Lecture 4 Summary Statistics

<u>Data Visualization · 1-DAV-105</u> Lecture by Broňa Brejová More details in the <u>notebook version</u>

Introduction

Summary statistics (popisné charakteristiky / štatistiky):

Quantities that summarize basic properties of a single variable (a table column), such as the mean.

We can also characterize dependencies between pairs of variables.

Together with simple plots, they give us the first glimpse at the data when working with a new data set.

Data set for today

- The same data set as in group tasks 04.
- The data set describes 2049 movies.
- Originally downloaded from <u>https://www.kaggle.com/rounakbanik/the-movies-dataset</u> and preprocessed, keeping only movies with at least 500 viewer votes.

```
url = 'https://bbrejova.github.io/viz/data/Movies_small.csv'
movies = pd.read_csv(url)
display(movies.head())
```

| Led by Woody, Andy's toys live happily in his | 5415.0 | 7.7 | 1995-10-30 | 81.0 | en | 373554033.0 | 30000000.0 | 1995 | Toy Story |
|--|--------|-----|------------|-------|-----|-------------|------------|------|-----------|
| When siblings Judy and Peter discover an encha | 2413.0 | 6.9 | 1995-12-15 | 104.0 | en | 262797249.0 | 65000000.0 | 1995 | Jumanji |
| Obsessive master thief, Neil McCauley leads a | 1886.0 | 7.7 | 1995-12-15 | 170.0 | en | 187436818.0 | 60000000.0 | 1995 | Heat |
| James Bond must unmask the mysterious head of | 1194.0 | 6.6 | 1995-11-16 | 130.0 | en | 352194034.0 | 58000000.0 | 1995 | GoldenEye |
| The life of the gambling paradise – Las | 1242.0 | 7.0 | 100E 11 22 | 170.0 | an. | 116112275.0 | F2000000 0 | 1005 | Cooine |

1995-11-22

7.8

1343.0

overview

Vegas ...

Led by Woody, Andy's toys live happily in

revenue original_language runtime release_date vote_average vote_count

178.0

title year

budget

Casino 1995 52000000.0 116112375.0

Measures of central tendency (miery stredu / polohy)

These represent a **typical value** in a sample $x = x_1, ..., x_n$ (one numerical column).

Mean (priemer)
$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

This is the arithmetic mean, there are also geometric and harmonic means.

Median (medián) is the middle value when the values are ordered by size.

For even *n* usually defined as the average of the two middle values.

Example:

Median of 10, 12, 15, 16, 16 is 15.

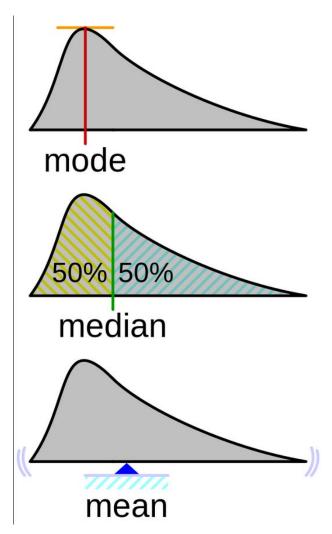
Median of 10, 12, 15, 16, 16, 20 is 15.5.

Measures of central tendency (cont.)

- Mean (priemer)
- Median (medián)
- Mode (modus) is the most frequent value (for a discrete variable).
 Mode of 10,12,15,16,16 is 16.

For continuous variables, we may look for a mode in a histogram.

This is sensitive to bin size.



Mean vs median

Mean can be heavily influenced by outliers

800, 1000, 1100, 1200, 1800, 2000, 30000: mean 5414.3, median 1200 800, 1000, 1100, 1200, 1800, 2000, 10000: mean 2557.1, median 1200

Therefore we often **prefer median** (e.g. median salary).

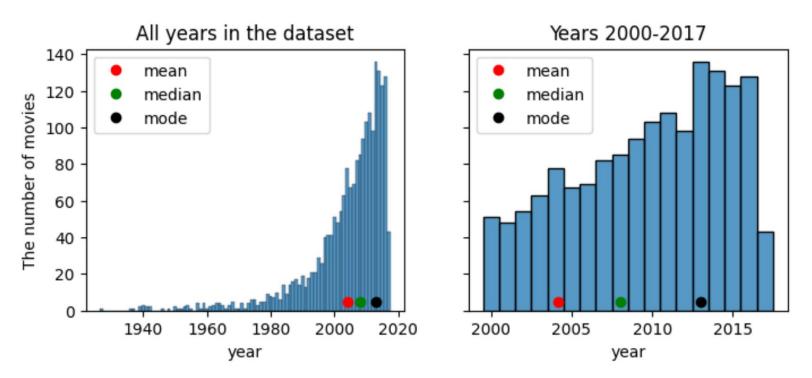
Computation in Pandas

```
display(Markdown("**Properties of the column `year` in our table:**"))
print(f"Mean: {movies['year'].mean():.2f}")
print(f"Median: {movies['year'].median()}")
print(f"Mode:\n{movies['year'].mode()}")
```

Properties of the column year in our table:

```
Mean: 2004.14
Median: 2008.0
Mode:
0 2013
Name: year, dtype: int64
```

Shown in a histogram (whole and detail)



What causes the difference between the mean and the median?

