STA 275 Practice Problems 3

1. Consider the problem of estimating the population mean height of 5-year old boys. Suppose a sample of 45 boys yields that following results (X_i = height of the i^{th}th boy in the sample):

$$\sum_{i=1}^{45} X_i = 1845 \text{ inches} \quad \text{and} \quad \sum_{i=1}^{45} X_i^2 = 76184 \text{ square inches.}$$

a. The sample mean is often called a point estimator of the population mean. Using the information provided, compute a point estimate of μ.
b. What is the sampling distribution of the point estimator of the population mean? Did you use the CLT to answer this question? Explain.
c. The sample standard deviation is often called a point estimator of the population standard deviation. Using the information provided, compute a point estimate of σ.
d. How many five-year old boys need to be sampled in order to be 90% certain that the population mean height is estimated to within 0.25 inches?

2. The lifting capacities of industrial workers are assumed to be normally distributed with mean = 65 lbs and standard deviation = 10 lbs.
a. What is the probability a randomly selected worker can lift more than 80 lbs?
b. Suppose random samples of 16 workers are chosen. What is the distribution of the mean lifting capacity of these samples? Did you use the CLT to answer this question? Explain.
c. What is the probability that a sample of workers in part b. has a mean lifting capacity that is between 65 and 70 lbs?

3. Suppose that a particular candidate for a public office is in fact favored by 48% of all registered voters in the district. A polling organization will take a random sample of 500 voters and will use $\hat{p}$, the sample proportion, to estimate p. What is the approximate probability that $\hat{p}$ will be greater than 0.5, causing the polling organization to incorrectly predict the result of the upcoming election?

4. By measuring the heights of 62 six-year old girls selected at random, an investigator determined that a 95% CI for the population mean height μ of six-year-old girls was (42.2 inches, 46.1 inches). Answer the following questions with "Yes", "No", or "Can't Tell" and give a brief explanation.
a. Does the population mean lie in the above CI?
b. Is the probability the population mean is in the confidence interval 0.95?
c. Does the sample mean lie in the above CI?
d. For a future sample of 62 six-year-old girls, will the sample mean lie in the above CI?
e. Using the same sample, will an interval having a 99% confidence level be narrower than (42.2 in, 46.1 in)?

5. Referring to the previous problem, if (42.2 inches, 46.1 inches) is a 95% CI for μ, find:
a. The sample mean.
b. The sample standard deviation.

6. Discuss how each of the following factors affects the width of the confidence interval for p.
a. The confidence level.
b. The sample size.
c. The value of $\hat{p}$.

7. *USA Today* reported that 36% of adult drivers admit that they often or sometimes talk on a cell phone when driving. This estimate was based on data from a sample of 1004 adult drivers, and the half-width of a 95% confidence interval for p was computed to be 0.031. Do you agree with the statement that the half-width is 0.031? Explain.

8. Problem 8.171 on page 386.

9. Problems 9.1, 9.2, and 9.3 on page 394.

10. Problem 9.19 on page 395.