



## Deep survival analysis of searching for on-street parking in urban areas

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### ABSTRACT

Searching for parking is a significant contributor to urban road congestion leading to additional costs for the driver emerging from the increased time spent traveling and fuel consumption. The present work attempts to model the duration for searching for parking space monitored with smartphone sensing using the widespread parametric and semi-parametric survival models, as well as random survival forests and deep learning survival models. The available dataset consists of more than 48,000 driving trips conducted in the Region of Attica, Greece, and is enriched with exogenous variables, such as population density and land use in each trips' destination area. Findings reveal that the time of day in which the trip was performed, as well as trip duration and length, significantly affect parking searching duration. In addition, the land use of the destination area appears to be a significant factor for predicting parking searching duration. Although all survival models share similar results in terms of the significance of the parameters, deep survival neural networks noticeably improve the survival time predictions.

### 1. Introduction

Searching for on-street parking - seen either from the planning or the management perspective - is a long-standing complex problem influenced by users' attitudes and mobility patterns, including specific destination and arrival time needs, cost of travel and desired walking time (Du et al., 2019). Searching for on-street parking has been also considered as a major contributor to city traffic congestion with far-reaching economic, environmental and social impacts (Shoup, 2006; Arnott and Williams, 2017). Recent evidence shows that searching for parking may exceed 15% of the total traffic observed (Hampshire and Shoup, 2018). In a large-scale study in the Netherlands, van Ommeren et al. (2012), based on survey data, reported that 30% of car drivers cruise before finding a parking spot, yet the average searching time is only 36 s. Short searching times have also been found in Brooke et al. (2018). Furthermore, on-street searching for parking time has been found to be inversely proportional to the demand for paid parking services (Alemi et al., 2018). Most of the above results are based on an assumed relationship between parking search time and the on-street parking occupancy rate (Geroliminis, 2015; Arnott and Williams, 2017), as well as on data coming from stated preference surveys (Lee et al., 2017; Brooke et al., 2018) or other dedicated parking surveys (Alemi et al., 2018; Hampshire and Shoup, 2018; Dave et al., 2019).

Considering the advancements in Information and Communication Technologies that accelerate the deployment of novel intelligent and connected transport services dedicated to parking (Vlahogianni et al., 2016; Lin et al., 2017; Golias and Vlahogianni, 2018), experience shows that intelligent parking systems can reduce searching for on-street parking (Cao and Menendez, 2018). A growing

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body of literature utilizes GPS trajectory data to quantify the time spent searching for parking space (Kaplan and Bekhor, 2011; Montini et al., 2012; Van der Waerden et al., 2015; Mannini et al., 2017; Weinberger et al., 2017). Smartphone sensing has been also gaining popularity in parking studies for various reasons. First, by exploiting the different sensors, smartphones can be used to continuously monitor (via participatory or opportunistic sensing) real-time user behavior. Second, smartphones can be considered as a cost-effective non-intrusive manner to monitor users' activities and behavioral patterns. Third, the data granularity acquired by the sensors enables the use and reuse of these data in various transportation applications, from studying the mobility patterns (Mantouka et al., 2019) to analyzing and rating drivers behavior for road safety (Vlahogianni and Bampounakis, 2017), identifying extreme traffic conditions (Lu et al., 2017), as well as predicting parking space availability (Nawax et al., 2013; Xu et al., 2013; Rinne et al., 2014; Carnelli et al., 2017).

Sensing seems to overcome the problem of lack of objectivity met in questionnaire surveys or dedicated parking studies, yet entails difficulties, such as how to define the starting point of parking search process, how to ensure the sample representativeness, as well as how to collect the data in an as much as possible non-intrusive, yet attractive, manner, to ensure system's pervasiveness and sustainability (Arnott and Williams, 2017; Bock and Di Martino, 2017; Weinberger et al., 2017). Regardless of the uncertainties introduced when trying to measure or deduce searching for parking from real world sensing (Arnott and Williams, 2017), there is a certain merit in developing novel methodological constructs that can transform sensors' data to parking related information: This information can be further enriched with exogenous information, for example road type, land use, traffic demand etc., in order to develop predictive data-driven models for searching for parking duration or for calibrating existing simulation models.

The aim of this paper is twofold: first, to identify factors that affect searching for time duration on an area and then, to compare the performance of classical survival models to this of a deep learning survival implementation. Furthermore, it contributes to searching for parking identification using smartphone sensing. Several modeling strategies are employed ranging from classical parametric survival analysis to deep learning survival neural networks and tree-based survival analysis. The remainder of the paper is organized as follows: The next section briefly presents the methodology to detect searching for parking from real-world trajectory data. Next, the basic theoretical aspects of classical and machine learning survival analysis are presented. Then, the dataset used for this analysis as well as the results are thoroughly described. Finally, some discussion on the main findings and contribution of this work is provided and conclusions and future research steps are presented.

## 2. The mechanics of searching for parking

Several definitions of searching for parking have emerged over the years, mainly stemming from the different approaches of monitoring and quantifying the phenomenon (Weinberger et al., 2017). These differences reflect the complexity and uncertainty of identifying the onset of the parking search procedure. Some studies addressed searching for parking from a strategic perspective and assumed that users' behavior is dependent on prior knowledge of the area and the parking space availability and easiness to find a parking space (Thompson and Richardson, 1998). Others identify the onset of searching for parking space as the time when the driver enters an area of a certain radius near the final parking location (Kaplan and Bekhor, 2011) or first passes from his/her destination point (Jones et al., 2017). Some studies identify as searching for parking the last part of a trip, conducted at low speeds, indicative of

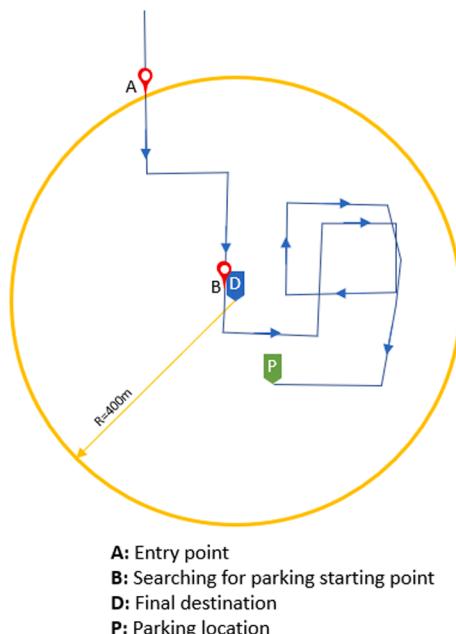


Fig. 1. Searching for parking trajectory and critical points.

their willingness to park (Benenson et al., 2008; Van der Waerden et al., 2015).

