Arrival Estimation in Emergency Departments

Context and Data

Emergency Room @ Maggiore Hospital

We will now consider a problem from the healthcare sector

We will use a dataset for the "Maggiore" hospital in Bologna

- In particular, we will focus on predicting arrivals
- ...To the Emergency Department (Pronto Soccorso)

A Look at the Dataset

We will start as usual by having a look at the dataset

In [59]:
```
data = util.load_ed_data(data_file)
data
```

Out[59]:

	year	ID	Triage	TkCharge	Code	Outcome
0	2018	1	2018-01-01 00:17:33	2018-01-01 04:15:36	green	admitted
1	2018	2	2018-01-01 00:20:33	2018-01-01 03:14:19	green	admitted
2	2018	3	2018-01-01 00:47:59	2018-01-01 04:32:30	white	admitted
51238	2018	51239	2018-01-01 00:49:51	NaT	white	abandoned
51240	2018	51241	2018-01-01 01:00:40	NaT	green	abandoned
...	...	...	...	...	...	...
95665	2019	95666	2019-10-31 23:26:54	2019-10-31 23:41:13	yellow	admitted
95666	2019	95667	2019-10-31 23:46:43	2019-11-01 09:30:25	green	admitted
108622	2019	108623	2019-10-31 23:54:05	NaT	green	abandoned
95667	2019	95668	2019-10-31 23:55:32	2019-11-01 00:18:46	yellow	admitted
108623	2019	108624	2019-10-31 23:59:21	NaT	green	abandoned

108625 rows × 6 columns

A Look at the Dataset

Dataset fields and there are four relevant fields:

In [60]:
```
data.iloc[:2]
```

Out[60]:

	year	ID	Triage	TkCharge	Code	Outcome
0	2018	1	2018-01-01 00:17:33	2018-01-01 04:15:36	green	admitted
1	2018	2	2018-01-01 00:20:33	2018-01-01 03:14:19	green	admitted

- `Triage` is the arrival time of each patient
- `TKCharge` is the time when a patient starts the first visit
- `Code` refers to the estimated priority (white < green < yellow < red)
- `Outcome` discriminates some special conditions (people quitting, fast tracks)

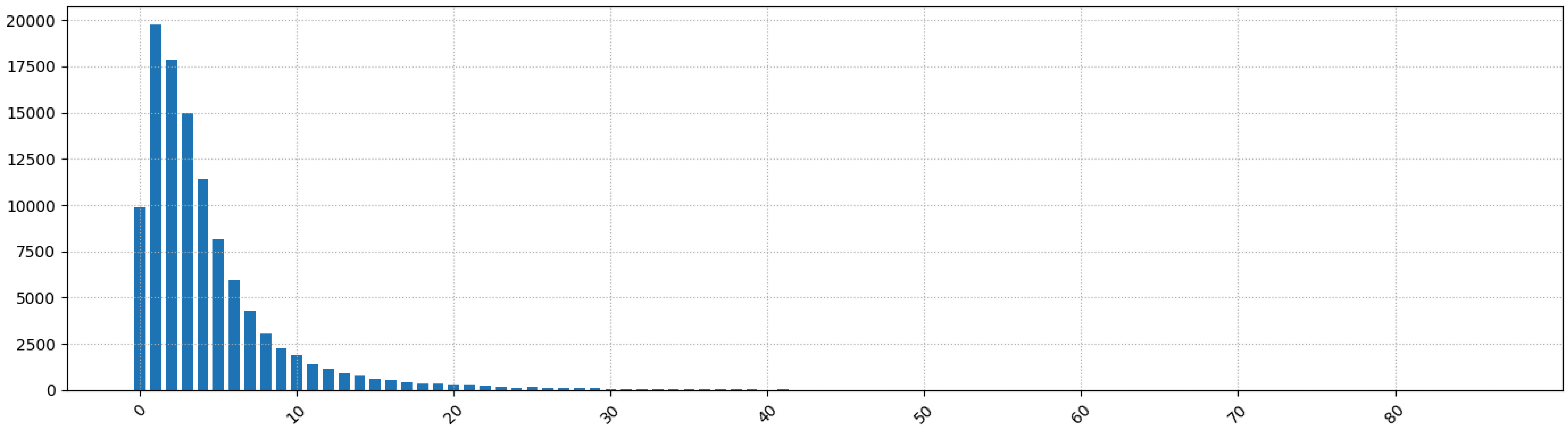
We'll sort the rows by increasing triage time:

In [61]:
```
data.sort_values(by='Triage', inplace=True)
```

Inter-Arrival Times

Let's check empirically the distribution of the inter-arrival times

In [62]:
```
delta = data['Triage'].iloc[1:].round('2min') - data['Triage'].iloc[:-1].round('2min')
tmp = delta.value_counts().sort_index().values
tmp = pd.Series(index=np.arange(len(tmp)), data=tmp)
util.plot_bars(tmp, tick_gap=10, figsize=figsize)
```

