Handout #10

Displaying a 95% Confidence Region for $\underline{\mu}$ in R when $p = 2$

Recall that a $100(1-\alpha)\%$ Confidence Region for $\underline{\mu}$ is given by the set of all vectors $\underline{\mu}$ that satisfy $(\overline{\underline{\mathbf{X}}} - \underline{\mu})'\mathbf{S}^{-1}(\overline{\underline{\mathbf{X}}} - \underline{\mu}) < c^2$ where $c^2 = \frac{(n-1)p}{n(n-p)} F_{p,n-p}(\alpha)$ is the critical value from the *F* distribution with *p* and *n-p* degrees of freedom and right-tail probability α.

Using the package "ellipse" in **R** and function `ellipse`, we can do this easily:

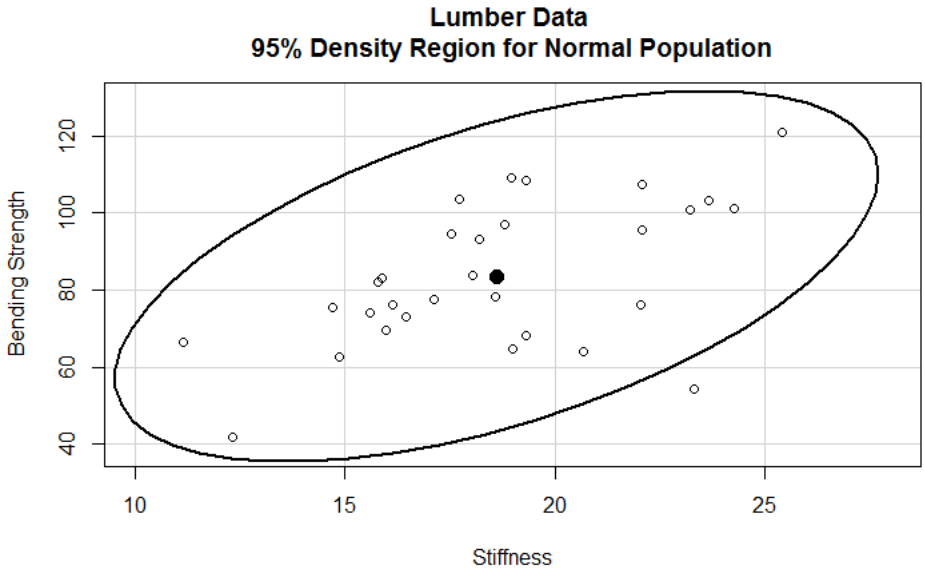
Lumber Example
In this example, $n = 30$ and $p = 2$, recall that the two variables are Stiffness and Bending Strength (of pieces of lumber).

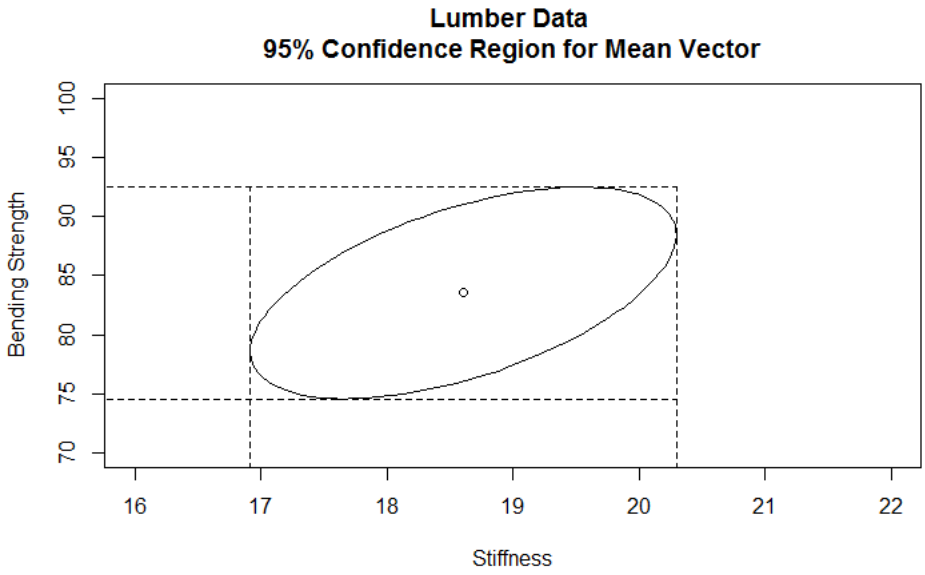
The code below generates a 95% density ellipse for the population and a 95% confidence region for $\underline{\mu}$.

R commands

First load the package "ellipse"

```
# Construct Scatterplot of Data with 95% Density Ellipse
#
library(car)
dataEllipse(Lumber$Stiffness,Lumber$Strength,level=0.95,xlim=c(10,28),ylim=c(38,130),main="Lum
ber Data \n95% Density Region for Normal Population",xlab="Stiffness",ylab="Bending
Strength",col=1)
#
# Construct 95% Confidence region for mean vector
#
library(ellipse)
plot(ellipse(S,centre=xbar,t=sqrt(((n-1)*p)/(n*(n-p))*qf(1-.05,df1=p,df2=n-p))),
type="l", xlim=c(16,22),ylim=c(70,100),main="Lumber Data \n95% Confidence Region for Mean Vect
or",xlab="Stiffness",ylab="Bending Strength")
points(xbar[1],xbar[2])

mu1.L=xbar[1]-sqrt((((n-1)*p)/(n-p))*qf(1-.05,df1=p,df2=n-p))*sqrt((1/n)*S[1,1])
mu1.U=xbar[1]+sqrt((((n-1)*p)/(n-p))*qf(1-.05,df1=p,df2=n-p))*sqrt((1/n)*S[1,1])

mu2.L=xbar[2]-sqrt((((n-1)*p)/(n-p))*qf(1-.05,df1=p,df2=n-p))*sqrt((1/n)*S[2,2])
mu2.U=xbar[2]+sqrt((((n-1)*p)/(n-p))*qf(1-.05,df1=p,df2=n-p))*sqrt((1/n)*S[2,2])

c(mu1.L,mu1.U)
c(mu2.L,mu2.U)

lines(c(mu1.L,mu1.L),c(-20,mu2.U),lty=2)
lines(c(mu1.U,mu1.U),c(-20,mu2.U),lty=2)
lines(c(-30,mu1.U),c(mu2.L,mu2.L),lty=2)
lines(c(-30,mu1.U),c(mu2.U,mu2.U),lty=2)
```






```
> c(mu1.L,mu1.U)
Stiffness Stiffness
16.91347  20.29653
> c(mu2.L,mu2.U)
Strength Strength
74.57413 92.50854
```

(Courtesy of Dr. Roy St. Laurent, modified on 10/22/2016)