Ex: See Example 6.9, p.304 of textbook.

$p=2$, $g=3$ $\qquad n_1=3$, $n_2=2$, $n_3=3$

$$\underset{2\times 3}{X_1'} = [\underset{\sim}{X}_{11}, \underset{\sim}{X}_{12}, \underset{\sim}{X}_{13}] = \left[\begin{bmatrix}9\\3\end{bmatrix}, \begin{bmatrix}6\\2\end{bmatrix}, \begin{bmatrix}9\\7\end{bmatrix} \right], \quad \bar{\underset{\sim}{X}}_1 = \begin{bmatrix}8\\4\end{bmatrix}$$

$$\underset{2\times 2}{X_2'} = [\underset{\sim}{X}_{21}, \underset{\sim}{X}_{22}] = \left[\begin{bmatrix}0\\4\end{bmatrix}, \begin{bmatrix}2\\0\end{bmatrix} \right], \quad \bar{\underset{\sim}{X}}_2 = \begin{bmatrix}1\\2\end{bmatrix}$$

$$\underset{2\times 3}{X_3'} = [\underset{\sim}{X}_{31}, \underset{\sim}{X}_{32}, \underset{\sim}{X}_{33}] = \left[\begin{bmatrix}3\\8\end{bmatrix}, \begin{bmatrix}1\\9\end{bmatrix}, \begin{bmatrix}2\\7\end{bmatrix} \right], \quad \bar{\underset{\sim}{X}}_3 = \begin{bmatrix}2\\8\end{bmatrix}$$

and $\bar{\underset{\sim}{X}} = \begin{bmatrix}4\\5\end{bmatrix}$.

$$B = \sum_{\ell=1}^{3} \sum_{j=1}^{n_\ell} (\bar{\underset{\sim}{X}}_\ell - \bar{\underset{\sim}{X}})(\bar{\underset{\sim}{X}}_\ell - \bar{\underset{\sim}{X}})' = \sum_{\ell=1}^{3} n_\ell (\bar{\underset{\sim}{X}}_\ell - \bar{\underset{\sim}{X}})(\bar{\underset{\sim}{X}}_\ell - \bar{\underset{\sim}{X}})'$$

$$= 3\left(\begin{bmatrix}8-4\\4-5\end{bmatrix} [8-4, 4-5] \right) + 2\left(\begin{bmatrix}1-4\\2-5\end{bmatrix} [1-4, 2-5] \right) + 3\left(\begin{bmatrix}2-4\\8-5\end{bmatrix} [2-4, 8-5] \right).$$

$$= 3\begin{bmatrix}16 & \overset{-4}{\cancel{-8}} \\ \underset{-4}{\cancel{-8}} & 1\end{bmatrix} + 2\begin{bmatrix}9 & 9\\9 & 9\end{bmatrix} + 3\begin{bmatrix}4 & -6\\-6 & 9\end{bmatrix} = \begin{bmatrix}78 & -12\\-12 & 48\end{bmatrix}.$$

Similarly,

$$W = \sum_{\ell=1}^{g} \sum_{j=1}^{n_\ell} (\underset{\sim}{X}_{\ell j} - \bar{\underset{\sim}{X}}_\ell)(\underset{\sim}{X}_{\ell j} - \bar{\underset{\sim}{X}}_\ell)'$$

$$= (\underset{\sim}{X}_{11} - \bar{\underset{\sim}{X}}_1)(\underset{\sim}{X}_{11} - \bar{\underset{\sim}{X}}_1)' + (\underset{\sim}{X}_{12} - \bar{\underset{\sim}{X}}_1)(\underset{\sim}{X}_{12} - \bar{\underset{\sim}{X}}_1)' + (\underset{\sim}{X}_{13} - \bar{\underset{\sim}{X}}_1)(\underset{\sim}{X}_{13} - \bar{\underset{\sim}{X}}_1)'$$

$$+ (\underset{\sim}{X}_{21} - \bar{\underset{\sim}{X}}_2)(\underset{\sim}{X}_{21} - \bar{\underset{\sim}{X}}_2)' + (\underset{\sim}{X}_{22} - \bar{\underset{\sim}{X}}_2)(\underset{\sim}{X}_{22} - \bar{\underset{\sim}{X}}_2)'$$

$$+ (\underset{\sim}{X}_{31} - \bar{\underset{\sim}{X}}_3)(\underset{\sim}{X}_{31} - \bar{\underset{\sim}{X}}_3)' + (\underset{\sim}{X}_{32} - \bar{\underset{\sim}{X}}_3)(\underset{\sim}{X}_{32} - \bar{\underset{\sim}{X}}_3)' + (\underset{\sim}{X}_{33} - \bar{\underset{\sim}{X}}_3)(\underset{\sim}{X}_{33} - \bar{\underset{\sim}{X}}_3)'$$

$$= \begin{bmatrix}10 & 1\\1 & 24\end{bmatrix}$$

$$= (3-1)S_1 + (2-1)S_2 + (3-1)S_3.$$

TABLE 6.3 DISTRIBUTION OF WILKS' LAMBDA, $\Lambda^* = |\mathbf{W}|/|\mathbf{B}+\mathbf{W}|$

No. of variables	No. of groups	Sampling distribution for multivariate normal data
$p = 1$	$g \geq 2$	$\left(\frac{\Sigma n_\ell - g}{g-1}\right)\left(\frac{1-\Lambda^*}{\Lambda^*}\right) \sim F_{g-1,\Sigma n_\ell - g}$
$p = 2$	$g \geq 2$	$\left(\frac{\Sigma n_\ell - g - 1}{g-1}\right)\left(\frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right) \sim F_{2(g-1),2(\Sigma n_\ell - g - 1)}$
$p \geq 1$	$g = 2$	$\left(\frac{\Sigma n_\ell - p - 1}{p}\right)\left(\frac{1-\Lambda^*}{\Lambda^*}\right) \sim F_{p,\Sigma n_\ell - p - 1}$
$p \geq 1$	$g = 3$	$\left(\frac{\Sigma n_\ell - p - 2}{p}\right)\left(\frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right) \sim F_{2p,2(\Sigma n_\ell - p - 2)}$