

# A data-driven hybrid control framework to improve transit performance



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## ABSTRACT

This paper presents a data-driven hybrid control (DDHC) framework that can arrange adaptive control strategies for vehicles to effectively improve the transit performance of the public transport system. The framework depicts a powerful combination of a data-driven control method that is used to imitate the control behaviour of dispatchers and a mathematical optimization method. Three components comprise the DDHC framework: a data-driven control module, a performance module, and an optimization module. The data-driven control module contains a random forest model which is adopted to justify whether to intervene in the operation of a bus line, and if so, which vehicles should be controlled and what type of control strategy should be taken – an acceleration strategy or deceleration strategy. The performance module including vehicle operation state models is used to describe the system evolution. The last component optimizes the specific control actions – which type of acceleration or deceleration strategy should be adopted – by minimizing total passenger travel time. The effectiveness of the proposed DDHC framework is evaluated with the data of a transit route in Urumqi, China. The results show that the DDHC framework with reasonable parameters can suit the needs of real-time control in complex traffic environments.

## 1. Introduction

### 1.1. Background

Public transport services are difficult to regularize in daily operations because operational vehicles are impacted by diverse disruptions (Newell and Potts, 1964; Chen et al., 2013; Liao and van Wee, 2017; Varga et al., 2018). The disruptions mainly include two aspects: the variability in travel time and the variations of passenger demand (Yao et al., 2019). These disruptions tend to induce ‘bus bunching’ or large intervals in service, resulting in an increase in passenger travel time. Since the length and reliability of travel time are the main criteria for users to evaluate the level of public transport service (Liao et al., 2014), headway variability will heavily impact the service level perceived by users.

To reduce the negative effects of headway variability, researchers have devoted numerous efforts to developing flexible control

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strategies. These strategies have been designed to allow operators to react dynamically to real-time system disturbances. The most studied operational control strategies include holding, stop skipping (deadheading, short turning or expressing), cruising speed control, overtaking and transit signal priority mechanisms (Eberlein et al., 2001; Hickman, 2001; Sun and Hickman, 2005; Yu and Yang, 2009; Daganzo and Pilachowski, 2011; Delgado et al., 2012; Yu et al., 2012; He, 2015; Guo et al., 2017; Wu et al., 2017, 2018; Berrebi et al., 2018; Huang et al., 2018; Gkiotsalitis et al., 2019). However, the operation of 3 public transport systems is complicated and time-varying (Liao et al., 2013). At a certain moment and at a specified station, it is difficult to determine whether to intervene in the operation of a route and which control strategy is applicable.

Actual operations still rely on the dispatchers' experience to manually control the operation of bus lines. Bus operators deploy professional dispatchers to control the operation of bus lines in real time to enhance the service level of the public transportation system. The experience of these dispatchers is robust. They are always able to make effective control decisions based on the known operational status information and their own experience. It is known that the decisions made by human can be suitable for practical applications, since the human brain has high flexibility and can include some random factors in the decision-making process. Therefore, this paper intends to combine the advantages of dispatchers' practical experience with a theoretical optimization method to design a control framework which is suitable for the complex operating context of the public transportation system.

## 1.2. Literature review

A number of studies focused on real-time control strategies have been developed to improve service regularity. One of the most studied strategies is holding. Holding is the process of delaying a bus at a station intentionally after passengers have boarded and alighted (Eberlein et al., 2001). The strategy commonly includes three types (Delgado et al., 2012; Hernández et al., 2015): schedule-based holding, headway-based holding and mathematical programming based holding. Schedule-based holding control tries to improve transit performance by adjusting the timetables of vehicles (Wu et al., 2016; Gu et al., 2017; Zhu et al., 2017). It is based on the analysis of the causes of bus 'bunching' (Furth and Muller, 2007, 2009), and the comprehending of the influence of schedule-based controls (Zhao et al., 2006). Then, with the investigation of the main causes of bus bunching, some scholars suggested shifting from a schedule-based (static) control to a headway-based (dynamic) control (Feng and Figliozzi, 2011). The vehicles operated with the headway-based strategy do not need to adhere to any schedule, which is suitable for the transit routes with high frequency (Abkowitz and Lepofsky, 1990; Fu and Yang, 2002). Nevertheless, instead of setting the regularization of headways as the target of a control strategy, it is also a common method to express the control problem as an optimization problem (Xuan et al., 2011; Tang et al., 2012). The objective functions mainly include passenger waiting times, passenger in-vehicle delays or both of them (Eberlein et al., 2001; Zolfaghari et al., 2004; Cortés et al., 2010). The control problem becomes more complicated, e.g., by considering capacity constraints (Jiang et al., 2003; Delgado et al., 2012; Sánchez-Martínez et al., 2016; Schmöcker et al., 2016; Yang et al., 2017), or by considering multiple routes operating in a same corridor (Hernández et al., 2015).

Coupled with holding strategy, another strategy commonly used is stop skipping (Furth, 1985; Eberlein et al., 1998; Yu et al., 2012). Stop skipping strategies permit partial vehicles to run through some stops without service to decrease the travel time and the interval between the current bus and its preceding bus. The common objective of the approach is minimizing passenger waiting times or total travel times, with the consideration of the extra waiting times and travel times of passengers whose destination has been skipped (Furth, 1985; Eberlein et al., 1999; Sun and Hichman, 2005, Yu et al., 2012; Chen et al., 2015). Early, stop skipping strategies, including short turning, deadheading and expressing were formulated as pre-planned strategy (Jordan and Turnquist, 1979; Furth, 1986). Then, with the availability of real-time information supplied by, for instance, APC and AVL systems, short turning strategies have been studied as a real-time control strategy (Fu and Liu, 2003; Chen et al., 2015).

There are also several works have been done in the field of control framework. Sáez et al. (2012) proposed a hybrid predictive control (HPC) framework which integrated holding and expressing strategies to solve a real-time public transport control problem with uncertain passenger demand. The HPC framework includes a predictive model of the bus system, written in discrete time and a dynamic objective function. Both the online and offline information about passenger behavior are utilized to predict the uncertain passenger demand. They implemented a Genetic Algorithm to optimize the HPC strategy. Sánchez-Martínez et al. (2016) presented an optimization framework to optimize holding times with dynamic running times and demand. They used a performance model to predict system evolution and compute the control input as the solution of an optimization problem. The model can be utilized to plan holding times based on both the current system state and expected changes in passenger demand and running times. Moreira-Matias et al. (2016) presented a control framework to combine travel time prediction and control actions. The travel time predicted by machine learning approach is used to infer the headway distribution. Then, they used the headway distribution to determine which action should be executed at a control station, no intervention, holding or stop skipping. Andres and Nair (2017) provided a predictive-control framework to address bus bunching. They combined the data-driven headway prediction and headway-based holding strategy. Continuous-time headways were interpreted as time series and predicted directly to decide the holding time of buses.

## 1.3. Contributions

Scholars have performed many studies on single control strategies or hybrid control strategies and have obtained many achievements. However, these theoretical results are either difficult to use, or not feasible in practical applications. Dispatcher interviews revealed that it was not difficult for the dispatchers to keep all the vehicles evenly distributed all the time, albeit at the expense of excessive intervention (multiple controls). Instead, their most difficult job was to identify the vehicles that truly required control and determine when and how control these vehicles, to improve the system service level as much as possible with the least

amount of intervention.

In the expected conditions (running time and dwelling time are assumed to be constant), disturbances are easily amplified and eventually cause bunching. However, the actual survey data showed that some disturbances gradually disappeared due to the system's self-recovery ability (e.g., the drivers of the vehicles behind the timetable usually tend to accelerate to narrow down the distance from the previous vehicle.). Therefore, it is important for the real-time control problem to estimate the impact of disturbances on system evolution and control the disturbances that may worsen (causing bunching) timely. Dispatchers require a method to determine the “control timing” for dynamic control based on the current operational state and the estimated future traffic state.

Furthermore, most traditional studies on real-time control problems are based on historical rules to infer future trajectories for vehicles. These historical rules are mainly derived from the data obtained by surveys or GPS. The research idea itself is correct. However, data obtained directly from surveys or GPS cannot accurately reflect the actual operational status of buses. For many cities (for example, most metropolises in China), dispatchers always perform some control operations on some vehicles according to traffic conditions or passenger demand. In other words, data obtained from surveys and GPS have usually been *corrected* by dispatchers. Thus, it needs to identify these man-made adjustments to see what is the initial operational state of a bus system and how dispatchers made these adjustments.

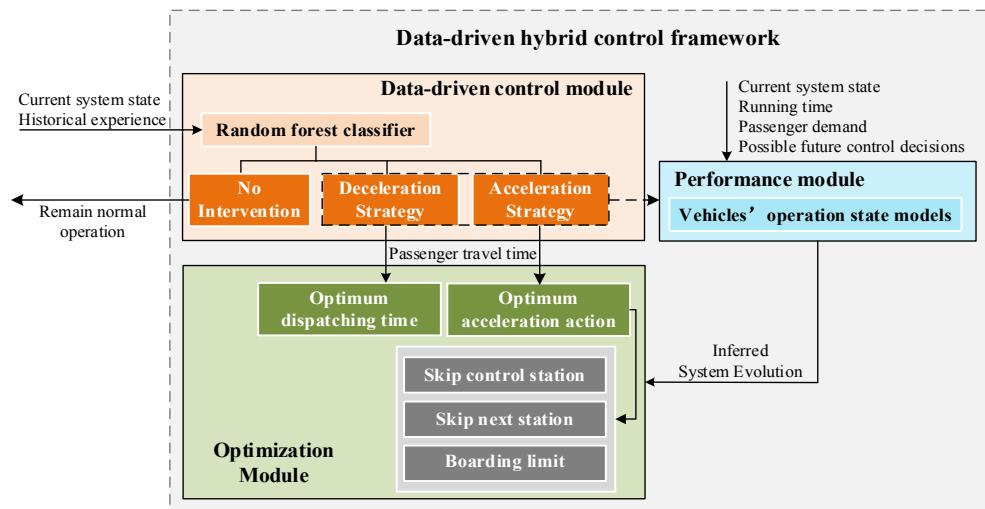
With this as motivation, this paper proposes a data-driven hybrid control framework that depicts a powerful combination of the rich practical experience of dispatchers with mathematical optimization models. Dispatchers can obtain real-time location and distribution information of vehicles through a visualized intelligent transportation system. Based on the information and their practical experience, dispatchers judge whether to intervene in the operation of a bus line. If the control is required, which vehicle should be controlled and which control strategy (acceleration strategy or deceleration strategy) should be applied. We abstract the decision behaviour of dispatchers as the data-driven control module in the DDHC framework. A machine learning approach is used to identify the rules dispatchers use to make control decisions. The effective control decisions made by dispatchers are collected to train the random forest (RF) model in the data-driven control module, so that the RF model can learn positive experiences from historical data. The trained RF model can determine whether to control a vehicle and assign an appropriate control strategy to the vehicle. After the control strategy to be taken is determined, specific to this strategy, a mathematical optimization model is established with the goal of minimizing passenger travel time to optimize the operable optimal control action. The superiority of a mathematical optimization method is that it can have a holistic view of a bus line and understand the overall impact that any control action might have on the entire bus line. Thus, the DDHC framework combines the advantages of dispatchers' practical experience and theoretical optimization models, which can meet the needs of real-time control in complex traffic environments.