Summarizing many columns or rows

- Functions mean and median can be applied to all numerical columns
- With axis=1 we get means or medians in rows

Quantiles, percentiles and quartiles (kvantily, percentily, kvartily)

Median is the middle value in a sorted order; about 50% of values are smaller and 50% larger.

For a percentage p, the **p-th percentile** is at position roughly $(p/100) \cdot n$ in the sorted order of values.

Similarly **quantile** (in Pandas), but we give fraction between 0 and 1 rather than percentage.

Quartiles are three values Q_1 , Q_2 and Q_3 that split input data into quarters. Therefore, Q_2 is the median.

Quantiles in Pandas

Median:

2008.0

Quantile for 0.5:

2008.0

All quartiles:

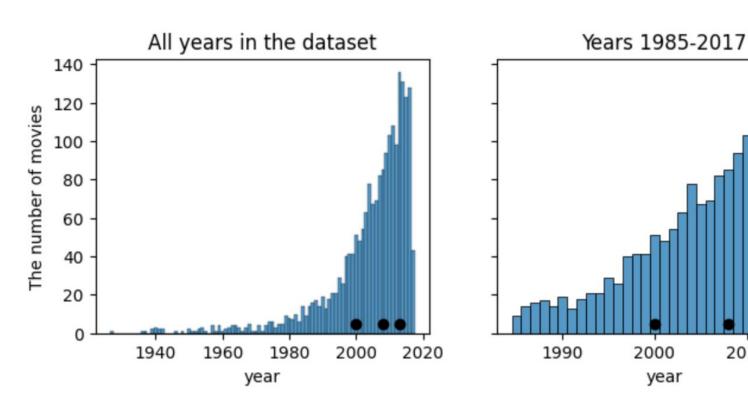
0.25 2,000.00 0.50 2,008.00 0.75 2,013.00

Name: year, dtype: float64

With step 0.1:

0.10 1,988.80 0.20 1,998.00 0.30 2,002.00 0.40 2,005.00 0.50 2,008.00 0.60 2,010.00 0.70 2,012.00 0.80 2,014.00 0.90 2,015.00 Name: year, dtype: float64

Quartiles in a histogram (whole and detail)



2010

Quantile interpolation

Optional parameter interpolation accepts values 'linear' (default), 'lower', 'higher', 'midpoint', 'nearest'.

Minimum is at quantile 0, maximum at quantile 1, the rest evenly spaced between.

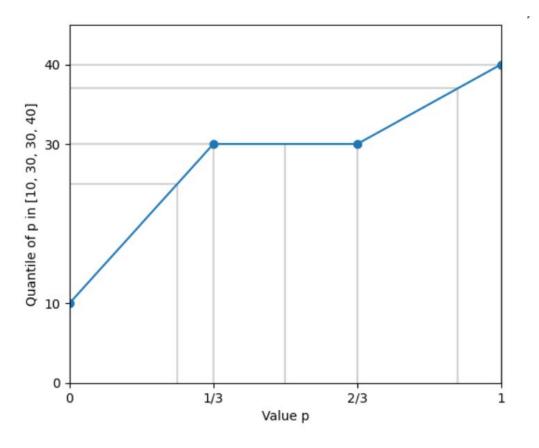
The quantile between two elements is influenced only by its two neighbors.

Example: list [10,30,30,40]

p=0: 10, p=1/3: 30, p=2/3: 30, p=1: 40

p=1/4: by default linear interpolation at 3/4 between 10 and 30, i.e. 25.

Linear interpolation is continuous as p changes from 0 to 1.



Quantiles for [10, 30, 30, 40]

0.00 10.00 0.25 25.00 0.33 30.00 0.50 30.00 0.67 30.00 0.90 37.00 1.00 40.00 dtype: float64

Quantiles for [10, 30, 30, 40] with interpolation='lower'

0.00 10 0.25 10 0.33 10 0.50 30 0.67 30 0.90 30 1.00 40 dtype: int64

Measures of variability (miery variability)

Values in the sample may be close to central values or spread widely.

