Optimal Portfolio Weights II: Rebalancing

In [1]:

```
# Working with data:
import numpy  as np                                        # For scientific computing
import pandas as pd                                        # Working with tables.

# Downloading files:
import requests, zipfile, io                                     # To access websites

# Specific data providers:
from tiingo import TiingoClient                            # Stock prices.
import quandl                                              # Economic data, futures p

# API keys:
tiingo = TiingoClient({'api_key':'XXXX'})
quandl.ApiConfig.api_key = 'YYYY'

# Plotting:
import matplotlib.pyplot as plt                             # Basic plot library.
plt.style.use('ggplot')                                     # Make plots look nice
```

Get data

Get ETF prices and returns (GLD: Gold ETF, TLT: 20+ year treasuries, IEF: 7-10 year treasuries, SHY: 1-3 year treasuries:

In [2]:

```
# start in 2005 since GLD not available earlier
PRICE = tiingo.get_dataframe(['SPY','GLD','TLT'], '2005-1-1', metric_name='adjCl
PRICE.index = pd.to_datetime(PRICE.index).tz_convert(None)
RET = PRICE.pct_change()
RET[:3]
```

Out[2]:

	SPY	GLD	TLT
2005-01-03	NaN	NaN	NaN
2005-01-04	-0.012219	-0.006509	-0.010480
2005-01-05	-0.006901	-0.001638	0.005352

Get federal funds rate and treasury yields:

In [3]:

```
RATES = quandl.get(['FRED/FEDFUNDS','FRED/DGS1','FRED/DGS5','FRED/DGS10','FRED/D
RATES.columns = ['FedFunds','Treasury_1', 'Treasury_5', 'Treasury_10', 'Treasury
RATES[-3:]
```

Out[3]:

	FedFunds	Treasury_1	Treasury_5	Treasury_10	Treasury_30
Date					
2021-04-02	NaN	0.0007	0.0097	0.0172	0.0235
2021-04-05	NaN	0.0006	0.0094	0.0173	0.0236
2021-04-06	NaN	0.0006	0.0088	0.0167	0.0232

Calculate margin rate:

In [4]:
```
RET = RET.join(RATES.FedFunds.rename('MarginLoan'), how='outer')
RET['MarginLoan'] = RET.MarginLoan.ffill()/252 + 0.01/252          # Assume mar
RET = RET.dropna(subset=['SPY'])
RET
```

Out[4]:

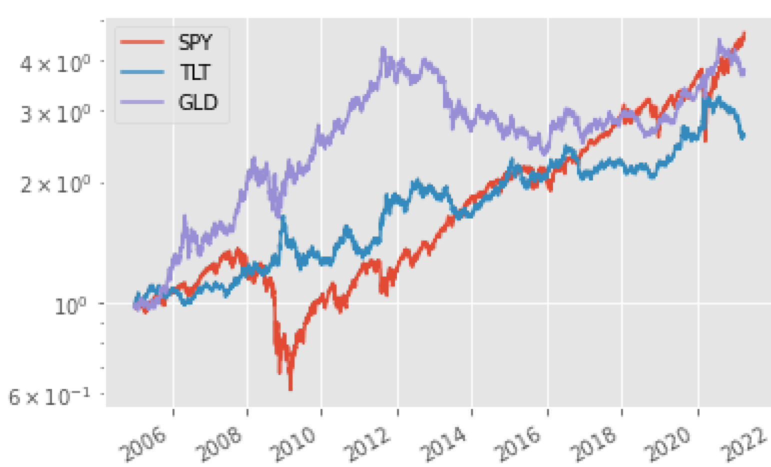
	SPY	GLD	TLT	MarginLoan
2005-01-04	-0.012219	-0.006509	-0.010480	0.000130
2005-01-05	-0.006901	-0.001638	0.005352	0.000130
2005-01-06	0.005084	-0.012187	0.000680	0.000130
2005-01-07	-0.001433	-0.007355	0.002264	0.000130
2005-01-10	0.004728	0.002629	0.001581	0.000130
...	...	...	...	...
2021-03-31	0.004053	0.015168	-0.005580	0.000042
2021-04-01	0.010799	0.012628	0.016575	0.000042
2021-04-05	0.014353	-0.000370	-0.004363	0.000042
2021-04-06	-0.000591	0.008029	0.006793	0.000042
2021-04-07	0.001157	-0.002818	-0.006965	0.000042