The reminder of this paper is structured as follow: [Section 2](#) describes the DDHC framework proposed in this study; [Section 3](#) presents the methodologies and formulations for each module in the framework; [Section 4](#) provides an algorithm to solve the DDHC problem; [Section 5](#) discusses the implementation of this control framework based on a BRT route operated in the city of Urumqi, China. Finally, [Section 6](#) concludes this study.

## 2. Problem description

### 2.1. The description of the DDHC framework

The DDHC framework proposed in this paper includes three modules: a data-driven control module, a performance module and an optimization module, as shown in [Fig. 1](#). Through these three modules, dispatchers' excellent experiences are combined with



[Fig. 1](#). The description of the DDHC framework.

mathematical optimization methods. First, the machine learning method in the data-driven control module is used to imitate the dispatchers' decision process. After analyzing the known operating status of a bus line and combining historical experience, it is to determine whether the vehicles arriving at control stations need to be controlled, and if so, which control strategy is appropriate. By classifying multiple control strategies, we find that all these strategies can be essentially divided into two categories: acceleration strategy (e.g. stop skipping, boarding limit) and deceleration strategy (e.g. holding). Therefore, in the data-driven control module, random forest classifier is adopted to classify the control categories for vehicles. Three control categories can be outputted by the random forest classifier: no intervention, acceleration strategy and deceleration strategy. If the result of the classification for a vehicle is no intervention, the vehicle remains its normal operation; if the classification result is acceleration strategy or deceleration strategy, then enter the performance module and optimization module.

The vehicle operational state models which used to describe the relationship between passenger demand and bus operation in the performance module use current system state, stochastic running times, possible future control decisions and passenger demand to infer how the system will evolve. The inferred variables about system state, such as headway, arrival time and loads of vehicles, are transmitted to the optimization module.

Based on the mathematical optimization method, the objective functions in the optimization module optimize specific control actions with the target of minimizing total passenger travel time. If the control category outputted by the data-driven control module is deceleration strategy, the dispatching time of a vehicle at a control station is optimized. This dispatching time is later than the inherent departure time of the vehicle. For the control category of acceleration strategy, three acceleration actions are set forth in this study: (i) Skip control station; (ii) Skip next station. That is, the next station of the control station is skipped and passengers destined for the next station get off at the control station; (iii) Boarding limit. That is, boarding is only allowed during the alighting period at the control station. After alighting process ends, the vehicle leaves immediately. This paper sets the third acceleration action because boarding processes generally take more time than alighting processes, especially for overcrowded vehicles at high demand stations. The benefit of this acceleration action is that there is no chance of a passenger arriving at his destination but unable to get off. Through optimization, an optimal acceleration action is selected for the vehicle under control.

## 2.2. The context and principle of the DDHC framework used

The DDHC framework is used in the context of a one-way loop network consisting of  $N$  stations, as shown in Figs. 2 and 3. According to the characteristics of the entire route and each station, a set of representative stations (e.g. passenger demand is too high or too low; if space is available for holding) are selected as the control stations,  $S_t = \{s_1, s_2, \dots, s_t\}$ , as the big points shown in Figs. 2 and 3.

The RF model in data-driven control module simulates the dispatchers' decision process. Because the speed of human brain is limited in dealing with large-scale information to obtain optimal solutions, in daily work, dispatchers judge control categories based on the local operational state information and their experience. They control vehicles based on the known system operational status information and the pre-judgment of the possibility of congestion along the road sections ahead.

Fig. 2 is used to describe how the data-driven control module works. Take control station  $s_1$  and vehicle  $i$  as an example. When vehicle  $i$  departs from station  $s_1 - 1$ , we evaluate the current operational status of the stations between two control stations (from station  $s_1$  to station  $s_2 - 1$ , called the impact set of control station  $s_1$  in data-driven control module) and the operational status of vehicles associated with vehicle  $i$ . The operational state of continuous  $R$  vehicles last passing through a station is used to reflect the operational state of this station. For example, vehicle  $u$  is the last vehicle that has served station  $s_2 - 1$  when vehicle  $i$  is under control, thus, the set of vehicles used to reflect the operational state of station  $s_2 - 1$  is from vehicle  $u$  to vehicle  $u-R+1$ , denoted as  $\Omega_{i,s_2-1}$ .  $\Omega_{i,s_2-1}$  can be described as the rolling horizon of station  $j$  when vehicle  $i$  is under control. When reacting to the operational state of vehicles, such as the headway between vehicle  $i$  and its subsequent vehicle  $i + 1$ , we apply the latest known headway information. As

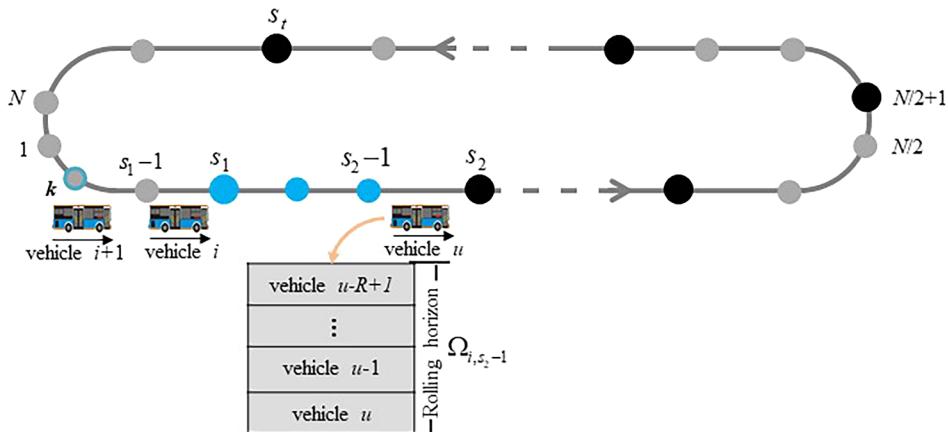
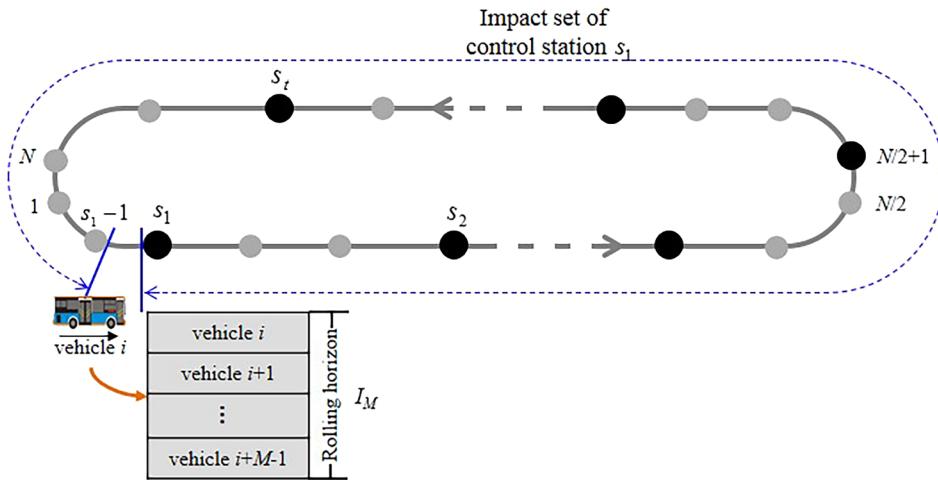


Fig. 2. The description of the data-driven control module.



**Fig. 3.** The description of the optimization module.

shown in Fig. 2, station  $k$  is the last station served by vehicle  $i + 1$ , then the headway information used is the headway between vehicle  $i$  and vehicle  $i + 1$  at station  $k$ . In summary, only known information about operational status is used in the data-driven control module.

The features used to describe the current system operational state from the perspective of both stations and vehicles, and the historical congestion level of the road segment between station  $s_1$  and station  $s_2$  are used as the explanatory variables in RF model to judge the control strategy for vehicle  $i$  at control station  $s_1$ .

Fig. 3 describes how the optimization module takes actions. A control decision making for a vehicle at a control station can clearly impact on the travel time of passengers at the control station and its downstream stations. In optimization module, since it hopes to have a holistic view of the bus line and understand the overall impact that any control action might have on the entire line, all the stations downstream the control station are included into the impact set. Take control station  $s_1$  as an example. We call the stations from station  $s_1$  to station  $N$  then to station  $s_1 - 1$  as the impact set of control station  $s_1$ . The total travel time of the passengers waiting at all the stations of the route is considered to optimize the specific control actions. Furthermore, given the real-time, dynamic nature of transit operations control, a finite rolling horizon is adopted for the optimization module. That is, when optimize the specific control actions in optimization module, we consider the impact of the control on a set of  $M$  consecutive vehicles,  $I_M = \{i, i + 1, \dots, i + M - 1\}$ .

### 3. Data-driven hybrid control framework

This section describes the methodology and formulation of each module in the DDHC framework in detail. According to the current system operating state and the expectation of future road congestion, the random forest classifier in the data-driven control module estimates control categories for vehicles. If the classification results are acceleration or deceleration strategies, the vehicles' operation state models in the performance module are used to infer how the single loop bus system will evolve. The features to describe system evolution, such as headway and load, are substituted into the optimization module. Based on the control categories output by the data-driven control module, the mathematical optimization models in the optimization module optimize the specific control actions for vehicles.

#### 3.1. Definitions of variables

The definitions of variables used throughout the DDHC framework are summarized in Table 1.

#### 3.2. Formulation of the data-driven control module

The major concept of the data-driven control module is to imitate dispatchers' decision processes to classify the control category for upcoming vehicles through proper features with effective classification method. Random forest, a popular and efficient algorithm for both classification and regression problems is selected for the data-driven control module to provide decision support services. Below, we introduce the methodology of the random forest (RF henceforth) model and the way it is used in this paper.

##### 3.2.1. Methodology of random forest model

RF model based on model aggregation ideas, is introduced by Breiman (2001). RF model belongs to the family of ensemble methods and is a kind of Classification and Regression Tree (CART) models. The statistical framework with the consideration of a

**Table 1**  
Definition of variables.

Variable	Definition
$M$	Number of vehicles in the rolling horizon
$N$	Total number of stations of a route
$I_M$	The set of vehicles in the rolling horizon associated with vehicle $i$ in optimization module
$J$	The number of stations in the impact set of control station $j$ ( $J \in S_I$ ) used in data-driven control module
$\Omega_{i,j}$	The set of vehicles used to describe the operational state of station $j$ when estimating control category for vehicle $i$
$cap$	Rated capacity of a standard vehicle
$tr_{i,j}$	Running time of vehicle $i$ from station $j - 1$ to station $j$
$tai_{i,j}$	Arrival time of vehicle $i$ at station $j$
$tid_{i,j}$	Inherent departure time of vehicle $i$ at station $j$
$tdi_{i,j}$	Dispatching departure time of vehicle $i$ at station $j$
$tg_{i,j}$	The gap between the dispatching departure time $tdi_{i,j}$ and the inherent departure time $tid_{i,j}$
$ts_{i,j}$	Dwelling time of vehicle $i$ at station $j$
$A_{i,j}$	The number of alighting passengers of vehicle $i$ at station $j$
$B_{i,j}$	The number of boarding passengers of vehicle $i$ at station $j$
$L_{i,j}$	Load of vehicle $i$ when it departs from station $j$
$\lambda_j$	Average passenger arrival rate at station $j$
$q_j$	Passenger alighting proportion at station $j$ , which is a constant obtained by history data for each station.
$\beta_{i-1,j}$	Number of passengers left by vehicle $i-1$ at station $j$ and wait for vehicle $i$
$w_{i,j}$	Number of passengers prevented to board vehicle $i$ at station $j$ . It is associated with the “Boarding limit” control action.
$h_{i,j}$	Headway between vehicle $i$ and vehicle $i - 1$ at station $j$
$y_{i,j}$	A binary variable for stop skipping actions in acceleration strategy.
$\Delta ta_{i,j}$	Arrival time deviation of vehicles in the rolling horizon
$P_j$	Passenger demand at station $j$ during last continuous $R$ vehicles passing through station $j$
$P_J$	Total passenger demand throughout the impact set of control station $j$ ( $P_J = \sum_{j \in J} P_j$ )
$D_{i,j}$	The control category of vehicle $i$ at control station $j$