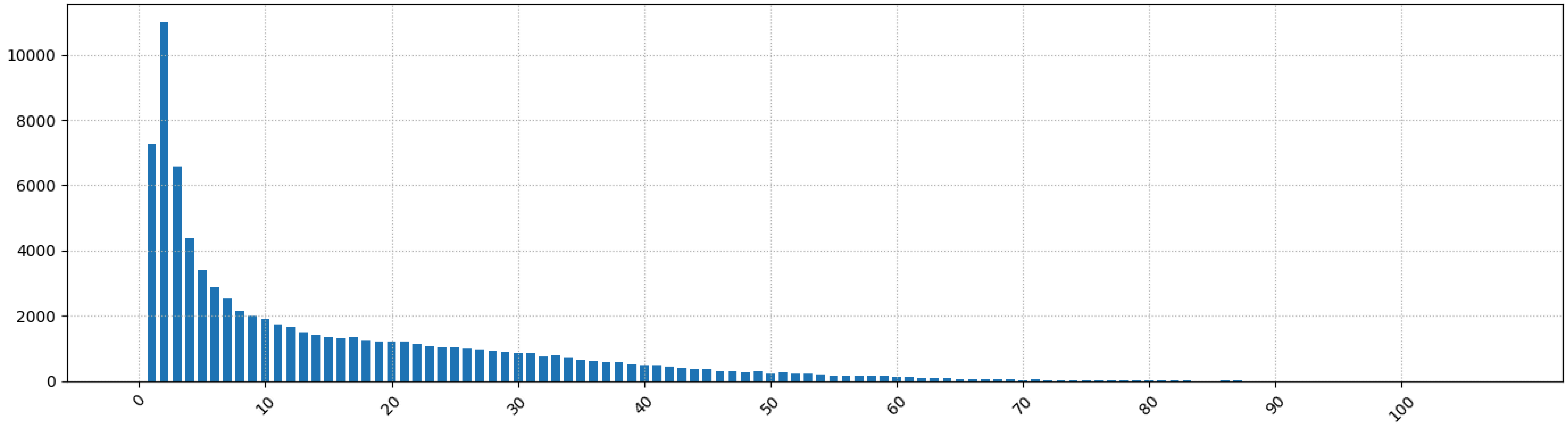


There is a number of very low inter-arrival times

Waiting Time

Here is the distribution of the waiting times

In [63]:
```
tmp = data[~data['TkCharge'].isnull()]
wait_time = tmp['TkCharge'].round('10min') - tmp['Triage'].round('10min')
tmp = wait_time.value_counts().sort_index().values
tmp = pd.Series(index=np.arange(len(tmp)), data=tmp)
util.plot_bars(tmp, tick_gap=10, figsize=figsize)
```



- The distribution is heavy-tailed (large waiting times are quite relatively likely)

Arrival Estimation in Emergency Departments

Number of Arrivals

Binning

We will be interested in estimating the number of arrivals in a hour

First, we use a one-hot encoding for the priority codes

In [64]:
```
codes = pd.get_dummies(data['Code'])
codes.set_index(data['Triage'], inplace=True)
codes.columns = codes.columns.to_list()
print(f'Number of examples: {len(codes)}')
codes.head()
```

```
Number of examples: 108625
```

Out[64]:

	green	red	white	yellow
Triage				
2018-01-01 00:17:33	True	False	False	False
2018-01-01 00:20:33	True	False	False	False
2018-01-01 00:47:59	False	False	True	False
2018-01-01 00:49:51	False	False	True	False
2018-01-01 01:00:40	True	False	False	False

Resampling

Then, we need to aggregate data with a specified frequency

In [65]:
```
codes_b = codes.resample('H').sum()
print(f'Number of examples: {len(codes_b)}')
codes_b.head()
```

```
Number of examples: 16056
```

Out[65]:

	green	red	white	yellow
Triage				
2018-01-01 00:00:00	2	0	2	0
2018-01-01 01:00:00	7	1	1	1
2018-01-01 02:00:00	4	1	4	3
2018-01-01 03:00:00	7	0	1	1
2018-01-01 04:00:00	3	0	2	0

We count the arrivals in each hour, for each code

Computing Totals

Then we compute the total arrival counts

In [66]:
```
cols = ['white', 'green', 'yellow', 'red']
codes_b['total'] = codes_b[cols].sum(axis=1)
codes_b
```

Out[66]:

	green	red	white	yellow	total
Triage					
2018-01-01 00:00:00	2	0	2	0	4
2018-01-01 01:00:00	7	1	1	1	10
2018-01-01 02:00:00	4	1	4	3	12
2018-01-01 03:00:00	7	0	1	1	9
2018-01-01 04:00:00	3	0	2	0	5
...	...	...	...	...	...
2019-10-31 19:00:00	3	1	0	4	8
2019-10-31 20:00:00	9	0	2	0	11
2019-10-31 21:00:00	3	0	0	2	5
2019-10-31 22:00:00	1	2	3	1	7
2019-10-31 23:00:00	5	0	0	2	7

16056 rows × 5 columns

Adding Time Information

Finally, we add time information (for later convenience)

In [67]:

```
codes_bt = codes_b.copy()
codes_bt['month'] = codes_bt.index.month
codes_bt['weekday'] = codes_bt.index.weekday
codes_bt['hour'] = codes_bt.index.hour
codes_bt
```

Out[67]:

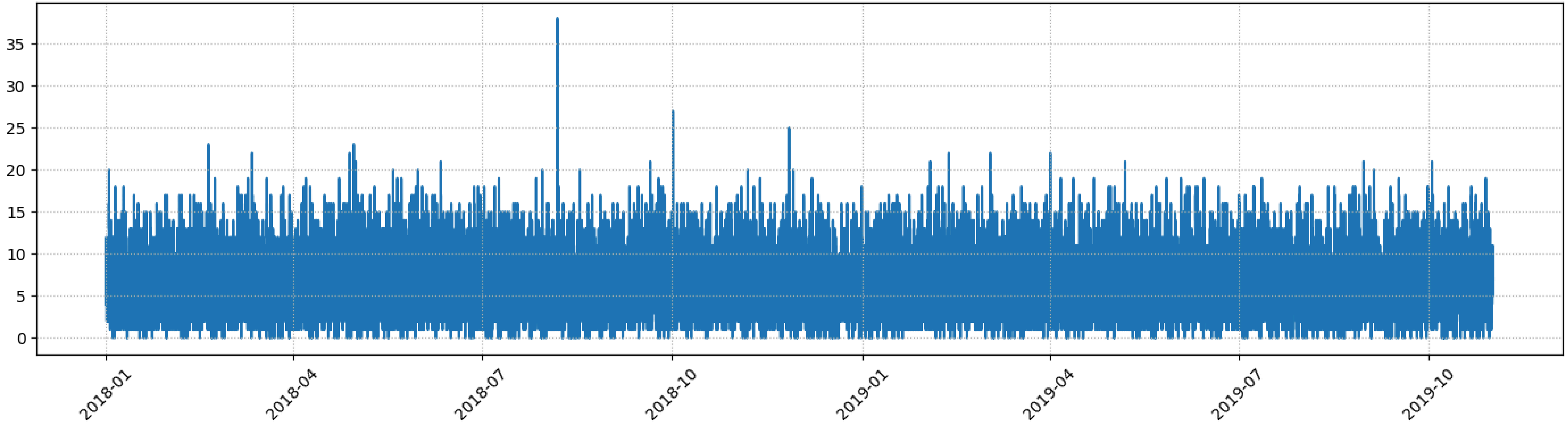
	green	red	white	yellow	total	month	weekday	hour
Triage								
2018-01-01 00:00:00	2	0	2	0	4	1	0	0
2018-01-01 01:00:00	7	1	1	1	10	1	0	1
2018-01-01 02:00:00	4	1	4	3	12	1	0	2
2018-01-01 03:00:00	7	0	1	1	9	1	0	3
2018-01-01 04:00:00	3	0	2	0	5	1	0	4
...	...	...	...	...	...	...	...	...
2019-10-31 19:00:00	3	1	0	4	8	10	3	19
2019-10-31 20:00:00	9	0	2	0	11	10	3	20
2019-10-31 21:00:00	3	0	0	2	5	10	3	21
2019-10-31 22:00:00	1	2	3	1	7	10	3	22

