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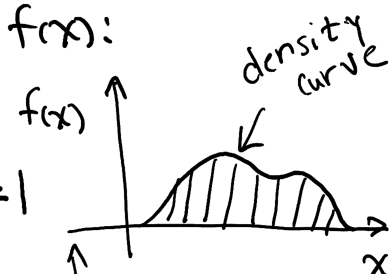
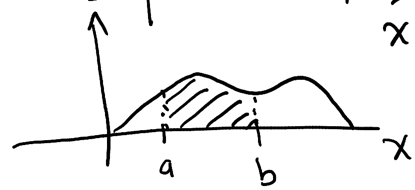
Chapter 6 — Continuous Probability Distributions

Def: A continuous rv X can take on any value in some interval on the real number line.

Def: A probability density function, denoted $f(x)$, describes the probability distribution of a continuous rv X. Properties of $f(x)$:

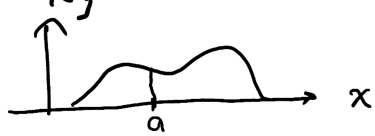


1. $f(x) \geq 0$ for all x-values
2. Area under the density curve $= 1$
3. $P(a < X < b)$ $=$ Probability



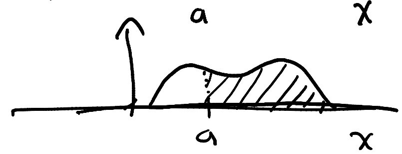
Remarks: For any continuous rv X,

1. $P(X=a) = 0$

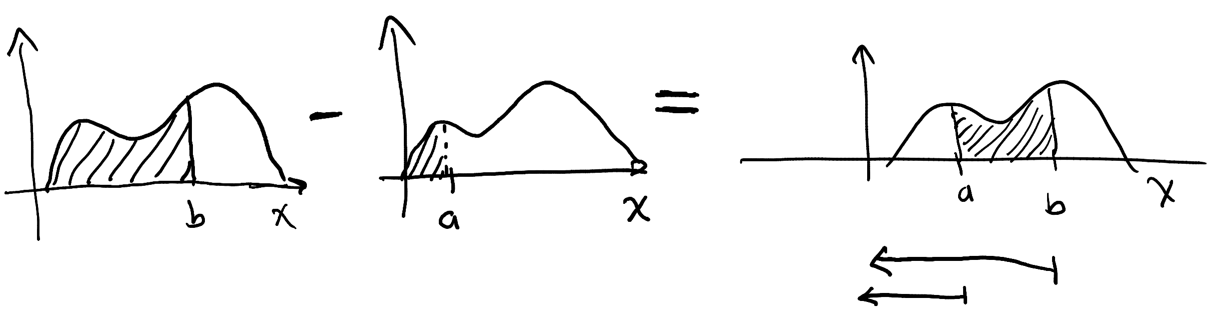


2. $P(X \leq a) = P(X < a) + P(X=a) = P(X < a)$

3. $P(X > a) = 1 - P(X \leq a) = 1 - P(X < a)$



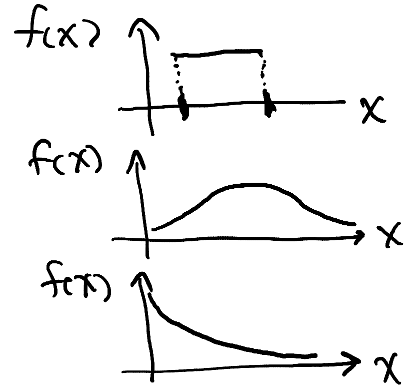
4. $P(a \leq X \leq b) = P(X \leq b) - P(X < a)$



Some continuous Distribution names

- Uniform
- Normal
- Exponential

⋮

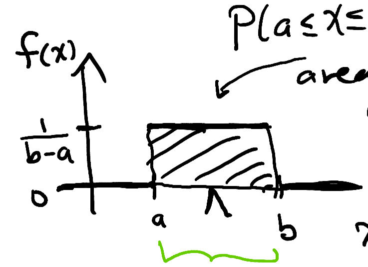


The Uniform Distribution

Def: The rv X has a uniform distribution on the interval $[a,b]$ if $f(x)$ looks like the following density curve

