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Chapter 8 – Hypothesis Testing

In hypothesis testing, there are two competing claims (or hypotheses) under consideration.

"H nought"

Null hypothesis: Denoted by H_0, is the claim that is initially assumed. i.e., the default claim.

sometimes H_a

Alternative hypothesis: Denoted by H_1, is a hypothesis considered as an alternative to H_0. H_1 is often referred to as the research hypothesis.

Ex: Proportion of auto mechanics who correctly identify an engine problem is 0.75. The owner of a local shop thinks this claim is too low.

$$H_0: p = 0.75$$
$$H_1: p > 0.75$$

Ex: RDA of iron for adults is 18 mg. A nutritionist believes that, on average, adults get less than 18 mg of iron per day.

$$H_0: \mu = 18 \text{ mg}$$
$$H_1: \mu < 18 \text{ mg}$$

Remark: H_0 will be rejected in favor of H_1 only if evidence from a sample strongly

Remark: H_0 will be rejected in favor of H_1 only if evidence from a sample strongly suggests that H_0 is false.

Ex: H_0: "Innocent"

H_1: "Guilty"

There are two possible outcomes or decisions

1. Reject H_0 in favor of H_1.
2. Fail to reject H_0.

Remark: When H_0 is not rejected, it does not mean strong evidence to support H_0. It just means there is not enough evidence against H_0.

Typically,

H_0: p = hypothesized value = p_0 $\qquad \mu = \mu_0$

and H_1 takes on one of the following

(left tail) $\quad H_1$: $p <$ hypothesized value $\qquad \mu < \mu_0$

(right tail) or $\quad H_1$: $p >$ hypothesized value $\qquad \mu > \mu_0$

(two tail) or $\quad H_1$: $p \neq$ hypothesized value $\qquad \mu \neq \mu_0$

Errors in Hypothesis Testing

Since a decision is made from sample information, it is possible the decision made is not correct.

Type of Errors

		Underlying Reality: H_0 is true	Underlying Reality: H_0 is false
Decision made	Reject H_0	Type I error	—
	Fail to reject H_0	—	Type II error

$$P(\text{reject } H_0 \text{ when } H_0 \text{ is true}) = P(\text{Type I error})$$
$$= \alpha$$
$$= \text{significance level of the test}$$

$$P(\text{fail to reject } H_0 \text{ when } H_0 \text{ is false}) = P(\text{Type II error})$$
$$= \beta$$

Ex: H_0: "Innocent"

H_1: "Guilty"

Type I error?

Type II error?

Remark: Error probabilities α and β cannot be controlled simultaneously. In fact, α and β are inversely related.

In practice, one selects an acceptable α-value

In practice, one selects an acceptable α-value and then conducts the hypothesis test.

0.01, 0.05, and 0.10 are commonly used values of α.

Q: How to select α-value? A: Up to researcher/investigator.

Test Statistic and P-values

Hypothesis test for p

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Based on CLT for $\hat{p}$

Hypothesis test for μ

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Based on CLT for $\bar{x}$ or normal population when $n \leq 30$.

<u>Def</u>: P-value = P(A sample yields a test statistic that is at least as contradictory to H_0 as the observed test statistic, assuming H_0 is true)

<u>Remark</u>: A small P-value is a measure of strength against H_0.

In general,

Reject H_0 if P-value $\leq \alpha$

Fail to reject H_0 if P-value $> \alpha$.

IMPORTANT : See Handout #7 in Canvas for main steps of conducting hypothesis tests.

Ex: Weight loss center has a new diet program.

Center claims participants can expect to lose over 22 pounds in a ten week period.

Test the center's claim.

μ = population mean
= mean weight loss of all participants in the diet program.

$H_0: \mu = 22$

$H_1: \mu > 22$ (right tail test)

Suppose $\alpha = P(\text{type I error})$
$= 0.01$