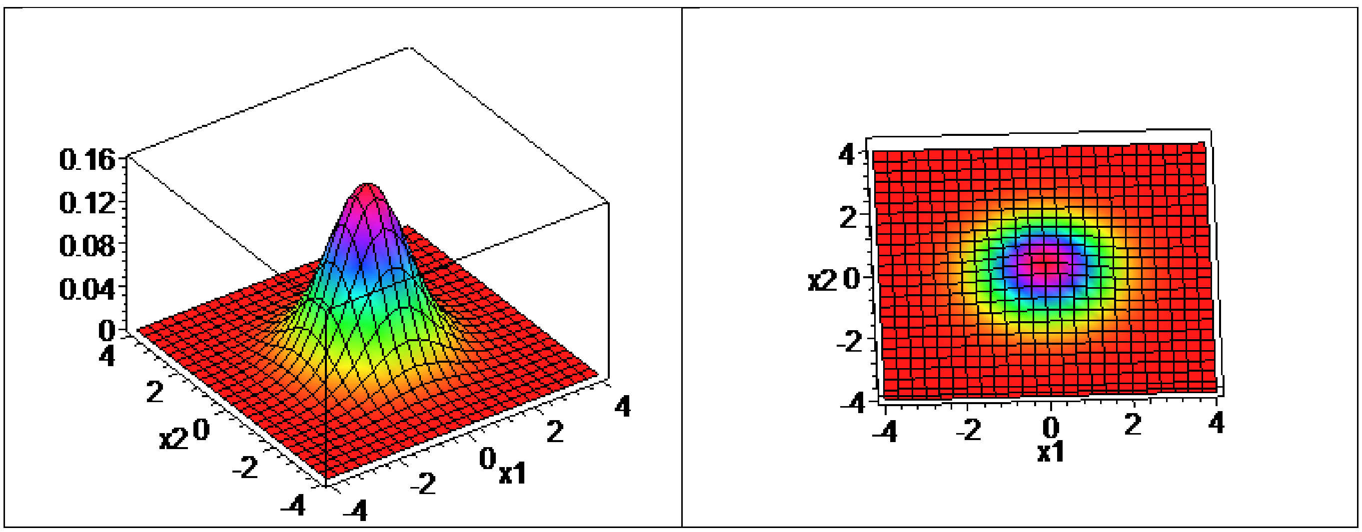
Handout #5

Bivariate Normal Density Function: Contours of Constant Density

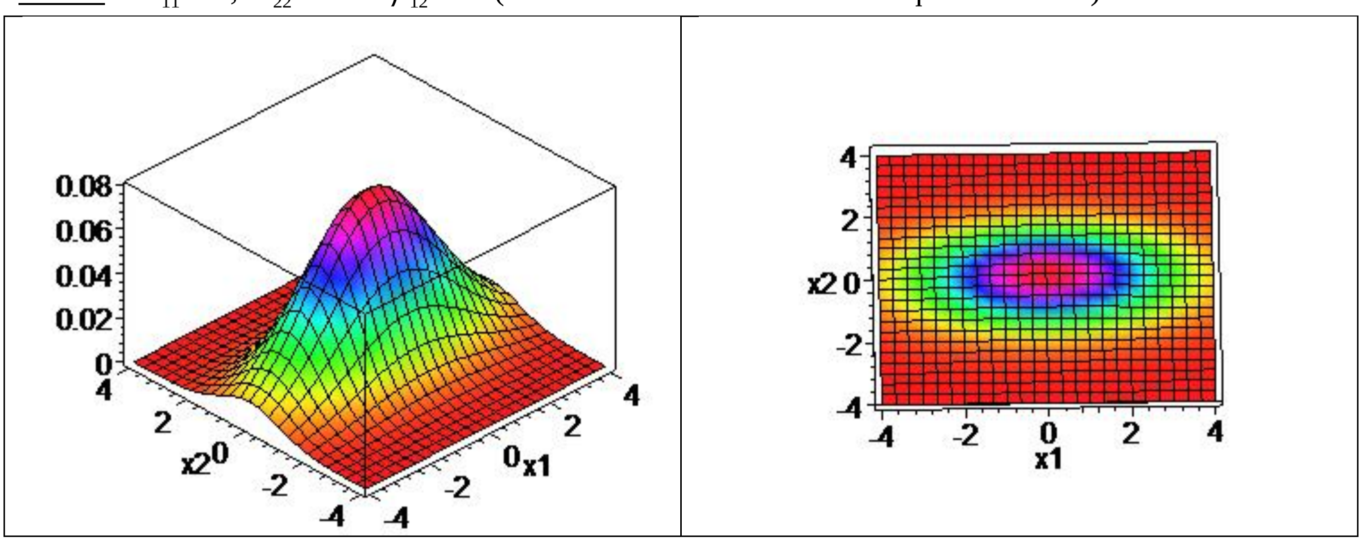
The multivariate normal density function is given by $f(\mathbf{x}) = \dfrac{e^{-\frac{1}{2}Q(\mathbf{x})}}{(2\pi)^{\frac{p}{2}}|\Sigma|^{\frac{1}{2}}}$ where the density depends upon $\mathbf{x}$ only through the quadratic form $Q(\mathbf{x}) = (\mathbf{x}-\mu)'\Sigma^{-1}(\mathbf{x}-\mu)$. This means that the height of the density function is constant whenever $Q(\mathbf{x})$ is equal to a constant. But the **x**-vectors that satisfy $Q(\mathbf{x}) = c^2$ define an ellipsoid centered at the mean vector μ.