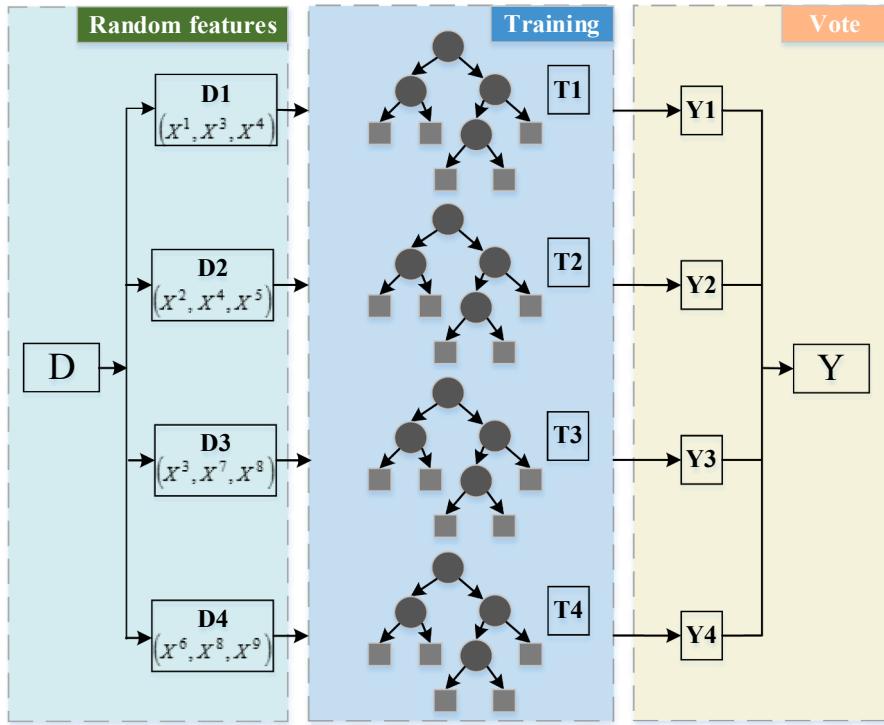
learning set  $L = \{(X_1, Y_1), \dots, (X_n, Y_n)\}$  made of  $n$  independent identically distributed (IID) observations of a random vector  $(X, Y)$  is recalled briefly. Vector  $X = (X^1, \dots, X^P)$  contains explanatory variables, say  $X \in R^P$ , and  $Y \in y$  where  $y$  is either a numerical response or a class label. For regression problems, it is assumed that  $Y = s(X) + \varepsilon$  with  $E[\varepsilon | X] = 0$  and  $s$  is the so called regression function, while for classification problems, a classifier  $t$  is a mapping  $t: R^P \rightarrow y$  (Genuer et al., 2010). RF is a model building strategy which provides estimators of either the regression function or the Bayes classifier, which is the mapping minimizing the classification error  $P(Y \neq t(X))$ .

The principle of RF is to combine a number of binary decision trees built with some bootstrap samples belonging to the learning sample  $L$  and at each node, selecting randomly a subset of explanatory variables  $X$ . With respect to the well-known Classification and Regression Tree (CART) model building strategy (Breiman et al., 1984) which performs a growing step followed by a pruning one, there are two differences between RF and CART. For RF, first, at each node, a given number (denoted as  $mtry$ ) of input variables are randomly selected and the best split is calculated within this subset only. Second, no pruning step is executed so all the trees of the forest are maximal trees (Genuer et al., 2010).

Bagging is used in RF to choose random features. In each set of bootstrap training, about one-third of the instances are left out. The observations which are not adopted to build the current tree are called out-of-bag samples. The out-of-bag samples can be used to evaluate the prediction error and then to estimate variable importance. RF model combines multiple decision trees. Each tree is an expert of regression or classification for a certain subset of features. The final result is yielded by the voting of all decision trees, which outperforms single classifier models (Liaw and Wiener, 2002). RF model becomes increasingly popular and proved to be potent in numerously different applications (Cutler et al., 2007; Genuer et al., 2010; Yang et al., 2017). There are few papers about the application of RF model in transport system. RF model in these studies is mainly used to predict traffic flows (Lesheim and Ritov, 2007; Hamner, 2010) and bus traveling time (Moreira, 2008; Gal et al., 2015; Yu et al., 2017).

2 major parameters are required for RF model:  $ntree$  (the number of trees in the forest) and  $mtry$  (the number of explanatory variables chosen randomly to generate each tree). The value of  $mtry$  impacts the strength of each individual classifier and the correlation between them. The smaller  $mtry$  is, the less strength and correlation are. To improve accuracy,  $mtry$  should be estimated critically to minimize the correlation while maintaining strength. A third of the total input variables is recommended for  $mtry$  (Ho, 2002; Ließ et al., 2012; Yu et al., 2017).

Because this paper applies the classification function of RF model, the classification principle of RF model is described in detail. Fig. 4 is used to visualize the RF algorithm for a classification problem. Assuming 9 predictors ( $X = (X^1, \dots, X^9)$ ) jointly determine the output  $Y$  (a class label). Set  $ntree = 4$  and  $mtry = 3$ . Then, RFs model generates a forest with 4 trees (T1, T2, T3, T4) and each tree is generated with 3 random predictors chosen from vector  $X$ , e.g., predictors  $X^1, X^3, X^4$  in D1 are used to generate tree T1. Each tree is



**Fig. 4.** Process of the RF model.

trained based on bootstrap samples and a class label of each tree is obtained. Then, these trees vote for the most popular class.  $Y$  in Fig. 4 denotes the final result. In voting, the weight of each tree is equivalent.

### 3.2.2. Application of the RF model in the data-driven control module

In the data-driven control module, the RF classifier described above is used to determine the control category for vehicles. The classification of the control category can be described as follow: when a vehicle departs from the previous station of a control station, it is to determine what type of control category should be issued to the vehicle at the control station (no intervention, deceleration strategy or acceleration strategy).

To improve the classification accuracy, it is essential to select appropriate features to describe the system operational state (Liu et al., 2010). After consulting professional dispatchers and reading the relevant literature, we concluded several types of operational features that affect control categories, including time-of-day, headway, arrival time, passenger demand, service reliability and load, which are used to describe the current operational status, and historical cruising speed, which is used to reflect the expectation of future road congestion. How these features are used as the explanatory variables will be described in detail below.

#### (i) Time-of-day

The feature time-of-day ( $T$ ) is mainly used to distinguish the peak hours and off-peak hours of a day. During peak hours, passenger demand is high and roads are prone to congestion (Sun and Elefteriadou, 2012, 2014). Therefore, the regularity of bus service is easily disrupted. After being disrupted, the self-repair capability of the system is poor. An effective control strategy is needed to help the system restore an adequate service level. In contrast, during the off-peak hours, the public transport service easily maintains regularity. Even if irregular service appears occasionally, the system can be self-healing in subsequent operation.

#### (ii) Headway

In previous studies, headway between sequential vehicles was an important indicator of whether to control a bus, such as the headway-based holding strategy studied by Andres and Nair (2017). In addition to headway-based control strategies, passenger waiting time associated with headway was also used as an objective function in many control-related studies. Thus, headways are used as explanatory variables in this study. Assume station  $k$  is the last station that has been served by vehicle  $i + 1$ . The headways used in the RF model include the headway between vehicle  $i$  and vehicle  $i + 1$  at station  $k$  ( $h_{i+1,k}$ ), and the headway between vehicle  $i$

and vehicle  $i-1$  at station  $j-1$  ( $h_{i,j-1}$ ). These two variables are used to describe the headways between the control vehicle  $i$  and its preceding and subsequent vehicles

(iii) Arrival time deviation of vehicles associated with the vehicle under control

Arrival time deviation is used to describe whether the arrival times of vehicles associated with vehicle  $i$  are behind or ahead of the scheduled arrival times.  $\Delta ta_{i,j-1}$  is calculated as Eq. (1), where  $ta_{i,j-1}^0$  denotes the scheduled arrival time of vehicle  $i$  at station  $j-1$ , and  $R$  is the total number of vehicles associated with vehicle  $i$  (from vehicle  $i$  to vehicle  $i-R+1$ ). Although there is no prescribed timetable in a high frequency public transport system, and vehicles are not required to operate in accordance with a timetable, the design of a fleet and its optimal frequency implies the arrival time for each vehicle at each station according to the ideal running time and dwell time, which is  $ta_{i,j-1}^0$  in this study. The selection of this feature is based on the assumption that the decision to accelerate or decelerate a vehicle cannot be based solely on the headway of vehicles and the short-sighted pursuit of headway regularity, but must also consider the overall system operation state. If the interval between vehicle  $i$  and vehicle  $i-1$  is small and the interval between vehicle  $i$  and vehicle  $i+1$  is large, vehicle  $i$  should be decelerated to make the headway more regular. At the same time, if  $\Delta ta_{i,j-1} > 0$ , the arrival times of vehicles associated with vehicle  $i$  are later than their scheduled arrival times overall. If a deceleration strategy is implemented for vehicle  $i$ , the deviation compared with  $ta_{i,j-1}^0$  will be exacerbated. In the short term, the service level may be raised but this is not conducive to the improvement of the system service level in the long term. Fleet resources have not been utilized effectively.

$$\Delta ta_{i,j-1} = \frac{\sum_{i \in \Omega_{i,j-1}} (ta_{i,j-1} - ta_{i,j-1}^0)}{R} \quad (1)$$

(iv) Service reliability associated with passenger demand

Service reliability has been regarded by many scholars as an important indicator to assess operational state (Yu et al., 2012; Yao et al., 2014; Fonzone et al., 2015). Therefore, this paper selects the service reliability of the impact set of control station  $j$  as one set of explanatory variables. Two service reliability parameters are included: the weighted average headway throughout the impact set ( $\Phi_i^{mean}$ ), which is used to reflect the concentrated trend of headways, and the deviation between each vehicle's headway and their average headway ( $\psi_i^{mean}$ ), which is used to describe the dispersion degree of the headways. The equations of  $\psi_i^{mean}$  and  $\Phi_i^{mean}$  are shown as follows. When deciding the control strategy, in addition to considering the operational state of control station  $j$ , the operational state of the entire impact set of station  $j$  should also be considered. The passenger demand during the last continuous  $R$  vehicles passing through a station services as the weight for the service reliability of the entire impact set.  $u_j^i$  indicates the last vehicle to provide service at station  $j$  when the control category is estimated for vehicle  $i$ .

$$\psi_i^{mean} = \sum_{j \in J} \frac{p_j}{P_j} \cdot \rho_{u_j^i, j}^{mean} \quad (2)$$

$$\rho_{u_j^i, j}^{mean} = \frac{\sigma_{u_j^i, j}^{mean}}{\bar{h}_{u_j^i, j}} \quad (3)$$

$$\sigma_{u_j^i, j}^{mean} = \sqrt{\frac{\sum_{u_j^i \in \Omega_{i,j}} (h_{u_j^i, j} - \bar{h}_{u_j^i, j})^2}{R}} \quad (4)$$

$$\Phi_i^{mean} = \sum_{j \in J} \frac{p_j}{P_j} \cdot \bar{h}_{u_j^i, j} \quad (5)$$

$$\bar{h}_{u_j^i, j} = \frac{\sum_{u_j^i \in \Omega_{i,j}} h_{u_j^i, j}}{R} \quad (6)$$

$$p_j = \lambda_j \times (tdd_{u_j^i, j} - tdd_{u_j^i - R, j}) \quad (7)$$

(v) Load

In the literature of control strategies with capacity constraints, load is a feature that judges whether to control a vehicle (Jiang et al., 2003; Delgado et al., 2012; Sánchez-Martínez et al., 2016). This paper considers that when the vehicle under control is overcrowded and reaches its capacity limit, it may be counterproductive to enact a deceleration strategy to the vehicle. By delaying the overcrowded vehicle at the control station, the vehicle cannot serve more passengers. On the other hand, if the occupancy rate of

vehicle  $i$  is low, and the occupancy rate of vehicle  $i + 1$  or/and vehicle  $i - 1$  is high, vehicle  $i$  can be controlled to support other vehicles effectively with an appropriate control strategy. Therefore, this paper selects,  $L_{i-1,j-1}$ ,  $L_{i,j-1}$  and  $L_{i+1,k}$  as explanatory variables in our RF model.