Recent literature suggests the existence of two different searching profiles: i. the *informed users*, who are usually familiar with the area where they are willing to park and have a predefined strategy for searching for on-street parking, and ii. the *uninformed users* who will attempt to reach their destination and start searching for an on-street parking space. In this work, we focus on the modeling of the latter behavior, namely we assume that each driver will first drive close to his destination and start searching for parking. However, to also address the first case, we assume that the proximity to the destination is variable but within a certain distance range, namely 400 m from the destination, that exceeds by far the average time a car driver is willing to walk from the parking spot to reach his destination (van der Waerden et al., 2017). Previous studies have also shown that a driver would try to find an available parking space as close to the final destination as possible, i.e. inside a radius of 200–500 m (Martens et al. 2010; Levy et al., 2013; Leclercq et al., 2017; Khaliq et al., 2018; van der Waerden & Agarad, 2019). Cases where an informed driver searches for parking space in an area far from his destination (more than 400 m) are not covered by the specific study.

To extract the desired searching for parking information from GPS trajectory data the following methodology was adopted: Let  $D_i$  and  $P_i$  be the destination and parking locations respectively.  $D_i$  may be considered to have an influence area, namely a wider parking search area,  $A_i$ .  $A_i$  is the area within a radius of 400 m from the destination's location, inside which  $P_i$  lies. Fig. 1 illustrates the above points and a typical trajectory of an uninformed driver.

After entering the search area ( $A_i$ ), approaching the destination ( $D_i$ ), the driver may, at some point, stop heading to it and start driving away, while he searches for an available parking space. That is a local minimum of the vehicle's distance to the destination and it is assumed as an indicator of the beginning of the searching process (point B of Fig. 1). Afterward, the vehicle may or may not get closer to the destination, stray from it again multiple times, in the driver's attempt to find an available parking space until he parks at some location, eventually. In case the driver exits the 400 m radius while searching for parking and later returns, searching metrics (duration, distance etc.) before exiting the radius are added to the data recorded after the car enters again the radius. Based on the customization of the specific algorithm, data recorded during the time the car is outside the radius are not considered as circulation. The rationale behind this decision lies in the fact that driving times outside the 400 m radius are considered not to influence the specific area.

We consider that the onset of searching for parking occurs at the first local minimum, within the area  $A_i$ , of the vehicle's distance from the destination point  $D_i$  with time and ends when the vehicle parks at location  $P_i$ . In this time period, the average travel speed is expected to be lower than a predetermined searching speed  $S_c$ , although this does not act as a unique and sufficient condition to determine searching for parking start. The overall idea is that a driver starts straying from the destination, creating a local minimum, when he realizes that there is no parking space near his destination and has to search somewhere else. The above is visualized in Fig. 2.

Since the recorded parking search times can take on both small values and larger ones, in the specific study, we introduce the Parking Straightness  $ST_i$  measure as a complementary feature to evaluate the recorded parking search times.  $ST_i$  aims to characterize the divergence of a “straight” trajectory at the emergence of a searching for available parking space event, which can be leveraged to identify atypical trip trajectories as an outcome of searching for parking. The rationale behind  $ST_i$  is as follows: We can assume, with a certain degree of realism, that a driver that reaches his destination and will not search for parking, will follow the shortest path route after entering the parking area. Consequently, the ratio of the shortest path to the actual distance covered between B and P will quantify the divergence from a typical trajectory (without searching for parking); the smaller the straightness metric, the larger the divergence, the more atypical the trajectory, meaning the larger the time a driver will travel before parking near his destination. The shortest driving path between two points is assumed as the “true” distance someone would cover to reach the specific destination if he

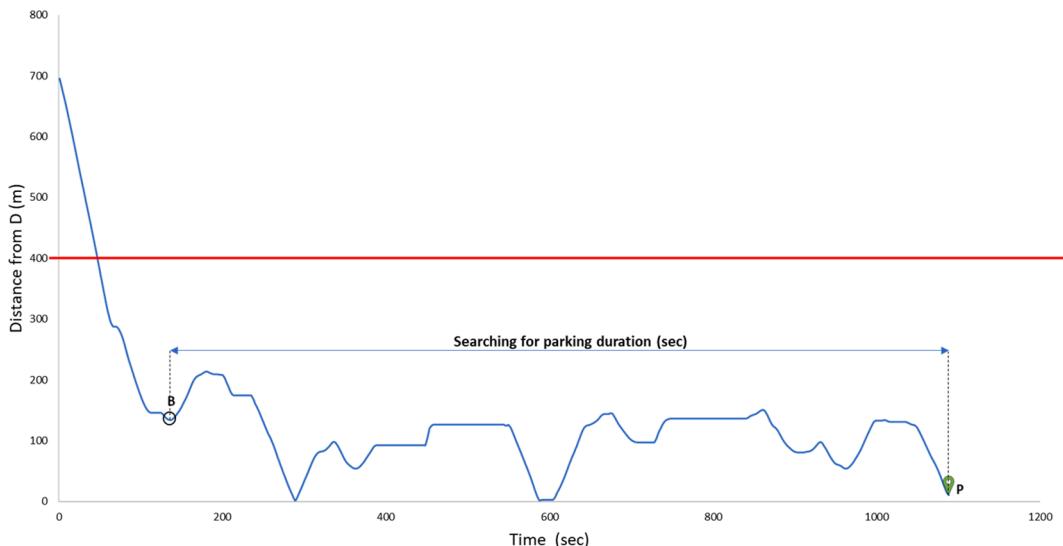


Fig. 2. Example trajectory of searching for parking.

was not searching for parking and can be easily and at a relatively low cost from various third-party map and navigation APIs. In the specific work, we define  $ST_i$  as the ratio of the shortest driving distance between searching for parking starting point (B) and parking location (P) and the actual distance covered, based on the recorded trajectory from B to P (see also Fig. 1):

$$\text{Straightness} = \frac{\text{Shortest driving distance between BandP}}{\text{distance covered from BtoP}} \quad (1)$$

Parking Straightness varies from 0 to 1. The estimation of parking straightness is considered useful since the time covered while searching for parking may, sometimes, be misleading. In the specific study, the  $ST_i$  measure complements the knowledge on true parking search times; it can help identify boundary parking search behavior conditions e.g. low parking search times associated with high  $ST_i$  values indicating easiness of finding a parking space, or high parking search times associated with high  $ST_i$  values indicating that the recorded search times (within the area of 400 m) were probably owned to other conditions (e.g. queue spillovers or congested road section) rather than a lack of free parking space.

### 3. Parametric and Semi-Parametric survival analysis

Survival analysis refers to the modeling of the time-to-event data, meaning the time that transpires until or between the occurrence of specific events. Time to event data are massively produced in transportation science. In this paper, searching for parking (time to find a parking space) is modeled using survival analysis.

The distribution of the time to an event is characterized by the survival function  $S(t) = Pr[T > t]$  for  $t > 0$ , which is the probability that the individual is still alive at time  $t$ , or - in our case - the vehicle still cruises for parking, where  $T$  is the actual time to event. Based on whether the event occurred or not before the end of the observation period, data are labeled as censored or uncensored.