Counts over Time

Our resampled series can be plotted easily over time

Let's see the total counts as an example:

In [68]:
```
util.plot_series(codes_b['total'], figsize=figsize)
```

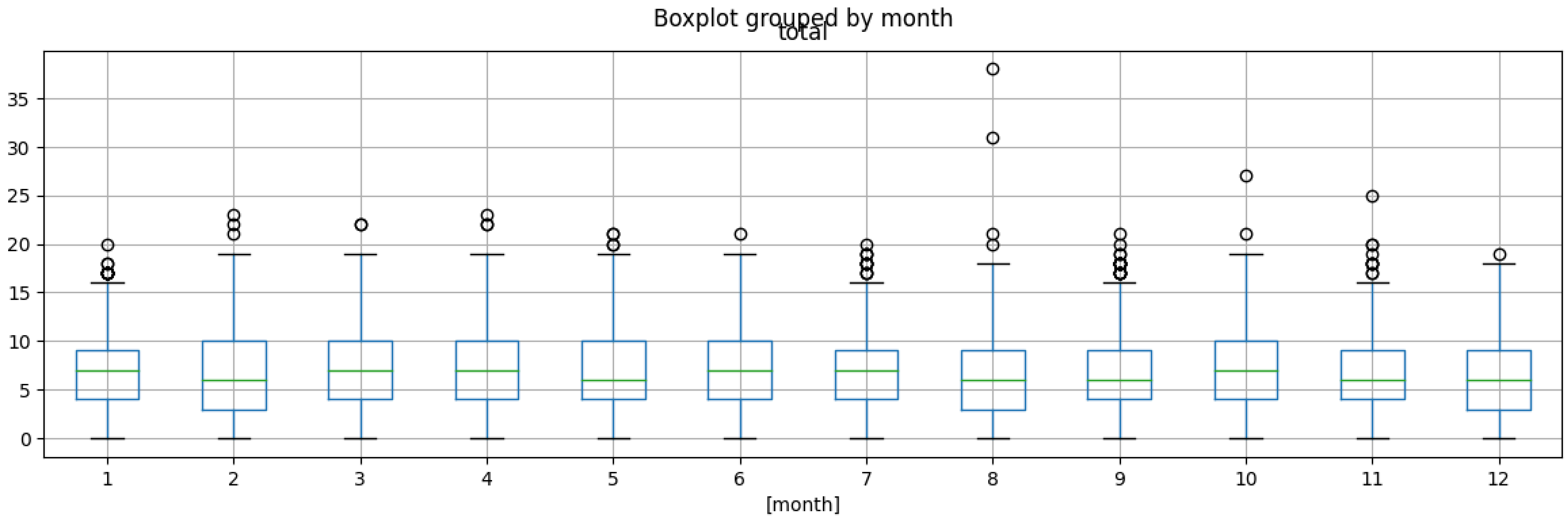


Variability

With our binned series, we can assess the count variability

Let's check it over different months:

```
In [69]: codes_bt[['month', 'total']].boxplot(by='month', figsize=figsize);
```

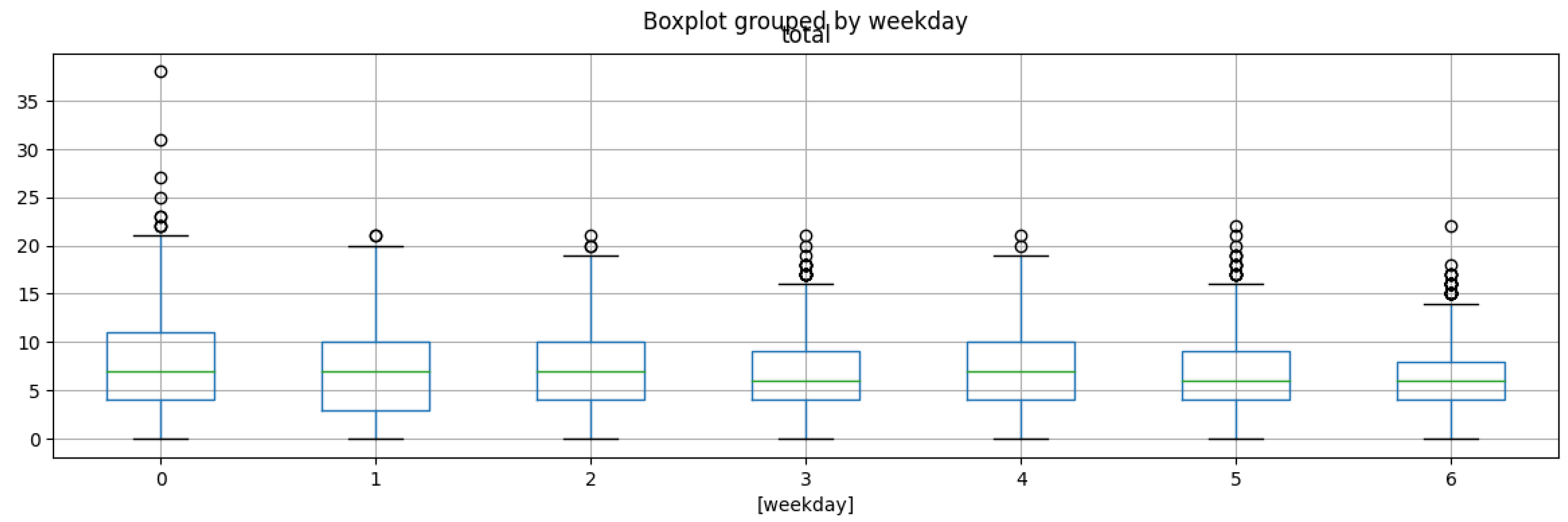


- The variability does not change much over different months

Variability

Here is the standard deviation over weekdays

In [70]:
```
codes_bt[['weekday', 'total']].boxplot(by='weekday', figsize=figsize);
```

