Example of Factor Analysis: Sparrow Data

Bumpus (1898) collected data on $n = 49$ moribund female sparrows after a severe storm. Measurements were made on $p = 5$ characteristics: X_1=total length (mm); X_2=alar (wing) length (mm); X_3=length of beak and head (mm); X_4=length of humerus (mm); and X_5=length of keel of sternum (longitudinal length of breast bone, mm). Subsequently, 28 of the 49 female sparrows died.

```
# Bumpus Sparrow Data from Manly
#
X<-matrix(data=TX,ncol=7,byrow=TRUE)
#
Xdata <-
data.frame(Bird=X[,1],TotalLength=X[,2],AlarExtent=X[,3],BeakandHead=X[,4],Humerus=X[,
5],KeelofSternum=X[,6],Survived=X[,7])
#
summary(Xdata[,2:6])
cov(Xdata[,2:6])
cor(Xdata[,2:6])
#
#PC method (assumes package psych is loaded)
library(psych)
#
FApc1 <- principal(Xdata[,2:6], nfactors = 2, rotate = "none", covar = TRUE)
FApc1
#
library(GPArotation)  # required for factor rotation
FApc2 <- principal(Xdata[,2:6], nfactors = 2, rotate = "varimax", covar = TRUE)
FApc2
#
#MLE method
#
FAmle1 <- factanal(Xdata[,2:6], factors = 2, rotation = "none")
FAmle1
#
FAmle2 <- factanal(Xdata[,2:6], factors = 2, rotation = "varimax")
FAmle2
############
> summary(Xdata[,2:6])
  TotalLength     AlarExtent      BeakandHead        Humerus        KeelofSternum
 Min.   :152    Min.   :230.0   Min.   :30.10    Min.   :17.20    Min.   :18.60
 1st Qu.:155    1st Qu.:238.0   1st Qu.:30.90    1st Qu.:18.10    1st Qu.:20.20
 Median :158    Median :242.0   Median :31.50    Median :18.50    Median :20.70
 Mean   :158    Mean   :241.3   Mean   :31.46    Mean   :18.47    Mean   :20.83
 3rd Qu.:161    3rd Qu.:245.0   3rd Qu.:32.00    3rd Qu.:18.80    3rd Qu.:21.50
 Max.   :165    Max.   :252.0   Max.   :33.40    Max.   :19.80    Max.   :23.10
> cov(Xdata[,2:6])
              TotalLength AlarExtent BeakandHead   Humerus KeelofSternum
TotalLength     13.353741  13.610969   1.9220663 1.3306122     2.1922194
AlarExtent      13.610969  25.682823   2.7136054 2.1977041     2.6578231
BeakandHead      1.922066   2.713605   0.6316327 0.3422662     0.4146471
Humerus          1.330612   2.197704   0.3422662 0.3184184     0.3393707
KeelofSternum    2.192219   2.657823   0.4146471 0.3393707     0.9828231
> cor(Xdata[,2:6])
              TotalLength AlarExtent BeakandHead   Humerus KeelofSternum
TotalLength     1.0000000  0.7349642   0.6618119 0.6452841     0.6051247
AlarExtent      0.7349642  1.0000000   0.6737411 0.7685087     0.5290138
BeakandHead     0.6618119  0.6737411   1.0000000 0.7631899     0.5262701
Humerus         0.6452841  0.7685087   0.7631899 1.0000000     0.6066493
KeelofSternum   0.6051247  0.5290138   0.5262701 0.6066493     1.0000000
```

```
> #PC method (assumes package psych is loaded)
> library(psych)
> #
> FApc1 <- principal(Xdata[,2:6], nfactors = 2, rotate = "none", covar = TRUE)
> FApc1
Principal Components Analysis
Call: principal(r = Xdata[, 2:6], nfactors = 2, rotate = "none", covar = TRUE)
Unstandardized loadings (pattern matrix) based upon covariance matrix
                PC1   PC2    h2     u2    H2      U2
TotalLength    3.19  1.78 13.34 0.0160 1.00 0.00120
AlarExtent     4.93 -1.18 25.68 0.0037 1.00 0.00014
BeakandHead    0.57  0.07  0.33 0.2975 0.53 0.47099
Humerus        0.44 -0.03  0.20 0.1221 0.62 0.38360
