NANYANG POLYTECHNIC

2022/2023 SEMESTER 1

Computing Mathematics Assignment

June 2022

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1. Answer **ALL** the questions from Sections A and B.
2. All answers must be handwritten on A4 paper.
3. Attach on front page a signed Declaration of Plagiarism (found on last page of this document) together with your answers.
4. **Scan into ONE PDF** document and submit in Brightspace.
5. Please **resubmit the WHOLE COMPLETE SET of ANSWERS if you want to change any part of your answers** and submit as another attempt in Brightspace.

Section A: Matrices-Theory

Question 1 **Maximum 20 marks**

(a) Given $D = \begin{pmatrix} -1 & -2 \\ 3 & 5 \end{pmatrix}$, find $D^{-1} + D^{T}$.

(b) Given $X = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$ and $Y = \begin{pmatrix} 7 & 1 & -1 \\ -2 & 4 & 6 \end{pmatrix}$, find Z where $3X - Z = Y$.

(c) Given $A = \begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 1 \\ 2 & 4 \end{pmatrix}$, find the inverse of AB.

(d) Given $X = \begin{pmatrix} 5 & -1 \\ 1 & 0 \end{pmatrix}$ and $Y = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & -2 \end{pmatrix}$, , state with reasons whether it is possible to evaluate XY and YX. Where possible, find the product of the matrices.

(e) Given that $\begin{vmatrix} 2 & \beta + 3 & \beta \\ 1 & 7 & 4 \\ 3 & 9 & 6 \end{vmatrix} = 0$, find β.

(f) Given $A = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$, show that $AB = BA$. What can you conclude the relationship between A and B?

(g) Given $A = \begin{pmatrix} 1 & 1 \\ 3 & -1) \end{pmatrix}$ find B if $(AB)^{T} = \begin{pmatrix} 4 & -3 \\ -2 & 14 \end{pmatrix}$.

(h) Let Y be a 2×2 matrix. If $4I - 5Y = \begin{pmatrix} -6 & -5 \\ 5 & -21 \end{pmatrix}$, determine $12I - 3Y^{T}$.

(i) Rewrite the following linear equations to the matrix equation of the form AX = B where A is the matrix consisting of coefficients, X is the column vector of unknowns and B is column vector of constants. Use the Inverse of A to solve for x and y.

$$-x - 4y = -40$$

$$3x + 2y = 5$$

j) Let $M = \begin{pmatrix} 1 & 0 \\ 99 & 1 \end{pmatrix}$. Find M^2 and M^3. Hence, deduce M^n .

Section B: Matrices – Applications

Question 2 **Maximum 10 marks**

System of Linear Equations: A system of linear equations consists of two or more linear equations made up of two or more variables such that all equations in the system are considered simultaneously. For example, a system of linear *m* equations and n variables $x_1, x_2 \ldots x_n$ is written as follows:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

System of linear equations allow us to model and solve real-world situations in fields such as engineering, business, education and sciences. The exercise below is one such example.

Required: Solve the following exercise on *System on Linear Equations*. Show all workings.

Founded in 1904, with headquarters in Paris, the Fédération Internationale de l'Automobile (FIA) is the governing body for world motor sport and the federation of the world's leading motoring organisations. This year, FIA is proposing a new scoring system with a base score 100 for all Formula 1 (F1) drivers by assigning different weightage to the points scored in the F1 Grand Prix races in the 3 Middle East countries of **Bahrain**, **Saudi Arabia** and **Abu Dhabi**. The scores obtained for each of the grand tour is multiplied with a weightage as shown in Table 1, to compute the final score of a pro cyclist at the end of year.

Grand Prix	Bahrain	Saudi Arabia	Abu Dhabi
Weightage	2	3	4

Table 1

Table 2 shows the component scores obtained by 3 F1 driver of **Charles Leclerc** (Ferrari), **Max Verstappen** (Red Bull Racing) and **Lewis Hamilton** (Mercedes).

F1 Driver	Bahrain Grand Prix	Saudi Arabia Grand Prix	Abu Dhabi Grand Prix
Charles Leclerc	85	80	65
Max Verstappen	60	80	90
Lewis Hamilton	70	90	q

Table 2

(a) Express Table 1 in a 1 x 3 Matrix W.

(b) Express Table 2 in a 3 X 3 Matrix S.

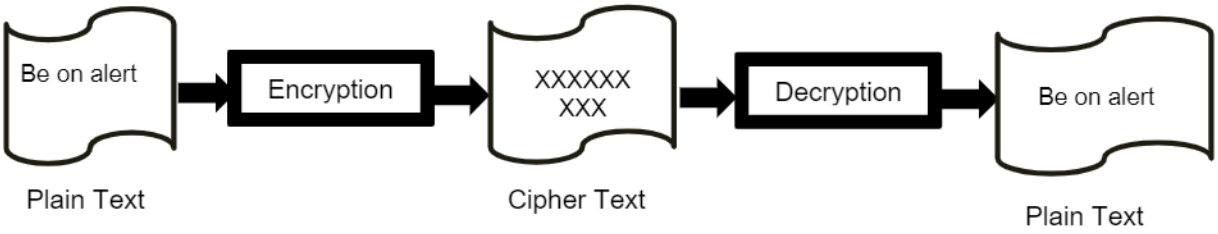
(c) Given $T = SW^T$, find T in terms of q. Explain the meaning of T.

