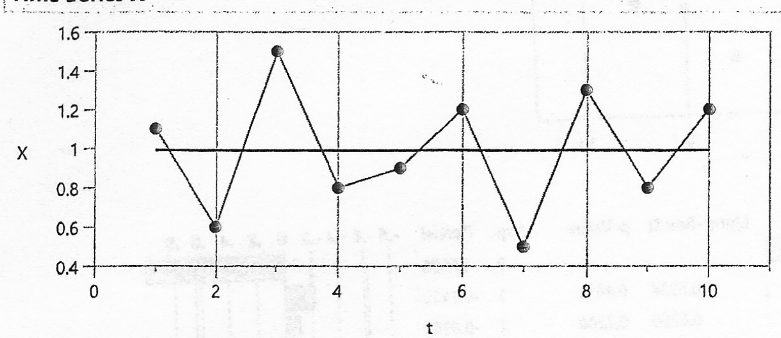
Example 14.1: Consider the following time series,

$$(x_1 \cdots, x_{10}) = (1.1, 0.6, 1.5, 0.8, 0.9, 1.2, 0.5, 1.3, 0.8, 1.2).$$

a) Construct the time series plot, sample autocorrelation and partial autocorrelation plots for the data.
b) Indentify an appropriate model for the data and fit the model to the data.
c) Check adequacy of the fitted model by residual analysis.
d) Is Z_t normally distributed? Justify your answer.
e) Does the model overfit the data? Justify your answer.

Note: Use the significance level $\alpha = 0.05$ for each hypothesis test.

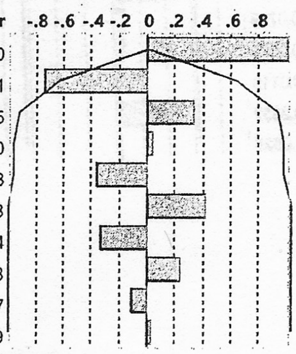
Time Series X

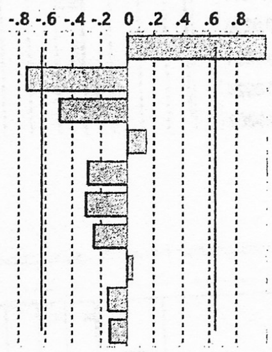


Mean	0.99
Std	0.304795
N	10
Zero Mean ADF	-0.799056
Single Mean ADF	-7.370218
Trend ADF	-6.793015

Time Series Basic Diagnostics

Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	-0.7471	7.4431	0.0064*	1	-0.7471
2	0.3356	9.1328	0.0104*	2	-0.5039
3	0.0470	9.1707	0.0271*	3	0.1447
4	-0.3653	11.8402	0.0186*	4	-0.2890
5	0.4193	16.0590	0.0067*	5	-0.3083
6	-0.3354	19.4341	0.0035*	6	-0.2571
7	0.2318	21.5825	0.0030*	7	0.0462
8	-0.1107	22.3172	0.0044*	8	-0.1440
9	0.0249	22.3914	0.0077*	9	-0.1216





Model Comparison

Report	Graph	Model	DF	Variance	AIC	SBC	RSquare	-2LogLH	Weights
☑	☐	ARMA(1, 1)	7	0.0264355	-1.965307	-1.057552	0.716	-7.965307	0.304948
☑	☐	MA(2)	7	0.0180752	-1.872603	-0.964848	0.692	-7.872603	0.291135
☑	☐	AR(2)	7	0.0368927	-0.699359	0.208396	0.686	-6.699359	0.161930
☑	☐	MA(1)	8	0.0368644	-0.460066	0.145104	0.589	-4.460066	0.143670
☑	☐	AR(1)	8	0.0467505	0.298580	0.903750	0.588	-3.70142	0.098317

Model: ARMA(1, 1)

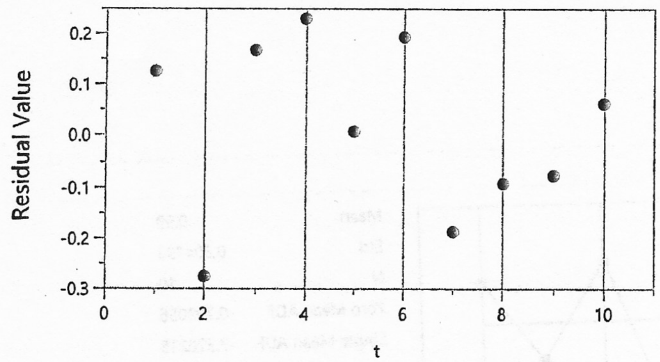
Model Summary

DF	7	Stable	Yes
Sum of Squared Errors	0.18504817	Invertible	Yes
Variance Estimate	0.02643545		
Standard Deviation	0.16258983		
Akaike's 'A' Information Criterion	-1.9653074		
Schwarz's Bayesian Criterion	-1.0575521		
RSquare	0.7155567		
RSquare Adj	0.63428719		
MAPE	17.3927319		
MAE	0.14228725		
-2LogLikelihood	-7.9653074		