4092 rows × 4 columns

Compare stocks, long-term bonds and gold:

In [5]:
```
RET[['SPY','TLT','GLD']].add(1).cumprod().plot(logy=True)
```

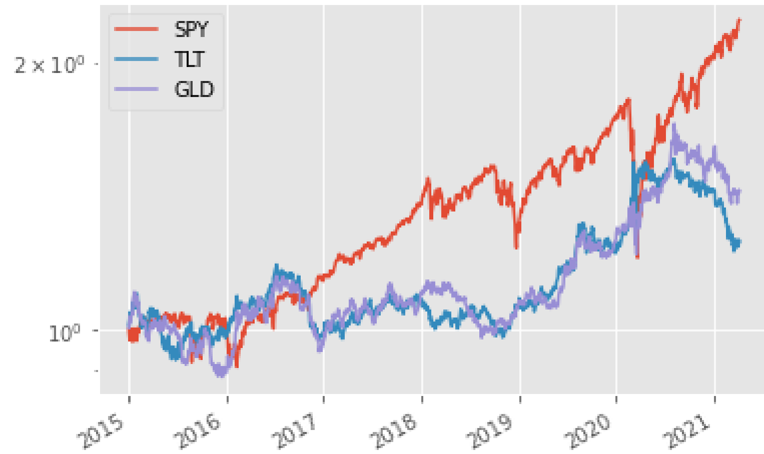
Out[5]: <AxesSubplot:>



Since 2015:

In [6]:
```
RET.loc['2015':,['SPY','TLT','GLD']].dropna().add(1).cumprod().plot(logy=True)
```

Out[6]: <AxesSubplot:>



Group data by year and get table of correlations (1 table for each year):

In [7]:
```
RET[['SPY','TLT','GLD']].groupby(RET.index.year).corr()  [:6]  # First 6 rows.
```

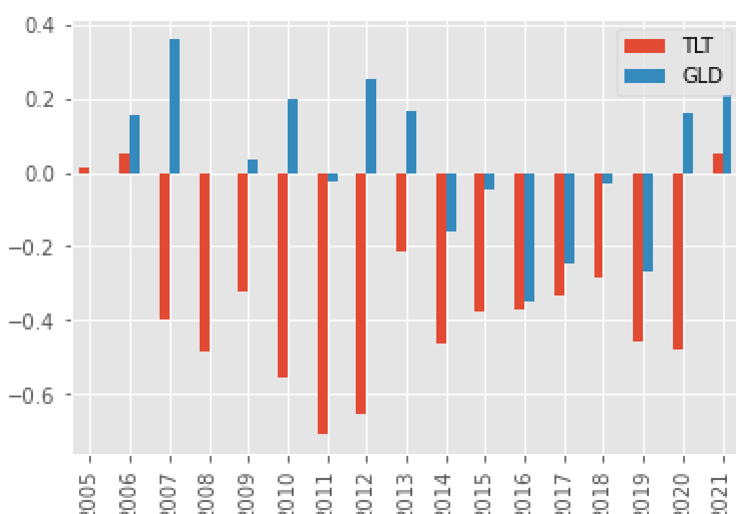
Out[7]:

		SPY	TLT	GLD
2005	**SPY**	1.000000	0.016146	-0.001293
	TLT	0.016146	1.000000	0.067286
	GLD	-0.001293	0.067286	1.000000
2006	**SPY**	1.000000	0.052858	0.153172
	TLT	0.052858	1.000000	0.038417
	GLD	0.153172	0.038417	1.000000

Daily correlations for each year for SPY-TLT and SPY-GLD:

In [8]:
```
RET[['SPY','TLT','GLD']].groupby(RET.index.year).corr().unstack().SPY[['TLT','GL
```

Out[8]: <AxesSubplot:>



Mean-variance optimization

Risk premiums and covariances:

In [9]:
```
cov     = RET[['SPY','TLT','GLD']].cov() * 252
cov_inv = pd.DataFrame(np.linalg.inv(cov), columns=cov.columns, index=cov.index)

r_annual       = RET[:'2020'].add(1).resample('A').prod().sub(1)
r_annual_Tbill = RATES.Treasury_1.resample('A').first()

rx_annual = r_annual.sub(r_annual_Tbill, 'rows').dropna()

meanx = rx_annual[['SPY','TLT','GLD']].mean()  # Risk premiums (average annual e
meanx
```

Out[9]:
```
SPY     0.093715
TLT     0.066050
GLD     0.089407
dtype: float64
```

Maximum Sharpe ratio weights:

In [10]:
```
w_maxSharpe = cov_inv.dot(meanx) /  cov_inv.dot(meanx).sum()
w_maxSharpe
```

Out[10]:
```
SPY     0.355471
TLT     0.467277
GLD     0.177252
dtype: float64
```

Minimum volatility portfolio weights:

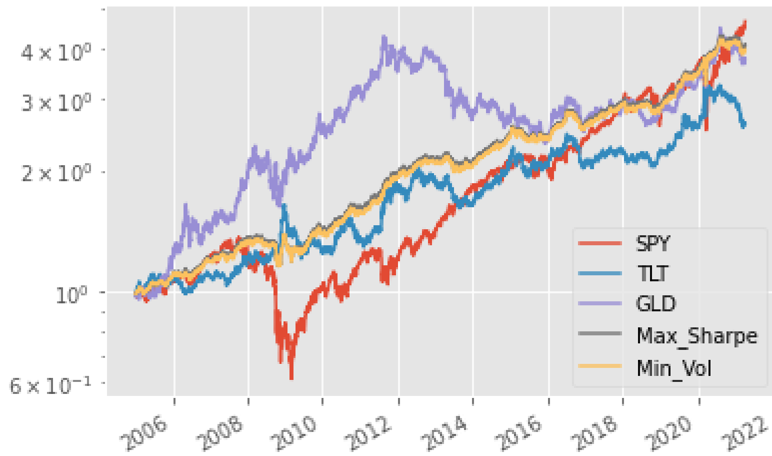
In [11]:
```
w_minVol = cov_inv.sum() / cov_inv.sum().sum()
w_minVol
```

Out[11]:
```
SPY     0.337197
TLT     0.519308
GLD     0.143494
dtype: float64
```

Compound returns of optimal portfolios

In [12]:
```
t = pd.DataFrame()
t['SPY']        = RET.SPY
t['TLT']        = RET.TLT
t['GLD']        = RET.GLD
t['Max_Sharpe'] = RET.multiply(w_maxSharpe).sum('columns')
t['Min_Vol']    = RET.multiply(w_minVol).sum('columns')

t.add(1).cumprod().plot(logy=True)
```

Out[12]: <AxesSubplot:>



Backtesting this strategy

In [13]:

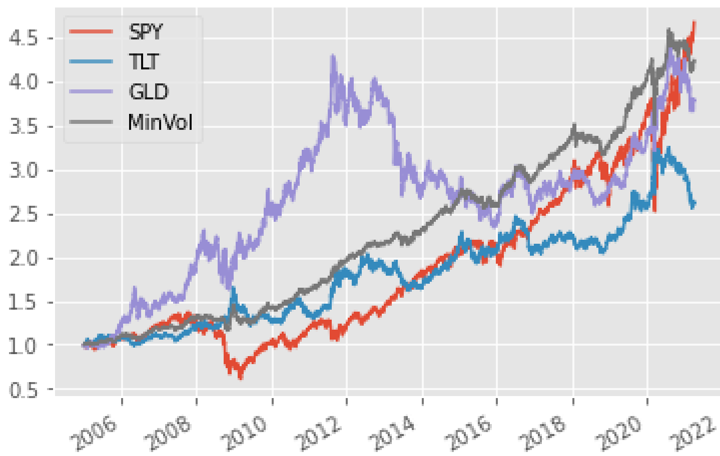
```
def get_rebalance_dates(frequency):
    group = getattr(PRICE.index, frequency)
    return PRICE[:1].index.union(PRICE.groupby([PRICE.index.year, group]).tail(1



def run_backtest(frequency):

    rebalance_dates = get_rebalance_dates(frequency)

    portfolio_value = pd.Series(1,                              index=[rebalance_dates[0
    weights         = pd.DataFrame(columns=RET.columns, index=[rebalance_dates[0
    trades          = pd.DataFrame(columns=RET.columns, index=[rebalance_dates[0

    previous_positions = weights.iloc[0]

    for i in range(len(rebalance_dates)-1):
        start_date = rebalance_dates[i]
        end_date   = rebalance_dates[i+1]

        cum_ret = RET[start_date:end_date][1:].add(1).cumprod()

        start_weights = select_weights(start_date)      # Call "select_weights()

        new_positions = portfolio_value.iloc[-1] * start_weights

        start_to_end_positions = new_positions   * cum_ret
        start_to_end_value     = start_to_end_positions.sum('columns')

        portfolio_value = portfolio_value.append(start_to_end_value)

        weights = weights.append(start_to_end_positions.div(start_to_end_value,'

        trades.loc[start_date] = new_positions - previous_positions
        previous_positions     = start_to_end_positions.iloc[-1]       # Previous

    return portfolio_value, weights, trades
```