Examples of measures:

Range of values from minimum to maximum (sensitive to outliers).

Interquartile range IQR (kvartilové rozpätie): range between Q_1 and Q_3 (contains the middle half of the data).

Variance and standard deviation (next)

Variance (rozptyl)

For each x_i square its difference from the mean

Squaring gives non-negative values (and squares are easier to work with mathematically than absolute values).

Variance is the mean of these squares, but we divide by n-1 rather than n:

$$s^{2} = \frac{\sum_{i=1}^{n} (x_{i} - \bar{x})^{2}}{n-1}$$

Division by n would underestimate the true variance of the underlying population (more in the statistics course).

Standard deviation (smerodajná odchýlka)

Square root of the variance

$$s = \sqrt{s^2}$$

It is in the same units as the original values (variance is in units squared).

Recall:

$$s^{2} = \frac{\sum_{i=1}^{n} (x_{i} - \bar{x})^{2}}{n-1}$$

Properties

Larger variance and standard deviation mean that data are spread farther from the mean.

These measures are strongly **influenced by outliers**:

800, 1000, 1100, 1200, 1800, 2000, 30000: st. dev. 10850.0, IQR 850

800, 1000, 1100, 1200, 1800, 2000, 10000: st. dev. 3310.5, IQR 850.

```
display(Markdown("**Minimum**"),
                                                 Minimum
        movies['year'].min())
                                                 1927
display(Markdown("**Maximum**"),
                                                 Maximum
        movies['year'].max())
                                                 2017
display(Markdown("**Mean**"),
                                                 Mean
        movies['year'].mean())
                                                 2004.1449487554905
display(Markdown("**Variance**"),
                                                 Variance
        movies['year'].var())
                                                 161.2714600681735
display(Markdown("**Standard deviation**"),
                                                 Standard deviation
        movies['year'].std())
q1 = movies['year'].quantile(0.25)
                                                 12.699270060447313
q3 = movies['year'].quantile(0.75)
                                                 Q1, Q3 and IQR:
display(Markdown("**Q1, Q3 and IQR:**"),
                                                 2000.0
                                                 2013.0
        q1, q3, q3-q1)
                                                 13.0
```

Outliers (odľahlé hodnoty)

Outliers are the values which are far from the typical range of values.

In data analysis, it is important to check outliers.

If they represent **errors**, we may try to correct or remove them.

They can also represent interesting anomalies.

Different definitions of outliers may be appropriate in different situations.

One possible definition of outliers

The criterion by statistician John Tukey:

Outliers are the values outside of the range $[Q_1 - k \cdot IQR, Q_3 + k \cdot IQR]$, e.g. for k = 1.5.

In our example 800, 1000, 1100, 1200, 1800, 2000, 30000:

$$Q_1 = 1050$$
, $Q_3 = 1900$, $IQR = 850$.

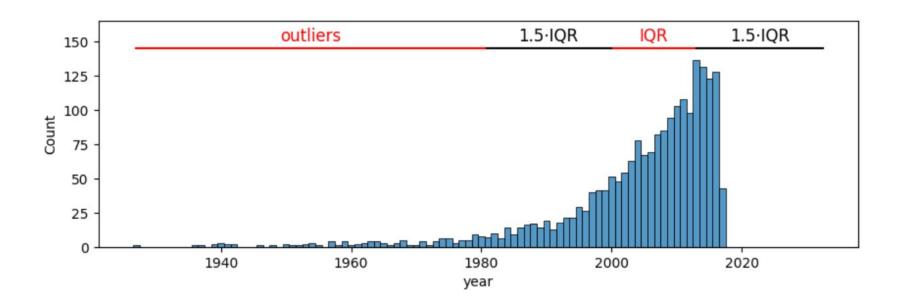
$$Q_1 - 1.5 \cdot IQR = -225$$
, $Q_3 + 1.5 \cdot IQR = 3175$.

Outliers are values smaller than -225 or larger than 3175; here only 30000.

The range of outliers is not influenced if we change the outliers.