KeelofSternum  0.60  0.21  0.40 0.5819 0.41 0.59206

                        PC1   PC2
SS loadings           35.33  4.62
Proportion Var         0.86  0.11
Cumulative Var         0.86  0.98
Proportion Explained   0.88  0.12
Cumulative Proportion  0.88  1.00


 Standardized loadings (pattern matrix)
              item  PC1    PC2   h2      u2
TotalLength      1 0.87   0.49 1.00 0.00120
AlarExtent       2 0.97  -0.23 1.00 0.00014
BeakandHead      3 0.72   0.09 0.53 0.47099
Humerus          4 0.78  -0.06 0.62 0.38360
KeelofSternum    5 0.60   0.22 0.41 0.59206

                  PC1  PC2
SS loadings      3.20 0.35
Proportion Var   0.64 0.07
Cumulative Var   0.64 0.71
Cum. factor Var  0.90 1.00

> library(GPArotation)  # required for rotation in principal
> FApc2 <- principal(Xdata[,2:6], nfactors = 2, rotate = "varimax", covar = TRUE)
> FApc2
Principal Components Analysis
Call: principal(r = Xdata[, 2:6], nfactors = 2, rotate = "varimax",
    covar = TRUE)
Unstandardized loadings (pattern matrix) based upon covariance matrix
               RC1  RC2    h2     u2    H2      U2
TotalLength   1.46 3.35 13.34 0.0160 1.00 0.00120
AlarExtent    4.64 2.05 25.68 0.0037 1.00 0.00014
BeakandHead   0.41 0.41  0.33 0.2975 0.53 0.47099
Humerus       0.37 0.24  0.20 0.1221 0.62 0.38360
KeelofSternum 0.34 0.53  0.40 0.5819 0.41 0.59206


                        RC1    RC2
SS loadings           24.04  15.91
Proportion Var         0.59   0.39
Cumulative Var         0.59   0.98
Proportion Explained   0.60   0.40
Cumulative Proportion  0.60   1.00
```

```
Standardized loadings (pattern matrix)
             item  RC1  RC2   h2      u2
TotalLength     1 0.40 0.92 1.00 0.00120
AlarExtent      2 0.91 0.40 1.00 0.00014
BeakandHead     3 0.52 0.51 0.53 0.47099
Humerus         4 0.66 0.43 0.62 0.38360
KeelofSternum   5 0.35 0.54 0.41 0.59206


                 RC1  RC2
SS loadings     1.82 1.74
Proportion Var  0.36 0.35
Cumulative Var  0.36 0.71
Cum. factor Var 0.51 1.00


> #MLE method
> FAmle1 <- factanal(Xdata[,2:6], factors = 2, rotation = "none")
> FAmle1

Call:
factanal(x = Xdata[, 2:6], factors = 2, rotation = "none")

Uniquenesses:
  TotalLength    AlarExtent   BeakandHead       Humerus KeelofSternum
        0.005         0.307         0.362         0.015         0.552

Loadings:
              Factor1 Factor2
TotalLength    0.983  -0.170
AlarExtent     0.792   0.257
BeakandHead    0.730   0.325
Humerus        0.766   0.631
KeelofSternum  0.646   0.176

              Factor1 Factor2
SS loadings     3.129   0.630
Proportion Var  0.626   0.126
Cumulative Var  0.626   0.752


> FAmle2 <- factanal(Xdata[,2:6], factors = 2, rotation = "varimax")
> FAmle2

Call:
factanal(x = Xdata[, 2:6], factors = 2, rotation = "varimax")

Uniquenesses:
  TotalLength    AlarExtent   BeakandHead       Humerus KeelofSternum
        0.005         0.307         0.362         0.015         0.552

Loadings:
              Factor1 Factor2
TotalLength    0.366   0.928
AlarExtent     0.631   0.543
BeakandHead    0.657   0.454
Humerus        0.937   0.326
KeelofSternum  0.486   0.460


              Factor1 Factor2
SS loadings     2.078   1.681
Proportion Var  0.416   0.336
Cumulative Var  0.416   0.752
```