of the
1473                                   hub
1474                                   %
1475                                   time-delay controller
1476                                   %
1477                                   % hydro plus steady wind forcing
1478                                   %
1479                                   % dxdt of the
1480                                   hub
1481                                   %
1482                                   time-delay controller
1483                                   %
1484                                   % hydro plus steady wind forcing
1485                                   %
1486                                   % dxdt of the
1487                                   hub
1488                                   %
1489                                   time-delay controller
1490                                   %
1491                                   % hydro plus steady wind forcing
1492                                   %
1493                                   % dxdt of the
1494                                   hub
1495                                   %
1496                                   time-delay controller
1497                                   %
1498                                   % hydro plus steady wind forcing
1499                                   %
1500                                   % dxdt of the
1501                                   hub
1502                                   %
1503                                   time-delay controller
1504                                   %
1505                                   % hydro plus steady wind forcing
1506                                   %
1507                                   % dxdt of the
1508                                   hub
1509                                   %
1510                                   time-delay controller
1511                                   %
1512                                   % hydro plus steady wind forcing
1513                                   %
1514                                   % dxdt of the
1515                                   hub
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1517                                   time-delay controller
1518                                   %
1519                                   % hydro plus steady wind forcing
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1533                                   % hydro plus steady wind forcing
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1536                                   hub
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1543                                   hub
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1547                                   % hydro plus steady wind forcing
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1549                                   % dxdt of the
1550                                   hub
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1552                                   time-delay controller
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1554                                   % hydro plus steady wind forcing
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1556                                   % dxdt of the
1557                                   hub
1558                                   %
1559                                   time-delay controller
1560                                   %
1561                                   % hydro plus steady wind forcing
1562                                   %
1563                                   % dxdt of the
1564                                   hub
1565                                   %
1566                                   time-delay controller
1567                                   %
1568                                   % hydro plus steady wind forcing
1569                                   %
1570                                   % dxdt of the
1571                                   hub
1572                                   %
1573                                   time-delay controller
1574                                   %
1575                                   % hydro plus steady wind forcing
1576                                   %
1577                                   % dxdt of the
1578                                   hub
1579                                   %
1580                                   time-delay controller
1
```

---

```

12 V_rel = V_hub(tindex)-dxdt_hub;
13 % Set values for CT_hub
14 if V_rel <= V_rated
15     CT_hub = CT_0;
16 else
17     CT_vrel = CT_0 *exp(-aCT*(V_rel-V_rated)^bCT);
18     CT_hub = q5;
19 end
20 % Set values for CT_10
21 if V_10 <= V_rated
22     CT_10 = CT_0;
23 else
24     CT_10 = CT_0 *exp(-aCT*(V_10-V_rated)^bCT);
25 end
26 % Calculate mean aero force
27 Fwind_m = 0.5*rho_air*A_r*CT_10*V_10^2;
28 % Calculate time-varying aero force
29 Fwind_t = 0.5*rho_air*A_r*CT_hub*V_rel^2;           % using the CT
               calculated previously using the update
30 % Calculate reduction factor
31 if V_10<V_rated
32     f_red = 0.54;
33 elseif V_10
34     f_red = 0.54 + 0.027*(V_10-V_rated);
35 end
36 Fwind = Fwind_m + f_red*(Fwind_t-Fwind_m);
37 Tauwind = Fwind*z_hub;
38 F_Tauwind=[Fwind;Tauwind];

1 function [F_Tauwind,CT_hub] = F_wind_timepoint(dxdt_hub,tindex)
2 %% Description
3 % This function calculates the forcing from wind, and can compute for
4 % steady wind or for unsteady time series.
5 % This function takes the V_10 time average, V_hub, a and b parameters,
   and
6 % the hub motion to calculate the relative velocity, corrected CT, and
7 % determine thrust.
8 % This function returns the forcing and torque for a given point in
   time.
9 %% Implementation
10 global rho_air A_r V_rated CT_0 V_10 V_hub aCT bCT z_hub
11 % Find V_rel
12 V_rel = V_hub(tindex)-dxdt_hub;
13 % Set values for CT_hub
14 if V_rel <= V_rated
15     CT_hub = CT_0;
16 else

```

---