If  $T$  has probability density function  $p(t)$ , then the hazard function is defined by  $\lambda(t) = p(t)/S(t)$ . The hazard function measures the instantaneous rate of failure and provides more insight into the failure mechanisms. The function  $\Lambda(t) = \int_0^t \lambda(u)du$  is the cumulative hazard function, so the survival function can be written as  $S(t) = e^{-\Lambda(t)}$ .

The hazard function may be selected by the researcher to follow various distributions, with the most widespread being the exponential (Washington et al., 2010). Moreover, most models implemented in transportation science assume proportionality of hazards (PH), meaning that a multiplicative effect of the covariates on the hazard function exists,

$$\lambda(t|x) = \lambda_0(t)e^{w^T x} \quad (2)$$

where  $\lambda(t|x)$  is the hazard function of an individual with covariates  $x$ ,  $\lambda_0(t)$  is the baseline hazard function (i.e., when  $x = 0$ ), which is typically also based on the exponential or the Weibull distributions,  $e^{w^T x}$  is the relative hazard function,  $w$  is a set of unknown regression parameters and  $w^T x = h(x)$  is called the risk function.

Cox proportional Hazards Model is a semi-parametric approach, which is one of the most widespread regression techniques for survival analysis. This model is used to relate time of survival to various risk factors (predictor variables). Parameter estimates in the PH model are obtained by maximizing Cox's partial likelihood (of the weights) (Cox, 1972):

$$L(w) = \prod_{T_i, \text{uncensored}} \frac{e^{w^T x_j}}{\sum_{T_j \geq T_i} e^{w^T x_j}} \quad (3)$$

Finally, (Austin, 2012) proposed a method which, given the distribution of the hazard function and an individual's characteristics, allows the estimation of the survival time for each individual, using different equations for each distribution.

#### 3.1. Random survival Forests

Machine learning methods for survival analysis have been introduced to overcome the restrictive assumption of proportionality of hazards (Wang et al., 2019). These methods are non-parametric and – at least conceptually – robust to nonlinearities. One such method is the Random Survival Forests. It is a generalization of Random Forests that introduces the concept of ensemble mortality defined based on a conservation-of-events principle which asserts that the sum of the estimated cumulative hazard function (CHF) over observed time (both censored and uncensored) equals the total number of deaths (Ishwaran et al., 2008). A high-level description of the algorithm provided by Ishwaran et al. (2008) is as follows: First, the algorithm creates  $B$  bootstrap samples from the original data. Note that each bootstrap sample excludes on average 30% of the data, called out-of-bag data (OOB data). Then, survival trees are grown, where each tree node randomly selects  $p$  candidate variables and is split based on the variables that maximize the survival difference between child nodes. The training lasts until the terminal nodes have no less than  $d_0$  greater than 0 unique deaths. Following, the cumulative hazard function (CHF) is calculated for each tree and averaged over all trees to obtain the ensemble CHF. Ensemble CHF can be estimated based on bootstrap and OOB data. The ensemble CHF is based on the terminal node CHF calculated as the Nelson–Aalen estimator:

$$\widehat{H}_h(t) = \sum_{t_{l,h} \leq t} \frac{d_{l,h}}{Y_{l,h}} \quad (4)$$

where  $d_{l,h}$  and  $Y_{l,h}$  are the number of deaths and individuals at risk at time  $t_{l,h}$  for a terminal node  $h$ .

The importance of variable  $x$  in the random survival forest is estimated based on the permutation scheme: when a predictor  $x$  is randomly permuted, and the remaining variables are used to predict the response on the test set observations, the prediction accuracy decreases if the original variable  $x$  is an important predictor (Strobl et al., 2008).

### 3.2. Deep learning survival analysis

Neural Networks is one of the most popular modeling techniques and can be adopted on numerous occasions in transportation literature due to their ability to fit non-linear, very complex functions and represent the corresponding relations between independent and dependent variables (Karlaftis and Vlahogianni 2011). Among many other tasks, such as traffic conditions forecasting, road traffic control, measurement of road traffic parameters, driving behavior and autonomous vehicles, and transport policy and economics, neural networks have already been used previously in transportation literature for modeling traffic incident duration (Vlahogianni and Karlaftis, 2013; Pamula, 2016; Hamad et al., 2019).

In the case of survival analysis, neural networks are also deployed lately, because they offer two additional advantages:

- They fit well the training data and can predict very accurately the time-to-event (or equivalently hazard rate or risk) for unseen data, and
- They can be used without worrying about any constraints that are required in other techniques, such as linear proportionality of the covariates in Cox regression.

The Deep Survival Neural Network extends the common multi-layer perceptron by reformulating the single output neuron to estimating the risk function  $\hat{h}_\theta(x)$  parameterized by the weights of the network  $\theta$ . The loss function is set to be the negative log partial likelihood (Katzman et al., 2018):

$$l(\theta) := - \sum_{i:E_i=1} \left( \hat{h}_\theta(x_i) - \log \sum_{j \in R(T_i)} e^{\hat{h}_\theta(x_j)} \right) \quad (5)$$

where  $E$  is the censored condition of each instance ( $i:E_i = 1$  means that only uncensored instances are taken into account) and  $R(T_i)$  is the set of instances that are still at risk at time  $T_i$ , i.e. event has not occurred at time  $T_i$ .

Furthermore, the Deep Neural Network used for Survival Analysis may consist of multiple hidden layers and units (of the user's choice, based on each problem), with their activation being, initially, the Rectified Linear Unit, but may also be changed by the user with a more suitable one, if the later leads to more accurate results.

### 3.3. Model evaluation

For the assessment of the survival models, the value of the concordance index (c-index) is usually estimated, which can be seen as a generalization of the area under the ROC curve (AUC) to regression problems (Harrell et al., 1982). The value of the c-index is equal to the percentage of the individuals whose times-to-event are ranked correctly after the prediction of the fitted model. A correctly ranked pair means that if the real time-to-event of the first individual is shorter than the second, the same applies for predicted times, regardless of the precision of the predicted values and their similarity to the real values. The last sentence highlights the adequacy of the c-index for judging the order of the predicted times (or hazard ratios) rather than the actual accuracy of the predicted values in terms of absolute error. C-index value of 0.6–0.7 is usually a sign of a well-fitted model, while a value closer to 0.5 means that the model is not predicting an outcome better than random chance (Steck et al., 2008).

