Friendliness between AIMD Algorithms

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Abstract

This paper aims to provide a robust grounding for the additive increase factor used in the 'TCP-Friendly' mode of the CUBIC congestion control algorithm.

1 Introduction

The first IETF RFC to define the CUBIC congestion control algorithm [RXH+18] was based on the original paper introducing CUBIC [HRX08]. For 'TCP-friendly' mode, both draw on an equation in an ACIRI technical report [FHP00] when they specify the additive increase factor. The derivation of the equation in that technical report assumes a deterministic dropping (or ECN-marking) algorithm at the bottleneck, which limits its applicability. Also the technical report attempts to validate the theoretical formula empirically by simulation using a RED gateway at the bottleneck, but it leads to flow rates that are significantly different (by a factor of more than 2×) when they should be the same.

Below, an equation for the additive increase factor is derived without the assumption of deterministic dropping. Instead it is assumed that drops are synchronized between flows, which is typically the case for tail-drop queues. The resulting equation turns out to be the same as that in the technical report [FHP00]. However, the derivation here is a straightforward geometric one. It relies on fewer assumptions and no approximations; it considers variation of the RTT explicitly and it does not use loss probability at all.

The present paper is not intended to be ambitious or insightful, just pedestrian and rigorous. [BB01] provides and analyses a wider set of TCP-friendly algorithms, but does not dwell on the simple linear cases analysed here.

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2 Terminology

Nowadays, TCP-friendly mode is more accurately known as Reno-friendly mode, given its flow rate is intended to match that of the Reno congestion control, and given that it is irrelevant which wire protocol is used, whether TCP, QUIC, SCTP, etc. The term C-Reno will be used for CUBIC in Reno-friendly mode.

This paper uses the variables defined below:

a : Additive increase factor;
b : Multiplicative decrease factor;
j : Round index;
J : Rounds per sawtooth cycle;
$R(j)$: Round trip time (RTT);
$W(j)$: Congestion window;
$\widehat{W}$: Maximum W;
$r(j)$: Packet rate;
X_r : Reno variant of any variable X;
X_c : C-Reno variant of any variable X.

3 AIMD-Friendliness

Consider two types of Additive Increase Multiplicative Decrease (AIMD) flow with parameters (a_r, b_r) and (a_c, b_c) competing at a bottleneck, under the following assumptions:

- The buffer is large enough not to drain completely, even if all flows reduce simultaneously (this assumption is relaxed later).
- All other factors of all the flows, particularly packet size and base RTT, are equal. When flows sharing the same bottleneck queue all have the same base RTT, they all have the same RTT, $R(j)$, at every stage, j, of their sawtooth cycles.
- The bandwidth-delay product (BDP) is low enough for all flows to remain in their AIMD mode throughout the cycle.
- All the flows have run long enough to converge to a steady state.
- All flows only respond to the presence of loss, not its extent.