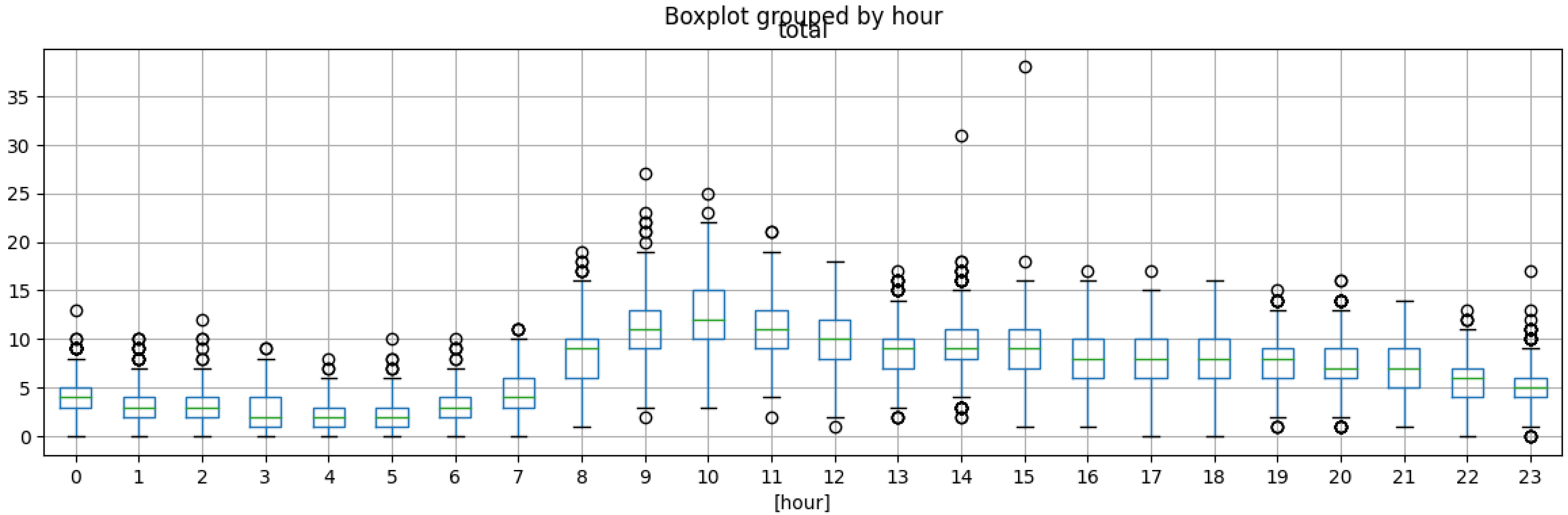


- There is a trend, but rather weak

Variability

...And finally over hours

In [71]:
```
codes_bt[['hour', 'total']].boxplot(by='hour', figsize=figsize);
```



- Variance and mean seem to be quite correlated

Arrival Estimation in Emergency Departments

Estimating Stochastic Quantities

Arrival Prediction

We can now frame our arrival prediction problem

We have some input information:

- Hour, day of the week, and month
- ...Plus possibly the observed arrivals in previous hours

We want to predict the number of arrivals in the next interval

Have we encountered similar tasks in other use cases?

Arrival Prediction

We can now frame our arrival prediction problem

We have some input information:

- Hour, day of the week, and month
- ...Plus possibly the observed arrivals in previous hours

We want to predict the number of arrivals in the next interval

Have we encountered similar tasks in other use cases?

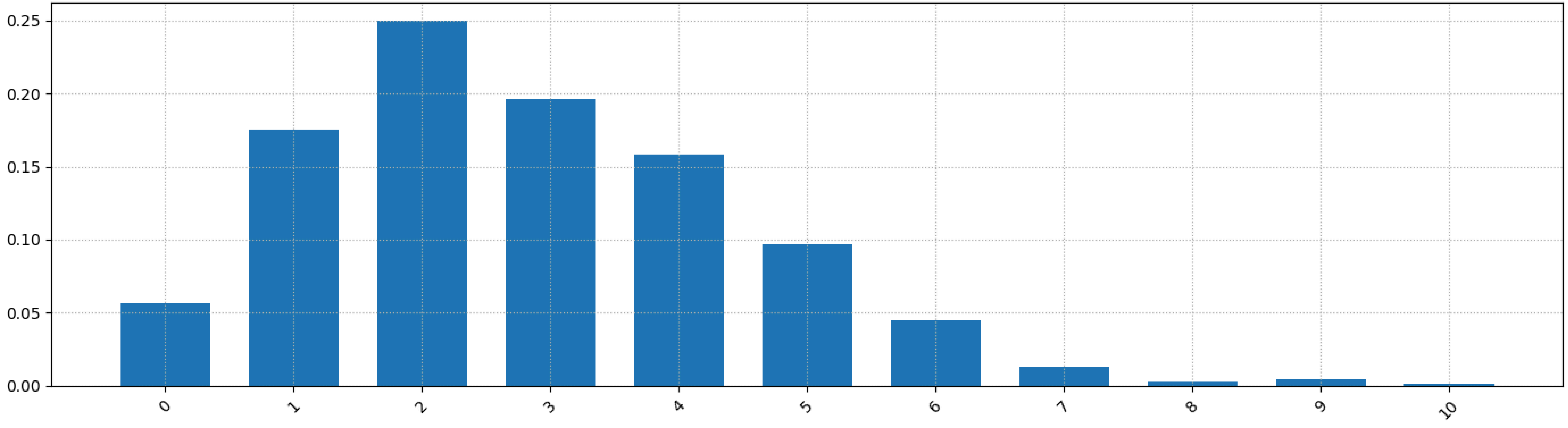
On the face of it, this is a regression problem
But there is a catch!

Prediction and Randomness

The number of arrivals is not subject to a lot of uncertainty!

Let's check its values against the most informative input, i.e. the hour of the day

```
In [72]: tmp = codes_b[codes_b.index.hour == 6]['total']
         tmpv = tmp.value_counts(sort=False, normalize=True).sort_index()
         util.plot_bars(tmpv, figsize=figsize)
```



- There isn't a single, very likely value: the number of arrival is stochastic!

