STA 674 Practice Problems #1

1. Suppose that $X_1, X_2 \overset{iid}{\sim} F(x)$ and

$$X_{(1)} = \begin{cases} X_1 & \text{, if } X_1 < X_2 \\ X_2 & \text{, if } X_2 < X_1. \end{cases}$$

Show that $F_{X_{(1)}}(x) = F(x)(2 - F(x))$.

2. Assume that $X_1, ..., X_n$ are independent and $X_i \sim N(\mu, \sigma_i^2)$, $i = 1, ..., n$.
Define $U = \sum_{i=1}^n (X_i/\sigma_i^2) / \sum_{i=1}^n (1/\sigma_i^2)$ and $V = \sum_{i=1}^n [(X_i - U)^2/\sigma_i^2]$.
a. Show that U and V are independent.
b. Find the distribution of U and the distribution of V.

3. Assume that $X_1, ..., X_n \overset{iid}{\sim} Exponential(\theta)$. Define $R = X_{(n)} - X_{(1)}$. Explain how you would find the mean and variance of R.

4. Assume that $X_1, ..., X_n \overset{iid}{\sim} Bernoulli(p)$.
a. Find a sequence of constants b_n and a constant a such that $b_n((\overline{X}_n)^2 - a) \xrightarrow{d} U$ where U has a non-degenerate distribution.
b. What is the asymptotic distribution of $(\overline{X}_n)^2$?

5. Problem 6.2