#### (vi) Cruising speed

The cruising speed of vehicles can reflect the degree of congestion of a road section. If the cruising speed is fast, then the road section is unblocked. In contrast, if the road section is congested, the regularity of the bus service is easily disturbed. In practice, when judging whether to control a vehicle, dispatchers anticipate the possibility of traffic congestion during the road ahead based on historical experience. To imitate dispatchers' consideration of the variable, the vehicles' historical average cruising speed between the two control stations is included in the classification model.  $v_{t_i,j}$  represents the historical average cruising speed of vehicles running from control station  $j$  to its next control station at 10 min after the time when determining the control category for vehicle  $i$ .

Let  $D_{i,j}$  denote the control category of vehicle  $i$  at control station  $j$ ; the decision model can be described as follow.

$$D_{i,j} = f(T, h_{i+1,k}, h_{i,j-1}, \Delta t_{i,j-1}, \Phi_i^{\text{mean}}, \psi_i^{\text{mean}}, L_{i-1,j-1}, L_{i,j-1}, L_{i+1,k}, v_{t_i,j}) \quad (8)$$

### 3.3. Formulation of performance module

If the control categories output by the data-driven control module are acceleration or deceleration strategies, the vehicles' operation state models in performance module use running time, passenger demand, future control decisions, and the current system state to infer how the system will evolve. The running times between two adjacent stations are stochastic variables. The arrival time of vehicle  $i$  at station  $j$  is yielded by adding the running time ( $tr_{i,j}$ ) to the dispatching departure time of vehicle  $i$  at station  $j-1$  ( $tdd_{i,j-1}$ ). To avoid overtaking, the arrival time of vehicle  $i$  at station  $j$ ,  $ta_{i,j}$ , can be achieved using Eq. (9).

$$ta_{i,j} = \max[tdd_{i,j-1} + tr_{i,j}, tdd_{i-1,j}] \quad (9)$$

The departure time of vehicle  $i$  at station  $j$  ( $tdd_{i,j}$ ) is associated with the arrival time, dwelling time and the control action for the vehicle. The inherent departure time  $tid_{i,j}$  is obtained by adding dwelling time  $ts_{i,j}$  to arrival time  $ta_{i,j}$ .

$$tid_{i,j} = ta_{i,j} + ts_{i,j} \quad (10)$$

$$tdd_{i,j} = tid_{i,j} + tg_{i,j} \quad (11)$$

If vehicle  $i$  does not execute a deceleration strategy,  $tg_{i,j} = 0$ , and  $tdd_{i,j} = tid_{i,j}$ . If the decision made for vehicle  $i$  in the data-driven control module is a deceleration strategy,  $tg_{i,j} > 0$ , and then its value is optimized in the optimization module.

The dwelling time of vehicle  $i$  at station  $j$  ( $ts_{i,j}$ ) can be modelled with boarding and alighting occurring in parallel or in series. Boarding and alighting times are linear functions of the number of passengers boarding and alighting at station  $j$ . The number of alighting passengers of vehicle  $i$  at station  $j$  can be obtained by Eq. (12).

$$A_{i,j} = y_{i,j} \times q_j \times L_{i,j-1} + (1 - y_{i,j+1}) \times q_{j+1} \times L_{i,j-1} \quad (12)$$

$$y_{i,j} + y_{i,j+1} \geq 1 \quad (13)$$

where  $y_{i,j}$  is a binary variable.  $y_{i,j} = 1$  indicates that station  $j$  is served by vehicle  $i$ . There are three scenarios included in Eq. (12). The scenario  $y_{i,j} = 0$  and  $y_{i,j+1} = 1$  denotes that vehicle  $i$  skips station  $j$  and serves station  $j + 1$ . That is, no passenger gets off vehicle  $i$  at station  $j$ ,  $A_{i,j} = 0$ . The scenario  $y_{i,j} = 1$  and  $y_{i,j+1} = 0$  shows that vehicle  $i$  serves station  $j$  and skips station  $j + 1$ . That is, passengers with destinations of station  $j$  and station  $j + 1$  will all get off at station  $j$ . The last scenario  $y_{i,j} = 1$  and  $y_{i,j+1} = 1$  indicates that vehicle  $i$  does not provide stop skipping service.

The number of boarding passengers of vehicle  $i$  at station  $j$  is comprised of three parts: the passengers  $\beta_{i-1,j}$  left by vehicle  $i - 1$ , the passengers  $g_{i,j}^1$  who arrive between vehicle  $i - 1$  departing from station  $j$  and vehicle  $i$  arriving at this station, and the passengers  $g_{i,j}^2$  who arrive during the dwelling of vehicle  $i$  at station  $j$ . Hence, when vehicle  $i$  arrives, the number of passengers intending to board this vehicle at station  $j$  is the sum of  $\beta_{i-1,j}$  and  $g_{i,j}^1$ .

$$g_{i,j}^1 = \lambda_j \times (ta_{i,j} - tdd_{i-1,j}) \quad (14)$$

A bus system in which boarding and alighting passengers use different doors are modelled with a parallel process, where dwelling time is the greater of the boarding and alighting times. The occupancy rate of a vehicle (measured by  $L_{i,j-1}/cap$ ) impacts the boarding and alighting time for each passenger. If the occupancy rate is low, the time needed by each passenger to board or alight a vehicle is short. If a vehicle is crowded, passengers will need more time to board and alight. Thus, the estimated dwelling time  $ts'_{i,j}$  is,

$$ts'_{i,j} = y_{i,j} \times c \times \max[\mu \times (g_{i,j}^1 + \beta_{i-1,j}), \eta \times A_{i,j}] \quad (15)$$

$$\begin{cases} c = 1 & \text{if } 0 \leq L_{i,j-1}/cap \leq \theta \\ c > 1 & \text{if } \theta < L_{i,j-1}/cap \leq 1 \end{cases} \quad (16)$$

where  $c$  is a constant that is associated with the occupancy rate of a vehicle.  $\theta$  denotes the threshold of the occupancy rate.  $\mu$  and  $\eta$  are constant parameters for the average boarding and alighting times per passenger respectively. Then, the number of passengers  $g_{i,j}^2$  who arrive during the estimated dwelling time  $ts_{i,j}'$  can be yielded with Eq. (17).

$$g_{i,j}^2 = \lambda_j \times ts_{i,j}' \quad (17)$$

Therefore, based on our control actions and the capacity constraint of vehicles, the number of boarding passengers  $B_{i,j}$  can be calculated.

$$B_{i,j} = \begin{cases} y_{i,j} \times (g_{i,j}^1 + \beta_{i-1,j} + g_{i,j}^2 - w_{i,j}) & \text{if } g_{i,j}^1 + \beta_{i-1,j} + g_{i,j}^2 \leq cap - (L_{i,j-1} - A_{i,j}) \\ y_{i,j} \times (cap - (L_{i,j-1} - A_{i,j}) - w_{i,j}) & \text{otherwise} \end{cases} \quad (18)$$

$w_{i,j}$  is the number of passengers prevented to board vehicle  $i$  at station  $j$ . Delgado et al. (2012) proposed a method to speed up the vehicle's operational speed by limiting the number of passengers boarding a vehicle, and confirmed that boarding limit method can indeed improve the system's performance under various operational conditions. The "Boarding limit" control action in this paper is that boarding is only allowed during the alighting period. Thus,  $w_{i,j}$  can be obtained with Eq. (19).

$$w_{i,j} = \begin{cases} \max[g_{i,j}^1 + \beta_{i-1,j} + g_{i,j}^2 - \eta \times A_{i,j}/\mu, 0] & \text{if } g_{i,j}^1 + \beta_{i-1,j} + g_{i,j}^2 \leq cap - (L_{i,j-1} - A_{i,j}) \\ \max[cap - (L_{i,j-1} - A_{i,j}) - \eta \times A_{i,j}/\mu, 0] & \text{otherwise} \end{cases} \quad (19)$$

If the "Boarding limit" control action is not executed, then  $w_{i,j} = 0$ .

Accordingly, the load of vehicle  $i$  when it departs from station  $j$  can be obtained.

$$L_{i,j} = L_{i,j-1} + B_{i,j} - A_{i,j} \quad (20)$$

Then, with the acquisition of the number of boarding passengers, the final dwell time of vehicle  $i$  at station  $j$  is obtained.

$$ts_{i,j} = y_{i,j} \times c \times \max[\mu \times B_{i,j}, \eta \times A_{i,j}] \quad (21)$$

The number of passengers left by vehicle  $i$  at station  $j$  is determined with Eq. (22).

$$\beta_{i,j} = \max[g_{i,j}^1 + \beta_{i-1,j} + \lambda_j \times ts_{i,j} - B_{i,j}, 0] \quad (22)$$

Therefore, the departure time of vehicle  $i$  from station  $j$  can be yielded according to Eqs. (10) and (11). The headway between vehicle  $i$  and vehicle  $i-1$  at station  $j$  is defined as the time gap between vehicle  $i-1$  leaving station  $j$  and vehicle  $i$  arriving at station  $j$ .

$$h_{i,j} = ta_{i,j} - td_{i-1,j} \quad (23)$$

### 3.4. Formulation of optimization module

When the control category is acceleration strategy or deceleration strategy, this optimization module is used to optimize specific control actions with the variables output by the performance module. Then, we formulate a mathematical programming problem for the optimization module to determine specific control actions assuming that the system will evolve as expected. The variables that need to be optimized include all future control actions consisting of acceleration and deceleration strategies that are considered to have impacts on the current control decision, (i.e., the decision for vehicle  $i$  at station  $j$ ). If the control category output by the RF model is the acceleration strategy,  $tg_{i,j}$  is restricted to 0. Correspondingly, in a deceleration strategy,  $y_{i,j}$  and  $y_{i,j+1}$  are restricted to 1 and  $w_{i,j}$  is 0.