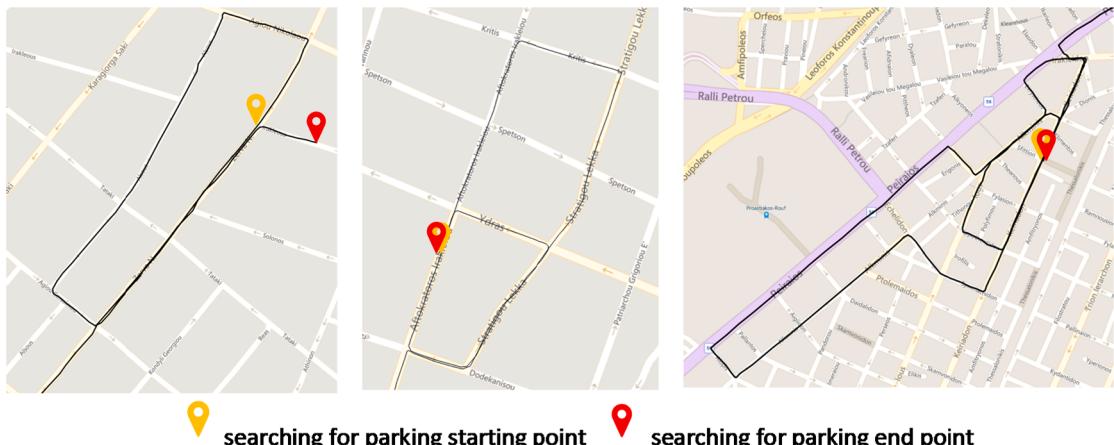


Fig. 3. Indicative searching for parking trajectories.

#### 4. The data

For the specific analysis, a dataset of more than 74,000 trips that encompass searching for parking in the Region of Attica was provided by OSeven Telematics, an insurance telematics company. Data recording is carried out through the OSeven smartphone application for both iOS and Android operational systems. The application exploits smartphone's embedded sensors to collect valuable data concerning among others trip characteristics, driving behavior, eco behavior and searching for parking behavior. The data recording process is activated automatically without requiring any user action. All data was provided by OSeven Telematics in a fully anonymized format.

These trips were annotated, meaning that users have confirmed that they have been searching for parking on the specific trip. The data cover a period of approximately one and a half years in 2018 and 2019. Using the detection mechanism presented in [Section 2](#), the *duration*, the *distance traveled* and the *straightness* estimated from 1 Hz corrected GPS trajectory data of the users were estimated. Searching for parking duration is the time estimated from the moment the vehicle passes from the local minimum of its distance to the destination until the user parks their vehicle. Searching for parking distance is the real distance covered at the above-described time. Moreover, for each trip, the easiness of finding an available on-street parking space was expressed through the measure of straightness. Three indicative searching for parking trajectories are presented in [Fig. 3](#). Although the process of GPS signal correction is not disclosed, the visual inspection shows that signals are matched to the road network. The further evaluation of the GPS signal falls beyond the scope of the specific work.

A significant number of trips were detected to include very small searching for parking duration, which may hinder various behaviors, from cruising for very short times to simple passing from the destination and parking in a private parking spot. The present work attempts to focus on searching for parking that has a substantial duration and can affect traffic conditions as well. To further investigate this issue and identify trips that do not include searching for parking, we performed clustering on the data, using the Expectation-Maximization Algorithm and the following variables: Arc distance (The distance between the first and the last point of the trip across the earth's surface), Searching duration, Parking straightness and number of detected distance minima inside the 400 m radius. The results, presented in [Table 1](#), indicate that there is a cluster (Cluster 3) whose trips are indeed connected to very small searching durations and high values of parking straightness and very few minima detected compared to the other clusters. It can be assumed that trips belonging to this cluster may refer to cases where the driver either uses a private parking slot or that too many spaces were available, consequently no searching for parking was necessary and the short searching duration detected was due to the network's topology or traffic conditions.

Based on the above findings, it was decided to exclude trips of the 3rd cluster from further analysis, as they do not involve searching for parking. Thus, the final dataset consists of 48,840 trips performed in the Region of Attica by more than 500 drivers. The exact parking locations of the above trips are presented in [Fig. 4](#).

As depicted in [Fig. 5](#), parking straightness takes high values for trips with short searching duration indicating the validity of the measure. In contrast, there are only a few trips with high searching for parking duration that are related to high straightness values, while numerous trips with searching duration less than one minute have straightness value over 0.8 and close to 1.0, as one can notice by the increased density of the points in [Fig. 5](#). The points of the lower-left corner represent searching for parking that took little time, but the driver covered a very long distance compared to the actual distance of points B and P.

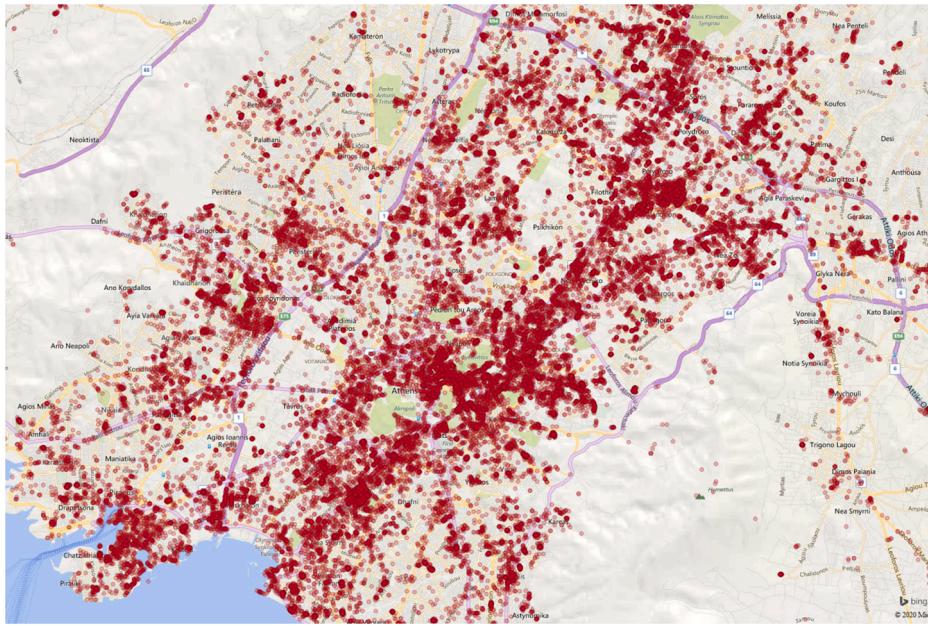
[Fig. 6](#) depicts the geographical distribution of parking straightness as an indicator of easiness to find an available parking space in each area. Areas in the southern and western suburbs of Athens as well as in the greater area of the center of Athens are found to relate to lower straightness values (areas that are orange or red), meaning that searching duration is expected to be relatively higher. These areas are densely populated and attract significant economic and commercial activities.

Finally, data gathered from the smartphone application were enriched with additional attributes, describing the characteristics of the area of the destination, namely land use, population density and road type in which the vehicle was parked. Area related information were gathered from Urban Atlas (<https://land.copernicus.eu/local/urban-atlas/urban-atlas-2012>), which provides pan-European high-resolution land uses maps and land cover data for urban zones with more than 100,000 residents as defined by the Urban Audit for the reference year 2012. The Copernicus system includes data collected by different sources, including Earth observation satellites and in-situ sensors. Land uses, population and other information of the study area were introduced as an additional layer of a GIS software. This layer was at that time matched with a layer containing all trips' ending points from the dataset. All trips having an ending point "off-road" or on a fast transit road were excluded from the dataset. The result of this process is the creation of a large dataset that contains all the variables presented in [Table 2](#).

