Chapter 8b

Saturday, January 7, 2023 1:07 PM

3/29/2023

Chapter 8 – Hypothesis Testing (continued)

Ex: Weight loss center has a new diet program.

Center claims participants can expect to lose over 22 pounds in a ten week period.

Test the center's claim.

Follow the steps on the handout!

μ = population mean
= mean weight loss of all participants in the diet program.

$H_0: \mu = 22$

$H_1: \mu > 22$ (right tail test)

Suppose $\alpha = P(\text{type I error}) = 0.01$

$n = 16$

$\bar{x} = 23.5$ lbs

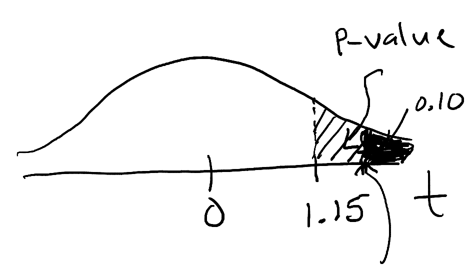
$s = 5.2$ lbs

test statistic $\quad t = \dfrac{\bar{x} - 22}{s/\sqrt{n}}$

$df = n - 1 = 15$

Value of observed test statistic is

$$t = \frac{23.5 - 22}{5.2/\sqrt{16}} = 1.15$$



$P\text{-value} = P(t \geq 1.15) > 0.10$ using table.

Since $P\text{-value} > \underset{\underset{0.01}{\shortparallel}}{\alpha}$ so fail to reject Ho.

The center's claim that weight loss participants can expect to lose more than 22 pounds is not substantiated by the sample information where $\alpha = 0.01$.

Ex: Heights of one-year old red pine seedlings. Does sample evidence suggest that the mean population height of red pine seedlings differ from 1.9 cm?

$\mu =$ "fill in"

$H_0: \mu = 1.9$ cm

$H_1: \mu \neq 1.9$ cm (two tail test)

Here, consider $\alpha = 0.05$

$n = 40$

$\bar{x} = 1.715$ cm

$s = 0.475$ cm

test statistic $t = \dfrac{\bar{x} - 1.9}{s/\sqrt{n}}$

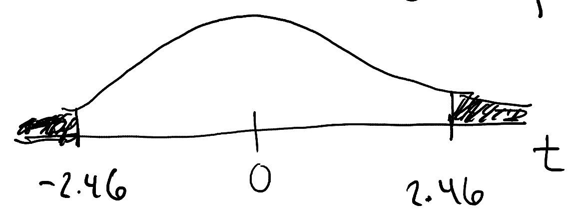
Assumptions?

- Random sample

- x-values are normal
  OR
  $n \geq 30$ so use CLT

Observed test statistic $\quad t = \dfrac{1.715 - 1.9}{0.475/\sqrt{40}} = -2.46$

df = 39



$$\text{P-value} = P(t \leq -2.46 \text{ or } t \geq 2.46)$$
$$= P(t \leq -2.46) + P(t \geq 2.46)$$
$$= 2 \times P(t \leq -2.46)$$

From t-table $\quad 0.005 < P(t \leq -2.46) < 0.01$

so $\quad 2(0.005) < \text{P-value} < 2(0.01)$

and $\quad 0.01 < \text{P-value} < 0.02$

Since P-value $\leq \alpha$, reject H_0.

Sample evidence suggests that the mean height of one year old red pine seedlings is different 1.9 cm using $\alpha = 0.05$.

<u>Ex</u>: Out of 72 mechanics who examined a car with a specific defect, 63 correctly identified the problem. Does this indicate that the true proportion who could correctly identify the problem exceeds 0.75?

p = "fill in" = population proportion

H_0: $p = 0.75$

H_1: $p > 0.75$ (right tail test)

Select $\alpha = P(\text{type I error}) = ?$

$n = 72$

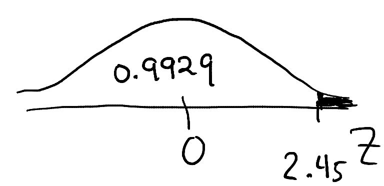
$\hat{p} = \frac{63}{72}$

test statistic $\quad Z = \dfrac{\hat{p} - 0.75}{\sqrt{\dfrac{(0.75)(1-0.75)}{n}}}$

NOTE: Using CLT,
$n(0.75) \geq 5$
$n(1-0.75) \geq 5$

Observed test statistic $\quad z = \dfrac{\frac{63}{72} - 0.75}{\sqrt{\dfrac{(0.75)(0.25)}{72}}} = 2.45$

P-value $= P(Z \geq 2.45) = 0.0071$.



Since P-value $\leq \alpha$ for any commonly used α-values so reject H_0. Sample evidence suggests the true proportion of mechanics who can correctly identify the problem is greater than 0.75 (using any commonly chosen α-value).

Ex: (revisited) μ = population mean height of one year old red pine seedlings.

$H_0: \mu = 1.9\text{cm}$

$H_1: \mu \neq 1.9\text{ cm}$

$\alpha = 0.05$ Reject Ho based on data.

A 95% CI for μ is

$$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

NOTE: $df = n-1$
$= 39$

so

$$1.715 \pm 2.023 \times \frac{0.475}{\sqrt{40}}$$

$t_{\frac{\alpha}{2}} = t_{0.025}$

or

$$(1.56 \text{ cm}, 1.87 \text{ cm})$$

NOTE: $\mu = 1.9\text{cm}$ is not in the above CI.
1.9cm is not a likely value μ.