Algorithms Test 1 Review

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1 EX 1

1.1 Prompt

1.2 Answer

2 EX 2

2.1 Prompt

2.2 Answer

3 EX 3

3.1 Prompt

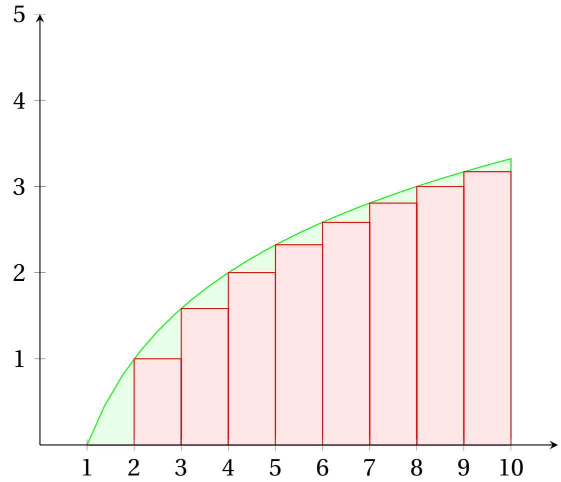
Use the technique of bounding definite integrals to find the Θ category for the function.

$$A(n) = log_2(1) + log_2(2) + log_2(3) + \ldots + log_2(n-1) + log_2(n) \tag{1}$$

Actually, you should use the integral bound technique for one equality, and use trivial analysis for the other.

3.2 Answer

3.2.1 Step 1: Draw a graph of the summation and Integral



3.2.2 Step 2: Deduce the Graph's implications

Following from the facts that:

1. The integral is the area under the function $log_2(x)$ from 1 to n.
2. If we chose to view the integral as the summation of all of its length one segments from 1 to n plus the integral of whatever is left over (if n is not an integer value), the resulting integral's value will be unaffected.
3. The value of each integral segment is greater than the value of its corresponding series segment because:
   (a) the value of the function and the series are equal at integer values
   (b) the value of the function increases between integers and the value of the series does not.

$\int_1^n log_2(x)$ is greater than $\sum_{i=1}^n log_2(i)$ for any n greater than 1. Thus, to find the upper bound or O of $\sum_{i=1}^n log_2(i)$ we merely need to find the integral of $\int_1^n log_2(x)$

3.2.3 Step 3: Calculate the Definite integral

So now I will calculate $\int_1^n log_2(x)$

1. Recognize that to take the integral of a logarithm, we will have to perform integration by parts. so we must chose u and dv values.

$$u = log_2(x) \qquad du = 1/xln(2) \qquad dv = 1 \qquad v = x \tag{2}$$

2. solve for the indefinite integral

$$\int_1^n log_2(x) = \frac{ln(x)*x}{ln(2)}\Big|_1^n - \int_1^n \frac{x}{xln(2)} = \frac{ln(x)*x}{ln(2))}\Big|_1^n - \int_1^n \frac{1}{ln(2)} = \frac{ln(x)*x}{ln(2)} - \frac{x}{ln(2)}\Big|_1^n = \frac{ln(x)*x-x}{ln(2)}\Big|_1^n \tag{3}$$

3. solve for the definite integral

$$\frac{ln(x)*x-x}{ln(2)}\Big|_1^n = \frac{ln(n)*n-n}{ln(2)} - \left(\frac{ln(1)*1-1}{ln(2)}\right) = \frac{ln(n)*n-n}{ln(2)} + \left(\frac{1}{ln(2)}\right) = \frac{ln(n)*n-n+1}{ln(2)}$$

Thus, The equation:

$$\frac{ln(n)*n-n+1}{ln(2)}$$

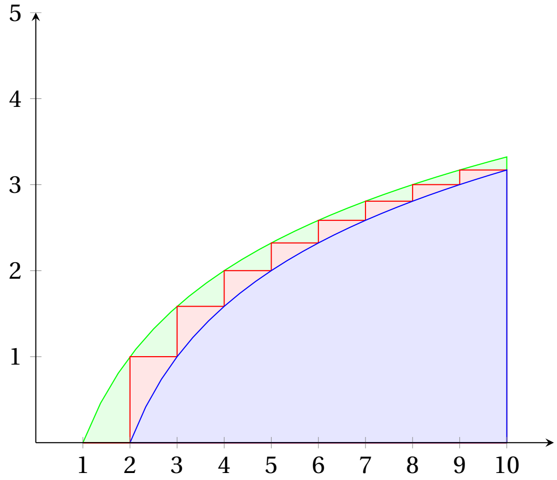
Acts as an upper bound for the summation.

3.2.4 Step 4: Prove Upper Bound is in $\Theta(nlog(n))$

1. prove equation is $O(nlog(n))$. According to our notes, an equation of the category $O(n)$ when it

$$ \tag{4} $$

2.



4 EX 4

4.1 Prompt

4.2 Answer

5 EX 5

5.1 Prompt

5.2 Answer

6 EX 6

6.1 Prompt

6.2 Answer

7 EX 7

7.1 Prompt

7.2 Answer