Computation in Pandas

```
# get quartiles and iqr
q1 = movies['year'].quantile(0.25)
q3 = movies['year'].quantile(0.75)
iqr = q3 - q1
# compute thresholds for outliers
lower = q1 - 1.5 * iqr
upper = q3 + 1.5 * iqr
# count outliers
count = movies.query('year < @lower or year > @upper')['year'].count()
```

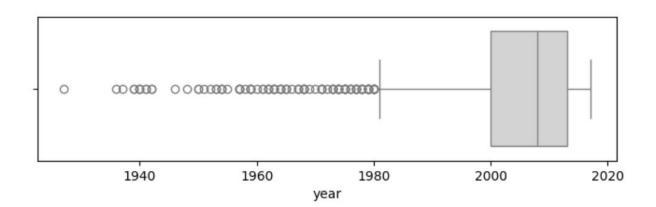


Boxplot (krabicový graf)

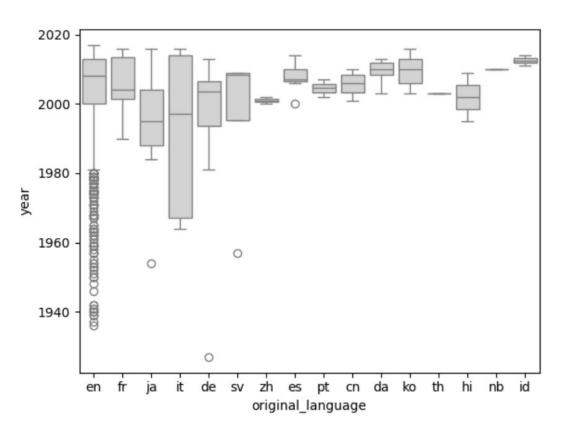
Developed by Mary Eleanor Hunt Spear and John Tukey.

It shows the **five-number summary:** min, Q1, median (Q2), Q3, max

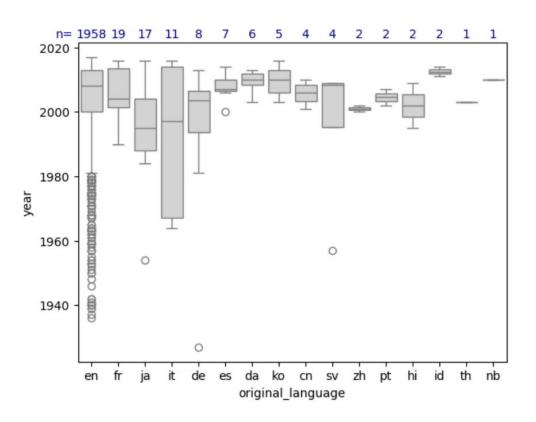
Q1 and Q3: a box, median: line through box, min and max: whiskers Outliers often excluded from the whiskers and shown as points.



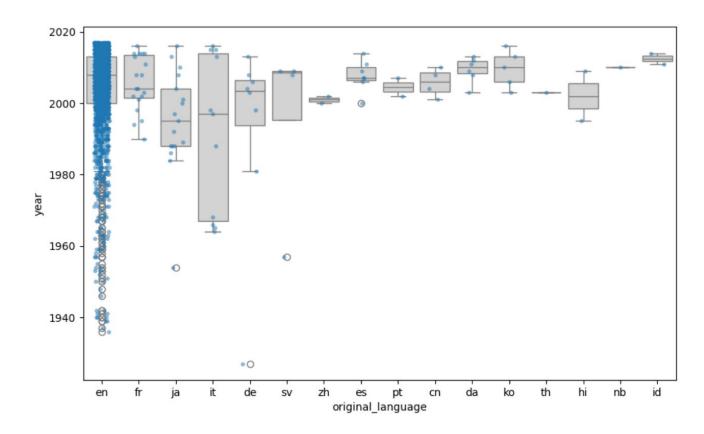
Boxplots are used for quick comparison



Beware: boxplots may hide small sets



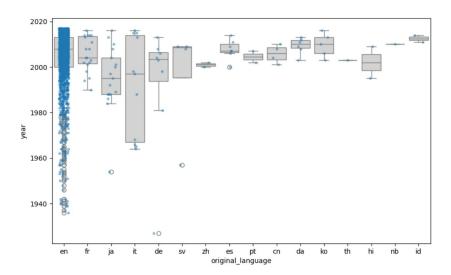
For small datasets we may add strip plot



Works well for smaller languages, mess for en.

What do we see for sv, pt, th, hi, nb, id?

Code for the plot



Quick overview of a data set: describe in Pandas

movies.describe()

| | year | budget | revenue | runtime | vote_average | vote_count |
|-------|----------|----------------|------------------|----------|--------------|------------|
| count | 2,049.00 | 1,959.00 | 1,965.00 | 2,049.00 | 2,049.00 | 2,049.00 |
| mean | 2,004.14 | 55,108,939.70 | 198,565,134.28 | 112.66 | 6.63 | 1,704.64 |
| std | 12.70 | 53,139,663.86 | 233,028,732.94 | 24.76 | 0.77 | 1,607.89 |
| min | 1,927.00 | 1.00 | 15.00 | 7.00 | 4.00 | 501.00 |
| 25% | 2,000.00 | 16,000,000.00 | 52,882,018.00 | 97.00 | 6.10 | 709.00 |
| 50% | 2,008.00 | 38,000,000.00 | 122,200,000.00 | 109.00 | 6.60 | 1,092.00 |
| 75% | 2,013.00 | 75,000,000.00 | 250,200,000.00 | 124.00 | 7.20 | 2,000.00 |
| max | 2,017.00 | 380,000,000.00 | 2,787,965,087.00 | 705.00 | 9.10 | 14,075.00 |

count unique top freq std min mean Beauty and 2018 title 2049 NaN NaN NaN the Beast 2,049.00 NaN NaN NaN 2,004.14 12.70 1,927.00 year