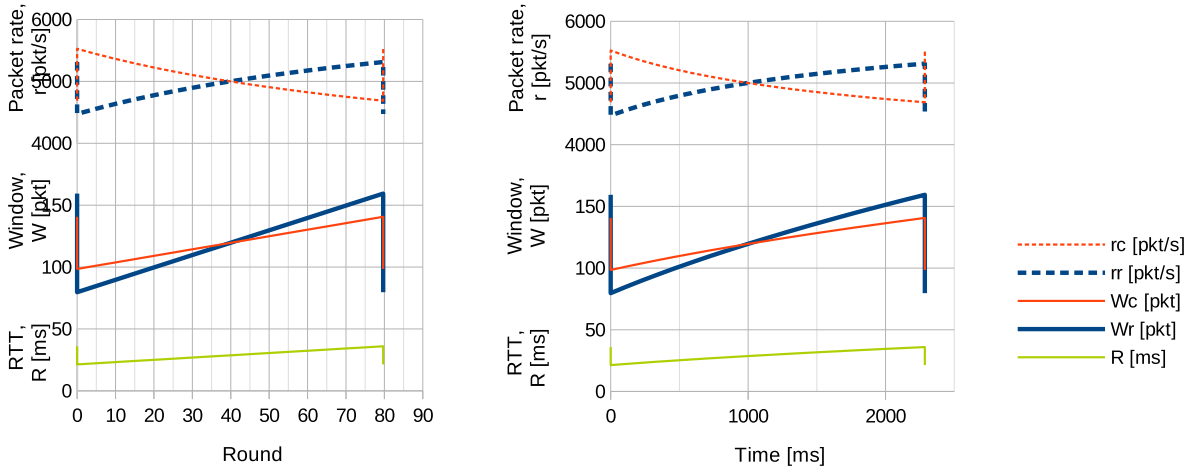

Figure 1: One synchronized sawtooth cycle of C-Reno and Reno plotted wrt. round trips (left) and wrt. time (right)

3.1 Synchronized (Tail-Drop) Case

For this case, it is additionally assumed that:

- All the flows are synchronized so that, whenever one flow experiences loss the others do too;
- All flows experience at least one loss at each congestion event (relaxation of this assumption is discussed later);

Steady state: For each flow, the additive increase of a cycle balances with its multiplicative decrease from the max, $\check{W}/b$, to the min, $\check{W}$.

$$\begin{aligned} a_r J &= \widehat{W}_r - b_r \widehat{W}_r \\ &= \widehat{W}_r (1 - b_r) \qquad (1) \\ a_c J &= \widehat{W}_c (1 - b_c) \qquad (2) \end{aligned}$$

Flow rate equality: Given the parameters a_r, b_r, b_c the aim is to derive a_c such that each flow's average rate is the same. This is equivalent to each flow transferring the same number of packets over a cycle.

As a cycle progresses, the RTT grows. So to derive the number of packets transferred over a cycle, the packet rate has to be weighted by the RTT in each cycle before being summed:

$$\begin{aligned} \sum_{j=0}^{J-1} r_c(j) R(j) &= \sum_{j=0}^{J-1} r_r(j) R(j) \\ \sum_{j=0}^{J-1} W_c(j) &= \sum_{j=0}^{J-1} W_r(j) \qquad (3) \\ \sum_{j=0}^{J-1} b_c \widehat{W}_c + a_c j &= \sum_{j=0}^{J-1} b_r \widehat{W}_r + a_r j \\ J b_c \widehat{W}_c + \frac{J^2 a_c}{2} &= J b_r \widehat{W}_r + \frac{J^2 a_r}{2} \end{aligned}$$

Dividing through by J and substituting from Equation 1 & Equation 2:

$$\begin{aligned} \widehat{W}_c \left(b_c + \frac{(1 - b_c)}{2} \right) &= \widehat{W}_r \left(b_r + \frac{(1 - b_r)}{2} \right) \\ \frac{\widehat{W}_c}{\widehat{W}_r} &= \frac{(1 + b_r)}{(1 + b_c)} \qquad (4) \end{aligned}$$

Returning to the steady state equations, we can divide Equation 2 by Equation 1, then substitute from Equation 4:

$$\begin{aligned} \frac{a_c}{a_r} &= \frac{\widehat{W}_c}{\widehat{W}_r} \frac{(1 - b_c)}{(1 - b_r)} \\ &= \frac{(1 - b_c)}{(1 + b_c)} \frac{(1 + b_r)}{(1 - b_r)} \qquad (5) \end{aligned}$$

Plugging in Reno's AIMD factors, $a_r = 1, b_r = 1/2$:

$$\boxed{a_c = \frac{3(1 - b_c)}{(1 + b_c)}} \qquad (6)$$

And plugging in the multiplicative decrease factor of C-Reno recommended in [RXH+18], $b_c = 0.7$:

$$\begin{aligned} a_c &= 9/17 \\ &\approx 0.53. \end{aligned}$$

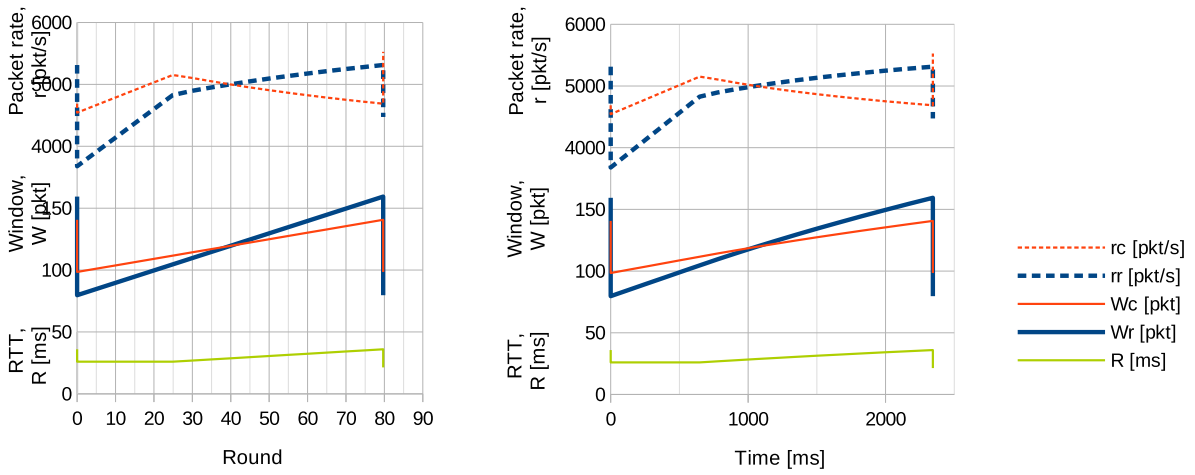

Figure 2: Similar synchronized sawtooth cycle of C-Reno and Reno to Figure 1, but with a 11 ms buffer that is too shallow to accommodate both sawteeth at once.

3.1.1 Geometric interpretation

Figure 1 shows one flow each of C-Reno and Reno competing over one synchronized sawtooth cycle. Superficially, the whole derivation of a_c above can be derived from simple triangle geometry, by drawing congestion window sawteeth that increase linearly wrt. round trips (mid-left) then setting the mid-points of the two ramps to the same height. Then Equation 4 gives the ratio between the heights of the bases of the triangles, and Equation 5 gives the ratio of the heights of the triangles themselves.

However, it is not enough to merely assert that the average heights of these sawteeth are equal, as [FHP00] does. Strictly, it is necessary to start from the goal of equal flow rates averaged over time, as the above analysis does. Given RTT grows throughout the cycle, the plots of flow-rate against time (top right in Figure 1) stretch out more to the right, forming concave curves. It is not at all obvious how to equate the averages of these two curves until they are weighted by round trip duration, which transforms them into the linear plots of window wrt. rounds (mid-left), as in Equation 3.

It is also interesting to note from Figure 1 that C-Reno's packet rate *decreases* as its window increases over the sawtooth. This is because the competing Reno flow causes the RTT to grow faster than would be the case with only C-Reno flows.

If the buffer is not deep enough to hold all the synchronized sawteeth, it will be empty during the early part of the sawteeth. Then both flows will under-perform during the early part of the cycle when C-Reno would have achieved its highest packet rate and Reno would have achieved its lowest, as shown in Figure 2. Nonetheless, the rate of both flows reduces proportionately, so the ratio between their flow rates remains unaltered.

3.1.2 Synchronized losses?

We will now question the assumption that each flow always catches at least one loss at each congestion event. We still consider two flows with equal RTTs: 1 Reno and 1 CReno. Between responses to losses, the queue grows inexorably by $(a_r + a_c \approx 1.53)$ pkt/RTT. Every time the buffer fills, one packet has to be dropped, but the queue continues to grow by 1.53 segments during the next round trip (until the resulting response reaches the queue). So it is likely that another packet will have to be discarded within the same RTT as the first.

The likelihood that a particular flow catches any one of the losses depends on its packet rate relative to the other.[1] That is,

$$\frac{p_r}{p_c} = \frac{\widehat{r_r}}{\widehat{r_c}}. \tag{7}$$

If the flows both have the same average window (the goal in Figure 1) then, by Equation 4 (or triangle geometry), the ratio between the packet rates of the two flows when they both reach their max is

$$\frac{\widehat{r_r}}{\widehat{r_c}} = \frac{(1+b_c)}{(1+b_r)} = \frac{1.7}{1.5} \approx 1.13. \tag{8}$$

When a loss occurs, from Equation 7 & Equation 8, we can say that:

$$p_r/p_c = 17/15 \tag{9}$$

$$\text{and} \quad p_r + p_c = 1, \tag{10}$$

[1] Irrespective of how rapidly the flow's own window grows.

because one or the other flow was hit with certainty.

