



PROBABILITY DISTRIBUTIONS

Properties:
C: Convolution
F: Forgetfulness
I: Inverse
L: Linear combination
M: Minimum
P: Product
R: Residual
S: Scaling
V: Variate generation
X: Maximum

Relationships:
→ Special cases
→ Transformations
--> Limiting
...> Bayesian

Distributions and their parameters/relationships:

- Zipf(α, n)**: $\alpha = 0, \alpha = 1$; $b = n$; $n \rightarrow \infty$ leads to **Zeta(α)**.
- Discrete uniform(a, b)**: R, V.
- Rectangular(n)**: V.
- Beta-binomial(a, b, n)**: $a = 0, a = 1$; $b = n$; $p \sim \text{beta}$ leads to **Poisson(μ)**.
- Negative hypergeometric(n_1, n_2, n_3)**: $n = n_1, a = n_2$; $n_3 \rightarrow \infty$, $n_1 \rightarrow \infty$, $n_2 = n$ leads to **Hypergeometric(n_1, n_2, n_3)**.
- Logarithm(c)**: $A(c) = -\log(1-c)$.
- Power series($c, A(c)$)**: $A(c) = e^c, \mu = c$; $\mu \sim \text{gamma}$ leads to **Poisson(μ)**.
- Poisson(μ)**: C; $\mu = np$, $n \rightarrow \infty$ leads to **Binomial(n, p)**.
- Binomial(n, p)**: C_p ; $\beta = 0$ leads to **Polya(n, p, β)**.
- Hypergeometric(n_1, n_2, n_3)**: $p = n_1/n_3$, $n_3 \rightarrow \infty$, $n_1 \rightarrow \infty$, $n_2 = n$ leads to **Bernoulli(p)**.
- Bernoulli(p)**: M, P, X.
- Geometric(p)**: F, M, V.
- Pascal(n, p)**: C_p ; $\sum X_i$ (iid); $\beta = 1$ leads to **Discrete Weibull(p, β)**.
- Gamma-Poisson(α, β)**: $\alpha = (1-p)/p$, $\beta = n$; $\mu = n(1-p)$, $n \rightarrow \infty$ leads to **Normal(μ, σ^2)**.
- Normal(μ, σ^2)**: L; $(X-\mu)/\sigma$ leads to **Standard normal**.
- Standard normal**: $\sum X_i^2/\sigma^2$ leads to **Noncentral chi-square(n, δ)**.
- Noncentral chi-square(n, δ)**: C; $\delta = 0$ leads to **Chi-square(n)**.
- Chi-square(n)**: C; \sqrt{X} leads to **Chi-square(n)**.
- Inverted gamma(α, β)**: $\log X$ leads to **Log gamma(α, β)**.
- Gamma(α, β)**: $C_{\alpha, S}$; X_1/X_2 , $\alpha = 1$ leads to **Inverted beta(β, γ)**.
- Beta(β, γ)**: $\beta = \gamma = 1$, $\beta = \gamma = \frac{1}{2}$ leads to **Arcsin V**.
- Arctangent(λ, ϕ)**: S, V; zero truncate leads to **Hyperbolic-secant V**.
- Hyperbolic-secant V**: $\log |X|/\pi$ leads to **Cauchy(α, α)**.
- Cauchy(α, α)**: C, I, S, V; $\alpha = 0$, $\alpha + \alpha X$, $\alpha = 1$ leads to **Standard Cauchy**.
- Standard Cauchy**: I, S, V; $\delta = 0$ leads to **Noncentral t(n, δ)**.
- Exponential(α)**: F, M, S, V; $\alpha = 1$, X_1/X_2 leads to **Hypoexponential($\vec{\alpha}$)**.
- Hypoexponential($\vec{\alpha}$)**: C; $d = \alpha$ leads to **Erlang(α, n)**.
- Erlang(α, n)**: S; $\beta = r$ leads to **Gomperts(δ, κ)**.
- Gomperts(δ, κ)**: V; $\gamma = 0$ leads to **Exponential power(λ, κ)**.
- Exponential power(λ, κ)**: V; $[\log(1-\log(1-X))]/\lambda^{1/\kappa}$ leads to **Minimummax(β, γ)**.
- Minimummax(β, γ)**: M $_g$, V; $\gamma = 1$ leads to **Standard power(β)**.
- Standard power(β)**: V, X; $a + (b-a)X$ leads to **Uniform(a, b)**.
- Uniform(a, b)**: R, V; $n = 1$, $\kappa \rightarrow 0$ leads to **von Mises(κ, μ)**.
- von Mises(κ, μ)**: S.
- Kolmogorov-Smirnov(n)**: V $_{1-4}$.
- Triangular(a, b, m)**: V; $a = -1$, $b = 1$, $m = 0$ leads to **TSP(a, b, m, n)**.
- TSP(a, b, m, n)**: V.
- Benford V**: $[10^X]$ leads to **Benford V**.
- Lomax(λ, κ)**: V; $\kappa = 1$, $\kappa = 1$ leads to **Lomax(λ, κ)**.
- Generalized Pareto(δ, κ, γ)**: V; $\log(X/\lambda)$ leads to **Pareto(λ, κ)**.
- Pareto(λ, κ)**: M, V; $\lambda X^{-1/\kappa}$ leads to **Pareto(λ, κ)**.
- Rayleigh(α)**: M, S, V; $\beta = 2$ leads to **Weibull(α, β)**.
- Weibull(α, β)**: M $_g$, S, V; $\log X$ leads to **Extreme value(α, β)**.
- Extreme value(α, β)**: V, M $_g$.
- IDB(δ, κ, γ)**: $\delta = 2/\alpha$, $\gamma = 0$ leads to **Rayleigh(α)**.
- Hyperexponential($\vec{\alpha}$)**: $d = \alpha$ leads to **Exponential(α)**.
- Doubly noncentral t(n, δ, γ)**: $\gamma = 0$ leads to **Noncentral t(n, δ)**.
- Noncentral F(n_1, n_2, δ)**: I; $F(n_1, n_2)$ leads to **Noncentral F(n_1, n_2, δ)**.
- Standard Wald(λ)**: S; $\mu = 1$, $\lambda(X-\mu)^2/(\mu^2 X)$ leads to **Chi-square(n)**.
- Inverse Gaussian(λ, μ)**: $L_{\lambda, (\mu^2 \alpha)}$ leads to **Chi-square(n)**.

Figure 1. Univariate distribution relationships.