EXAMPLE 9.6.3. The accidental death series $X_1, \ldots, X_{72}$ is plotted in Figure 1.6. Application of the operator $(1-B)(1-B^{12})$ generates a new series $\{Y_t\}$ with no apparent deviations from stationarity as seen in Figure 1.17. The sample autocorrelation function $\hat{\rho}(\cdot)$ of $\{Y_t\}$ is displayed in Figure 9.21. The values $\hat{\rho}(12) = -.333$, $\hat{\rho}(24) = -.099$ and $\hat{\rho}(36) = .013$ suggest a moving average of order 1 for the between-year model (i.e. $P = 0$, $Q = 1$). Moreover inspection of $\hat{\rho}(1), \ldots, \hat{\rho}(11)$ suggests that $\rho(1)$ is the only short-term correlation different from zero, so we also choose a moving average of order 1 for the between-month model (i.e. $p = 0$, $q = 1$). Taking into account the sample mean (28.831) of the differences $Y_t = (1-B)(1-B^{12})X_t$, we therefore arrive at the model,

$$Y_t = 28.831 + (1 + \theta_1 B)(1 + \Theta_1 B^{12})Z_t, \qquad \{Z_t\} \sim \mathrm{WN}(0, \sigma^2),$$

for the series $\{Y_t\}$. The maximum likelihood estimates of the parameters are,

$$\hat{\theta}_1 = -.479,$$

$$\hat{\Theta}_1 = -.591,$$

and

$$\hat{\sigma}^2 = 94240,$$

with AICC value 855.53. The fitted model for $\{X_t\}$ is thus the

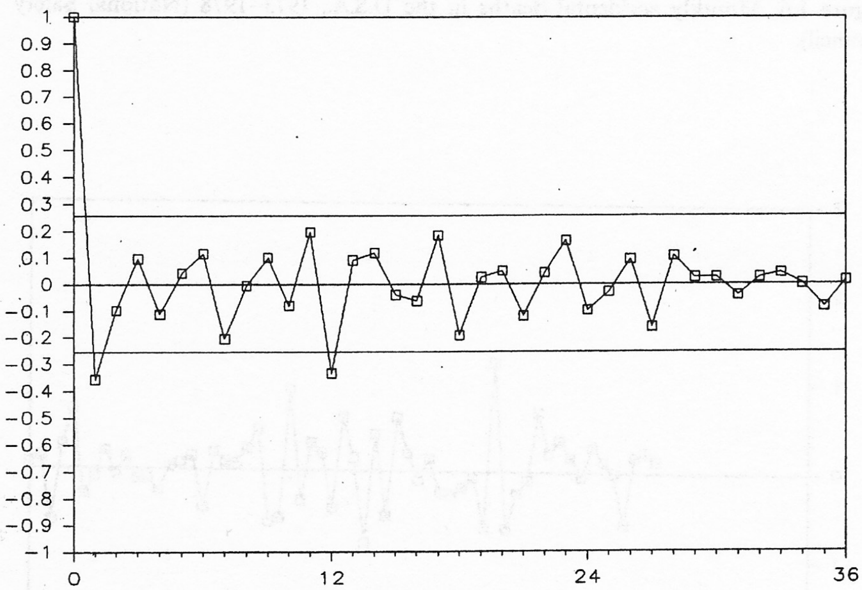

Figure 9.21. The sample ACF of the differenced accidental deaths $\{\nabla\nabla_{12}X_t\}$.

$\hat{\rho}(1) - \hat{\rho}(12)$:	−.36	−.10	.10	−.11	.04	.11	−.20	−.01	.10	−.08	.20	−.33
$\hat{\rho}(13) - \hat{\rho}(24)$:	.09	.12	−.04	−.06	.18	−.19	.02	.05	−.12	.04	.16	−.10
$\hat{\rho}(25) - \hat{\rho}(36)$:	−.03	.09	−.16	.11	.02	.03	−.04	.03	.04	.00	−.09	.01

p d q   P D Q

SARIMA(0, 1, 1) × $(0, 1, 1)_{12}$ process

$$(1-B)(1-B^{12})X_t = Y_t = 28.831 + (1 - .479B)(1 - .591B^{12})Z_t, \tag{9.6.5}$$

where $\{Z_t\} \sim \mathrm{WN}(0, 94240)$.

