Chapter 9b

Saturday, January 7, 2023 1:21 PM

04/05/2023

Chapter 9 - Inferences from Two Samples (continued)

CI for $\mu_1 - \mu_2$ using Matched Pairs Data

No longer have two independent random samples.

Subjects or objects are chosen in pairs. They have more in common with one another than they do with members of other pairs.

Ex: Compare Drug A and Drug B

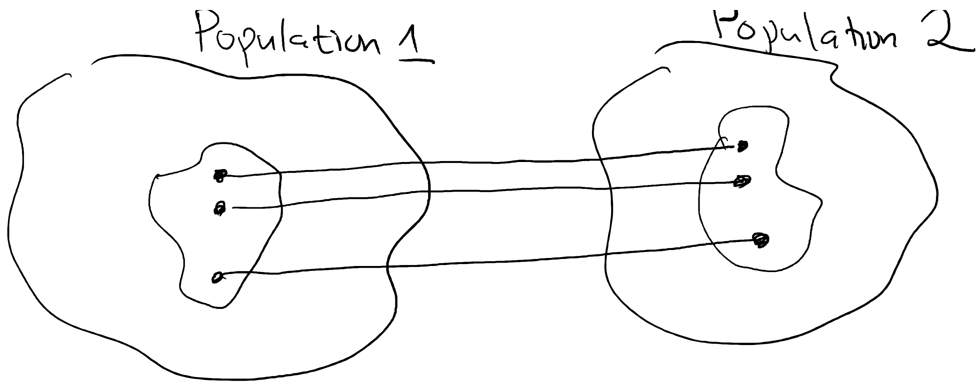
5 pairs of identical twins

Pair	Twin #1 (Drug A)	Twin #2 (Drug B)	Difference
1	✓	✓	d_1
2	✓	✓	d_2
3			d_3
4			d_4
5			d_5

"matched"

Not independent samples

Population 1 Population 2



In general,

n = number of pairs in sample

Pair	Treatment 1		Treatment 2	Difference
1		$\longleftrightarrow$		d_1
2		$\longleftrightarrow$		d_2
$\vdots$				$\vdots$
n		$\longleftrightarrow$		d_n

Focus on the differences

$d_1, d_2, \ldots, d_n$ can be regarded as a random sample of differences from a population of differences.

Notationally,

μ_d = mean value of population of differences

σ_d = standard deviation of population of differences

Estimate of μ_d using

$$\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n}$$

Estimate of σ_d^2 using

$$S_d^2 = \frac{\sum_{i=1}^{n} (d_i - \bar{d})^2}{n-1}$$

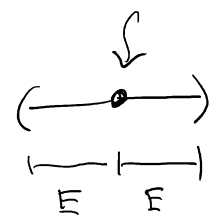
IMPORTANT: $\mu_d = \mu_1 - \mu_2$

mean of diffrences = difference of means

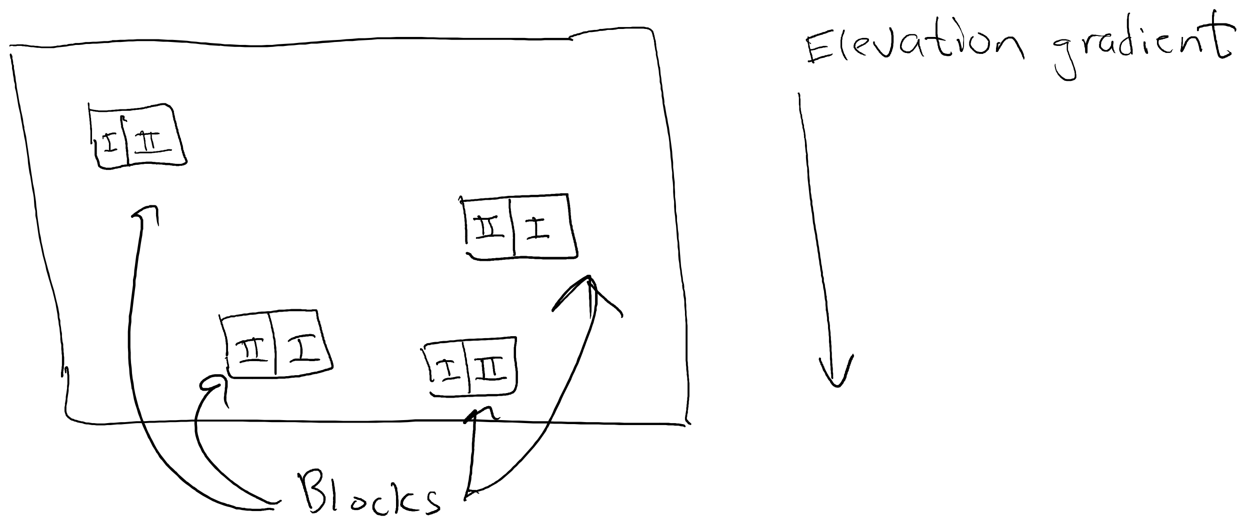
$$\bar{d} = \bar{x}_1 - \bar{x}_2$$

If $d_1, d_2, \ldots, d_n$ is a random sample from a normally distribution populations of differences OR $n \geq 30$, then a $100(1-\alpha)\%$ CI for μ_d is

$$\bar{d} \pm t_{\frac{\alpha}{2}} \frac{S_d}{\sqrt{n}}$$



Ex: See class handout.



Construct a 95% CI for $\mu_d = \mu_1 - \mu_2$

$n=4$, $\bar{d}=-1.575$, $S_d=\sqrt{\dfrac{\sum_{i=1}^{4}(d_i-\bar{d})^2}{4-1}}=0.7411$

$t_{0.025}=3.182$ using $df=4-1=3$.

$$-1.575 \pm 3.182 \times \frac{0.7411}{\sqrt{4}}$$

$$-1.575 \pm 1.179$$

or $(-2.75, -0.40)$

Conclusions? Comments?

Zero is not a likely value of $\mu_d=\mu_1-\mu_2$.

$H_0: \mu_d=0 \quad (\mu_1-\mu_2=0)$

$H_1: \mu_d\neq 0 \quad (\mu_1-\mu_2\neq 0)$

$\alpha=0.05$

test statistic $t=\dfrac{\bar{d}-0}{S_d/\sqrt{n}}$

⋮

Reject Ho and conclude that sample evidence suggests that $\mu_1\neq\mu_2$. Rewrite using everyday language.

↑ Exam 3