$P(a \le X \le b) = 1$

area under curve



Notation: $X \sim$ Uniform $[a, b]$ (distributed)

Area = width x height

= (b-a) x height

= 1

so height $= \frac{1}{b-a}$

Remark: $\mu = \frac{a+b}{2}$

$\sigma^2 = \frac{(b-a)^2}{12}$

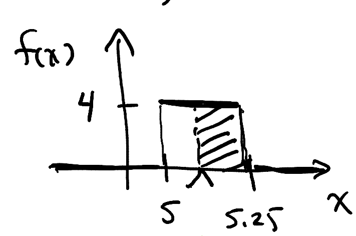
Ex: X = weight of a Major League Baseball (oz)

Suppose $X \sim$ Uniform$[5, 5.25]$

i.) Find probability a randomly selected baseball's weight exceeds the mean weight.

$\frac{1}{b-a} = \frac{1}{0.25} = 4$

$\mu = \frac{5 + 5.25}{2} = 5.125$ oz



$P(X > 5.125) = 0.5 = \frac{1}{2}$

= base x height

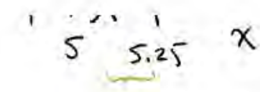
= (0.125) x 4

$P(X > 5.125)$

$= \text{base} \times \text{height}$

$= (0.125) \times 4$

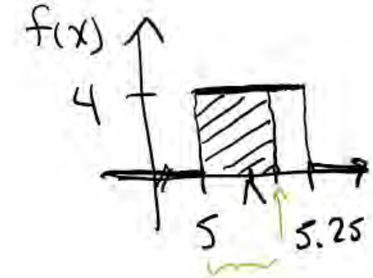
$= \frac{1}{2}$



ii.) Find probability randomly selected baseball does not exceed 5.1875 oz.

$P(X \le 5.1875) = (0.1875) \times 4$

$= 0.75$



The Normal Distribution

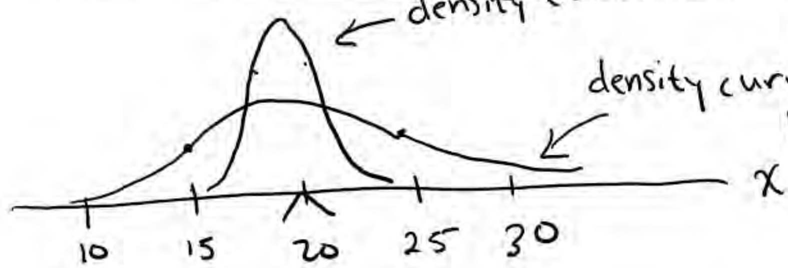
<u>Def</u>: A continuous rv X has a <u>normal distribution</u> with parameter μ and σ if

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

<u>Notation</u>: $X \sim N(\mu, \sigma^2)$

$\leftarrow$ density curve for $N(\underset{\mu}{20}, \underset{\sigma^2=1}{1})$ $\sigma = 1$

<u>Ex</u>:

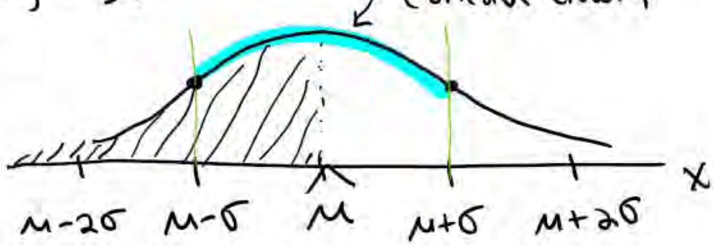


density curve for $N(20, 25)$
$\uparrow \mu \quad \uparrow \sigma^2$

$\sigma = 5$

<u>Ex</u>: $X \sim N(\mu, \sigma^2)$.