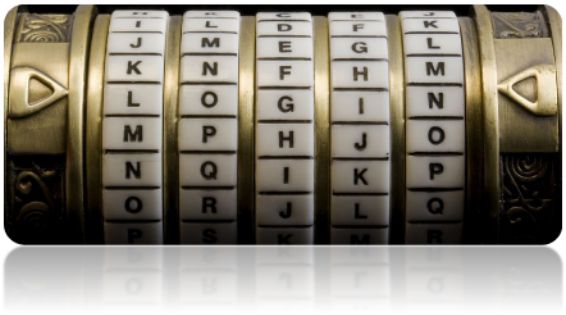
(d) Find the minimum points **Lewis Hamilton** must score at **Abu Dhabi Grand Prix** to achieve a total score of 750.

(e) The FIA decided to adjust the weightage of the F1 Grand Prix races in Middle east counties due to the disruption of Ukraine war. Based on the new weightages, **Charles Leclerc** (Ferrari), **Max Verstappen** (Red Bull Racing) and **Lewis Hamilton** (Mercedes) now have final scores of 800, 700 and 750 respectively. Assuming **Lewis Hamilton** has a score of 75 for **Abu Dhabi Grand Prix**, express the above information on the component scores, new weightages and final scores in a matrix form $SX=C$. Find the new weightages using Cramer's rule.

Question 3 **Maximum 10 marks**

Cryptography: Cryptography is a method of protecting information and communications using codes, so that only those for whom the information is intended can read and process it. Data that can be read and understood without any special measures is called plaintext or cleartext. The method of disguising plaintext in such a way as to hide its substance is called encryption. Encrypting plaintext results in unreadable information called ciphertext. You use encryption to ensure that information is hidden from anyone for whom it is not intended, even those who can see the encrypted data. The process of reverting ciphertext to its original plaintext is called decryption. Figure below illustrates this process.





Part A: (6 marks)

Required: Design a process of encrypting and decrypting a message using matrices by illustrating with an example of your own. Show all workings.

Steps:

1. Think of a plain text message by filling up in the table below. To keep this exercise manageable, limit your text to no more than 10 characters.

   eg

B	E		A	L	E	R	T		

   Yours :

2. Design an Alphabet to Numeric Substitution Table

   Fill in numbers 1 to 26 in a random order for A to Z, reserving 0 for <SPACE>.

A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	2	3	4	5	6	7	8	9	10	11	12	13	14
O	P	Q	R	S	T	U	V	W	X	Y	Z	<SPACE>	
15	16	17	18	19	20	21	22	23	24	25	26	0	

3. Replace each character of your plain text message using the Alphabet to Numeric Substitution Table, by filling in the corresponding numbers below:

   eg

2	5	0	1	12	5	18	20		

   Yours :

4. Design a 2 x 2 Encoding Matrix, **C**. The Encoding Matrix **must** be invertible. Show your working for computing $\mathbf{C}^{-1}$. (2 marks)

Message Encryption

5. Convert your result from Part 3 into a *m x 2* matrix, **M**, where m depends on the length of the message. (1 mark)

   For example, the following row of numbers

13	5	18	2	24	8

   is rewritten as

$$\begin{bmatrix} 13 & 5 \\ 18 & 2 \\ 24 & 8 \end{bmatrix}$$

6. Encrypt your message **S** by multiplying **M** by **C**. This cyphertext is sent to the receiver. (1 mark)

$$\mathbf{S} = \mathbf{MC}$$

Message Decryption

7. Since S = MC and C is invertible, obtain the original the original result in part 3 in m x 2 matrix form, by performing $M = SC^{-1}$. (1 mark)

8. Convert your results in part 7 to original plain text using the Alpha to Numeric Substitution Table. (1 mark)

eg

M=	2	5	0	1	12	5	18	20	

A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	2	3	4	5	6	7	8	9	10	11	12	13	14
O	P	Q	R	S	T	U	V	W	X	Y	Z	<SPACE>	
15	16	17	18	19	20	21	22	23	24	25	26	0	

eg

M=	B	E		A	L	E	R	T	

Yours :

Part B: (4 marks)

As an upgrade to your system in Part A, you are to make the following improvements to it:

(a) Replace the *2 x 2* Encoding Matrix **C** with a *3 x 3* Matrix. Find the inverse of the Encoding Matrix, $\mathbf{C^{-1}}$.

(b) Instead of having numeric equivalent of your message **M** in the form of a *m x 2* matrix, rearrange it in the form of *3 x 1* matrices. . For example, the sample message M matrix in Step (5)

13	5	18	2	24	8

can be rewritten as 2 matrices of size 3 x 1:

$$M_1 = \begin{bmatrix} 13 \\ 5 \\ 18 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 2 \\ 24 \\ 8 \end{bmatrix}$$

The ciphertext can then be obtained as follows :

$S_1 = CM_1$, $S_2 = CM_2$ etc

Required: Based on the above new requirements, rework Steps 4 to 8 as in Part A. Showing all workings. To keep the exercise manageable, you only need to perform encryption and decryption for only the **first 3 characters** of your plain text message.

END OF SECTION B

2022/2023 SEMESTER 1

Computing Mathematics Assignment

PLAGIARISM DECLARATION

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2. I acknowledge that copying someone else's assignment, or part of it, constitutes a form of plagiarism.
3. I have not allowed anyone to copy my work or part of it, with the intention of passing it off as their own work.

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Signature: _____ Date: _____