NaN

NaN

NaN

NaN

NaN

Led by Woody,

Andy's toys

live happily in his ...

2014-12-25

en

NaN

NaN

1958

NaN

NaN

NaN

6

55,108,939.70

198,565,134.28

NaN

NaN

6.63

NaN

1.704.64

112.66

53,139,663.86

233,028,732.94

NaN

24.76

NaN

0.77

NaN

1.607.89

25%

NaN

NaN

97.00

NaN

6.10

NaN

709.00

2,000.00

16,000,000.00

52,882,018.00

1.00

15.00

NaN

7.00

NaN

4.00

NaN

501.00

50%

NaN

NaN

NaN

6.60

NaN

1.092.00

109.00

2,008.00

38,000,000.00

122,200,000.00

movies.describe(include='all').transpose()

1,959.00

1.965.00

2.049.00

2.049.00

2.049.00

2049

2049

2049

NaN

NaN

16

NaN

1740

NaN

NaN

2049

budget

revenue

original_language

runtime

release date

vote_average

vote count

overview

Linear transformation of variables

Assume we apply **linear transformation** $a \cdot x_i + b$ with the same a and b to all x_i .

This corresponds e.g. to the change in the units of measurement

- grams to kilograms a=0.001, b=0
- degrees C to degrees F a=9/5, b=32

What happens to various statistics, e.g. mean?

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Linear transformation of variables

Assume we apply **linear transformation** $a \cdot x_i + b$ with the same a and b to all x_i .

What happens to various statistics, e.g. mean?

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\frac{(ax_1 + b) + (ax_2 + b) + \dots + (ax_n + b)}{n} = \frac{a(x_1 + \dots + x_n) + nb}{n} = a\bar{x} + b.$$

Linear transformation of variables

Assume we apply **linear transformation** $a \cdot x_i + b$ with the same a and b to all x_i .

Mean, median, mode are transformed in the same way

Neither variance nor standard deviation change with b. Variance is multiplied by a^2 , standard deviation by |a|.

$$s^{2} = \frac{\sum_{i=1}^{n} (x_{i} - \bar{x})^{2}}{n-1}$$

$$s = \sqrt{s^{2}}$$

Correlation (korelácia)

We are often interested in **relationships** among variables (data columns)

Next: two correlation coefficients that measure strength of such relationships

Beware: correlation does not imply causation

Correlation does not imply causation

If electricity consumption grows in a very cold weather, there might be **cause-and-effect** relationship: the cold weather is causing people to use more electricity for heating.

If healthier people tend to be happier, which is the cause and which is effect?

Both studied variables can be also influenced by some third, **unknown factor**. For example, within a year, deaths by drowning increase with increased ice cream consumption. Both increases are spurred by warm weather.

The observed correlation can be just a **coincidence**, see the <u>Redskins rule</u> and a specialized webpage <u>Spurious Correlations</u>. You can easily find such "coincidences" by comparing many pairs of variables.

Pearson correlation coefficient

It measures linear relationship between two variables.

Consider pairs of values $(x_1, y_1), \dots, (x_n, y_n)$, where (x_i, y_i) are two different features of the same object.

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{n} (y_i - \bar{y})^2}}$$
$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s_x}\right) \left(\frac{y_i - \bar{y}}{s_y}\right).$$

where s_x is the standard deviation of variable x.

Pearson correlation coefficient

$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right).$$

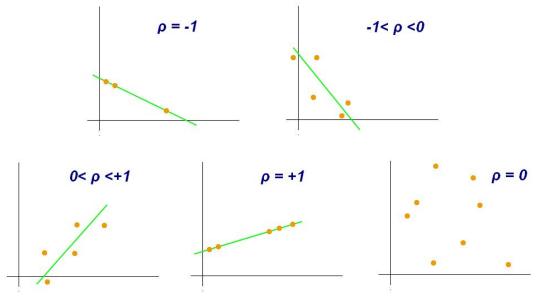
Expression $(x_i - \bar{x})/s_x$ is called the **standard score** or **z-score**: how many standard deviations above or below the mean is x_i ?