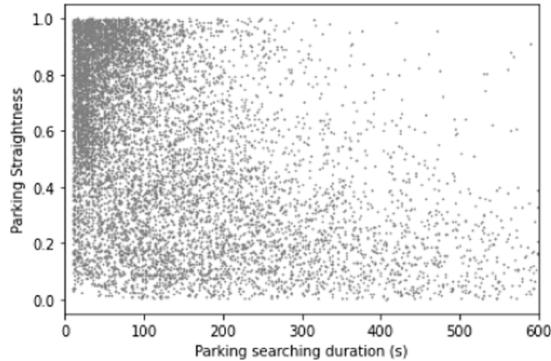
The distribution of the variables that are associated with each trip and the parking process for the cleaned dataset is presented in [Table 3](#). As depicted in Table 3, 75% of the sample have cruised for <2.5 min to find an available parking space, quite similarly to

**Table 1**  
Expectation-Maximization clustering results used for data cleaning.

	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5
Arc distance	28.86	4.19	3.97	14.29	2.89
Searching duration	24.38	145.02	9.13	477.25	32.84
Number of minima	2.47	5.84	1.31	18.28	5.95
Parking straightness	0.67	0.26	0.73	0.13	0.62
Number of trips	8948	13,794	25,713	6712	19,386



**Fig. 4.** Dataset coverage of the Region of Attica.



**Fig. 5.** Searching for parking duration versus parking straightness.

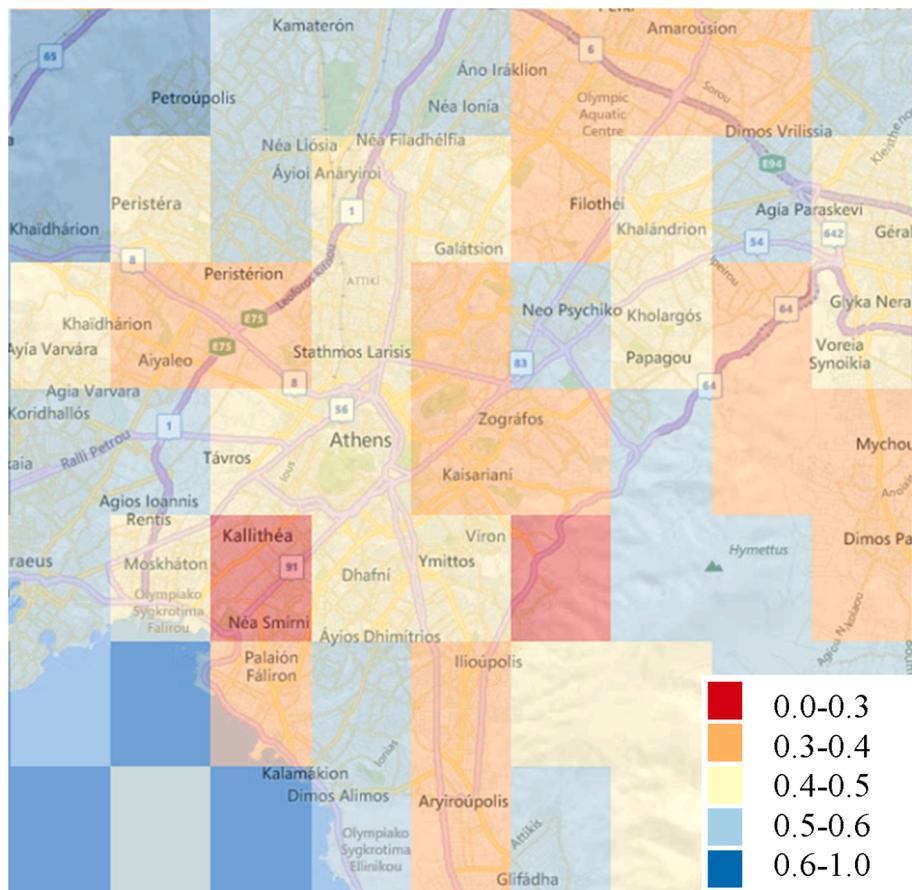
literature findings of short searching times discussed earlier. Interestingly, half of the drivers covered a distance two times longer than the Euclidean to eventually reach their parking location, after started searching for an available parking space, as the straightness metric suggests. The mean duration of the trips was about 20 min which means that more than 10% of the total trip duration is dedicated to searching for parking.

## 5. Implementation and results

Different survival models are developed and compared: a non-parametric Random Forest model, a semi-parametric Cox Proportional Hazards model, a parametric model following the Weibull distribution and a Deep Neural Network. In all four approaches, survival time is estimated leveraging the rest variables of each trip and the model fit is being assessed using the concordance index. More specifically, all models developed use as dependent variable searching for parking duration and as independent variables the following: peak hour, work/weekend, land use, population density, total trip duration, and the arc distance between the origin and the destination of the trip. The data were divided randomly into a training and test set, containing the 80% and 20% of the data respectively. The test set was used to evaluate the performance of each model and was not involved in the training process.

### 5.1. Cox Proportional Hazards model

First, a Cox Proportional Hazards model was fitted to investigate the effect of several variables on survival, e.g. the time until an individual finds a free on-street parking space. The dataset used for the development of the model includes 48,840 trips conducted in



**Fig. 6.** Geographical Distribution of Parking Straightness Measure.

**Table 2**  
Description of variables used in Survival Analysis.

Variables	Description	Type/Unit
<b>Parking attributes</b>		
Searching duration	Duration of searching until finding an available parking space	Continuous (sec)
Searching distance	Distance covered while searching for an available parking space	Continuous (km)
<b>Trip attributes</b>		
Trip duration	The total duration of the trip	Continuous (sec)
Arc distance	The distance between the first and the last point of the trip across the earth's surface	Continuous (km)
<b>Exogenous parameters</b>		
Peak hour	Whether the trip was conducted during the morning peak (6:00 – 10:00), during afternoon peak (16:00 – 20:00), or any other time during the day	Discrete
Work weekend	Whether the trip was conducted during the weekend or not	Discrete
Land use	<ul style="list-style-type: none"> <li>• Whether the vehicle was parked in an area characterized as:           <ul style="list-style-type: none"> <li>Continuous urban fabric (more than 80% of the land coverage by impermeable features like buildings, roads and artificially surfaced areas).</li> <li>Discontinuous urban fabric (30 to 80% land coverage by impermeable features.)</li> <li>Discontinuous urban fabric of lower density</li> <li>Industrial and commercial uses (under industrial or commercial use or for public service facilities.)</li> </ul> </li> </ul>	Discrete
Population Density	Population divided by area	Continuous (residents/km <sup>2</sup> )

the Region of Attica, where the minimum searching duration was 10 s. The training set (80% of the data) was used for fitting the model and the test set for evaluating its performance. The null hypothesis  $H_0 : \beta = 0$  is strongly rejected based on the results of three tests: Likelihood ratio test, Wald test and score test. In the final model, all significant covariates are included, and the corresponding results

**Table 3**

Descriptive statistics of the dataset.

