STA 674 Practice Problems #2

1. Problem 6.20

2. Let $T(\underline{\mathbf{X}})$ be a sufficient statistic for the family of pdf's/pmf's indexed by θ. If a unique maximum likelihood estimator of θ exists, then prove that the MLE is a function of $\underline{\mathbf{X}}$ only through the sufficient statistic $T(\underline{\mathbf{X}})$.

3. Assume that $X_1, ..., X_n \overset{iid}{\sim} N(0, \sigma^2)$.
a. Find an unbiased estimator of σ^2.
b. Find the MLE of σ^2.
c. Find a method of moments estimator of σ^2.
d. Using the prior distribution for σ^2 given by $\pi(\sigma^2) = \frac{1}{\Gamma(\alpha)\beta^\alpha(\sigma^2)^{\alpha+1}} e^{-1/(\beta\sigma^2)} I_{(0,\infty)}(\sigma^2)$, find the Posterior Bayes Estimator of σ^2.

4. Prove the Rao-Blackwell Theorem.

5. Assume that $X_1, ..., X_n \overset{iid}{\sim} Exponential(\beta)$. Find the UMVUE of $1/\beta$.