```

17     CT_hub = CT_0 *exp(-aCT*(V_rel-V_rated)^bCT);
18 end
19 % Set values for CT_10
20 if V_10 <= V_rated
21     CT_10 = CT_0;
22 else
23     CT_10 = CT_0 *exp(-aCT*(V_10-V_rated)^bCT);
24 end
25 % Calculate mean aero force
26 Fwind_m = 0.5*rho_air*A_r*CT_10*V_10^2;
27 % Calculate time-varying aero force
28 Fwind_t = 0.5*rho_air*A_r*CT_hub*V_rel^2;
29 % Calculate reduction factor
30 if V_10<V_rated
31     f_red = 0.54;
32 elseif V_10
33     f_red = 0.54 + 0.027*(V_10-V_rated);
34 end
35 Fwind = Fwind_m + f_red*(Fwind_t-Fwind_m);
36 Tauwind = Fwind*z_hub;
37 F_Tauwind=[Fwind;Tauwind];

1 function Fvec = hydroforcing(t_index,q3,q4)
2 %% Description
3 % This function can evaluate the forces and moments from hydrodynamic
4 % forcing for still water or for a given sea state. This function is
5 % designed for use with dqdtsparbuoy and ode4.
6 %% Inputs:
7 % t is the current time state and will be used to pull the pre-
   determined
8 % sea state
9 % q3 is dx0/dt
10 % q4 is d_theta/dt
11 %% Outputs:
12 % Hydrodynamic force and moment for a point in time
13 %% Implementation
14 global rhow Cm CD D_spar u udot z
15 A = D_spar^2*pi/4;
16 F = 0;
17 Tau = 0;
18 for i=1:length(z)
19     df = rhow*((Cm+1)*A*udot(i,t_index) + 0.5*CD*D_spar*(u(i,t_index)-
       q3-z(i)*q4)*abs(u(i,t_index)-q3-z(i)*q4));
20     dtau = df*z(i);
21     F = F + df;
22     Tau = Tau + dtau;
23 end

```

---

```

24 Fvec=[F;Tau];

1 function [u,a]=IrregularWaveKinematics(fvec,amp)
2 % This function calculates the velocity and acceleration at various
   heights
3 % for a floating spar for irregular waves
4 % Inputs
5 % fvec = wave frequency
6 % h = depth of water or spar
7 % g = gravity
8 % amp = wave amplitude
9 % rho = density of water
10 % U= Horizontal Velocity
11 global g z_bot z t
12 % Given constants
13 for gc = 1:1
14     h = -z_bot;
15 end
16 % Calculated Constants and Initializations
17 for cc = 1:1
18     k = zeros(size(fvec));
19     x = 0;
20     u = zeros(length(z),length(t));
21     a = zeros(length(z),length(t));
22 end
23 % Frequency domain inputs for wave number
24 for ifreq = 1:length(fvec)
25     k(ifreq) = wave_number(fvec(ifreq),g,h);
26 end
27 % random error
28 random = 2*pi*rand(1,length(fvec));
29 % calculating acceleration and velocity
30 for iz=1:length(z)
31     for it=1:length(t)
32         uj = 0;
33         aj = 0;
34         for ifreq = 2:length(fvec)
35             omega = 2*pi*fvec(ifreq);
36             uj = uj + amp(ifreq) *omega* cosh(k(ifreq)*(z(iz)+h)) /
                 sinh(k(ifreq)*h)*cos(omega*t(it)-(k(ifreq)*x)+(random(
                 ifreq)));
37             aj = aj - omega^2*amp(ifreq) * cosh(k(ifreq)*(z(iz)+h)) /
                 sinh(k(ifreq)*h) * sin(omega*t(it)-(k(ifreq)*x)+(random(
                 ifreq)));
38         end
39         u(iz,it) = uj;
40         a(iz,it) = aj;

```

---