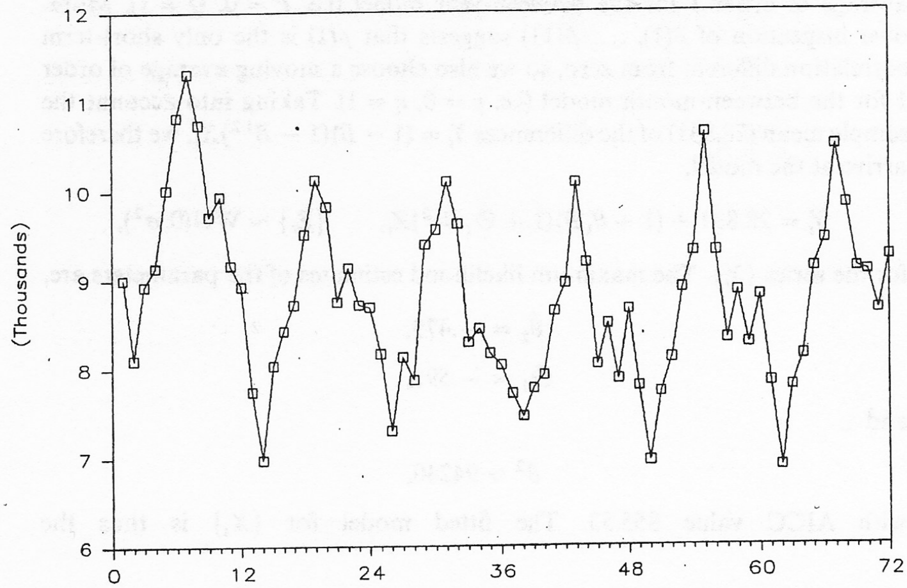

Figure 1.6. Monthly accidental deaths in the U.S.A., 1973–1978 (National Safety Council).

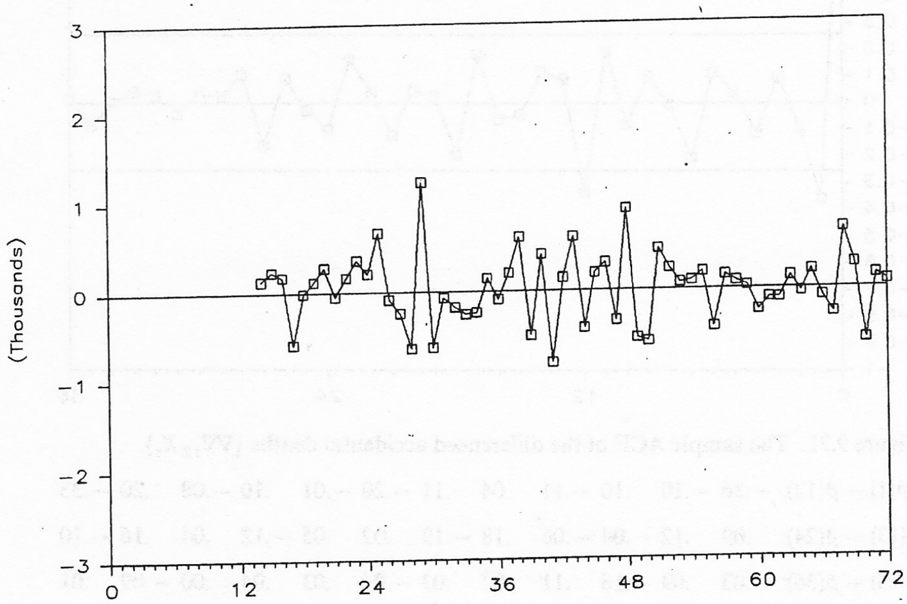

Figure 1.17. The differenced series $\{\nabla\nabla_{12}x_t, t = 14, \ldots, 72\}$ derived from the monthly accidental deaths $\{x_t, t = 1, \ldots, 72\}$.

From Time Series: Theory and Methods by Brockwell and Davis.