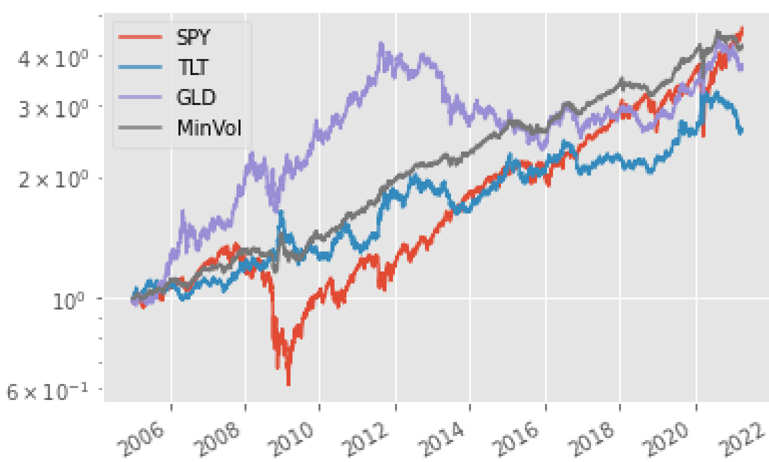
In [14]:

```
def select_weights(date):
        if not RET[:date].empty:                                              # If p
```

```
            cov     = RET[['SPY','TLT','GLD']][:date][-100:].cov() * 252  # Use
            cov_inv = pd.DataFrame(np.linalg.inv(cov), columns=cov.columns, inde

            w = cov_inv.sum() /  cov_inv.sum().sum()         # Minimum-volatilit
        else:
            w = pd.Series(1/3, index=['SPY','TLT','GLD'])    # If no previous da
        return w                                             # Return the select


min_vol, weights, trades = run_backtest('month')


t = RET[['SPY','TLT','GLD']].join( min_vol.pct_change().rename('MinVol') )
t.add(1).cumprod().plot()
t.add(1).cumprod().plot(logy=True)
```

Out[14]: <AxesSubplot:>





Compare statistics for this table:

In [15]:

```
annual_returns = t[:'2020'].add(1).resample('A').prod().sub(1)

x = pd.DataFrame()
x['Average_returns']   = annual_returns.mean()
x['Geometric_average'] = annual_returns.add(1).prod().pow(1/len(annual_returns))
x['Risk_premium']      = annual_returns.sub(r_annual_Tbill, 'rows').dropna().mea
x['Volatility']        = t[:'2020'].std() * 252**0.5
x['Sharpe_ratio']      = x.Risk_premium / x.Volatility
x
```

Out[15]:

	Average_returns	Geometric_average	Risk_premium	Volatility	Sharpe_ratio
SPY	0.109102	0.095103	0.093715	0.196220	0.477600
TLT	0.081438	0.071075	0.066050	0.142047	0.464988
GLD	0.104794	0.092954	0.089407	0.182457	0.490014
MinVol	0.100489	0.098068	0.085101	0.081891	1.039199