concave down



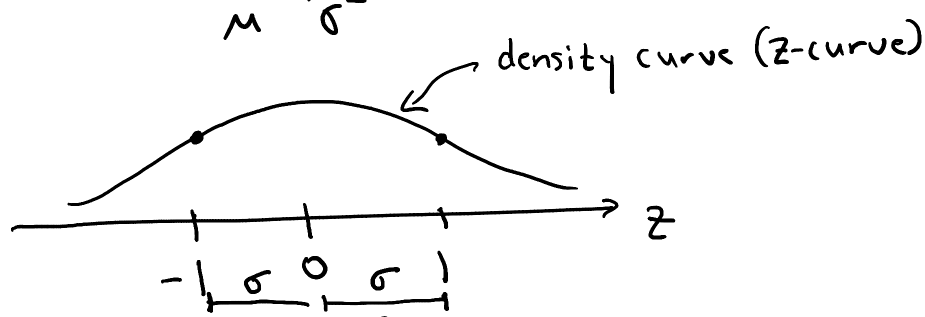
$P(X < \mu) = 0.5$

$P(X < \mu) = 0.5$

$P(X < \mu + \sigma) = ?$

<u>Def</u>: A continuous rv Z has a <u>standard normal distribution</u> if $\mu = 0$ and $\sigma^2 = 1$ $(\sigma = 1)$.

<u>Notation</u>: $Z \sim N(\underset{\mu}{\overset{\shortparallel}{0}}, \underset{\sigma^2}{\overset{\shortparallel}{1}})$



See handout (or Table A-2) for tabulated cumulative areas under the z-curve.

<u>Remark</u>: Always draw a picture of the density curve.

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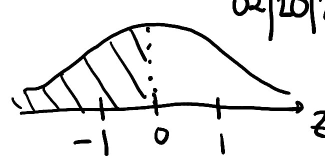
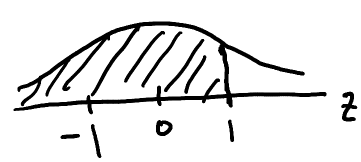
<u>Ex</u>: $Z \sim N(0,1)$

