Example 9.6

Linden [21] conducted a factor-analytic study of Olympic decathlon scores since World War II. Altogether, 160 complete starts were made by 139 athletes.[3] The scores for each of the 10 decathlon events were standardized and a sample correlation matrix was factor-analyzed by the methods of principal components and maximum likelihood. Linden reports that the "distributions of standard scores were normal or approximately normal for each of the ten decathlon events." The sample correlation matrix, based on $n = 160$ starts, is

	100-m run	Long jump	Shot put	High jump	400-m run	110-m hurdles	Discus	Pole vault	Javelin	1500-m run
	1.0	.59	.35	.34	.63	.40	.28	.20	.11	−.07
		1.0	.42	.51	.49	.52	.31	.36	.21	.09
			1.0	.38	.19	.36	.73	.24	.44	−.08
				1.0	.29	.46	.27	.39	.17	.18
					1.0	.34	.17	.23	.13	.39
R =						1.0	.32	.33	.18	.00
							1.0	.24	.34	−.02
								1.0	.24	.17
									1.0	−.00
										1.0

From a principal component factor analysis perspective, the first four eigenvalues, 3.78, 1.52, 1.11, .91, of **R** suggest a factor solution with $m = 3$ or $m = 4$. A subsequent interpretation of the factor loadings reinforces the choice $m = 4$.

TABLE 9.4

Variable	Principle component: Estimated factor loadings F_1	F_2	F_3	F_4	Specific variances $\tilde{\psi}_i = 1 - \tilde{h}_i^2$
1. 100-m run	.691	.217	−.520	−.206	.16
2. Long jump	.789	.184	−.193	.092	.30
3. Shot put	.702	−.535	.047	−.175	.19
4. High jump	.674	.134	.139	.396	.35
5. 400-m run	.620	.551	−.084	−.419	.13
6. 100-m hurdles	.687	.042	−.161	.345	.38
7. Discus	.621	−.521	.109	−.234	.28
8. Pole vault	.538	.087	.411	.440	.34
9. Javelin	.434	−.439	.372	−.235	.43
10. 1500-m run	.147	.596	.658	−.279	.11
Cumulative proportion of total variance explained	.38	.53	.64	.73	

Interpretation of factors

F_1: General athletic ability

F_2: Contrast of running and throwing ability

F_3: Contrast of endurance and speed

F_4: Contrast of jumping and running ability

TABLE 9.4

Variable	Principle component: Estimated factor loadings F_1	F_2	F_3	F_4	Principle component: Specific variances $\tilde{\psi}_i = 1 - \tilde{h}_i^2$	Maximum likelihood: Estimated factor loadings F_1	F_2	F_3	F_4	Maximum likelihood: Specific variances $\hat{\psi}_i = 1 - \hat{h}_i^2$
1. 100-m run	.691	.217	−.520	−.206	.16	−.090	.341	.830	−.169	.16
2. Long jump	.789	.184	−.193	.092	.30	.065	.433	.595	.275	.38
3. Shot put	.702	−.535	.047	−.175	.19	−.139	.990	.000	.000	.00
4. High jump	.674	.134	.139	.396	.35	.156	.406	.336	.445	.50
5. 400-m run	.620	.551	−.084	−.419	.13	.376	.245	.671	−.137	.33
6. 100-m hurdles	.687	.042	−.161	.345	.38	−.021	.361	.425	.388	.54
7. Discus	.621	−.521	.109	−.234	.28	−.063	.728	.030	.019	.46
8. Pole vault	.538	.087	.411	.440	.34	.155	.264	.229	.394	.70
9. Javelin	.434	−.439	.372	−.235	.43	−.026	.441	−.010	.098	.80
10. 1500-m run	.147	.596	.658	−.279	.11	.998	.059	.000	.000	.00
Cumulative proportion of total variance explained	.38	.53	.64	.73		.12	.37	.55	.61	

Interpretation of Factors

Factor 1: Endurance

Factor 2: Arm strength

Factor 3: Leg speed

Factor 4: Jumping Ability