Regardless of whether an acceleration strategy or deceleration strategy is deployed, the impact of control actions on passengers includes positive and negative aspects. In an acceleration strategy, positive aspects include the reduction of waiting time and in-vehicle time for some passengers, and negative aspects include the extra travel time of passengers whose destination is skipped or who are prevented from boarding. In a deceleration strategy, the positive aspect is the reduction of some passengers' waiting time, and the negative aspect is the increase of some passengers' in-vehicle time. Therefore, the objective function of this paper contains both positive and negative aspects, minimizing the sum of them. These components can be written as follows:

(a) Waiting time experienced by passengers as they wait for vehicle  $i$  at station  $j$

$$W_{wait} = \sum_{i \in I_M} \sum_{j \in N} \left( \frac{1}{2} \cdot \lambda_j \cdot h_{i,j}^2 + \beta_{i-1,j} \cdot h_{i,j} \right) \quad (24)$$

(b) In-vehicle time experienced by passengers

$$W_{in-vehicle} = \sum_{i \in I_M} \sum_{j \in N} \{(tr_{i,j} + ts_{i,j}) \cdot L_{i,j-1} + tg_{i,j} \cdot L_{i,j}\} \quad (25)$$

(c) Penalty for passengers who cannot get off at destinations under acceleration strategy

$$W_{Penalty} = (1 - y_{i,j}) \cdot q_j \cdot L_{i,j-1} \cdot (tr_{i,j+1} + h_{e,N-j+1}) + (1 - y_{i,j+1}) \cdot q_{j+1} \cdot L_{i,j-1} \cdot h_{i+1,j} \quad (26)$$

where  $tr_{e,N-j+1}$  and  $h_{e,N-j}$  denote that if station  $j$  is skipped, passengers with the destination of station  $j$  get off at station  $j+1$ , then wait and board vehicle  $e$  at station  $N-j$  to return to their destination. The second part of Eq. (26) indicates that, if station  $j+1$  is skipped, passengers with a destination of station  $j+1$  have to get off at station  $j$  and then take another bus to station  $j+1$ , suffering from an extra delay of a headway. Then, the objective function is obtained.

$$\min W = \theta_1 \cdot W_{wait} + \theta_2 \cdot W_{in-vehicle} + \theta_3 \cdot W_{penalty} \quad (27)$$

where  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  are the weights of different components. In the deceleration strategy,  $\theta_3$  does not work. The greater the ratio of  $\theta_1$  and  $\theta_2$ , the longer vehicles might be held. The holding strategy reduces some passengers' waiting time at the cost of increasing other passengers' in-vehicle time. In the acceleration strategy, it can be observed that a sufficiently high value of  $\theta_3$  prevents the system from the stop skipping strategy. On the other hand, if passengers whose destination is skipped fully accept to travel with other vehicles of the route,  $\theta_3$  is set to 0.

### 3.5. Optimization based hybrid control framework

Eqs. (9)–(27) in the performance module and the optimization module comprise the control problem based only on an optimization method, which is called Optimization based Hybrid Control (OBHC hereafter) framework. Compared with the DDHC framework, the control category of the arriving vehicle under the OBHC framework is not predetermined, but needs to be optimized with its magnitude simultaneously in the optimization module. The values of decision variables  $tg_{i,j}$ ,  $y_{i,j}$ ,  $y_{i,j+1}$ , and  $w_{i,j}$  are not restricted. If  $tg_{i,j} = 0$ ,  $y_{i,j} = y_{i,j+1} = 1$ , and  $w_{i,j} = 0$  are satisfied, then vehicle  $i$  is not controlled at station  $j$ . If  $tg_{i,j} = a$  ( $a > 0$ ), then vehicle  $i$  is decelerated at station  $j$ , and its holding time is  $a$  seconds. If  $tg_{i,j} = 0$ ,  $y_{i,j} + y_{i,j+1} = 1$ , and  $w_{i,j} = 0$ , vehicle  $i$  executes the stop skipping strategy in the acceleration category. If  $tg_{i,j} = 0$ ,  $y_{i,j} = y_{i,j+1} = 1$ , and  $w_{i,j} = b$  ( $b > 0$ ), vehicle  $i$  performs the boarding limit strategy in the acceleration category, and the number of passengers who are restricted from boarding vehicle  $i$  is  $b$ .

## 4. Solution algorithm

For the implementation of the DDHC framework proposed in this paper, random forest package in Matlab is used in the data-driven control module to classify the control category. And two algorithms are established to solve the optimization problems in the optimization module. When local vehicles and control stations are involved (i.e., the control decisions of two vehicles on two control stations, DDHC-TT), an enumeration method is used. When the vehicles and control stations to be included are greatly increased (i.e., the control decisions of all the vehicles in the rolling horizon on all the control stations, DDHC-AA), a genetic algorithm (GA) is used to solve the problem.

### 4.1. Solution algorithm for the DDHC-TT problem

In the DDHC-TT problem, we consider that due to the time-varying nature of traffic conditions, current control decisions are difficult to have an obvious impact on the operational status of vehicles and stations that are far away. By the same token, current decisions are also difficult to be influenced by the far future decisions. Therefore, we only consider the control decisions of the arriving vehicle and its next vehicle at the current control station and the next control station (total 4 decisions). However, only the optimal control action for the arriving vehicle at the current control station is applied. All other control actions suggested by the optimization model are discarded, which is an approach applied to a rolling horizon framework. The calculation method for the DDHC-TT problem is shown in [Algorithm 1](#).

**Algorithm 1.** (The general procedure of the DDHC-TT problem).

---

1 **Input:** the length of the rolling horizon for data-driven control module ( $R$ ) and optimization module ( $M$ ), the parameters in RF model ( $ntree$  and  $mtry$ ), the average passenger arrival rate ( $\lambda_j$ ), value of  $\theta_1$ ,  $\theta_2$  and  $\theta_3$

2 Train RF model

- Set  $num\_data$  = number of initial dataset,  $num\_training$  = number of training set for RF model,  $num\_validation$  = number of validation set
- Normalization of data
- Stochastically select the dataset into  $num\_training$  for training set,  $num\_validation$  for validation set. Surplus dataset is for testing set
- Bootstrap selection for the  $num\_training$  samples
- Calibration with  $num\_validation$  samples
- $RF\_model$  = machine learning
- $prediction\_result$  = classification (voted by all the individual trees)

3 **for** each vehicle  $i$  arriving at control station  $j$  **do**

4   Update vehicles in rolling horizon for each station and the controlled vehicle.

5   Compute the explanatory variables used in the RF model, according to Eq. (1), (2), (5)

6   Determine the control category for bus  $i$  at station  $j$  with the RF model

7   **if**  $D_{ij}$  = acceleration strategy, **then**

8     **Input:** total number of acceleration actions ( $AA$ ), the value of objective function without control ( $W(acc_{best})$ )

9     **while**  $acc_{i,j} \leq AA$  **do** ( $acc_{i,j}$  denotes the sequence number of acceleration actions for bus  $i$  at station  $j$ )

10      **for** each  $acc_{i,j}$  **do**

11       **for** each  $Cact_{i,k}$  **do**

12         (Station  $k$  is the next control station of station  $j$ , and  $Cact_{i,k}$  denotes the control action of bus  $i$  at control station  $k$ ,  $g_{i,k}$  denotes the gap time between dispatching departure time and inherent departure time,  $Cact_{i,k} \in \{no\ intervention, acc_{i,k}, (g_{i,k} = 10s; g_{i,k} \leq 180s; g_{i,k} = g_{i,k} + 10s)\}$ )

13         **for** each  $Cact_{i+1,j}$  **do**

14           **for** each  $Cact_{i+1,k}$  **do**

15             Optimize and record the best objective  $W(acc_{i,j})$  and the solution combination  $\{acc_{i,j}, Cact_{i,k}, Cact_{i+1,j}, Cact_{i+1,k}\}$ .

16             **if**  $W(acc_{best}) > W(acc_{i,j})$ ,  $W(acc_{best}) = W(acc_{i,j})$ , update  $acc_{best}$

17         **Output:**  $acc_{best}$

18     **if**  $D_{ij}$  = deceleration strategy, **then**

19       **Input:** The value of objective function without control ( $W(g_{best})$ )

20       **while**  $g_{i,j} \leq 180s$  **do**

21         **for** ( $g_{i,j} = 10s; g_{i,j} = g_{i,j} + 10s$ ), **do**

22         **for** each  $Cact_{i,k}$  **do**

23           **for** each  $Cact_{i+1,j}$  **do**

24             **for** each  $Cact_{i+1,k}$  **do**

25               Optimize and record the best objective  $W(g_{i,j})$  and the solution combination  $\{g_{i,j}, Cact_{i,k}, Cact_{i+1,j}, Cact_{i+1,k}\}$ .

26               **if**  $W(g_{best}) > W(g_{i,j})$ ,  $W(g_{best}) = W(g_{i,j})$ , update  $g_{best}$

27         **Output:**  $g_{best}$

28     **else**

29       Terminate

30     **if** exceeding the maximum time, **then**

31       Terminate

---

#### 4.2. Solution algorithm for the DDHC-AA problem

When the vehicles and control stations to be included greatly increase, we use a heuristic algorithm to solve the problem. **Algorithm 2** introduces the solution process using the genetic algorithm (GA). The GA is used to optimize all decision variables, related to future control actions consisting of acceleration and deceleration strategies for all vehicles included in the rolling horizon, on all future control stations until they return to their current control station in the next cycle. The individual in the GA represents a possible control action sequence, where each control action at a control station is a gene, and the individual length corresponds to the number of vehicles in the rolling horizon and the number of control stations in the impact set.

**Algorithm 2.** (The general procedure of the DDHC-AA problem).

---

```

1   Input: the length of the rolling horizon for data-driven control module ( $R$ ) and optimization
      module ( $M$ ), the parameters in RF model ( $ntree$  and  $mtry$ ), the average passenger arrival rate ( $\lambda_i$ ),
      value of  $\theta_1$ ,  $\theta_2$  and  $\theta_3$ , the crossover rate  $p_c$  and the mutation rate  $p_m$ , the number of generation
      (Gen).
2   Train RF model
    Set  $num\_data$  = number of initial dataset,  $num\_training$  = number of training set for RF model,
     $num\_validation$  = number of validation set
    Normalization of data
    Stochastically select the dataset into  $num\_training$  for training set,  $num\_validation$  for
    validation set. Surplus dataset is for testing set
    Bootstrap selection for the  $num\_training$  samples
    Calibration with  $num\_validation$  samples
    RF_model = machine learning
    prediction_result = classification (voted by all the individual trees)
3   for each vehicle  $i$  arriving at control station  $j$  do
4     Update vehicles in rolling horizon for each station and the controlled vehicle.
5     Compute the explanatory variables used in the RF model, according to Eq. (1), (2), (5)
6     Determine the control category for bus  $i$  at station  $j$  with RF model
7     if  $D_{i,j}$  = acceleration or deceleration strategy, then
8       Go to Genetic algorithm
9       Step 1. Set  $gen=0$ , and create the initial population:  $Pop_0 = \{U_z^0 | z=1,2,\dots,Z\}$  ,  $Z$  is the
          generation size,  $U_z^0$  is the  $z$ th set of control actions in generation
          0,  $U = \{u_{r,l} | r=1,2,\dots,M; l=1,2,\dots,L\}$  .  $r$  is the identifier of vehicles in the rolling
          horizon,  $l$  is the identifier of control stations. The possible solutions of  $u_{1,1}$  are
          acceleration or deceleration actions according to the output from RF model, and others
          are acceleration and deceleration actions.
10      Step 2. Calculate fitness value of  $Pop_0$  according to the objective function based on  $u_{r,l}$  .
11      Step 3. Implement selection, crossover and mutation with elitism, then get child
          generation  $Pop'_0$ 
12      Step 4.  $gen = gen + 1$ , go to step 2, until Gen.
13     else
14       Terminate
14   if exceeding the maximum time, then
15     Terminate

```

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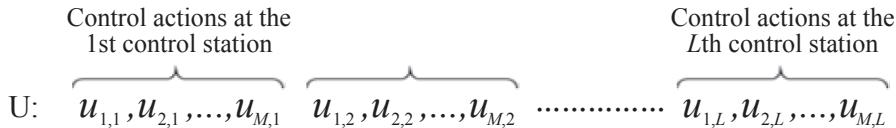
**Fig. 5** is used to describe the chromosome and gene of the GA. The chromosome consists of  $L$  parts, and each part represents the control actions at a control station. The gene in every part represents the control actions for all the vehicles in the rolling horizon at the control station.