Identifying the Distribution

Instead of predicting a value, we can predict the probability of every possible value

Formally, our goal is estimating a conditional distribution

$$P(Y \mid X)$$

- X is the observable input information we choose to employ
- Y is the number of arrivals in the next hour

Identifying the Distribution

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Formally, our goal is estimating a conditional distribution

$$P(Y \mid X)$$

- X is the observable input information we choose to employ
- Y is the number of arrivals in the next hour

We can think of training a parameterized model on this purpose

$$\hat{f}(x; \theta) \simeq P(Y \mid X)$$

- We will see one viable approach to achieve that
- ...Provided that we know the type of distribution we want to predict

Poisson Distribution

Many arrival process are well described by Poisson distributions

The Poisson distribution is defined by a single parameter λ

λ is the rate of occurrence of the events

- The distribution has a discrete support
- The Probability Mass Function is:

$$p(k, \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

- Both the mean and the standard deviation have the same value (i.e. λ)

The distribution is a good choice provided that the events we are counting are:

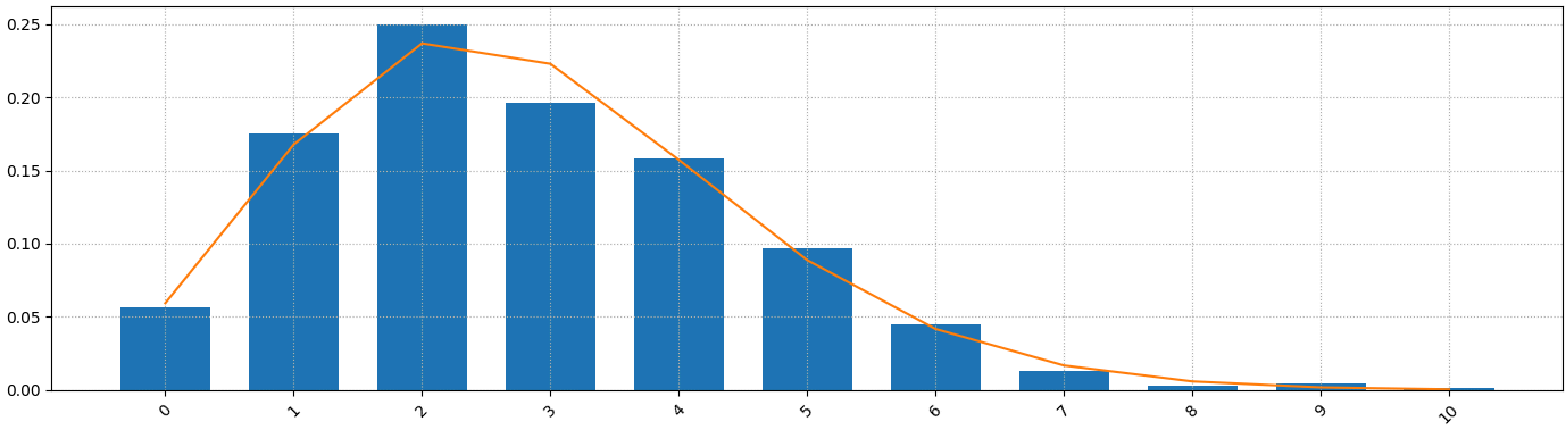
- Independent
- Happening with a costant rate

Fitted Poisson Distribution

Let's try to fit a Poisson distribution over our target

In [74]:

```
mu = tmp.mean()
dist = stats.poisson(mu)
x = np.arange(tmp.min(), tmp.max()+1)
util.plot_bars(tmpv, figsize=figsize, series=pd.Series(index=x, data=dist.pmf(x)))
```

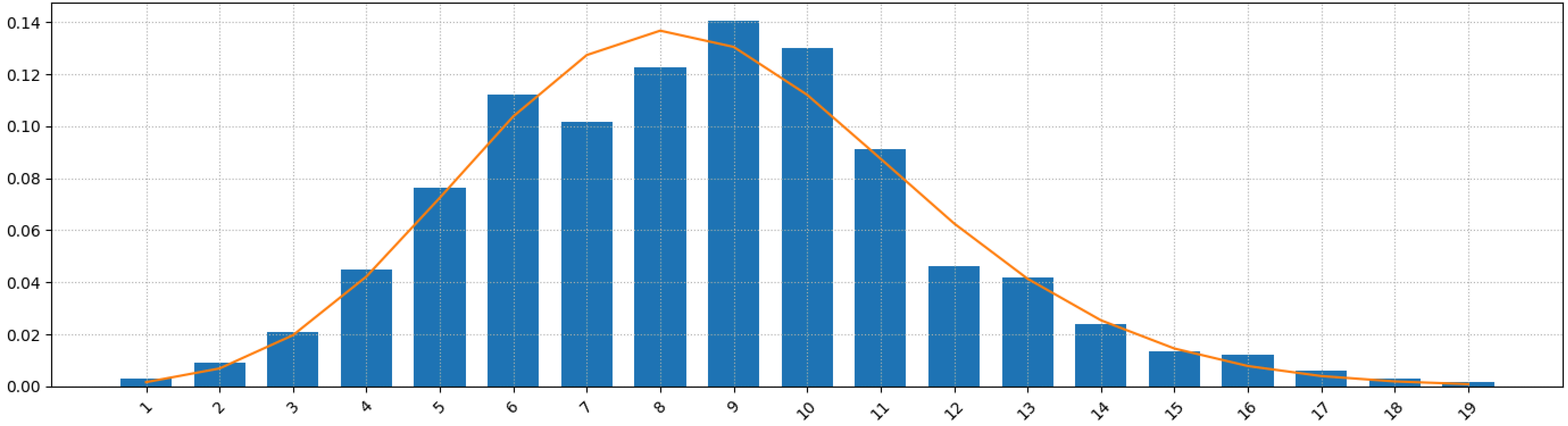


It's a very good match!

Fitted Poisson Distribution

Let's try for 8AM (closer to the peak)

