STA 673 Practice Problems #3

1. Please refer to Problem 3 on Homework #10.

a. Are R and θ independent random variables? Please explain.

b. Find the marginal distribution of R, the mean of R, and the standard deviation of R.

c. Find the marginal distribution of θ, the mean of θ, and the standard deviation of θ.

2. Suppose that $X, Y \overset{iid}{\sim} Exponential(\beta = 1)$. Let $(U, V) = (X, X + Y)$.

a. Find the marginal pdfs of U and V.

b. Find the joint pdf of (U, V).

c. Find the conditional pdf of V given $U = u$.

d. Find the conditional expectation and conditional variance of V given $U = u$.

3. Suppose that (X, Y) has a bivariate normal distribution with parameters $\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2$, and $\rho \neq 0$. Show that the bivariate moment generating function of (X, Y) is

$$M_{X,Y}(s,t) = exp\Big(\mu_X s + \mu_Y t + \frac{1}{2}(\sigma_X^2 s^2 + \sigma_Y^2 t^2 + 2\rho\sigma_X\sigma_Y st)\Big).$$

4. Suppose $X_1, ..., X_n \overset{iid}{\sim} N(\mu, \sigma^2)$. Let $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

a. Find $E(\overline{X})$ and $Var(\overline{X})$.

b. Do the solutions to part **a.** depend on normality?

c. Find the distribution of $\overline{X}$.