The RF model in the data-driven control module is first used to determine whether to control the arriving vehicle and its control category. If the intervention is needed, then the genetic algorithm is run. Thus, some possible control actions for the first vehicle at the first control station are discarded by the RF model. As shown in **Fig. 5**, we take the control action of the current vehicle at its upcoming control station as  $u_{1,1}$ . Under the DDHC framework, the control category of  $u_{1,1}$  is deterministic, i.e.

$$u_{1,1} \in \left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 10s \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 20s \\ 1 \\ 1 \\ 0 \end{bmatrix}, \dots, \begin{bmatrix} 180s \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\} \text{ or } \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

where the first row represents the holding action, the second one represents skipping control station, the third one represents skipping next station and the last one represents boarding limit.

The possible control actions of other genes in the chromosome (except  $u_{1,1}$ ), are shown as follow.



**Fig. 5.** The chromosome of the GA.

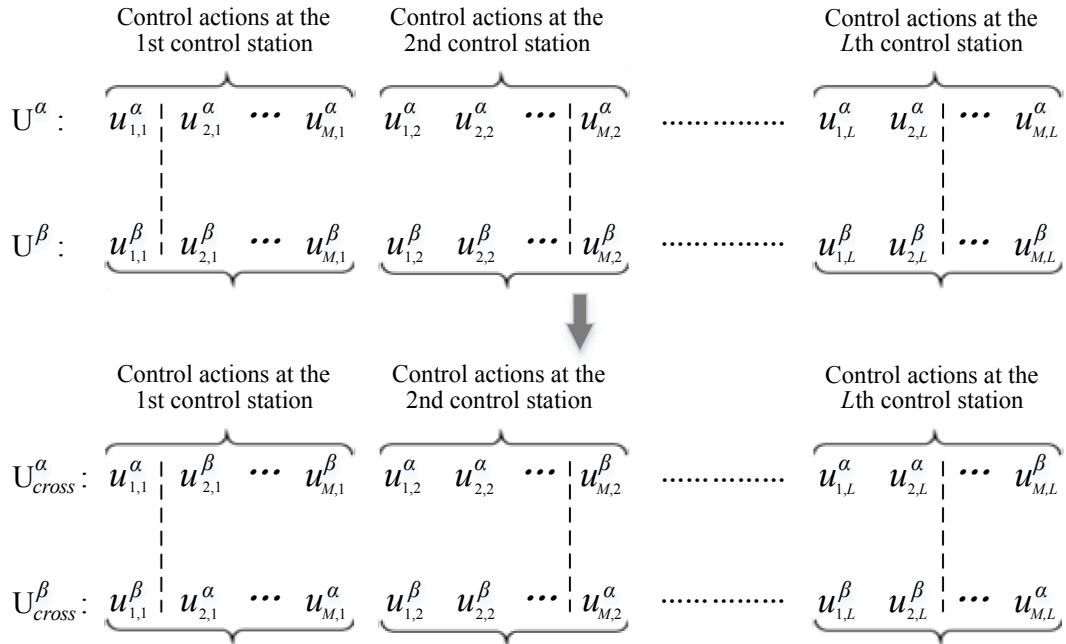


Fig. 6. Example of the chromosome and crossover operation.

$$u_{m,l} \in \left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 10s \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 20s \\ 1 \\ 1 \\ 0 \end{bmatrix}, \dots, \begin{bmatrix} 180s \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\} \quad (m + l \neq 2)$$

In order to reduce the calculation time, the crossover/mutation operations happened at one point in different control stations, the crossover/mutation point for each control station may be different, but the crossover/mutation rates are the same. Fig. 6 shows the crossover operation.

## 5. Numerical test

### 5.1. Background and data collection

#### 5.1.1. Background of the test route

The proposed DDHC framework is tested with the data of the route BRT1 in Urumqi city, China. The route runs north from Machinery Plant, south to Railway Station, with total 21 stations and 16.5 km per direction (Fig. 7). The scheduled headway is 3 min and the total number of operating vehicles is 89. The route runs through most of the heartlands of Urumqi and the daily passenger demand is about 200,000. The corridor of route BRT1 is not always specialized for BRT system. Some sections and all intersections are collinear with other lines and private vehicles. Therefore, the service of BRT1 will be affected by traffic congestion or incidents and cannot guarantee its regularity, especially during peak hours.

For the availability of data, we use the current control stations selected by Urumqi operators. From Machinery Factory to Railway station is direction 1 and the opposite direction is direction 2. The red stations shown in Fig. 7 are the control stations of direction 1, and the control stations of direction 2 are station 22, 26, 29, 34. All these stations have the space for vehicles to execute holding strategy. Along the driving direction of vehicles, the white stations downstream of each red station are the impact set of control stations used in the data-driven control module. Total 42 stations of the route are the impact set of control stations adopted in the optimization module. The control categories and specific control actions are made for each vehicle arriving at each red station based on the overall operational state of a control station and its impact set.

Here we need to explain why we choose to control dispatching time, service stations and number of boarding passengers at stations to achieve acceleration or deceleration control. Controlling at stations and controlling throughout the route both are commonly used real-time control methods. Most cities in China, such as Urumqi city, choose to control vehicles at stations, since this method is easier to implement in practice. Considering both methods can achieve similar effects, we used the method of controlling at stations.

#### 5.1.2. Data collection and processing

The data collected mainly includes two parts. The first one is the historical data of vehicle operational state and the corresponding

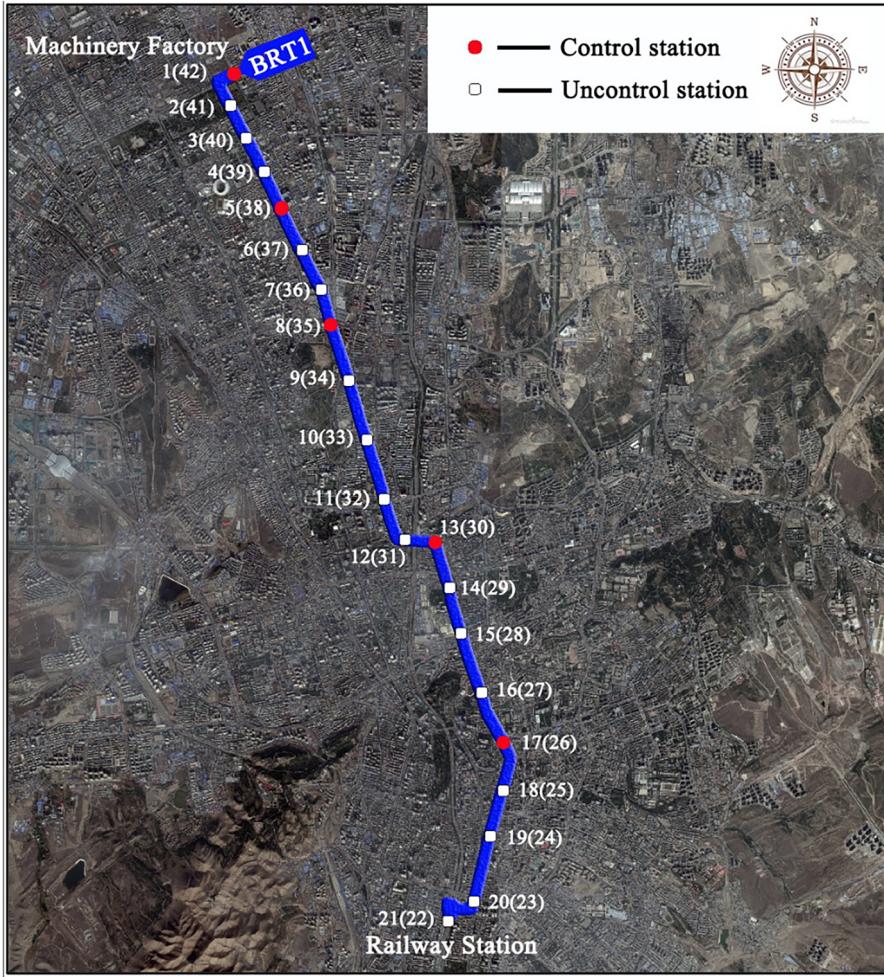


Fig. 7. The trajectory of BRT1 in Urumqi.

control decisions issued by dispatchers, which is mainly used to train the RF model. The second one is the data of features about vehicle operational state used to test our DDHC framework.

**5.1.2.1. Data for the data-driven control module.** To collect the first kind of data, this paper combined the methods of data statistics and field survey. Investigators observed and recorded dispatchers' control strategy and the inherent arrival and departure time for each vehicle at the bus command center. Based on the information (e.g., passenger number gathered in stations and uniformity of the vehicle distribution after control) presented by the video monitoring system, the effect of the control strategy issued by dispatchers was estimated preliminarily. Then, the vehicle operational data stored in the Global Position System (GPS) was exported. These data was processed with a SQL database to extract the data needed for this study, such as headway, running time, arrival time. Afterwards, these processed data and field observation records were combined to screen out effective control strategies. The intervals between vehicles without control recorded manually and the intervals after control provided by the GPS data were compared. The control decisions which make the vehicle distribution more uniform are retained as part of the samples. Two parts are included. On the one hand, after a dispatcher issued an acceleration or deceleration instruction to a vehicle, the transit performance of BRT1 was indeed improved. On the other hand, the dispatchers did not control a vehicle. But the transit performance of BRT1 did not decrease, even in the subsequent operation, the transit performance improved due to the self-recovery capability of the system. The screened out data is processed according to the form required by the RF model.

In addition, we calculated the average cruising speed of vehicles between each two control stations during 10 working days for every 10 min. For example, if arrange a control category for a vehicle at 8:45am, we apply the average cruising speed of the time period 8:50–9:00 in the RF model, to reflect the dispatchers' expectations of future road congestion. It is the next period of the time period 8:45 belonging to (8:40–8:50). Hence, each set of these explanatory variables corresponds to a control strategy, and the learning set for the RF model is obtained. It needs to be explained that the learning set is not distinguished for each control station. That is, we consider all control stations are homogeneous and can be controlled according to the same rules.