Therefore

$$\begin{aligned} p_r &= 17/32 & &\approx 53\% \\ p_c &= 15/32 & &\approx 47\% \end{aligned}$$

Then, for example, if there are two losses during a congestion event, the probabilities of each combination of two losses are:

$$\begin{aligned} p_{rr} &= (17/32)^2 & &\approx 28\% \\ p_{rc} \vee p_{cr} &= 17 * 15/32^2 + 15 * 17/32^2 & &\approx 50\% \\ p_{cc} &= (15/32)^2 & &\approx 22\%, \end{aligned}$$

where p_{ij} is the probability of a loss from flow i then j.

When there are two losses in a round and the same flow is hit twice, it doesn't reduce any more than if it's hit once, but the other flow doesn't reduce at all. So CReno is somewhat more likely than Reno to not get hit in some round. In such cases, only the Reno flow would reduce, then the queue would continue to grow by $(a_r + a_c \approx 1.53)$ pkt/RTT, so the next cycle would be shorter and the CReno flow would be much more likely to be hit when it next filled the buffer — and more likely to be hit twice.

It would be possible to calculate the average rate of each type of flow by calculating the probabilities of each chain of events programmatically. However, such precision is unnecessary. For the case of tail-drop buffers, it will be sufficient to say:

- either that the AI factor of CReno should be slightly lower than that derived from Equation 6 to make CReno and Reno flow rates more precisely equal;
- or that the average rate of CReno flows will be slightly higher in comparison with Reno flows, if Equation 6 is used.

Here, "slightly" means roughly within a 10% margin of error.

3.1.3 Empirical Results: Tail Drop

Bob: TBA

3.2 Desynchronized (AQM) Case

For this case, instead of the assumption of synchronization, it is assumed that:

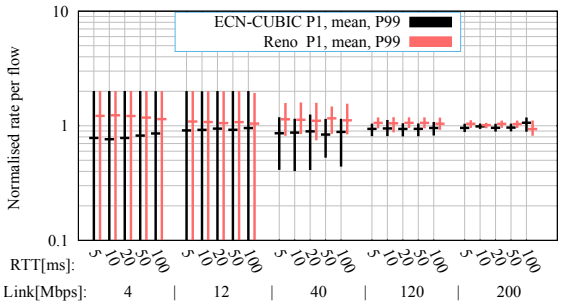

Figure 3: Empirical results comparing the flow rates of CUBIC and Reno competing 1:1 over a tail-drop queue, with CUBIC's Reno-friendly algorithm using AI factor $a_c = 0.53$ as calculated from Equation 6 with $b_c = 0.7$ (the defaults in Linux).

- As the queue grows, Active Queue Management (AQM) at the bottleneck selects single packets to drop or mark, so that the congestion responses of each flow tend not to coincide.[2]

The AQM case is harder to analyse than the synchronized case with tail-drop. Superficially, one could use the transformation from equal average flow rates (wrt. time) into equal window (wrt. rounds). However, there is no guarantee that the number of rounds per cycle, J is the same in each case.

If we assumed it was, we would end up with Equation 6 for C-Reno's additive increase factor a_c. Then, as shown in Figure 4, the phasing between the sawteeth would evolve so that the queue reached roughly the same depth before each reduction, i.e. the tips of the RTT sawteeth will all align at roughly the same level—the operating point of the AQM.

However, although Figure 4 shows the sawtooth reductions alternating Reno – C-Reno – Reno, this need not be the case. In the cycle after 400 ms in the top-right plot, the ratio between C-Reno's and Reno's packet rates is about 51:49. So it is nearly as likely that the AQM will hit a Reno packet as a C-Reno packet, causing Reno to reduce twice in a row. If the AQM did hit C-Reno around 400 ms, at around 600 ms the ratio would be about 42:58, making Reno more likely to be hit. Overall, if a_c is set as for the synchronized case (Equation 6), an AQM is likely to hit Reno somewhat more often than C-Reno.