The product of z-scores for x_i and y_i is positive iff they lie on the same side of the respective means of x and y.

Properties of Pearson correlation coefficient

The value of r is always from interval [-1,1].

- 1 if y grows linearly with x,
- -1 if *y* decreases linearly
- 0 means no correlation.

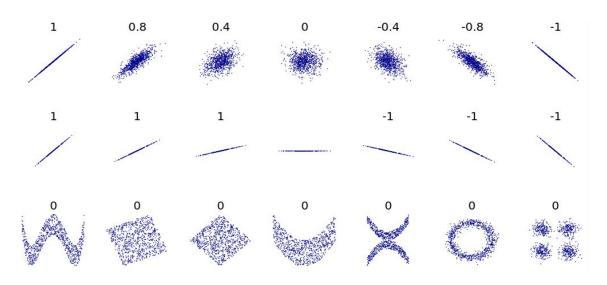


https://commons.wikimedia.org/wiki/File:Correlation_coefficient.png

Some cautions

Pearson correlation measures only linear relationships (bottom row)

Pearson correlation does not depend on the slope of the best-fit line (middle row)



https://commons.wikimedia.org/wiki/File:Correlation_examples2.svg

Properties of Pearson correlation

What if we linearly scale each variable, i.e. ax_i+b , cy_i+d ? What if a>0, a=0, a<0?

What if we switch x and y?

$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right).$$

Properties of Pearson correlation

Pearson correlation does not change if we linearly scale each variable, i.e. ax_i+b , cy_i+d (for a,c>0).

Pearson correlation is symmetric wrt. *x* and *y*.

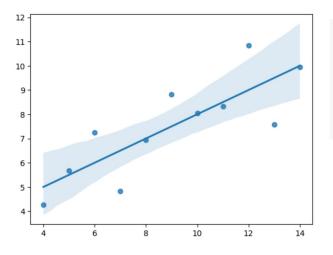
Due to reliance on mean and std.dev. it is sensitive to outliers.

$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right).$$

Linear regression

The process of finding the line best representing the relationship of x and y. In higher dimensions we can predict one variable as a linear combination of many. You will study linear regression in later courses.

We may draw regression lines in some plots.



Spearman's rank correlation coefficient

It can detect non-linear relationships.

We first convert each variable into ranks:

Rank of x_i is its index in the sorted order of x_1, \dots, x_n .

Equal values get the same (average) rank.

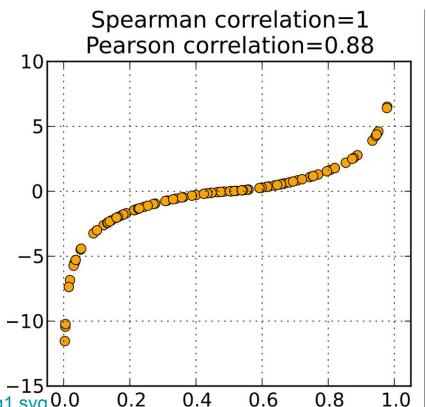
For example, the ranks of 10, 0, 10, 20, 10, 20 are 3, 1, 3, 5.5, 3, 5.5.

Then we compute **Pearson correlation coefficient of the two rank sequences**.

Properties of Spearman's rank correlation coeficient

Values of 1, -1 if *y* monotonically increases or decreases with *x*.

It is less sensitive to outliers (actual values of *x* and *y* are not important).



https://commons.wikimedia.org/wiki/File:Spearman fig1.svg 0.0 0.2 0.4 0.6 0.8 1.0

Computation in Pandas

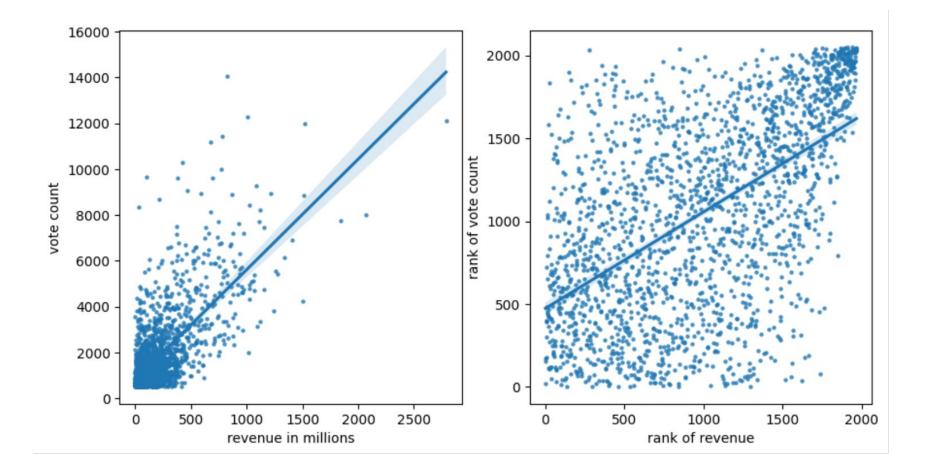
movies.corr(numeric_only=True)