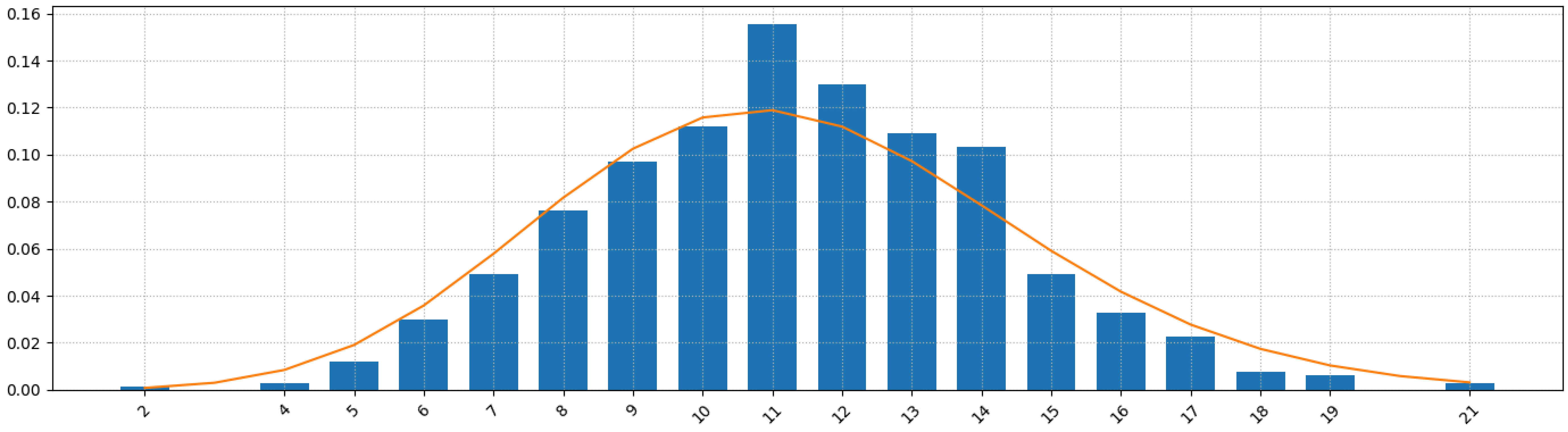
In [75]:
```
tmp = codes_b[codes_b.index.hour == 8]['total']
tmpv = tmp.value_counts(sort=False, normalize=True).sort_index()
mu = tmp.mean()
dist = stats.poisson(mu)
x = np.arange(tmp.min(), tmp.max()+1)
util.plot_bars(tmpv, figsize=figsize, series=pd.Series(index=x, data=dist.pmf(x)))
```



Fitted Poisson Distribution

...And finally for the peak itself (11am)

In [76]:
```
tmp = codes_b[codes_b.index.hour == 11]['total']
tmpv = tmp.value_counts(sort=False, normalize=True).sort_index()
mu = tmp.mean()
dist = stats.poisson(mu)
x = np.arange(tmp.min(), tmp.max()+1)
util.plot_bars(tmpv, figsize=figsize, series=pd.Series(index=x, data=dist.pmf(x)))
```



Learning and Estimator

How can we build an estimator for our problem?

Learning and Estimator

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We could build a table

For example, we could compute average arrivals for every hour of the day

- These correspond to λ for that hour, so we target the correct distribution
- ...But the approach has trouble scaling to multiple features

Learning and Estimator

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For example, we could compute average arrivals for every hour of the day

- These correspond to λ for that hour, so we target the correct distribution
- ...But the approach has trouble scaling to multiple features

We could train a regressor as usual

For example a Linear Regressor or a Neural Network, with the classical MSE loss

- If we do this, it's easy to include multiple input features

Neuro-Probabilistic Models

In practice there is an alternative

Let's start by build a probabilistic model of our phenomenon:

$$y \sim \text{Pois}(\lambda(x))$$

- The number arrivals in a 1-hour bin (i.e. y)
- ...Is drawn from a Poisson distribution (parameterized with a rate)
- ...But the rate is a function of known input, i.e. $\lambda(x)$

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- ...Is drawn from a Poisson distribution (parameterized with a rate)
- ...But the rate is a function of known input, i.e. $\lambda(x)$

Then we can approximate lambda using an estimator, leading to:

$$y \sim \text{Pois}(\lambda(x, \theta))$$

- $\lambda(x, \theta)$ can be any model, with parameter vector λ

This is a hybrid approach, combining statistics and ML

Neuro-Probabilistic Models

How do we train this kind of model?

Just as usual, i.e. for (empirical) maximum log likelihood:

$$\operatorname*{argmin}_\theta - \sum_{i=1}^{m} \log f(\hat{y}_i, \lambda(\hat{x}_i, \theta))$$

- Where $f(\hat{y}_i, \lambda)$ is the probability of value $\hat{y}_i$ according to the distribution
- ...And $\lambda(\hat{x}_i, \theta)$ is the estimate rate for the input $\hat{x}_i$

In detail, in our case we have:

$$\operatorname*{argmin}_\theta - \sum_{i=1}^{m} \log \frac{\lambda(\hat{x}_i, \theta)^{\hat{y}_i} e^{-\lambda(\hat{x}_i, \theta)}}{\hat{y}_i!}$$

...Which is differentiable and can be solved via gradient descent!

Building a Neuro-Probabilistic Model

We can build this class of models by using custom loss functions

...But it's easier to use a library such as TensorFlow Probability

- TFP provides a layer the abstracts a generic probability distribution:

```
tfp.layers.DistributionLambda(distribution_function, ...)
```

- And function (classes) to model many statistical distributions, e.g.:

```
tfp.distributions.Poisson(log_rate=None, ...)
```

About the `DistributionLambda` layer

- Its input is a symbolic tensor (like for any other layer)
- Its output is tensor of probability distribution objects
- ...Rather than a tensor of numbers

Building a Neuro-Probabilistic Model

The `util` module contains code to build our neuro-probabilistic model

```
def build_nn_poisson_model(input_shape, hidden, rate_guess=1):
    model_in = keras.Input(shape=input_shape, dtype='float32')
    x = model_in
    for h in hidden:
        x = layers.Dense(h, activation='relu')(x)
    log_rate = layers.Dense(1, activation='linear')(x)
    lf = lambda t: tfp.distributions.Poisson(rate=rate_guess * tf.math.exp(t))
    model_out = tfp.layers.DistributionLambda(lf)(log_rate)
    model = keras.Model(model_in, model_out)
    return model
```

- An MLP architecture computes the `log_rate` tensor (corresponding to $\log \lambda(x)$)
- Using a log, we make sure the rate is strictly positive
- A `DistributionLambda` yield the output (a distribution object)

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    model_out = tfp.layers.DistributionLambda(lf)(log_rate)
    model = keras.Model(model_in, model_out)
    return model
```

- The `DistributionLambda` layer is parameterized with a function
- The function (`lf` in this cse) constructs the distribution object
- ...Based on its input tensor (called `t` in the code)

Building a Neuro-Probabilistic Model

We need to be careful about initial parameter estimates

