(A Quick Review)

Principal Component Method of Estimation

Choice of $S_{p\times p}$ or $R_{p\times p}$.

Now $\tilde{L}_{p\times m} = \left[\sqrt{\hat{\lambda}_1}\, \underset{p\times 1}{\underset{\sim}{e}_1}, \ldots, \sqrt{\hat{\lambda}_m}\, \underset{p\times 1}{\underset{\sim}{e}_m} \right] = \begin{bmatrix} \tilde{l}_{11} & \tilde{l}_{12} & \cdots & \tilde{l}_{1m} \\ \tilde{l}_{21} & \tilde{l}_{22} & \cdots & \tilde{l}_{2m} \\ \vdots & \vdots & & \vdots \\ \tilde{l}_{p1} & \tilde{l}_{p2} & \cdots & \tilde{l}_{pm} \end{bmatrix}_{p\times m}$

$$\tilde{\Psi}_i = S_{ii} - \sum_{j=1}^{m} \tilde{l}_{ij}^{\,2}, \quad i=1,\ldots,p$$
$$= S_{ii} - \tilde{h}_i^2, \quad i=1,\ldots,p$$

if use $S_{p\times p}$.

Remarks 1. Contribution to the total sample variance from the j^{th} factor $= (\sqrt{\hat{\lambda}_j}\, \underset{\sim}{\hat{e}_j})'(\sqrt{\hat{\lambda}_j}\, \underset{\sim}{\hat{e}_j})$

$$= \hat{\lambda}_j\, \underset{\sim}{\hat{e}_j}'\, \underset{\sim}{\hat{e}_j}$$
$$= \hat{\lambda}_j$$
$$= \sum_{i=1}^{p} \tilde{l}_{ij}^{\,2}$$

2. Proportion of total sample variance due to j^{th} factor $= \dfrac{\sum_{i=1}^{p} \tilde{l}_{ij}^{\,2}}{S_{11}+S_{22}+\cdots+S_{pp}} = \dfrac{\hat{\lambda}_j}{S_{11}+S_{22}+\cdots+S_{pp}}$

ML Method of Estimation

Choice of using $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$ or standardizing $\underset{\sim}{X}$.

Now $\hat{L}_{p\times m} = \text{MLE of } L$

$\hat{\Psi}_{p\times p} = \text{MLE of } \Psi$

$\hat{\Sigma} = \text{MLE of } \Sigma = \hat{L}\hat{L}' + \hat{\Psi}$

if use $\underset{p\times 1}{\underset{\sim}{X}}$.

Remark: Proportion of total sample variance due to j^{th} factor $= \dfrac{\sum_{i=1}^{p} \hat{l}_{ij}^{\,2}}{S_{11}+S_{22}+\cdots+S_{pp}}$