STA 674 Practice Problems #3

1. Suppose that $X_1, ..., X_n \overset{iid}{\sim} N(\theta, \theta)$, $\theta > 0$.

a. Show that $T = \frac{1}{n} \sum\limits_{i=1}^{n} X_i^2$ is a complete sufficient statistics for the family of distributions indexed by θ. What is the expectation of T?
b. There is some function of θ, say $\tau(\theta)$, for which the UMVUE is "obvious." Identify $\tau(\theta)$.
c. What is the limiting distribution of T?
d. Find a consistent estimator of θ.
e. Is the variance of the asymptotic distribution of the estimator in part **d.** equal to the CRLB? Please explain.

2. Suppose that $X_1, ..., X_n \overset{iid}{\sim} f(x|\theta) = \frac{1}{1-\theta} I_{(\theta,1)}(x)$, where $0 < \theta < 1$.

a. What is the distribution of $X_{(1)}$?
b. Does an UMVUE of θ exist? If so, justify and find it.
c. For $Uniform(0, 1)$ prior, find the Posterior Bayes Estimator of θ.
d. In testing $H_0 : \theta \le 1/2$ versus $H_1 : \theta > 1/2$, the following test was used: Reject H_0 if and only if $X_{(1)} > c$. Find c so that the test has size α and then find the power function of the test.

3. Using a random sample of size n from a normal distribution with unknown mean μ and unknown variance σ^2, derive the LRT procedure of size α for the test $H_0 : \sigma^2 \le \sigma_0^2$ versus $H_1 : \sigma^2 > \sigma_0^2$.

4. For a random sample of size n from a normal distribution with mean μ known, derive an equal-tailed $100(1-\alpha)\%$ confidence interval estimator of σ^2 using the pivotal quantity $\sum_{i=1}^{n} (X_i - \mu)^2/\sigma^2$. Compare the length of this interval with the length of the equal-tailed $100(1-\alpha)\%$ confidence interval estimator of σ^2 using the pivotal quantity $\sum_{i=1}^{n} (X_i - \overline{X})^2/\sigma^2$.