

## MAT 690 ADV TOPICS IN MATH: LINEAR STATISTICAL MODELS

### Practice Problems #1

1. Standard notation for a linear statistical model (with only fixed effects) is  $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ , where  $\mathbf{Y}$  is a  $n \times 1$  random vector,  $\mathbf{X}$  is a  $n \times p$  matrix,  $\boldsymbol{\beta}$  is a  $p \times 1$  vector, and  $\boldsymbol{\epsilon}$  is a  $n \times 1$  random vector.
  - a. Why is the phrase “only fixed effects” used above? Please explain.
  - b. Under what conditions is  $E(\mathbf{Y}) = \mathbf{X}\boldsymbol{\beta}$ ?
  - c. Describe the subspace of  $\mathbb{R}^n$  in which the mean vector of  $\mathbf{Y}$  resides.
  - d. Based on the linear statistical model above, explain how one may form an estimator of the mean vector of  $\mathbf{Y}$ .
  - e. What sampling conditions give rise to  $Cov(\mathbf{Y}) = \sigma^2\mathbf{I}$ ? Please explain.
  - f. Is  $\frac{1}{n-1}\mathbf{y}'(\mathbf{I} - \frac{1}{n}\mathbf{J})\mathbf{y} > 0$  for all possible observed samples  $\mathbf{y}' = (\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_n)'$ ? Please explain.
2. Using matrix  $\mathbf{A}$  in Problem 2.75, please answer the following.
  - a. Are the columns of  $\mathbf{A}$  linearly independent? Please explain.
  - b. After normalizing the columns of  $\mathbf{A}$ , is the resulting matrix an orthogonal matrix? Please explain.
  - c. Is  $\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{U}'$ , where  $\mathbf{U}$  is an orthogonal matrix and  $\mathbf{D}$  is a diagonal matrix? Please explain.
  - d. Is  $\mathbf{A}$  a projection matrix? Please explain.
  - e. Is  $\mathbf{A}^- = \mathbf{A}^{-1}$ ? Please explain.
3. Answer Problem 2.77 without using a calculator or software.
4. Problem 3.21