STA 673 Practice Problems #2

1. Suppose that X is a discrete random variable whose support is the nonnegative integers.

a. Show that $E(X) = \sum_{k=0}^{\infty} (1 - F_X(k))$ where $F_X(k)$ is the cdf of X.

b. Show that $\frac{d}{dt} M_X(t)|_{t=0} = E(X)$ where $M_X(t)$ is the mgf of X.

2. Suppose that the moment generating function of a random variable X, given by $M_X(t)$, exists for t in a neighborhood about zero. Does the mgf of X uniquely identify the distribution of X? Please explain.

3. Suppose that X is a nonnegative random variable and that $Y = a + bX^2$. Describe in detail how you would find the moment generating function of Y.

4. Assume $X_i \sim Binomial(n, p_i)$, $i = 1, 2$, where $p_1 < p_2$.
a. Are X_1 and X_2 members of an exponential family? Please explain.
b. Show that $P(X_1 \leq k) \geq P(X_2 \leq k)$ for $k = 0, 1, ..., n$.
(Hint: Consider using the regularized incomplete beta function)

5. Derive the mean, variance, and mgf (if they exist) of a commonly used random variable.

6. Jensen's Inequality states that if $g(x)$ as a convex function of x, then $E(g(X)) \geq g(E(X))$ for random variable X. Use Jensen's Inequality to prove that $Var(X) \geq 0$.