```

41     end
42 end

1 function [fvec,a,S_JS] = jonswap(Hs,Tp,df,fHighCut,gammaJS)
2     % This function calculates the JONSWAP distribution for waves,
3     % frequency and amplitude
4     % Inputs: Hs, Period, frequency step, max frequency considered,
5     %         gamma
6     % Outputs: time-varying frequency, time-varying wave amplitude,
7     %         Jonswap
8     % frequency spectra
9     fvec = [0 : df : fHighCut];
10    fp= 1/Tp;
11    for i =1: length(fvec)
12        if fvec(i) <= fp
13            sigma = 0.07;
14        else
15            sigma = 0.09;
16        end
17        gammaexp = exp(-0.5*(((fvec(i)/fp)-1)/sigma)^2);
18        S_JS(i) = 0.3125* Hs^2 *Tp * (fvec(i)/fp)^(-5)* exp(-1.25*(fvec
19            (i)/fp)^(-4))*(1-0.287*log(gammaJS))*gammaJS^gammaexp;
20        a(i) = sqrt(2*S_JS(i)*df);
21    end
22 return

1 function [S_W,V,f] = Kaimal_Timeseries(I,L,fHigh)
2 %% Description
3 % This function calculates wind power density using the Kaimal spectrum
4 % . It
5 % takes wind parameter inputs:
6 % I: turbulence intensity
7 % V_10: 10-minute average wind speed [m/s]
8 % L: turbulence length scale [m]
9 % fHigh: cut-off frequency
10 % t: time space [s]
11 % and returns:
12 % the spectral density function, S_W
13 % the velocity time series, V, and
14 % the frequency vector, f.
15 %% Implementation
16 % initializations
17 global t V_10
18 df = 1/t(end);
19 f = [df:df:fHigh];
20 rng(1)
21 ep = 2*pi*rand(1,length(f));

```



---

```

21 bp = zeros(1,length(f));
22 wp = 2*pi*f;
23 S_W = zeros(size(t));
24 V = zeros(size(t));
25 % calculate spectrum
26 for p = 1:length(f)
27     S_W(p) = 4*I^2*V_10*L/(1+6*(f(p)*L/V_10))^(5/3);
28     bp(p) = sqrt(2*S_W(p)*df);
29 end
30 % velocy time series
31 for i = 1:length(t)
32     V(i) = V_10 + sum(bp.*cos(wp*t(i)+ep));
33 end

1 function [u,a]=LinearWaveKinematics()
2 % This function calculates the kinematics of regular waves
3 % Inputs
4 % f= wave frequency
5 % h= depth of water or spar
6 % g= gravity
7 % rho = density of water
8 % U= Horizontal Velocity
9 global g z_bot Hs Tp z t
10 % Given constants
11     h = -z_bot;
12     f = 1/Tp;
13     H = Hs;
14 % Calculated Constants
15     w=2*pi*f;
16     k=wave_number(f,g,h);
17 % Pre calculations
18     x3=0;
19     u=zeros(length(z),length(t));
20     a=zeros(length(z),length(t));
21     for j=1:length(z)
22         for i=1:length(t)
23             u(j,i) = w*H/2 * cosh(k*(z(j)+h)) / sinh(k*h) * cos(w*t(i)-
                k*x3);
24             a(j,i) = -w^2*H/2 * cosh(k*(z(j)+h)) / sinh(k*h) * sin(w*t(
                i)-k*x3);
25         end
26     end

1 function Y = ode4(odefun, tspan, y0, varargin)
2 %ODE4 Solve differential equations with a non-adaptive method of order
   4.
3 % Y = ODE4(ODEFUN,TSPAN,Y0) with TSPAN = [T1, T2, T3, ... TN]

```

---

```

    integrates
4 %   the system of differential equations  $y' = f(t,y)$  by stepping from
    T0 to
5 %   T1 to TN. Function ODEFUN(T,Y) must return  $f(t,y)$  in a column
    vector.
6 %   The vector Y0 is the initial conditions at T0. Each row in the
    solution
7 %   array Y corresponds to a time specified in TSPAN.
8 %
9 %   Y = ODE4(ODEFUN,TSPAN,Y0,P1,P2...) passes the additional parameters
10 %   P1,P2... to the derivative function as ODEFUN(T,Y,P1,P2...) .
11 %
12 %   This is a non-adaptive solver. The step sequence is determined by
    TSPAN
13 %   but the derivative function ODEFUN is evaluated multiple times per
    step.
14 %   The solver implements the classical Runge–Kutta method of order 4.
15 %
16 %   Example
17 %       tspan = 0:0.1:20;
18 %       y = ode4(@vdp1,tspan,[2 0]);
19 %       plot(tspan,y(:,1));
20 %       solves the system  $y' = \text{vdp1}(t,y)$  with a constant step size of
    0.1,
21 %       and plots the first component of the solution.
22 %
23
24 if ~isnumeric(tspan)
25     error('TSPAN should be a vector of integration steps.');
```

---

