STA 474 Practice Problems 1

1. Suppose Y_1 and Y_2 are independent and identically distributed random variables with $E(Y_1) = E(Y_2) = \mu$ and $V(Y_1) = V(Y_2) = \sigma^2$. Suppose $U_1 = Y_1 + Y_2$ and $U_2 = Y_1 - Y_2$.
a. Find $E(U_1)$, $V(U_1)$, $E(U_2)$, and $V(U_2)$.
b. Find the covariance between $U_1 = Y_1 + Y_2$ and $U_2 = Y_1 - Y_2$.
c. How would your result in part **b.** change if Y_1 and Y_2 were not independent? Please explain.

2. In the previous problem, suppose Y_1, Y_2 is a random sample of size 2 from a normally distributed population with mean zero and variance one.
a. What is the distribution of U_1?
b. What is the distribution of U_2?
c. What is the joint distribution of U_1 and U_2?

The following three exercises are from the textbook.

3. Problem 6.1. Parts **a.**, **b.**, and **c.**
4. Problem 6.23
5. Problem 6.37

6. Suppose $Y_1, ..., Y_{100}$ are independent identically normally distributed with unknown mean μ and variance $\sigma^2 = 100$.
a. What is the distribution of $\overline{Y}$?
b. What is the probability that the sample mean differs from the population mean by no more than 2.0?
c. Did you use the Central Limit Theorem to answer part **b.**? Please explain.

The following two exercises are from the textbook.

7. Problem 7.26
8. Problem 8.12