```
def build_nn_poisson_model(input_shape, hidden, rate_guess=1):
    ...
    lf = lambda t: tfp.distributions.Poisson(rate=rate_guess * tf.math.exp(t))
    ...
```

- Assuming standardized/normalized input, under default weight initialization
- ...The `log_rate` tensor will be initially close to 0
- Meaning out rate λ would be initially close to $e^0 = 1$

We need to make sure that this guess is meaningful for our target

- In the code, this is achieve by scaling the rate
- ...With a guess that must be passed at model construction time

Training a Neuro-Probabilistic Model

Training the model requires to specify the loss function

...Which in our case is the negative log-likelihood

- So, it turns out we do need a custom loss functions
- ...But with TFP this is easy to compute

In particular, as loss function we always use:

```
negloglikelihood = lambda y_true, dist: -dist.log_prob(y_true)
```

- The first parameter is the observed value (e.g. actual number of arrivals)
- The second is the distribution computed by the `DistributonLambda` layer
- ...Which provides the method `log_prob`

Data Preparation

Let's see the approach in practice

We will start by preparing our data:

- As input we will use the field `weekday` in natural form
- ...And the field `hour` using a one-hot encoding

Let's perform the encoding:

In [77]:
```
np_data = pd.get_dummies(codes_bt, columns=['hour'], dtype='int32')
np_data.iloc[:2]
```

Out[77]:

	green	red	white	yellow	total	month	weekday	hour_0	hour_1	hour_2	...	hour_14	hour_15	hour_16	hour_
Triage															
2018-01-01 00:00:00	2	0	2	0	4	1	0	1	0	0	...	0	0	0	0
2018-01-01 01:00:00	7	1	1	1	10	1	0	0	1	0	...	0	0	0	0

Data Preparation

Now we can separate the training and test data

In [78]:
```
sep = '2019-01-01'
np_tr = np_data[np_data.index < sep]
np_ts = np_data[np_data.index >= sep]
```

...And then the input and output

In [79]:
```
in_cols = [c for c in np_data.columns if c.startswith('hour')] + ['weekday']
out_col = 'total'

np_tr_in = np_tr[in_cols].copy()
np_tr_in['weekday'] = np_tr_in['weekday'] / 6
np_tr_out = np_tr[out_col].astype('float64')

np_ts_in = np_ts[in_cols].copy()
np_ts_in['weekday'] = np_ts_in['weekday'] / 6
np_ts_out = np_ts[out_col].astype('float64')
```

Data Preparation

The input data need to be standardized/normalized as usual

In our case, we do this only for weekday (the hours are already $\in \{0, 1\}$)

```
np_tr_in['weekday'] = np_tr_in['weekday'] / 6
```

The output does not require standarization

...But we need to represent it using floating point numbers

```
np_tr_out = np_tr[out_col].astype('float64')
```

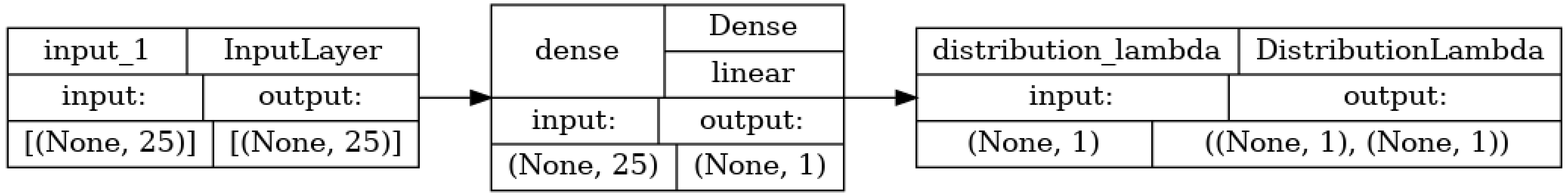
- This is an implementation requirement for TensorFlow

Building the Model

We can now build the Neuro-Probabilistic model

In [80]:
```
nnp = util.build_nn_poisson_model(input_shape=len(in_cols), hidden=[], rate_guess=np_t
util.plot_nn_model(nnp)
```

Out[80]:



As a rate guess we use the average over the training set

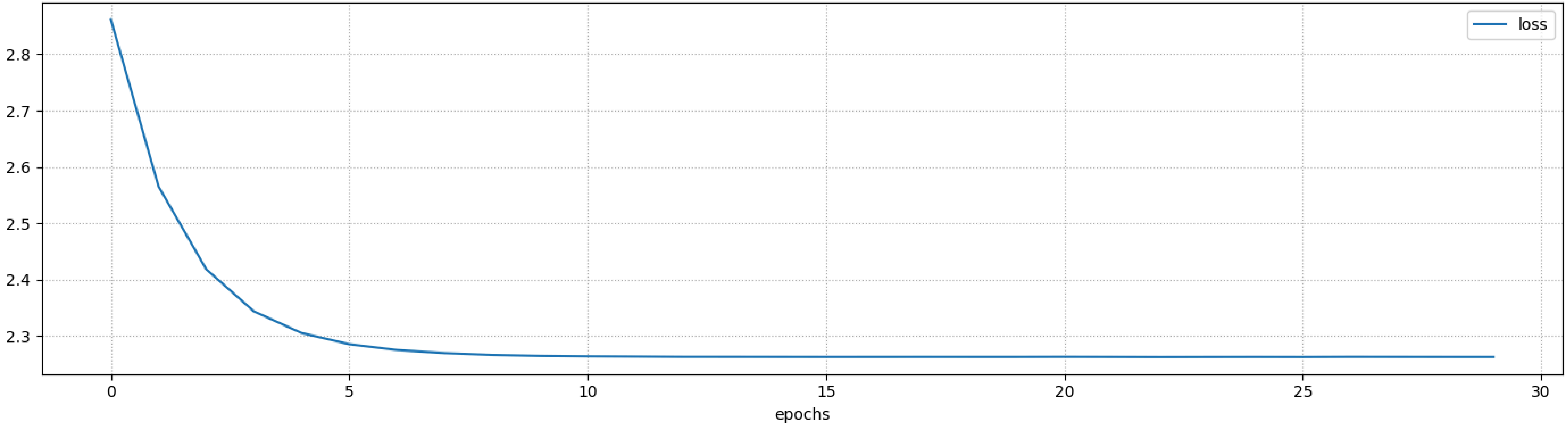
- This is easy to compute
- ...And will provide a better starting point for gradient descent

Training the Model

We can train the model (mostly) as usual

...Except that we need to use the mentioned custom loss function

In [81]:
```
negloglikelihood = lambda y_true, dist: -dist.log_prob(y_true)
nnp = util.build_nn_poisson_model(input_shape=len(in_cols), hidden=[], rate_guess=np_t
history = util.train_nn_model(nnp, np_tr_in, np_tr_out, loss=negloglikelihood, validat
util.plot_training_history(history, figsize=figsize)
```