Parameter Estimates

Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate
AR1	1	-0.5448764	0.2440603	-2.23	0.0607	1.50370317
MA1	1	0.9999414	0.3239964	3.09	0.0177*	
Intercept	0	0.9733485	0.0093310	104.31	<.0001*	

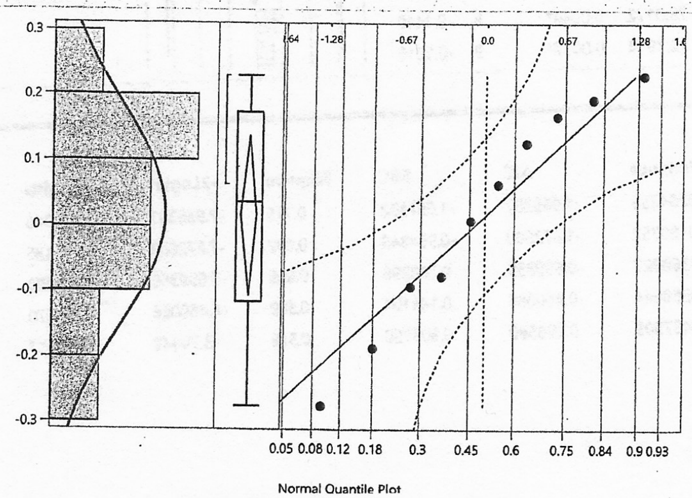
Residuals



Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	-0.2110	0.5934	0.4411	1	-0.2110
2	-0.0494	0.6299	0.7298	2	-0.0982
3	-0.0616	0.6949	0.8744	3	-0.0991
4	-0.3726	3.4720	0.4821	4	-0.4422
5	0.1611	4.0946	0.5359	5	-0.0789
6	0.0206	4.1072	0.6622	6	-0.0696
7	0.0851	4.3971	0.7331	7	-0.0119
8	-0.0924	4.9095	0.7672	8	-0.2857
9	0.0202	4.9584	0.8379	9	-0.0329

Distributions

Residual X



— Normal(0.01606,0.17051)

Fitted Normal

Parameter Estimates

Type	Parameter	Estimate	Lower 95%	Upper 95%
Location	μ	0.0160578	-0.105919	0.1380348
Dispersion	σ	0.1705121	0.1172842	0.3112886

-2log(Likelihood) = -8.00021317937994

Goodness-of-Fit Test

Shapiro-Wilk W Test

W	Prob<W
0.949422	0.6617

Note: Ho = The data is from the Normal distribution. Small p-values reject Ho.

	Actual X	t	Predicted X	Std Err Pred	Residual X	Upper CL	Lower CL
1	1.1	1	0.9733485092	0.1625898281	0.1266514908	1.2920187165	0.6546783018
2	0.6	2	0.8755180579	0.1625898281	-0.275518058	1.1941882652	0.5568478505
3	1.5	3	1.3322231005	0.1625898281	0.1677768995	1.6508933078	1.0135528931
4	0.8	4	0.5695364136	0.1625898281	0.2304635864	0.888206621	0.2508662062
5	0.9	5	0.8910019902	0.1625898281	0.0089980098	1.2096721976	0.5723317828
6	1.2	6	1.0060158417	0.1625898281	0.1939841583	1.3246860491	0.6873456343
7	0.5	7	0.6866848392	0.1625898281	-0.186684839	1.0053550466	0.3680146318
8	1.3	8	1.3923576017	0.1625898281	-0.092357602	1.711027809	1.0736873943
9	0.8	9	0.8765867492	0.1625898281	-0.076586749	1.1952569566	0.5579165418
10	1.2	10	1.136148857	0.1625898281	0.063851143	1.4548190644	0.8174786496
11	.	11	0.7922027029	0.1625898281	.	1.1108729103	0.4735324955
12	.	12	1.072050589	0.2992033755	.	1.6584784289	0.485622749
13	.	13	0.9195680726	0.3290176914	.	1.564430898	0.2747052472
14	.	14	1.0026522013	0.3373624037	.	1.6638703623	0.3414340404
15	.	15	0.9573816181	0.3398004184	.	1.6233782	0.2913850361
16	.	16	0.9820484918	0.3405208809	.	1.6494571543	0.3146398292

Example 14.1: (Continued) Using the fitted model

$$(X_t - 0.9733) = -0.5449(X_{t-1} - 0.9733) + Z_t - Z_{t-1}$$

f) Find $\hat{X}_{10}(1)$ and $\hat{X}_{10}(2)$.

g) Find a 95% prediction interval for X_{12} and interpret the interval.