Two-stage (or Multistage) Sampling

(Adapted from Cochran 1977)

1. The principal advantage to multistage sampling is that it is more flexible then single-stage sampling. For example, two-stage sampling reduces to one-stage sampling when $m_i = M_i$. The actual number of secondary units selected depends on a balance of statistical precision and cost.

2. Two-stage sampling requires lists of the primary units (groups) and the secondary units (elements) in each selected primary unit. It is often cheaper to implement than a simple or stratified sample of the same size.

3. Cluster sampling is a special case of two-stage sampling in which all the secondary units (elements) of each primary unit (group) are selected in the second stage. Cluster sampling is desirable when the group sizes and the group means are approximately equal.

4. Other things being equal, two-stage sampling should be the preferred to stratified sampling when the population consists of groups that are similar to one another and to the population at large. Stratified sampling should be the preferred method when the population consists of approximately homogeneous groups with respect to the variable of interest.

EX: Two-stage sampling

A forester wants to estimate the total number of trees in a certain county that are infected with a particular disease. There are ten well-defined forest areas in the county; these areas can be subdivided into plots of approximately the same size. Four crews are available to conduct the survey, which must be completed in one day. Hence two-stage cluster sampling is used. Four areas (clusters) are chosen with six plots (elements) randomly selected from each. Each crew can survey six plots in one day. The data are given in the accompanying table. Estimate the total number of infected trees in the county, and compute the estimated variance of this estimator.

Area	# plots	# of plots sampled	Number of infected trees per plot	$\overline{y}_i$	S_i^2
1	12	6	15, 14, 21, 13, 9, 10	$13.\overline{6}$	$18.2\overline{6}$
2	15	6	4, 6, 10, 9, 8, 5	7	5.6
3	14	6	10, 11, 14, 10, 9, 15	11.5	5.9
4	21	6	8, 3, 4, 1, 2, 5	$3.8\overline{3}$	$6.1\overline{6}$