	Trip duration (sec)	Searching duration (sec)	Straightness
mean	1175.39	130.74	0.52
std	1031.44	212.41	0.31
25%	466	25	0.23
50%	935	57	0.52
75%	1607	140	0.81

are presented in [Table 4](#).

Results indicate that searching for parking duration is significantly different when the trip is conducted during peak hours than off-peak hours. More specifically, negative coefficients of afternoon peak and off-peak hour indicate lower searching duration than on morning peak, as one may have expected. In addition, long trips appear to be related to longer searching durations. This could probably hint to a behavioral pattern arising from the lack of familiarity to the place of visit. However this is only a reasonable guess and cannot be confirmed by the available data as they are completely anonymized. We could safely reach this conclusion only if data were available on trip purpose, frequency of visits by a driver to a specific area, home and work locations etc., which is not the case here. Moreover, driving in discontinuous areas, as well as commercial and industrial areas, results in searching for an available parking space longer when compared to continuous urban fabric areas. The same applies also to areas with higher population density. Besides, searching for parking at the weekend is related to lower searching times, as it was expected. The concordance index of the model is equal to 60.3%, which is a decent value and implies a relatively well-fitted model. Furthermore, the Root Mean Squared Error is estimated at 91.05 and the Mean Absolute Error at 44.54.

One of the main assumptions of the Cox Proportional Hazards model is the *proportionality* of the covariates. This means that for any two individuals the ratio of hazards remains constant over time. In our case, the proportional hazards assumption is checked based on scaled Schoenfeld residuals. For each of the variables included in the model, a function that correlates the corresponding set of scaled Schoenfeld residuals with time was implemented to test independence between residuals and time. The same test applies to the model as a whole. The results indicate that the proportionality assumption is violated. As shown in [Fig. 7](#), the proportionality assumption is violated.

## 5.2. Weibull distribution parametric model

Further, several parametric duration models were assessed and compared based on the AIC criterion. Weibull distribution was found to best fit the data. The results of the parametric model are presented in [Table 5](#). The signs of the coefficients of the variables in this model are opposite from the ones in Cox Proportional Hazards model because the output of the model is the hazard rate and not the time to the event. In fact, the results of the two models are quite similar, since higher hazard is connected to shorter time and vice versa as mentioned earlier ( $S(t) = e^{\Lambda(t)}$ , where  $S(t)$  is the time to event and  $\Lambda(t)$  is the hazard rate). The results of the model's fitting are presented in [Table 5](#). The duration and the distance of the trip and the time and day in which it was performed are the parameters that mostly affect searching for parking. On the other hand, the area's density is the least significant parameter. The concordance index of the model is also quite similar to that of the previous one, 0.601, as well as the Root Mean Squared Error and Mean Absolute Error, 96.23 and 47.89 respectively.

**Table 4**

Cox Proportional Hazards Model results and metrics.

Variables	Coefficient	Pr(> z )	HR	Lower 95%	Upper 95%
Peak hour			1		
Morning peak	Ref.		1		
Afternoon peak	-0.217	<2e-16	0.805	0.785	0.826
Off peak	-0.312	<2e-16	0.732	0.715	0.749
Land use			1		
Continuous urban fabric	Ref.		1		
Discontinuous urban fabric – medium density	0.018	0.167	1.018	0.993	1.043
Industrial and commercial areas	0.227	<2e-16	1.255	1.212	1.301
Discontinuous urban fabric – low density	0.188	<2e-16	1.207	1.166	1.250
Work/Weekend	-0.033	0.002	0.967	0.947	0.988
Trip duration	-0.0005	<2e-16	0.1	0.1	0.1
Arc distance	0.03	<2e-16	1.030	1.029	1.031
Density	0.953	0.04	2.593	1.047	6.423
<b>Concordance (se = 0.001)</b>	0.603				
<b>Log-rank test (on 9 df)</b>	4756				
<b>Wald test (on 9 df)</b>	6035				
<b>Likelihood ratio test (on 9 df)</b>	6523				