To see what this looks like, we consider the bivariate case $(p = 2)$ for $\mu = \mathbf{0}$ and various values of $\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix}$. Note that since $\sigma_{12} = \rho_{12}\sqrt{\sigma_{11}\sigma_{22}}$, if we specify σ_{11}, σ_{22} and ρ_{12} that will completely determine the covariance matrix Σ for the problem.

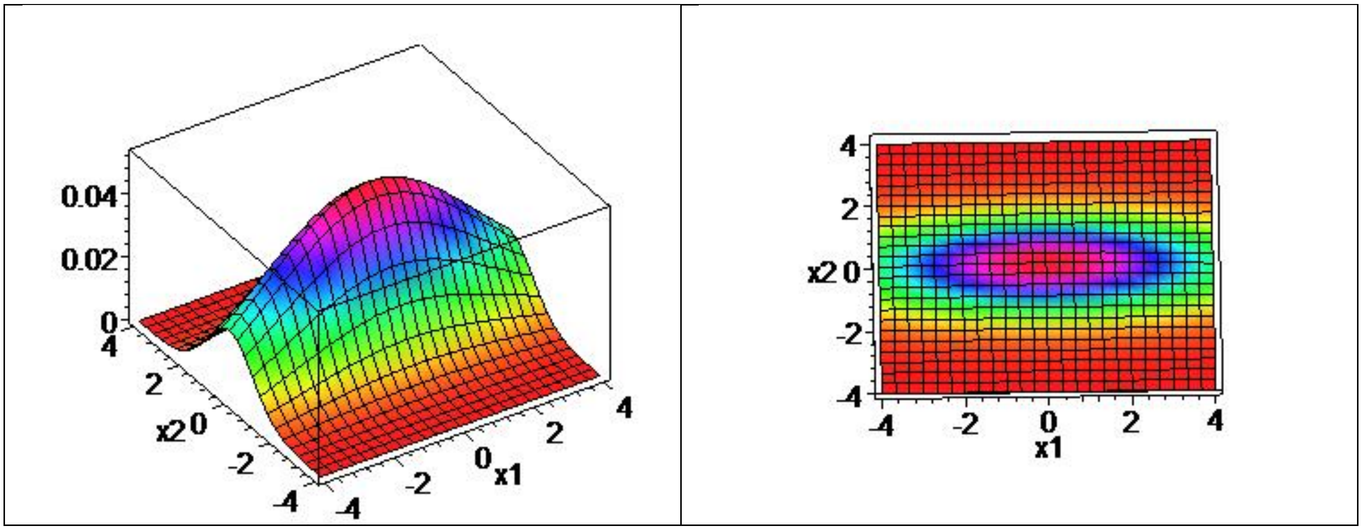
Case 1: $\sigma_{11} = \sigma_{22} = 1$ and $\rho_{12} = 0$ (uncorrelated variables with equal variance).