Since the GPS data cannot be used directly, we conducted a five-day (8:40–11:40am from March 6 to March 10, 2017) field

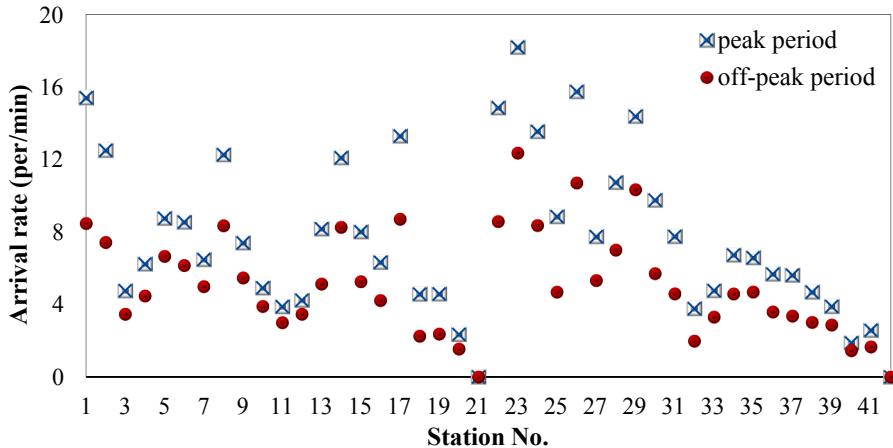


Fig. 8. Passenger arrival rates during peak and off-peak period.

survey. After careful screening, 386 valid samples were acquired. Then, we used these data to train multiple classification models, including support vector machines, artificial neural networks, and random forests. The experimental results showed that the RF model can be used to obtain accurate enough classification results with the 386 valid samples. Therefore, this paper used the RF model as the classification model in the data-driven control module. 8: 40–10: 10am is the early peak period of Urumqi, and 10: 10–11: 40am is the off-peak period. These samples are randomly divided into 3 subsets: 70% for training, 20% for validation and 10% for testing.

**5.1.2.2. Data for the performance and optimization module.** Then, the collection and processing of the second kind of data are illustrated, mainly including running time and passenger demand. By the statistics of vehicles' running times between each two stations, we find that they basically conform to the log-normal distribution. Since the distance and traffic conditions between each two stations are different, the mean and coefficient of variation of the running times are distinguished for each section. Stochastic running times between each two stations during peak and off-peak periods used in this paper are shown in Appendix A. The use of log-normal distributions to model running times is common practice (Zhao et al., 2003, Cats et al., 2011, Delgado et al., 2012, Sánchez-Martínez et al., 2016). These changeable running times on each road segment can reflect the changes in traffic conditions within a given period (peak or off-peak). Different running times will lead to different system evolutions. Then, the control decisions made for vehicles based on the different derived future system evolution could be different.

The other data needed to explain is passenger demand. All stations of route BRT1 are equipped with safety gates with counting functions which can automatically record the number of inbound passengers. These data was exported and used to calculate the arrival rates of each station during peak and off-peak periods, respectively. According to the survey, the passenger demand of route BRT1 is stable which is higher during the peak period and lower during the off-peak period, but always remains at a high level. This paper assumes that passengers arrive evenly at each station but the arrival rates during peak hours and off-peak hours are different. Fig. 8 shows the statistical passenger arrival rates at each station during peak and off-peak hours respectively.

The boarding and alighting parameters are taken as  $\mu = 3$  s,  $\eta = 1.5$  s, and if the occupancy rate is higher than 0.8 ( $\theta = 0.8$ ),  $c = 2$ . The real operational state at 8: 40am from Match 13 to 17, 2017 is used as the initial state for 5 testing days. Thus, based on the statistical passenger arrival rates, running time, and the equations of system evolution given in the performance module, all the data of system features can be obtained. The length of the rolling horizon is taken as  $R = M = 5$ , referring to the analysis by Eberlein et al. (2001) and Yu et al. (2012).

## 5.2. Tested control strategies

Five different control strategies are tested and compared in this paper. The first three (no control, HCBH and OBHC) are used for comparison purposes. The last two are DDHC-TT strategy and DDHC-AA strategy. The two algorithms introduced in Section 4 are used to calculate the effects of these two control strategies respectively. Comparing these two last strategies will allow us to understand the impact of future decisions with different scopes on the optimization results.

In summary, we compare the following five control strategies:

*No control.* The bus system evolves spontaneously, where vehicles are dispatched from the terminal at a pre-designed headway, without taking any control actions during the operation.

*Hybrid Control Based on Headway* (HCBH hereafter). This control strategy cites the study by Moreira-Matias et al. (2016), which used a headway-based approach to determine whether a holding or stop skipping strategy should be used. Their control principle is shown in Fig. 9. When the headway between bus  $i$  and bus  $i - 1$  is regular, and the headway between bus  $i$  and bus  $i + 1$  is short, bus  $i + 1$  is holding. If the headway between bus  $i - 1$  and bus  $i$  is very large, and the headway between bus  $i$  and bus  $i + 1$  is short, bus  $i$  executes the stop skipping strategy.

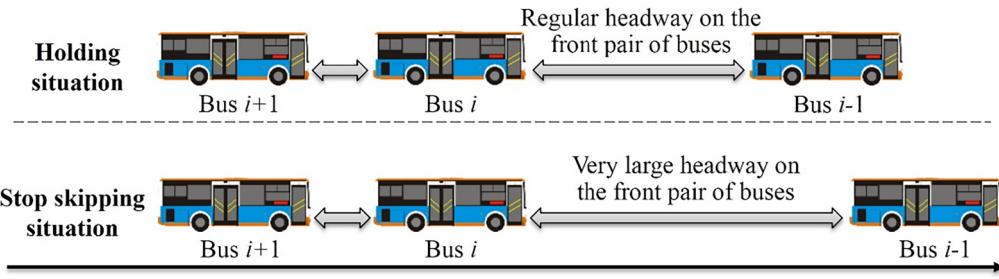


Fig. 9. The description of the control strategy HCBH.

OBHC. This control strategy has been described in Section 3.5 and is based solely on the optimization method. It simultaneously determines the control category and magnitude for all the vehicles in the rolling horizon at all the control stations using optimization models and the heuristic algorithm. The OBHC method is designed based on the research by Cortés et al. (2010).

DDHC-TT, where we only consider the control decisions of the arriving vehicle and its next vehicle at the current control station and the next control station (total 4 decisions).

DDHC-AA, in which all the decision variables, related to future control actions consisting of acceleration and deceleration strategies for all the vehicles included in the rolling horizon, on all future control stations until they return to their current control station in the next cycle are optimized.

It should be noted that the application of the genetic algorithm in the OBHC strategy is different from that in the DDHC-AA strategy. Under the OBHC strategy, whenever a vehicle departs from the previous station of the control station, the genetic algorithm is executed once. The control category  $u_{l,1}$  is not predetermined. That is

$$u_{m,l} \in \left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 10s \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 20s \\ 1 \\ 1 \\ 0 \end{bmatrix}, \dots, \begin{bmatrix} 180s \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\} \quad (m = 1, 2, \dots, M; L = 1, 2, \dots, L)$$

### 5.3. Results analysis

The two parameters of the RF model,  $n\text{tree}$  and  $m\text{try}$ , are obtained through sensitivity analysis and set to be 1000 and 3 respectively. The sensitivity analysis will be discussed later. The weights of the three components of the objective function are set as  $\theta_1 = 1$ ,  $\theta_2 = 0.5$  and  $\theta_3 = 2$ , which refers to the work by Delgado et al. (2012). For the heuristic algorithm described in Section 4, the crossover rate and mutation rate are set to 0.8 and 0.2 respectively. The number of generations is 600, and the population size is 300.

#### 5.3.1. The reduction of passenger travel time

The reduction of passenger travel time yielded by the five control strategies is presented in this subsection. The travel time experienced by passengers generally includes waiting time, dwelling time, running time, and extra travel time caused by control decisions. In these components, vehicle running time cannot be reduced by control strategies. Moreover, even if the headways between vehicles are perfectly regular, passengers will still have to wait on average half the headway. In Fig. 10, this paper reports only the passenger travel time that can be reduced due to control strategies, to highlight the difference between each control strategy.

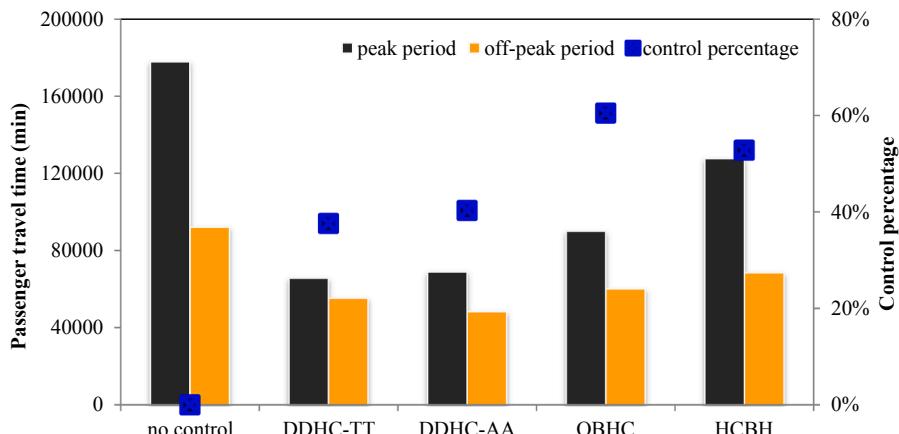


Fig. 10. Passenger travel time can be reduced under the five control strategies.

The control percentage of each control strategy is also shown in Fig. 10. Control percentage denotes the percentage of the times that vehicles are controlled in the total times that vehicles arrive at control stations. It reflects the frequency of control. The shown control percentages in Fig. 10 are the percentage of the sum of the control times during peak and off-peak periods in the total arrival times.

Obviously, the performance of the DDHC framework proposed in this study is the best among the five control strategies, regardless of the perspective of reducing passenger travel time or control percentage. The passenger travel time can be reduced under the DDHC-TT strategy is 55.20% less than the no control, 19.46% less than the OBHC strategy, and 38.32% less than the HCBH strategy. Under the DDHC-AA strategy, the passenger travel time can be reduced is 56.61% less than the no control, 22.01% less than the OBHC strategy, and 40.27% less than the HCBH strategy.

The performance of the DDHC-TT and DDHC-AA strategies outperformed the OBHC strategy. The DDHC framework judges whether to control vehicles based on the combination of dispatcher's rich practical experience and mathematical optimization method, rather than using optimization models only. This is why the control percentage with DDHC strategy is low. In the expected conditions (running time and dwelling time are assumed to be constant), the solution of OBHC strategy should not be worse than the solution of DDHC-AA strategy. However, the traffic conditions are highly random in practice. Thus, when the control decisions made based on the expected conditions are executed, overreaction (e.g. a greater holding time than needed) often occurs (Delgado et al., 2012). For DDHC framework, the RF method can mimic the dispatchers' decision behavior and take into account some random factors of traffic conditions in the real-time control problem. The RF model can estimate future conditions based on historical data and experience, so that overreaction can be reduced.

Compared to HCBH strategy, DDHC strategy not only considers the regularity of headways between three sequential vehicles, but also considers the service reliability of the entire impact set, the change of headways and loads etc., to make comprehensive judgments. For example, full load vehicles will not be allowed to provide a deceleration strategy. Thus, the control action outputted by DDHC strategy is more accurate than that by HCBH strategy. This is mainly due to similar reasons why the performance of OBHC strategy is better than that of HCBH strategy. The optimized control actions by the GA are more accurate than the control actions judging from headway alone, and are more effective in reducing total passenger travel time. Of course, the time needed by HCBH strategy to obtain a control action is quick, within 1 s.