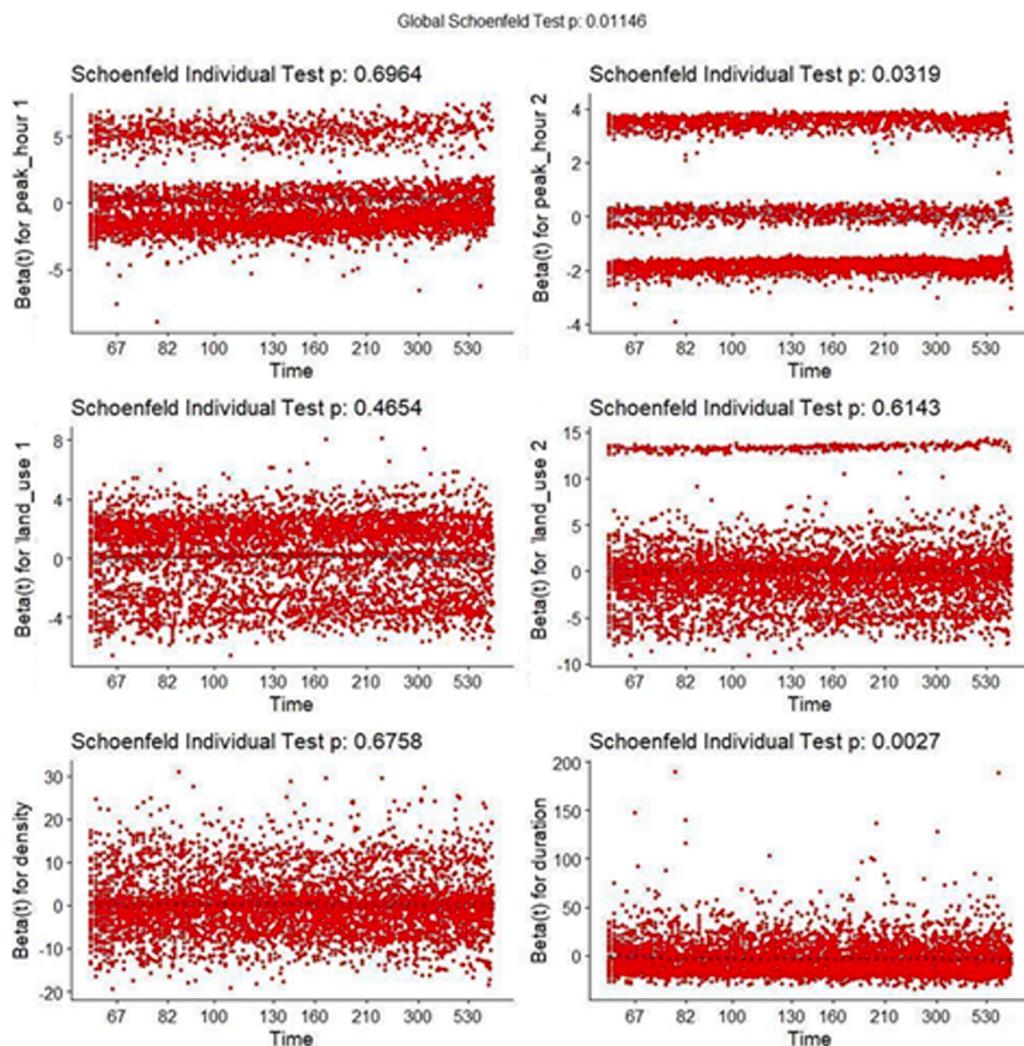


Fig. 7. Schoenfeld residuals versus time for each of the variables of the Cox model.

**Table 5**  
Weibull duration model results.

Variables	Coefficient	Std. Error	Pr(> z )
Peak hour			
Morning peak	Ref.		
Afternoon peak	0.259	0.014	<2e-16
Off peak	0.378	0.013	<2e-16
Land use			
Continuous urban fabric	Ref.		
Discontinuous urban fabric – medium density	-0.028	0.014	0.041
Industrial and commercial areas	-0.272	0.019	<2e-16
Discontinuous urban fabric – low density	-0.222	0.019	<2e-16
Work/Weekend	0.043	0.012	0.0002
Trip duration	0.001	7.5e-6	<2e-16
Arc distance	-0.037	4.3e-4	<2e-16
Density	-1.611	0.495	0.001
Concordance (se = 0.001)	0.601		
Number of Newton-Raphson Iterations	5		
Scale	1.07		

### 5.3. Random survival Forest

Next, a Random Survival Forest (RSF) model was built having 30 trees with 2 variables tried at each split. The default splitting criterion Log-rank was used. The minimum node size is 3 and the total number of variables is 6. In line with the results of both the Cox and the Parametric model, the important variables according to RSF with a descending order of importance are the trip duration, arc distance, the population density in the destination, whether the driver conducted the trip during peak hour and the land use in the final destination of the trip (Fig. 8). The trip duration is found to be the most influential factor; similar to the cox model, this behavioral pattern should be further analyzed with respect to other demand characteristics, such as the trip purpose and the frequency of visits by a driver to a specific area.

Each variable's importance is hereby estimated by measuring how the model's score decreases when the specific variable is not available, a method known as permutation importance (Altmann et al., 2010).

The c-index equals 0.561, implying that the model is not very reliable for predicting the searching for parking time, though it gives an insight into the effect of each of the independent variables. The error metrics are also indicative of the not very well fitting of the model: The Root Mean Squared Error is about 160.94 and the Mean Absolute Error 75.22.

### 5.4. Deep learning survival analysis

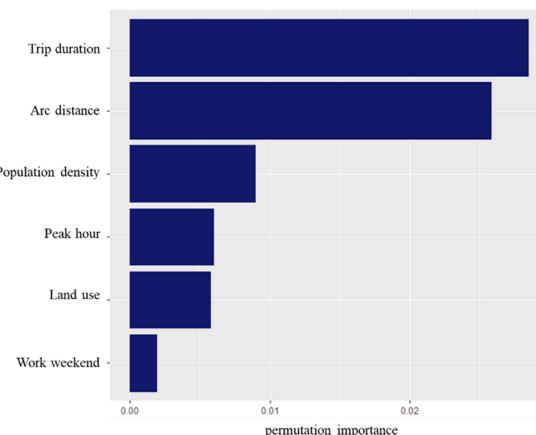
To achieve a better fit to the training data and improved predicting accuracy, a deep learning approach was adopted. A deep artificial neural network was developed based on the *DeepSurv* and *DeepSurv\_Keras* (<https://github.com/jaredleekatzman/DeepSurv>) python packages in order to simulate the searching for parking duration depending on the values of the independent variables. After an extensive grid search and cross-validation, the following structure of the neural network has proven to produce optimal performance:

- An input layer that consists of 32 neurons.
- 2 hidden layers consisting of 16 neurons each.
- An output layer of one neuron.

The rectifier linear unit activation function (ReLU) has been used for the neurons of the input and hidden layers and the linear unit for the neuron of the output layer, which were chosen among other activations, namely sigmoid and tanh because they lead to smaller values of prediction error. The model was fitted to the training data using the Adam optimizer with a learning rate equal to 0.001 for a training period of 80 epochs and a Dropout layer was employed after each of the hidden layers, in order to avoid overfitting. L2 regularization was also applied to the weights of the neurons of all layers. The batch size was set to 100. To conclude to the above choices, 5-fold cross-validation was conducted. Specifically, the above architecture as well as the values of the hyperparameters were the result of extensive grid search based exclusively on the average accuracy of the cross-validation scheme.

The output of the model is the predicted time of the event, e.g. the predicted searching for parking duration, in order to compare with the other models, as all outputs are transformed into duration using the equations proposed by Austin (2012). Furthermore, the knowledge of survival times allows for the formation of the survival curves.

Using the samples of the test set, the concordance index of the fitted model can be estimated. In this case, it is approximately equal to 65.6%, which is a high enough value to confirm the appropriateness of the variables used to predict the searching duration, as well as of the model's structure, and is noticeably higher than any other model presented in this paper. The Root Mean Squared Error was calculated at 51.66 and the Mean Absolute Error 33.53, which are adequate for the purposes of this paper and lower than the results of the rest of the methods tested.



**Fig. 8.** Feature importance of Random Forest.

### 5.5. Model comparison

The fitting of each of the four survival models is commonly assessed by the concordance index (Harrel's c-index). We compare the performance of the CoxPH, the parametric model, the RSF and the deep NN on the test set. From the four models already presented, Random Survival Forest has the poorest fit to the data, with a c-index value equal to 0.56. The parametric model and the Cox Proportional Hazard model perform similarly, having a c-index value close to 0.60, which is a decent value, as already mentioned. The Deep Neural Network seems to be better fitted than the other three, as the c-index value of 0.66 proves. This model is reliable and can produce predictions of competent accuracy, proving the suitability of Deep Neural Networks to modeling time-to-event. Specifically, recent studies suggest that a 0.06 increase of the c-index is quite significant (Chen et al., 2013; Luck et al., 2017).

Finally, by predicting the duration of searching for parking of each individual in the test set, it is possible to generate the predicted survival curve (graphic representation of the survival function  $S(t)$ ) for each model, which are presented in Fig. 9 alongside the real one. The y-axis represents the percentage of the vehicles that are expected to have been parked after the corresponding time of the x-axis.