[2] In desynchronized cases, the RTT varies less than in synchronized cases—on average the amplitude is respectively $1/\sqrt{n}$ vs. n times that of a single flow, where n is the number of flows [AKM04]. Therefore, the first assumption listed in § 3 (that the buffer is large enough not to drain completely) is much more likely to hold in desynchronized cases.

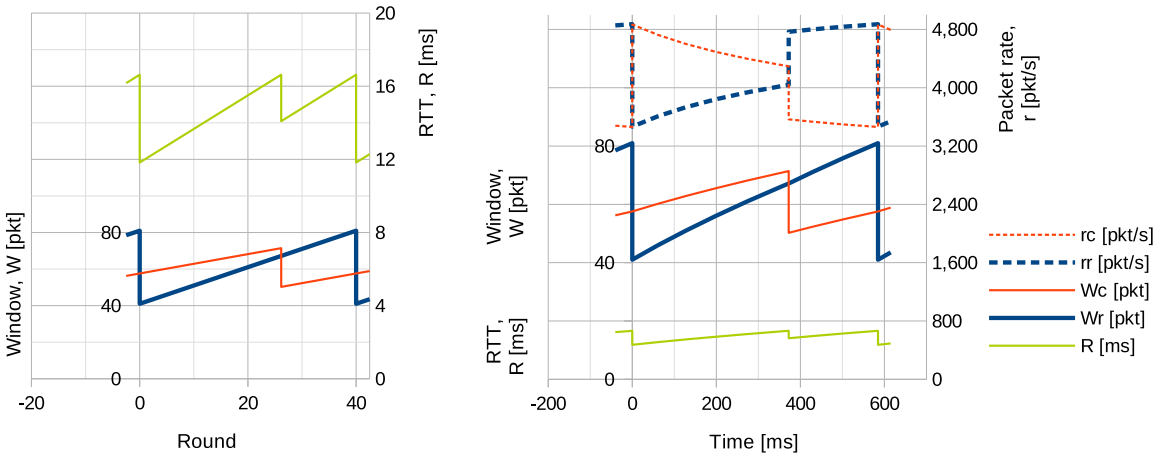

Figure 4: One desynchronized sawtooth cycle of C-Reno and Reno plotted wrt. round trips (left) and wrt. time (right)

In the absence of a tractable analytical model for the AQM case, in the next section we test empirically whether the model in Equation 6 applies to the AQM case.

3.2.1 Empirical Results: PIE AQM

A single long-running CReno flow with $b_c = 0.7$ and AI factor $a_c = 1.53$ was tested against a single Reno flow over a PIE AQM at the bottleneck of a range of 25 different paths with five different link rates and five different RTTs, as shown on the x-axis of **??**.[3] Also, various combinations of numbers of each flow type were tested, as shown along the x-axis of Figure 6. For instance, in the top plot of Figure 6, A2-B8 means 2 ECN-CUBIC flows vs. 8 (non-ECN) CUBIC flows. And in the bottom plot it means 2 ECN-CUBIC flows vs. 8 (non-ECN) Reno flows. All the different combinations of flow numbers were run over a path with base RTT 10 ms at the 40 Mb/s bottleneck, which was chosen to have low enough BDP for CUBIC to remain in its Reno-friendly mode.

In all cases the results are shown as whisker plots that show 1st %-ile (P1), mean and 99th %-ile (P99) normalized flow rate. Normalized flow rate is defined as the flow rate relative to $1/N$ of the capacity, where N is the total number of flows.

In the 1 vs. 1 tests (**??**), it can be seen that CReno and Reno share the bandwidth roughly equally except for cases with higher BDP. Thus, although an analytical model of CUBIC's Reno-friendly mode has not been produced for the AQM case, the empirical results show that, at least in Reno-friendly mode, CUBIC's rate is indeed equivalent to Reno's.

