02/01/2023

Chapter 4 - Probability

Def: An *experiment* is a situation whose outcomes depend on chance (or randomness). The result cannot be predicted with perfect certainty.

Def: A *sample space* of an experiment consists of all possible outcomes of the experiment.

Def: A *simple event* is an individual outcome of an experiment.

Def: An *event* is a collection of simple events.

Ex: Toss a *fair* coin 3 times and record head (H) or tail (T) after each toss.

Simple events: HHH, HHT, HTH, THH, THT, TTH, HTT, TTT

$P(H) = 0.5$

} equally likely since coin is fair

Aside: $\frac{2}{}\ \frac{2}{}\ \frac{2}{}$

Ex: (continued)

Sample Space = {HHH, HHT, HTH, THH, THT, TTH, HTT, TTT}

Event A = 2 heads occur in 3 tosses of fair coin

= {HHT, HTH, THH}

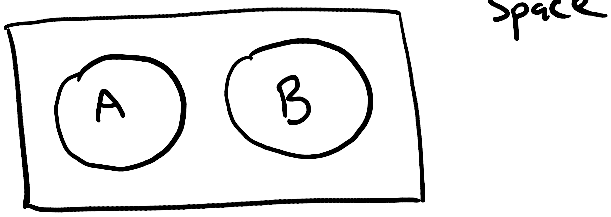
$$P(A) = \frac{3}{8}$$

B = 2 tails occur in 3 tosses of coin

= {THT, TTH, HTT}

P(A and B) = 0

Venn Diagram for this example

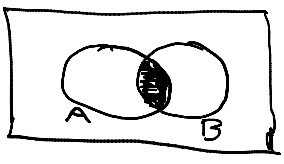


Def: 1. $\bar{A}$ is called the complement of A and consists of all simple events not included in A.

2. Event A or B is the event that at least one of A or B occurs.

3. Event A and B is the event that both A and B occur.

A and B



New picture

Ex: (continued)

C = [illegible] 1st two tosses

Ex: (continued) picture

C = 2 heads occur on 1st two tosses

$= \{HHH, HHT\}$

A and C $= \{HHT\}$ $\quad P(A \text{ and } C) = \frac{1}{8}$

Def: A and B are disjoint (or mutually exclusive) events if the have no simple events in common.

A and B $= \{\ \}$ = empty set

Axioms of Probability

1. $0 \le P(A) \le 1$ for any event A.
2. P(Sample Space) = 1

Ex: Coin example (continued)

$P(B) = \frac{3}{8}$, $P(C) = \frac{2}{8} = \frac{1}{4}$

$P(A \text{ and } B) = 0$, $P(A \text{ or } B) = \frac{6}{8} = \frac{3}{4}$

$P(A \text{ or } C) = \frac{4}{8} = \frac{1}{2} = P(\{HHT, HTH, THH, HHH\})$

Def: 1. If A and B are disjoint events, then

$P(A \text{ or } B) = P(A) + P(B)$

2. For any two events A and B,

2. For any two events A and B,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

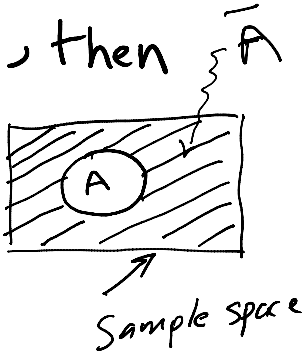
Ex: (continued)

$$P(A \text{ or } C) = P(A) + P(C) - P(A \text{ and } C)$$
$$= \frac{3}{8} + \frac{3}{8} - \frac{1}{8} = \frac{5}{8}.$$

Def: If $\bar{A}$ is the complement of event A, then

$$P(A) + P(\bar{A}) = 1.$$

$$P(A) = 1 - P(\bar{A}).$$



Ex: Toss a fair coin 3 times.

A = At least one tail in 3 tosses of coin.

$$P(A) = 1 - P(\bar{A})$$
$$= 1 - P(HHH)$$
$$= 1 - \frac{1}{8}$$
$$= \frac{7}{8}$$

Conditional Probability (and Independent Events)

Def: The conditional probability of event A given event B already occurred is

given event is already occurred is

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

(B: given)

provided $P(B) > 0$.

Remarks: 1. $P(A \text{ and } B) = P(B)\,P(A|B)$

$= P(A)\,P(B|A)$

$= P(A) \cdot \frac{P(B \text{ and } A)}{P(A)}$

2. $0 \le P(A|B) \le 1$

3. $P(B|B) = \frac{P(B \text{ and } B)}{P(B)} = 1$.

Ex: Toss balanced die once.

$A = \{2\}$, $\bar{A} = \{1,3,4,5,6\}$

B = even number $= \{2,4,6\}$

$P(A|B) = \frac{1}{3} = \frac{P(A \text{ and } B)}{P(B)} = \frac{1/6}{3/6} = \frac{1}{3}$.

$P(\bar{A}|B) = \frac{2}{3} = \frac{P(\bar{A} \text{ and } B)}{P(B)}$

Fact: $P(A|B) + P(\bar{A}|B) = 1$.

↑ Exam 1

EXTRA

Not on Exam 1

Def: Two events A and B are <u>independent</u> if the probability that either occurs is not affected by whether or not the other occurred. In this case, any one of the following is true.

i.) $P(A \text{ and } B) = P(A) \cdot P(B)$

ii.) $P(A|B) = P(A)$

iii.) $P(B|A) = P(B)$.

Ex: Toss a balanced 6-sided die once

$A = \{2\}$

$B = \{\text{even number}\} = \{2,4,6\}$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{1}{3}$$

$$P(A) = \frac{1}{6}$$

$P(A|B) \neq P(A)$ so A and B are not independent.

↑ EXTRA