```

43
44 y0 = y0(:); % Make a column vector.
45 if ~isequal(size(y0),size(f0))
46     error('Inconsistent sizes of Y0 and f(t0,y0).');
47 end
48
49 neq = length(y0);
50 N = length(tspan);
51 Y = zeros(neq,N);
52 F = zeros(neq,4);
53
54 Y(:,1) = y0;
55 for i = 2:N
56     ti = tspan(i-1);
57     hi = h(i-1);
58     yi = Y(:,i-1);
59     F(:,1) = feval(odefun,ti,yi,varargin{:});
60     F(:,2) = feval(odefun,ti+0.5*hi,yi+0.5*hi*F(:,1),varargin{:});
61     F(:,3) = feval(odefun,ti+0.5*hi,yi+0.5*hi*F(:,2),varargin{:});
62     F(:,4) = feval(odefun,tspan(i),yi+hi*F(:,3),varargin{:});
63     Y(:,i) = yi + (hi/6)*(F(:,1) + 2*F(:,2) + 2*F(:,3) + F(:,4));
64 end
65 Y = Y.';

1 function [] = PSD(t,signal,fHighCut,ylabelstr)
2 %% Description
3 % This function takes a timeseries (t) and the signal response for that
4 % time series as an input, and return a plot of the time series and
5 % frequency domain. It also takes the cutoff frequency for plotting.
6 % The dimension of the signal tells the function how many subplots to
   make.
7 % The user also designates the y-axis labels for the signal being
   plotted
8 % with a matrix of strings, size 2 x (number of signals).
9     %% Important PSD information for plotting only steady-state
       response
10     % when plotting time decay, set timestartpos to 1. when plotting
11     % forced response, set timestartpos to 10001
12     timestartpos = 1; % steady state time position 1
13 %% Implementation
14 % get the number of signals to be plotted, and the relevant direction
   of
15 % the signal input matrix
16 global w1 w5
17 [numbersignals,mindim] = min(size(signal));
18 % if needed, transpose the signal matrix
19 if mindim==1

```

---

```

20     signal=signal';
21 end
22 % create subplots
23 for numbersubplots=1:numbersignals
24     % Plot timeseries
25     subplot(numbersignals,2,2*numbersubplots-1), plot(t,signal(:,
        numbersubplots),'LineWidth',1.25), grid on
26     hold on
27     % xlabel only if the last plot
28     if numbersubplots==numbersignals
29         xlabel('Time [s]')
30     end
31     ylabel(ylabelstr(1,numbersubplots))
32
33     df = 1/(t(end)-t(timestartpos)); % Frequency resolution
34     fpsd = df*(0:length(t)-timestartpos); % Frequency vector starts
        from 0 for length t
35
36     signalhat = fft(signal(timestartpos:end,numbersubplots))/length(t(
        timestartpos:end)); % Fourier amplitudes
37     signalhat(1) = 0; % Discard first value (
        mean)
38     signalhat(round(length(fpsd)/2):end) = 0; % Discard all above
        Nyquist fr.
39     signalhat = 2*signalhat; % Make amplitude one-
        sided
40     psd = abs(signalhat).^2/2/df; % Calculate spectrum
41
42     % Plot frequency domain
43     subplot(numbersignals,2,2*numbersubplots), plot(fpsd,psd,'LineWidth
        ',1.25),
44     xline(w1,'—r')
45     xline(w5,'—m')
46     grid on
47     hold on, xlim([0 0.12])
48     legend('','f_{s}=0.0083 Hz','f_{p}=0.0326 Hz')
49     if numbersubplots==numbersignals
50         xlabel('Frequency [Hz]')
51     end
52     ylabel(ylabelstr(2,numbersubplots))
53 end

1 function k = wave_number(f,g,h)
2 % this function calculates the wave number from frequency (f) and depth
    (h)
3 % w = radian frequency
4 %

```

---

```
5 % w=2*pi*f;  
6  
7 fun = @(k) (2*pi*f)^2 - g*k*tanh(k*h);  
8  
9 k=fzero(fun,[0 3]);
```