Our best model's curve (Deep Neural Network) and the Kaplan-Meier share common characteristics while retaining one slight difference: Generally, the Deep Neural Network model has a higher gradient that results in more events occurring at an earlier time than they do, while the Kaplan-Meier curve is smoother. The similarity of the curves shows that a c-index value of about 66% is indeed a sign of a good fit. The Random Survival Forest model's curve significantly differs from the previous two, as indicated by its low concordance index. The Deep Survival Network, however, does not outperform the Cox PH model, as the two curves are both close to the real one. Similar findings are also reported in previous research studies (Ching et al., 2018, Gensheimer and Narasimhan, 2019). The Deep Survival Network performs clearly better for survival times  $< 100$  sec, which represent about 70% of the sample. Additionally, it is more flexible, it can capture non-linear dependencies between the data (unlike the Cox model which is linear) and it is not limited by restrictions, such as the proportionality of hazards.

## 6. Conclusions

Predicting searching for parking duration is vital, especially for core locations in the urban road network, as literature strongly suggests that searching for an available on-street parking space is related to congestion levels in the specific area.

In this paper trajectory data gathered from a smartphone application, consisting of real-world naturalistic driving trajectory data which include searching for parking, are exploited in order to develop searching for parking duration prediction models. Several trip attributes and parking searching characteristics were available for each trip. Furthermore, the notion of parking straightness was introduced, as a metric to characterize the trajectory at the emergence of a searching for parking event. In addition, data were enriched with valuable information about the basic characteristics of the area of ending point, such as population density and land use.

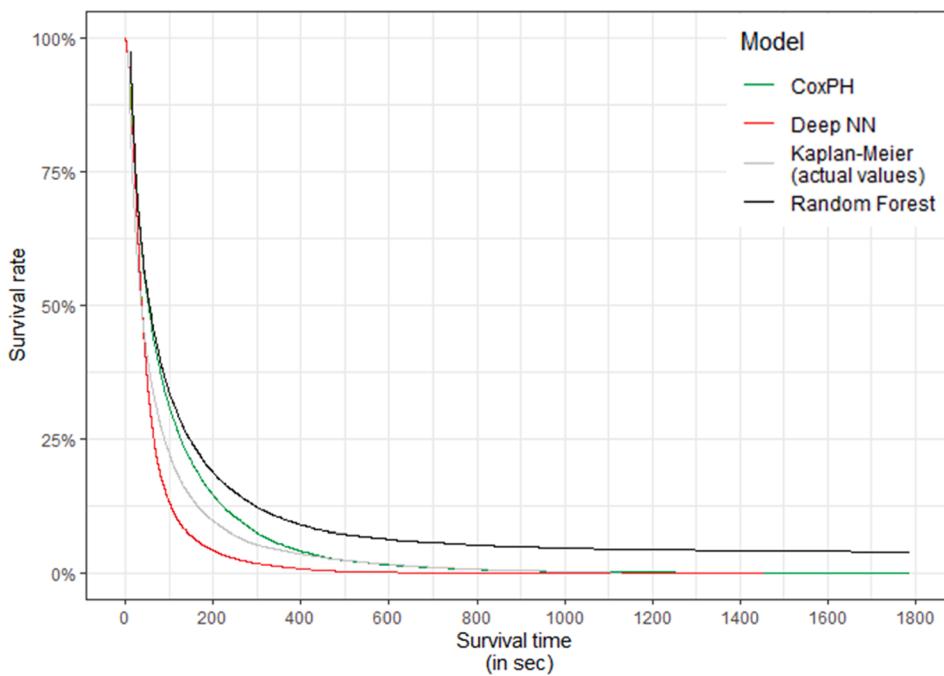
Several well-known survival models' performances were compared to a Deep Survival Neural Network. The fitting of the models was assessed using the widespread measure of c-index. Results indicated that Deep Survival Models improve survival time prediction accuracy when compared to non- and semi-parametric models. Regarding the influential factors, it was found that the time of day (e.g. traveling during peak hours or off-peak hours) and area characteristics, namely nearest land uses and population density have been found to significantly affect the time spent for finding an on-street parking space. The length and duration of the trip were also found as two of the most important parameters, possibly implying a relationship with the driver's degree of familiarity with the area, without however having here the ability to delve into the underlying behavioral patterns that may explain this output due to the lack of the appropriate personal information. The relationship between these two measures and parking behavior leaves room for further investigation if the data needed to identify additional features of the overall mobility patterns of each driver were available (e.g., purpose of travelling, frequency of visits per destination, etc.).

The investigation of the parameters that lead to more accurate predictions of searching duration is important for two main reasons: First, it allows policymakers and authorities to better understand the dynamics of the phenomenon and take appropriate actions towards the alleviation of areas facing parking problems. Secondly, knowing the parameters that lead to increased searching durations is also useful for drivers themselves who can take more informed decisions regarding their trip, such as avoiding peak hours.

Furthermore, the proposed methodology that accurately predicts the difficulty of finding available parking spaces in different areas, can be exploited for the development of smarter and more sophisticated parking management with pricing mechanisms, based on the predicted searching time, time of day, land use of the destination etc. The latter would lead to more efficient use of a city's space and decrease discomfort for the drivers, as well as the induced congestion coming from the searching for parking space. The methodology is also easily transferable to other areas, detached from third party location services as soon as similar trajectory data become available.

This study entails some limitations that emerge from the data quality and the complexity of the phenomenon of searching for parking in large metropolitan areas. First, since the data stem from a commercial application, tracing back to the accuracy of sensing was not possible. This is a typical concern in many similar applications. Thus, we think that such data-driven modeling should be evaluated based on empirical evidence and intuitive knowledge and act complementary to existing well-founded principles of transportation and traffic engineering. Second, searching for parking is highly influenced by driver's behavior and their familiarity with the visited area. Such data are not usually available especially for a large number of driving trips. Nevertheless, searching for parking behavior is investigated based on area characteristics that sufficiently describe the parking searching phenomenon.

To this end, our future research will focus on explicitly describe the parking area by enriching the dataset with more detailed land uses, demand rates and other significant attributes. From a modeling perspective, since deep learning seems to be a promising



**Fig. 9.** Predicted Survival curves of models.

alternative, more sophisticated approaches such as convolutional and recurrent neural networks should be tested in comparison to the traditional survival models. Finally, an obvious direction for future research is to provide an integrated treatment of searching for parking and traffic congestion, since searching for parking affects traffic congestion, and traffic congestion affects the speed at which cars cruise for parking.

#### Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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#### Author contributions statement

The authors confirm contribution to the paper as follows: study conception and design: E. Vlahogianni; E. Mantouka, P. Fafoutellis; analysis and interpretation of results: P. Fafoutellis, E. Mantouka, E. Vlahogianni; draft manuscript preparation: E. Mantouka, P. Fafoutellis, E. Vlahogianni. All authors reviewed the results and approved the final version of the manuscript.

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