| | year | budget | revenue | runtime | vote_average | vote_count |
|--------------|-------|--------|---------|---------|--------------|------------|
| year | 1.00 | 0.28 | 0.12 | -0.07 | -0.34 | 0.12 |
| budget | 0.28 | 1.00 | 0.69 | 0.22 | -0.18 | 0.47 |
| revenue | 0.12 | 0.69 | 1.00 | 0.25 | 0.06 | 0.69 |
| runtime | -0.07 | 0.22 | 0.25 | 1.00 | 0.31 | 0.25 |
| vote_average | -0.34 | -0.18 | 0.06 | 0.31 | 1.00 | 0.33 |
| vote_count | 0.12 | 0.47 | 0.69 | 0.25 | 0.33 | 1.00 |

Computation in Pandas (cont.)

movies.corr(method='spearman', numeric_only=True)

| | year | budget | revenue | runtime | vote_average | vote_count |
|--------------|-------|--------|---------|---------|--------------|------------|
| year | 1.00 | 0.21 | 0.02 | -0.03 | -0.27 | 0.14 |
| budget | 0.21 | 1.00 | 0.68 | 0.24 | -0.28 | 0.37 |
| revenue | 0.02 | 0.68 | 1.00 | 0.21 | -0.08 | 0.56 |
| runtime | -0.03 | 0.24 | 0.21 | 1.00 | 0.32 | 0.27 |
| vote_average | -0.27 | -0.28 | -0.08 | 0.32 | 1.00 | 0.29 |
| vote_count | 0.14 | 0.37 | 0.56 | 0.27 | 0.29 | 1.00 |



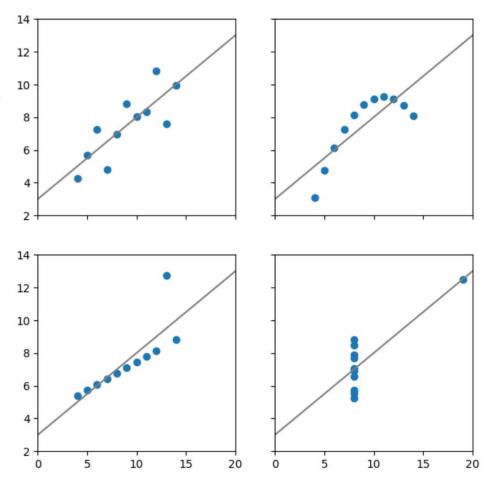
```
figure, axes = plt.subplots(1, 2, figsize=(10,5))
# plot of values
sns.regplot(x=movies['revenue'] / 1e6, y=movies['vote count'],
            ax=axes[0], scatter kws={'alpha':0.7, 's':5})
axes[0].set xlabel('revenue in millions')
axes[0].set ylabel('vote count')
# compute ranks
revenue rank = movies['revenue'].rank()
vote count rank = movies['vote count'].rank()
# plot of ranks
sns.regplot(x=revenue rank, y=vote count rank,
            ax=axes[1], scatter kws={'alpha':0.7, 's':5})
axes[1].set xlabel('rank of revenue')
axes[1].set ylabel('rank of vote count')
```

1500 2000 2500

Anscombe's quartet

Four **artificial data sets** designed by Francis Anscombe.

The same or very similar values of: means and variances of *x* and *y*, Pearson correlation coefficient (0.816) and linear regression line.



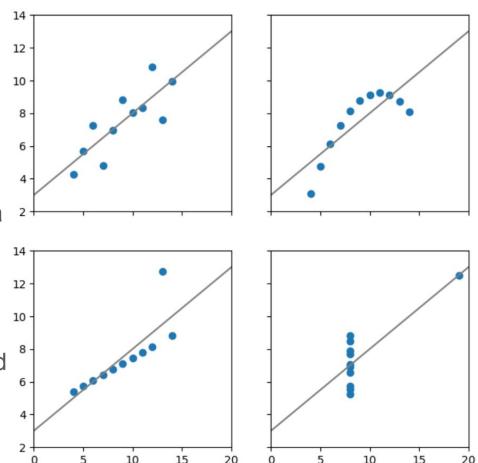
Anscombe's quartet

The same summary statistics, but visually very different

Importance of visualization:

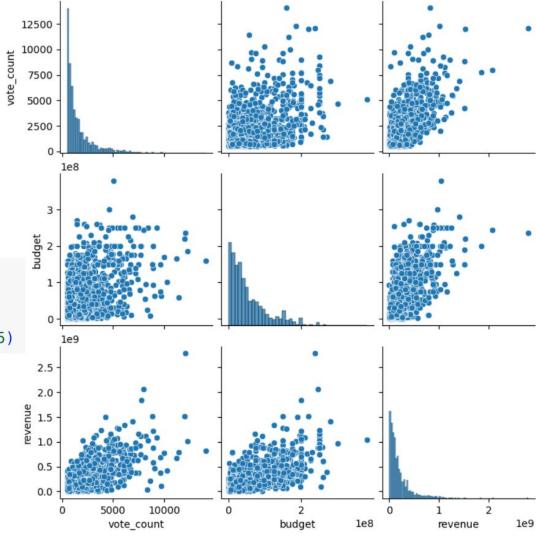
Plots often give us a much better idea of the properties of a data set than simple numerical summaries.

The bottom row illustrates the influence of outliers on correlation and regression.

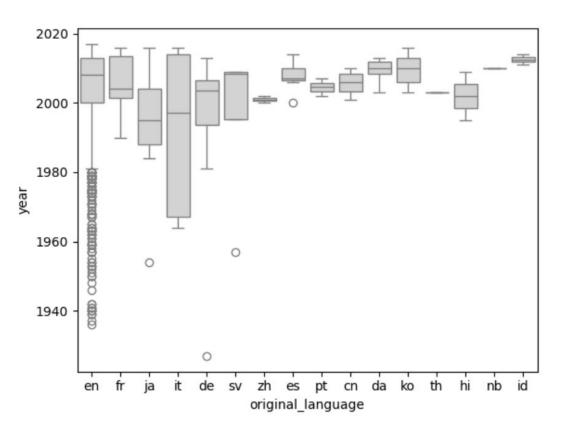


Visual overview of a data set: pairplot in Seaborn

All histograms + scatterplots



How to compute summaries for groups?



How to compute summaries for groups?

Method groupby splits the table into groups based on values of some column.

We can apply a summary statistics function or describe on each group.

movies.groupby('original_language').median(numeric_only=True).head()

40,000,000.00

2,000,000.00

year

2,008.00

2,007.00

original language

en

es

budget

| ogatag.aage | | | | | | |
|-------------|----------|---------------|---------------|--------|------|--------|
| cn | 2,006.00 | 12,902,809.00 | 39,388,380.00 | 108.50 | 7.20 | 762.50 |
| da | 2,010.00 | 10,000,000.00 | 16,740,418.00 | 119.00 | 6.80 | 867.50 |
| de | 2,003.50 | 6,250,000.00 | 70,000,000.00 | 129.00 | 7.60 | 669.00 |

126,397,819.00

30,448,000.00

revenue runtime vote_average vote_count

6.60

7.60

1,126.00

797.00

109.00

118.00

```
subset = movies.loc[:, ['original_language', 'year', 'budget']]
```

| subset. | <pre>groupby('original_la</pre> | nguage').desc | cribe().head(| 3).transpo | se() |
|---------|---------------------------------|---------------|---------------|------------|------|
| | original_language | cn | da | de | |
| year | count | 4.00 | 6.00 | 8.00 | |
| | mean | 2,005.75 | 2,009.33 | 1,992.50 | |
| | std | 4.03 | 3.61 | 28.13 | |
| | min | 2,001.00 | 2,003.00 | 1,927.00 | |
| | 25% | 2,003.25 | 2,008.25 | 1,993.75 | |

| | std | 4.03 | 3.61 | 28.13 |
|-------|-------|----------|----------|----------|
| ıdget | min | 2,001.00 | 2,003.00 | 1,927.00 |
| | 25% | 2,003.25 | 2,008.25 | 1,993.75 |
| | 50% | 2,006.00 | 2,010.00 | 2,003.50 |
| | 75% | 2,008.50 | 2,011.75 | 2,006.50 |
| | max | 2,010.00 | 2,013.00 | 2,013.00 |
| | count | 3.00 | 5.00 | 8.00 |
| | | | | |

mean

14,872,795.67 13,440,000.00 18,223,718.75

Summary

Summary statistics:

- mean, median, mode
- percentiles, quantiles, quartiles
- min, max, interquartile range, variance, standard deviation
- Pearson and Spearman correlation

More details in a statistics course

Visualization:

- boxplot
- scatter plots with regression lines
- pairplot

Pandas:

- functions for computing statistics, describe
- groupby

Next week: more Pandas