$P(Z < 0) = 0.5$

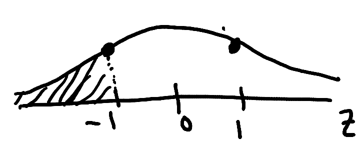


$P(Z < 1) = 0.8413$

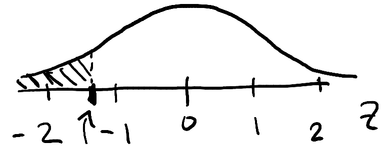
$P(Z \le 1) = 0.8413 = P(Z < 1)$



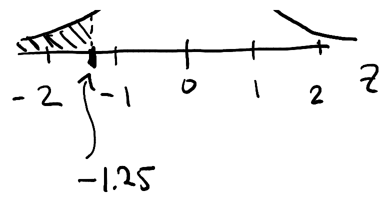
$P(Z < -1) = 0.1587$



$P(Z < -1.25) = 0.1056$



$P(Z < -1.25) = 0.1056$

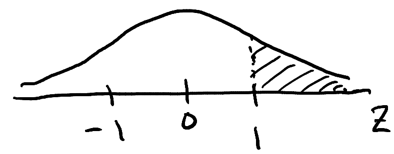


$P(Z > 1) = 1 - P(Z \leq 1)$

$= 1 - 0.8413$

$= 0.1587$

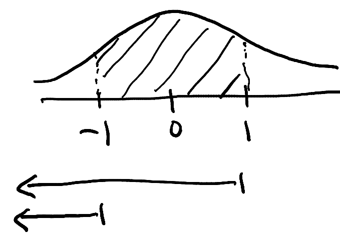
$= P(Z < -1)$

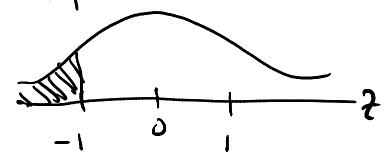


$P(-1 < Z < 1) = P(Z < 1) - P(Z \leq -1)$

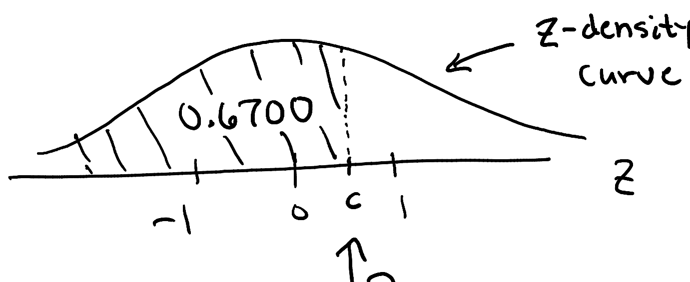
$= 0.8413 - 0.1587$

$= 0.6826$



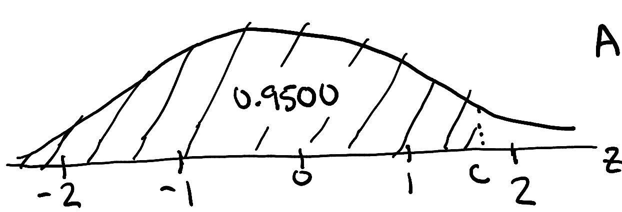


Ex: i.) For what value of c on the z-axis is the area under the z-curve to the left of c equal to 0.67?



Solution: $c = 0.44$
(see Z-table)

ii.) Find c such that area under the z-curve to the left of c is 0.95.

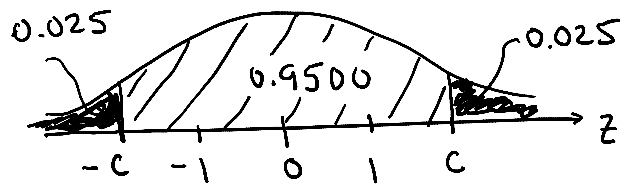


Area:	0.9495	0.9505
Z-value:	1.64	1.65

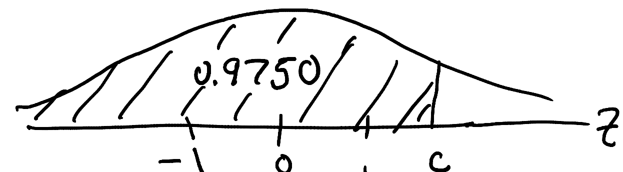
c = 1.64 yields 0.9495
c = 1.65 yields 0.9505 so use c = 1.645

OR use information at bottom of the Z-table.

iii) Find c where



i.e., find c such that $P(-c < Z < c) = 0.9500$.

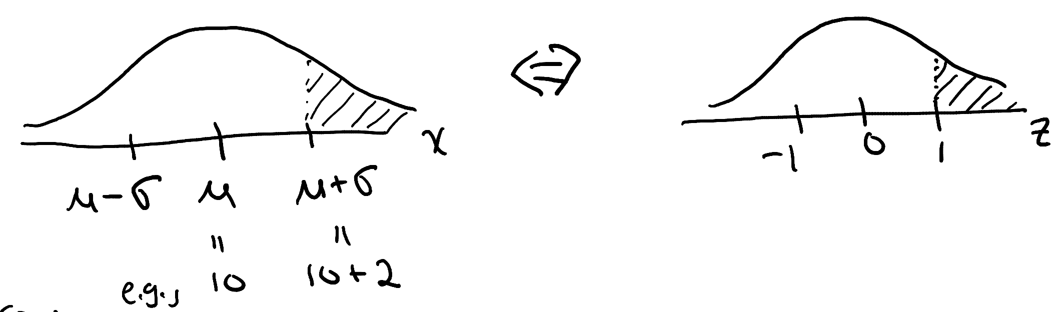


From Z-table, $c = 1.96$ and $-c = -1.96$.

i.e., $P(-1.96 < Z < 1.96) = 0.9500$

<u>IMPORTANT</u>: If $X \sim N(\mu, \sigma^2)$ and $Z \sim N(0,1)$, then

$$Z = \frac{X-\mu}{\sigma}.$$



<u>Remarks</u>:

1. $P(X \le b) = P(X-\mu \le b-\mu) = P\left(\frac{X-\mu}{\sigma} \le \frac{b-\mu}{\sigma}\right) = P\left(Z \le \frac{b-\mu}{\sigma}\right)$.

2. $P(a \le X \le b) = P\left(\frac{a-\mu}{\sigma} \le Z \le \frac{b-\mu}{\sigma}\right)$

3. $P(x > a) = P\left(z > \frac{a-\mu}{\sigma}\right)$

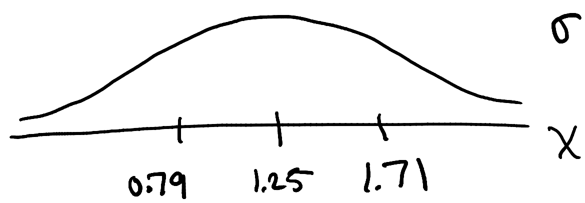
4. $x = \mu + z\sigma$

<u>Ex</u>: x = Driver's reaction time to brake lights ahead of them.