```
Final loss: 2.2627 (training)
```

Predictions

When we call the `predict` method on the model we obtain samples

This means that the result of `predict` is stochastic

In [82]:
```python
print(str(nnp.predict(np_tr_in, verbose=0)[:3]).replace('\n', ' '))
print(str(nnp.predict(np_tr_in, verbose=0)[:3]).replace('\n', ' '))
```

```
[[1.]  [7.]  [4.]]
[[3.]  [4.]  [5.]]
```

We can obtain the distribution object by simply calling the model

In [83]:
```python
nnp(np_tr_in.values)
```

Out[83]:
```
<tfp.distributions._TensorCoercible 'tensor_coercible' batch_shape=[8760, 1] event_s
hape=[] dtype=float32>
```

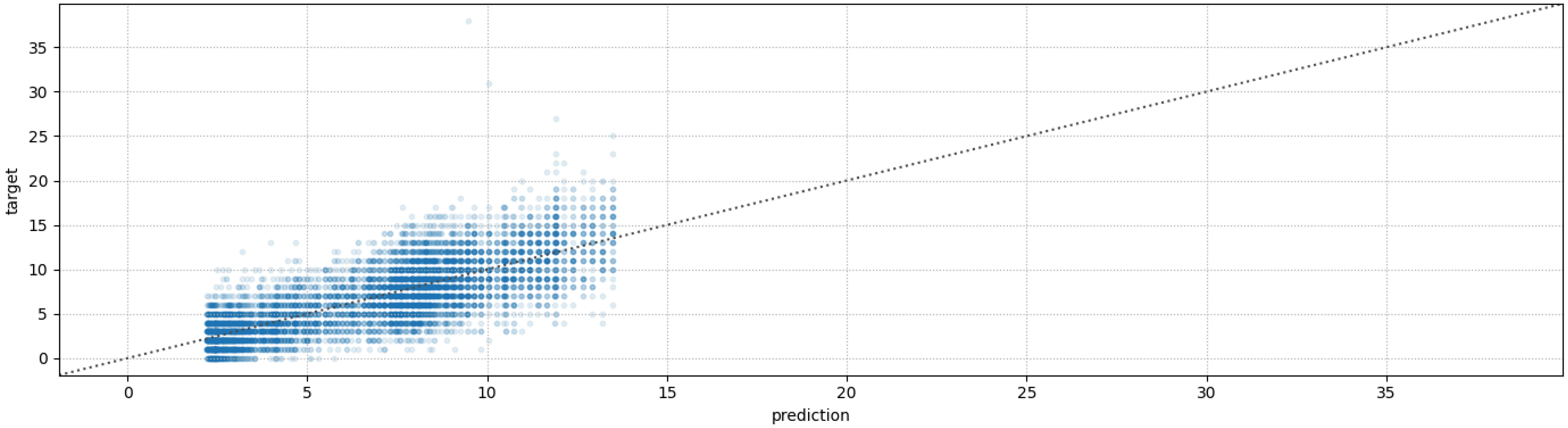
Then we can call methods over the distribution objects

Evaluation

Using the predict means, let's check the quality of our results

```
In [84]: tr_pred = nnp(np_tr_in.values).mean().numpy().ravel()
         util.plot_pred_scatter(np_tr_out, tr_pred, figsize=figsize)
```

```
R2: 0.60
MAE: 1.93
```



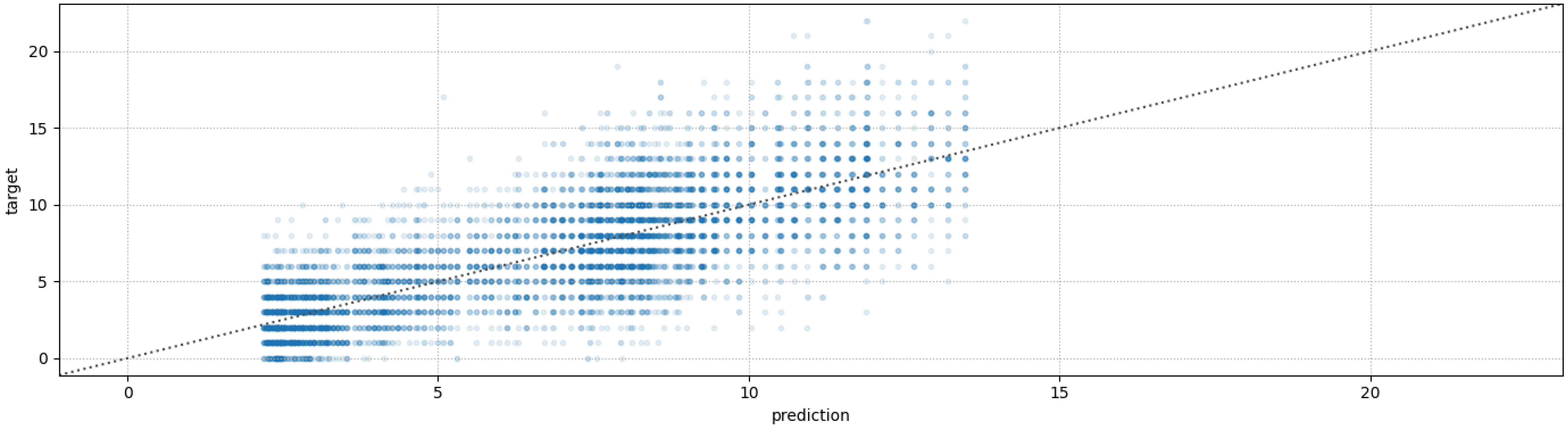
- This is a stochastic process, making this R^2 value very good

Evaluation

Let's repeat the exercise on the test set

```
In [85]: ts_pred = nnp(np_ts_in.values).mean().numpy().ravel()
         util.plot_pred_scatter(np_ts_out, ts_pred, figsize=figsize)
```

```
R2: 0.60
MAE: 1.94
```



- No overfitting, which is again very good

Confidence Intervals

Since our output is a distribution, we have access to all sort of statistics

Here we will simply show the mean and stdev over one week of data:

In [89]:
```
ts_pred_std = nnp(np_ts_in.values).stddev().numpy().ravel()
util.plot_series(pd.Series(index=np_ts_in.index[:24*7], data=ts_pred[:24*7]), std=pd.S
plt.scatter(np_ts_in.index[:24*7], np_ts_out[:24*7], marker='x');
```