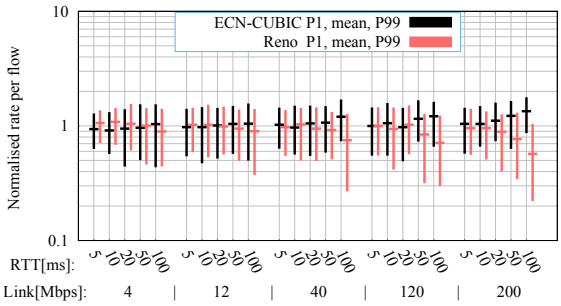

Figure 5: Empirical results with set up similar to Figure 3 except over a PIE AQM.

As BDP increases, CUBIC can be seen to start taking a greater share of capacity, as it increasingly operates beyond its Reno-friendly mode. The switch-over value closely matches that predicted from equation (6) in [Bri21] with 10 ms added to the base RTT for the average height of the sawtooth against PIE's target queue (as explained in § 3.4 and plotted in Figure 5, both referring to that paper).[4]

In the multi-flow tests over the PIE AQM (Figure 6), it can be seen that CReno gives roughly the same rate as Reno over the full range of tests. Indeed, if anything, CReno seems to be slightly less aggressive than Reno. However, the discrepancy is hardly visible, and more than an order of magni-

[3] In each run flow rate measurement was delayed until the flows had converged, then taken every second for 250 s. The resulting 250 measurements were analysed to calculate the percentiles. The two types of flow were sent from two sending hosts to two receiving hosts via a bridge and a bottleneck AQM node. All machines were running Linux kernel v5.10.31. All CCs and AQMs were configured with default parameters.

[4] Further discussion of pure CUBIC mode is outside the scope of the present paper, which focuses on AIMD.

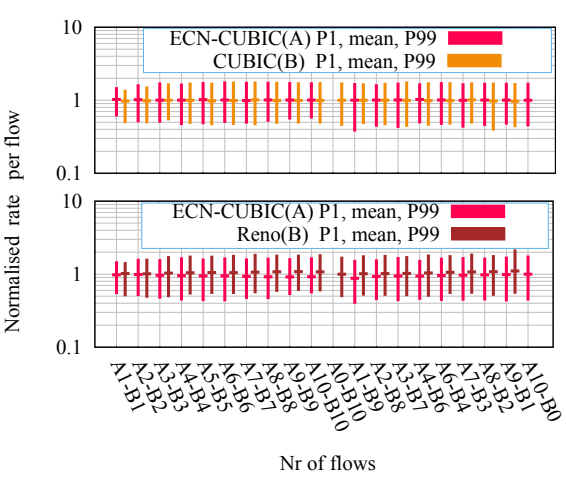

Figure 6: Average flow rates of different numbers of long-running CUBIC and Reno flows, normalized to an equal share of the capacity. AQM: PIE; link capacity: 40 Mb/s; Base RTT: 10 ms. All other parameters the same as in Figure 5.

tude smaller than the range of the ratio's natural variability between its P1 and P99. Note that in this test the BDP is always low enough to keep TCP CUBIC in its Reno-friendly mode.

4 Conclusion

In the simulations using the RED AQM in Floyd's report [FHP00], CReno throughput was 70% of that of competing Reno flows when C-Reno used $b_c = 7/8$ and a_c was set according to Equation 6. No-one has been able to explain those results.

Nonetheless, the present report shows that, with the widely deployed decrease factor of $b_c = 0.7$, the Reno-Friendly mode in CUBIC is sufficiently well modelled by Equation 6 that the Additive Increase factor it produces ($a_c = 0.53$) ensures that TCP CUBIC competes roughly equally with Reno across its intended operating range whether with a tail-drop queue or a single-queue AQM (PIE) at the bottleneck.

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Document history

Version	Date	Author	Details of change
00A	29 Sep 2021	Bob Briscoe	First draft.
00B	01 Oct 2021	Bob Briscoe	Added geometric interpretation and deterministic case.
00C	03 Aug 2022	Bob Briscoe	Added shallow buffer case, empirical results over PIE and discussion of the applicability of the synchronized loss model. Added Acks section. Altered analysis to use max window not min.
00D	08 Aug 2022	Bob Briscoe	Added explicit step at Equation 7 that was previously implicit.