Suppose $x \sim N(\underset{\mu}{\underset{\shortparallel}{1.25 \text{ sec}}}, \underset{\sigma^2}{\underset{\shortparallel}{(0.46 \text{ sec})^2}})$

$\sigma^2 = (0.46)^2$

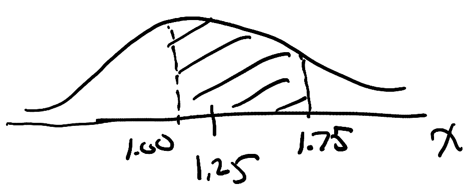
$\sigma = 0.46$ sec



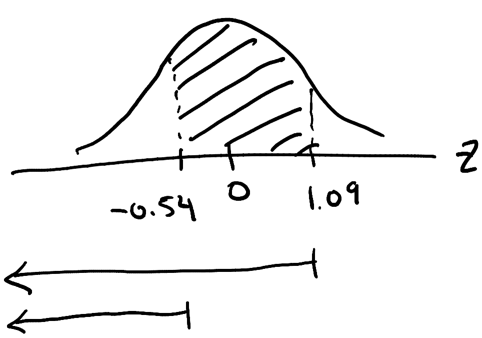
i.) Find probability reaction time is between 1 sec and 1.75 sec.

$$P(1.00 < x < 1.75) = P\left(\frac{1.00-1.25}{0.46} < Z < \frac{1.75-1.25}{0.46}\right)$$

$$= P(-0.54 < Z < 1.09)$$



$\Leftrightarrow$



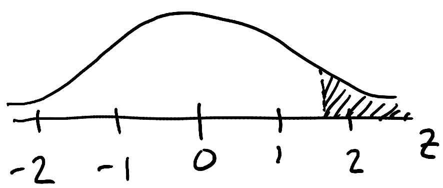
$$= P(Z < 1.09) - P(Z < -0.54)$$

$$= 0.8621 - 0.2946$$

$= 0.5675$

ii) If 2 seconds is viewed as a critically long reaction time, what proportion of reactions exceed this critical value?

$$P(x > 2) = P\left(z > \frac{2-1.25}{0.46}\right) = P(z > 1.63)$$



$$= 1 - P(z \le 1.63)$$

$$= P(z \le -1.63)$$

$$= 0.0516$$

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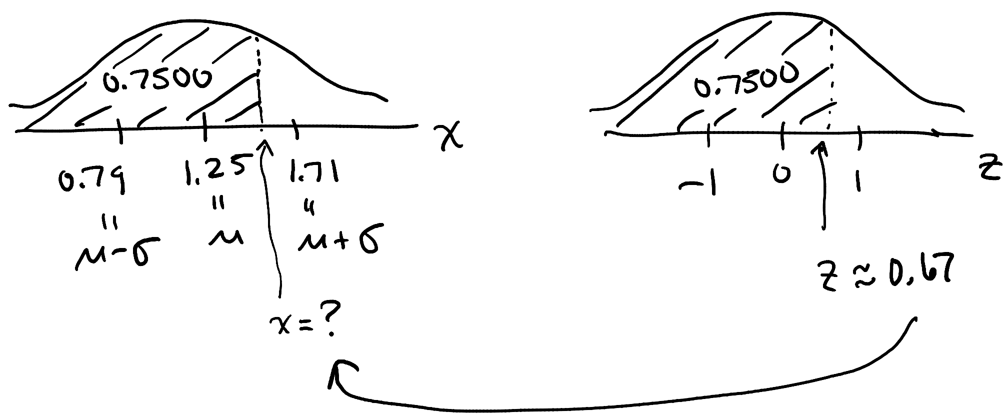
iii) What reaction time separates the fastest 75% of all times from the others?



$$x = \mu + z\sigma$$

$$= 1.25 + (0.67)(0.46)$$

$$= 1.56 \text{ sec}$$