MAT 690 ADV TOPICS IN MATH: LINEAR STATISTICAL MODELS

Practice Problems #2

1. Is there a quadratic form in the joint pdf of $\mathbf{Y} \sim N_n(\boldsymbol{\mu}, \boldsymbol{\Sigma})$? Please explain.

2. Suppose $\mathbf{Y} \sim N_n(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ and $\mathbf{W} = \mathbf{BY} + \mathbf{b}$ where $\mathbf{B}$ is a matrix of constants and $\mathbf{b}$ is a vector constants. Explain how moment generating functions may be used to find the distribution of $\mathbf{W}$.

3. Problem 4.9

4. Consider the quadratic form $\mathbf{Y}'\mathbf{AY}$ where $\mathbf{Y}$ is a $n \times 1$ random vector and $\mathbf{A}$ is a symmetric matrix.
a. Does $E(\mathbf{Y}'\mathbf{AY})$ depend on the assumption that $\mathbf{Y}$ is multivariate normally distributed? Please explain.
b. Does $Var(\mathbf{Y}'\mathbf{AY})$ depend on the assumption that $\mathbf{Y}$ is multivariate normally distributed? Please explain.

5. Problem 5.17

6. Suppose $\mathbf{Y} \sim N_n(\mu\mathbf{1}, \sigma^2\mathbf{I})$.
a. Find an unbiased estimator of σ^2. Please show your work.
b. What test statistic could be used to conduct the test $H_o : \sigma^2 = 1$? Please explain.

7. Consider the simple linear regression model discussed in class. In addition to the basic assumptions, suppose that $Y_1, ..., Y_n$ are normally distributed and are independent.
a. Under what conditions do Y_i and Y_j have the same distribution?
b. Develop a confidence interval for β_o.
c. Develop a hypothesis test for β_o.
d. Find $E(\widehat{Y}_i)$.
e. Find $Var(\widehat{Y}_i)$.
f. What is the distribution of $\widehat{Y}_i$?
g. Develop a confidence interval for the mean of Y_i.
h. Develop a hypothesis test for the mean of Y_i.