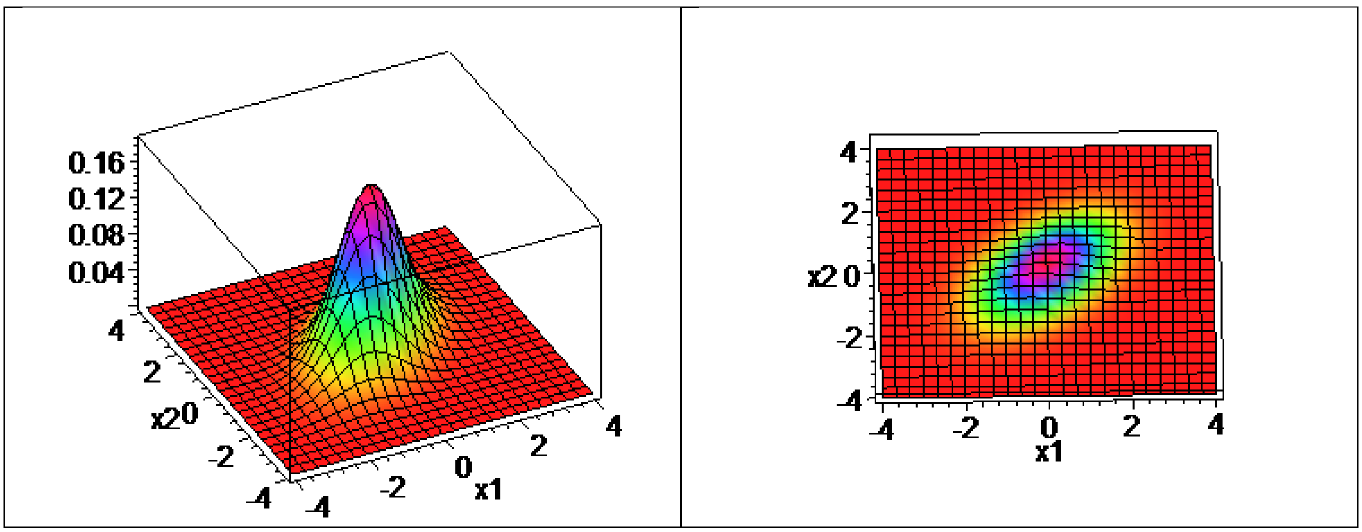
Case 2: $\sigma_{11} = 4$, $\sigma_{22} = 1$ and $\rho_{12} = 0$ (uncorrelated variables with unequal variances).



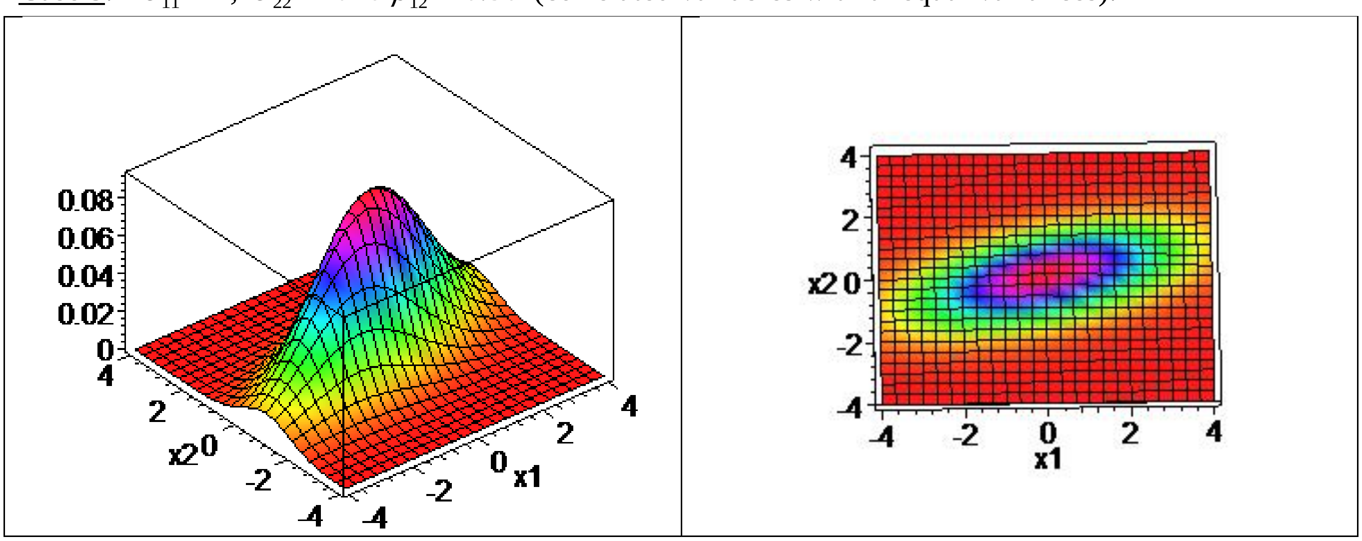
Case 3: $\sigma_{11}=9,\ \sigma_{22}=1$ and $\rho_{12}=0$ (uncorrelated variables with unequal variances).



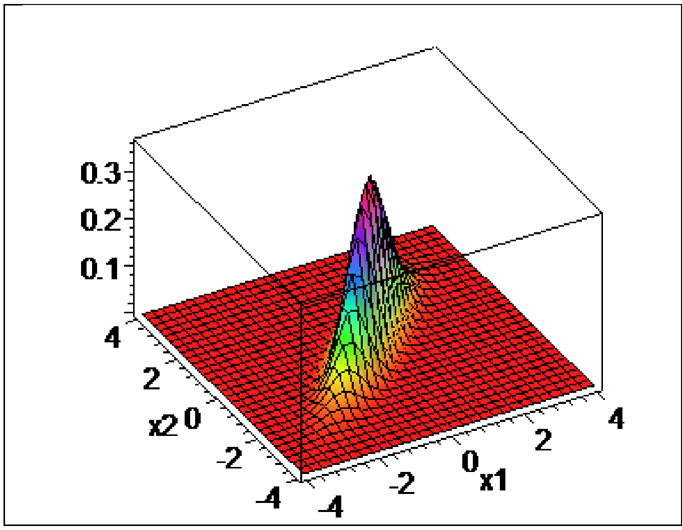
Case 4: $\sigma_{11}=\sigma_{22}=1$ and $\rho_{12}=0.50$ (correlated variables with equal variances).

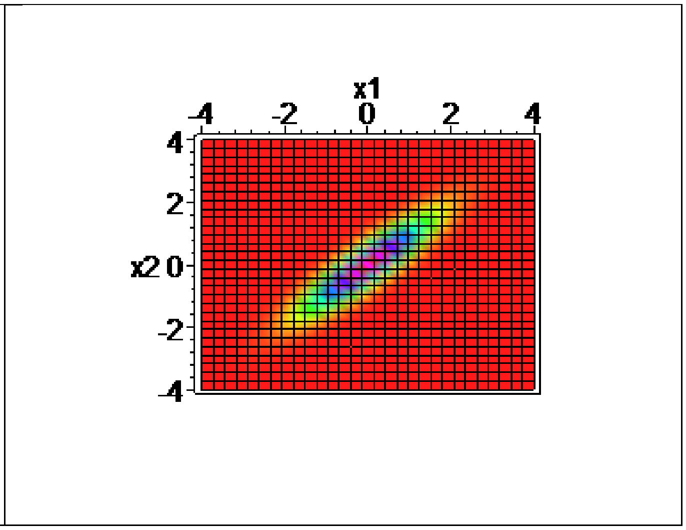


Case 5: $\sigma_{11}=4,\ \sigma_{22}=1$ and $\rho_{12}=0.50$ (correlated variables with unequal variances).

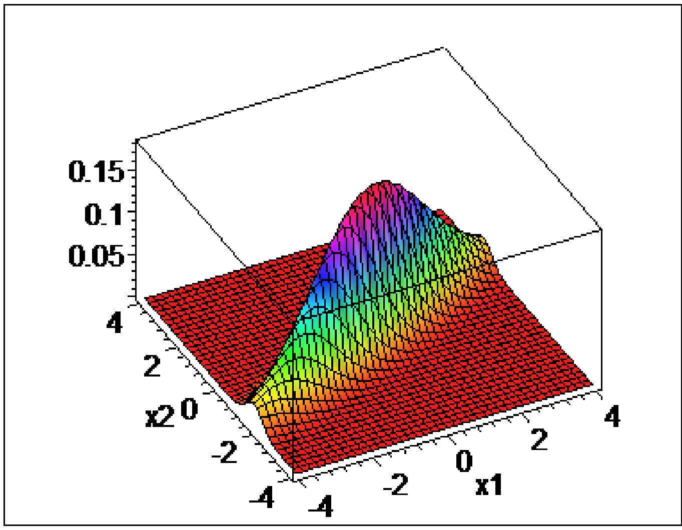


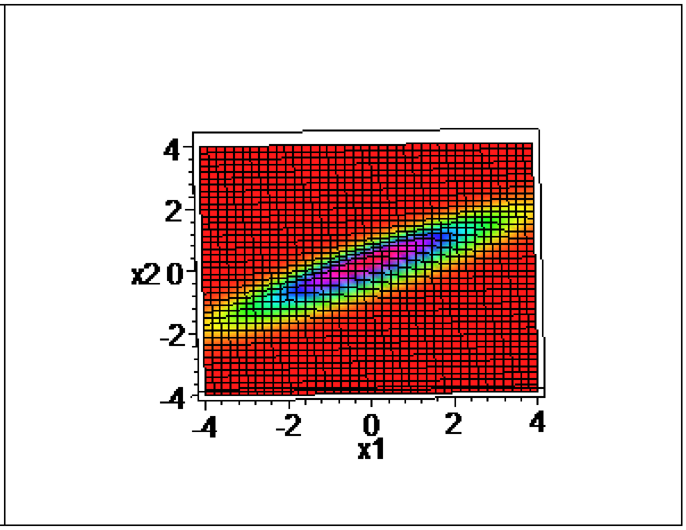
Case 6: $\sigma_{11} = \sigma_{22} = 1$ and $\rho_{12} = 0.90$ (correlated variables with equal variances).





Case 7: $\sigma_{11} = 4$, $\sigma_{22} = 1$ and $\rho_{12} = 0.90$ (correlated variables with unequal variances).





(Courtesy of Dr. Roy St. Laurent)