In our problem, the effect of the DDHC-AA strategy considering both the peak and off-peak periods is 3.26% better than that of DDHC-TT strategy. This improvement is mainly contributed by the control during the off-peak period. During the peak period, the DDHC-TT strategy performs 1.42% better than the DDHC-AA strategy. That is, DDHC-TT strategy performed better during peak periods and DDHC-AA strategy performed better during off-peak periods. During peak periods, in many cases, the future control decisions (i.e. the control decisions for the subsequent vehicles at the subsequent control stations) obtained based on the deduced system operational state were inconsistent with the decisions that these vehicles actually executed in the future. This is because, in the random traffic environment, the system operational state inferred based on the expected running time was likely to be different from the real system operational state (especially for the operational state of the vehicles and stations far away). During the peak periods, the consideration of multiple decisions which involve control decisions far away is difficult to play a role. Therefore, DDHC-AA strategy did not achieve better results. Instead, DDHC-TT strategy, which focused on short-term future decisions, performed better. During the off-peak periods, traffic conditions are relatively smooth, and the system operational state is easier to close to the expected state. Therefore, synergies of multiple control decisions could take effect, making the DDHC-AA strategy work better in off-peak periods.

From the results of this paper, the DDHC-AA strategy performs better overall, while the DDHC-TT strategy performs better during peak periods. However, for public transportation systems, the effect of real-time control strategies is affected by the operational environment (e.g. traffic conditions, passenger demand). This paper only applied the data of route BRT1 in Urumqi for testing. The conclusion that the DDHC-AA strategy is better than the DDHC-TT strategy could only provide a reference for bus operational control in other cities.

The orange bar in Fig. 10 shows the passenger travel time can be reduced during off-peak hours and the black bar shows the travel time can be reduced during peak hours. Control actions have a greater effect on reducing passenger travel time during peak hours than off-peak hours. Compared with the no control strategy, the decrease of travel time can be reduced under the other four control strategies during peak hours is 63.09%, 61.27%, 49.38% and 28.20% respectively. Relatively, the effect of reducing passengers' travel time during off-peak hours is small and the difference between the four control strategies is slight, at 39.98%, 47.63%, 34.73% and 25.76% respectively.

### 5.3.2. Control categories, control actions and accuracy

In this subsection, the control categories and the control actions of the acceleration strategy outputted by the DDHC-TT strategy and OBHC strategy at each control station are analyzed. During the peak hours of 5 workdays, a total of 7.5 h, both control strategies are used 345 times at each control station to classify control categories and optimize control actions for vehicles. During off-peak hours (also 7.5 h), the strategies are used 375 times. Table 2a summarizes the control categories and accuracy rate at each control station under the DDHC-TT strategy, and Table 2b shows the control categories under OBHC strategy. The utilization rate of the three control categories is different at different control stations, because of the discriminating passenger demand and running time volatility.

Tables 2a and 2b show that the number of vehicles that perform acceleration or deceleration strategies during peak hours is higher than that during off-peak hours. Under the DDHC-TT strategy, the control percentage during peak periods is approximately 50%, and the percentage is approximately 30% during off-peak periods. Under the OBHC strategy, the control percentages during peak and off-peak periods are approximately 80% and 45%, respectively. Compared with off-peak hours, the distribution of headway

**Table 2a**

The implementation of control categories and actions and accuracy rate at each control station under DDHC-TT strategy.

Control station No.	1		5		8	
Period	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak
Deceleration Strategy (%)	36.81	20.27	35.94	20.58	37.10	17.07
Acceleration Strategy (%)	13.04	9.60	17.10	8.00	14.78	8.27
Skip current station (%)	4.63	2.93	4.63	1.86	2.89	1.07
Skip next station (%)	8.41	6.67	4.64	1.87	6.67	3.73
Boarding limit (%)	0	0	7.83	4.27	5.22	3.47
No control (%)	50.14	70.13	46.96	71.42	48.12	74.67
Accuracy Rate (%)	95.07	93.87	92.75	91.47	93.04	92.80
Control station No.	13		17		22	
Period	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak
Deceleration Strategy (%)	28.12	13.33	40.29	22.93	36.81	17.33
Acceleration Strategy (%)	20.29	10.40	6.96	4.80	15.65	9.60
Skip current station (%)	5.51	2.93	1.45	1.06	7.53	4.80
Skip next station (%)	4.93	2.67	4.35	2.67	8.12	4.80
Boarding limit (%)	9.85	4.80	1.16	1.07	0	0
No control (%)	51.59	76.27	52.75	72.27	47.54	73.07
Accuracy Rate (%)	91.59	95.47	93.33	94.13	95.36	91.73
Control station No.	26		29		34	
Period	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak
Deceleration Strategy (%)	38.84	20.00	36.23	15.73	32.75	14.93
Acceleration Strategy (%)	8.70	5.07	13.33	8.00	13.33	8.27
Skip current station (%)	1.74	0.53	2.89	0.80	2.93	1.33
Skip next station (%)	4.06	2.67	6.38	4.00	3.73	2.67
Boarding limit (%)	2.90	1.87	4.06	3.20	6.67	4.27
No control (%)	52.46	74.93	50.43	76.27	53.91	76.80
Accuracy Rate (%)	94.20	93.60	93.62	92.53	92.53	93.60

**Table 2b**

The implementation of control categories and actions at each control station under OBHC strategy.

Control station No.	1		5		8	
Period	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak
Deceleration Strategy (%)	61.16	32.53	54.20	29.87	57.10	25.06
Acceleration Strategy (%)	19.71	13.34	23.48	12.80	21.45	13.87
Skip current station (%)	6.38	4.27	5.51	3.74	3.77	2.67
Skip next station (%)	13.33	9.07	5.80	2.93	9.28	6.13
Boarding limit (%)	0	0	12.17	6.13	8.40	5.07
No control (%)	19.13	54.13	22.32	57.33	21.45	61.07
Control station No.	13		17		22	
Period	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak
Deceleration Strategy (%)	49.86	22.13	65.51	32.26	60.87	28.27
Acceleration Strategy (%)	30.43	17.07	16.52	12.27	21.74	15.20
Skip current station (%)	8.70	5.07	3.18	2.67	11.02	7.73
Skip next station (%)	8.12	4.27	9.57	6.40	10.72	7.47
Boarding limit (%)	13.61	7.73	3.77	3.20	0	0
No control (%)	19.71	60.80	17.97	55.47	17.39	56.53
Control station No.	26		29		34	
Period	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak
Deceleration Strategy (%)	62.03	31.20	57.97	29.87	58.84	28.53
Acceleration Strategy (%)	19.13	12.80	21.45	13.07	18.26	12.80
Skip current station (%)	3.77	2.40	4.35	1.87	4.06	2.40
Skip next station (%)	9.85	7.20	10.43	6.67	5.50	3.73
Boarding limit (%)	5.51	3.20	6.67	4.53	8.70	6.67
No control (%)	18.84	56.00	20.58	57.06	22.90	58.67

and load among vehicles are more uneven and the service reliability is lower during peak hours. Furthermore, it is difficult to restore the service level through the self-regulation of the system when the regularity of service is disrupted during peak hours. Therefore, during peak periods, it is more necessary to improve the system service level through the adjustment of the control framework.

At all control stations and under both the DDHC-TT and OBHC strategies, the utilization rate of the deceleration strategy is higher than that of the acceleration strategy. This is because the deceleration strategy is simple and effective with few negative impacts. For example, if two vehicles are bunching, dispatchers tend to adopt a deceleration strategy for the latter vehicle, instead of letting the previous vehicle accelerate. The passenger demand at these 9 control stations can be divided into three types: first, passenger demand of the control station is higher than that of its next station (station 1, 8, 17, 26, 29); second, passenger demand of the control station is less than that of its next station (station 13, 22); and third, demand of the control station and its next station is similar (station 5, 34). For the first type, it would be more inclined to choose deceleration strategy, and if acceleration strategies are used, it would tend to adopt the control action “Skip next station”. Acceleration strategies are preferred for the second type stations and the control action “Skip current station” is desirable. The preference of the third type of station is not obvious.

Furthermore, the control action “Skip next station” in the acceleration strategy is more popular than the control action “Skip current station”. For example, under DDHC-TT strategy, at control station 8 (the first type station), the percentage of “Skip next station” is twice that of “Skip current station”. However, at control station 13 (the second type station), the percentage of “Skip current station” is only slightly higher than that of “Skip next station”. Though passenger demand of station 14 (the next station to station 13) is higher and station 13 should be skipped more times intuitively. This is because if the next station is skipped, the passengers with the destination of the next station can get off at the current control station. The additional travel time required is only waiting time. In contrast, if the current station is skipped, the passengers with the destination of the current station have to get off at the next station, the extra travel time is waiting time and twice the running time between the current station and its next station.

The application percentage of the control action “Boarding limit” is popular with all three types of stations, although this control action is less effective at reducing passenger travel time than the control actions “Skip current station” and “Skip next station”. The negative effects of this control action are minimal, especially when the headway between the controlled vehicle and its subsequent vehicle is short, this negative aspect even can be neglected. The application percentage of the “Boarding limit” is 0 at stations 1 and 22, because these two stations are origin stations and vehicles are all empty.

The ‘accuracy’ in Table 2a means that when the classifications output from the data-driven control module are acceleration or deceleration strategies, the optimized control actions output from the optimization module are indeed the corresponding control actions, instead of no intervention. The accuracy rate refers to dividing (the total number of classifications - the times of inaccurate acceleration or deceleration classifications) by the total number of classifications. The percentage of “no control” in Table 2a includes the number of times the RF model suggests no intervention and the cases where the optimization method suggests no intervention. The accuracy rate of the application of the RF model to classify the control category is over 90%. However, for the OBHC strategy, the optimization model is the only decision method adopted. Therefore, there is no way to calculate an indicator similar to the classification accuracy of the RF model for OBHC strategy. Although there are some erroneous classifications of the RF model, it can be seen from the analysis of the total passenger travel time in Section 5.2.1 that our control framework is effective overall.

### 5.3.3. Sensitivity analysis for parameters in the RF model

There are two parameters in the RF model: the number of trees in the forest (*ntree*) and the number of explanatory variables chosen randomly to generate each tree (*mtry*). The calibration of these two parameters is important to the performance of the RF model. To calibrate the parameters, an examination is conducted based on the valid samples. Fig. 11 shows the accuracy rate under different parameter values.

As shown in Fig. 11, regardless of the value *ntree*, the accuracy rate is the highest when *mtry* = 3. As *mtry* increases, the strength of

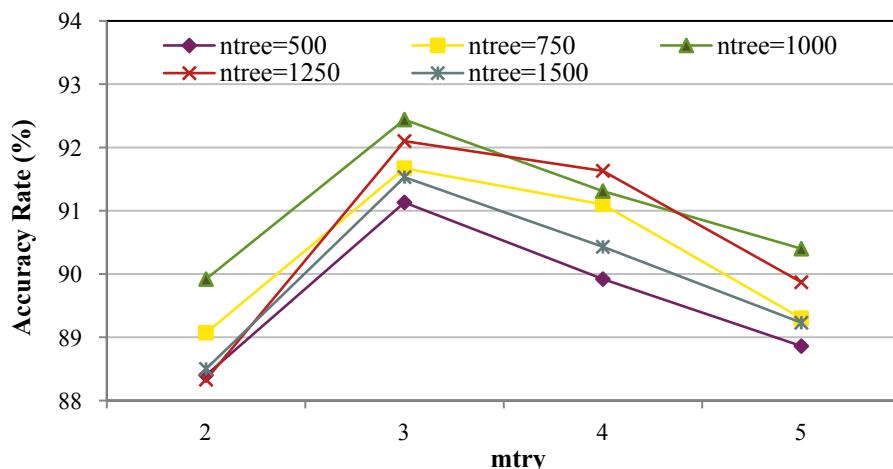


Fig. 11. Accuracy rate under different parameter values.

**Table 3**  
6 scenarios with different explanatory variables.

Scenario	Description	Input variables	Accuracy rate (%)
Sce0	Basic scenario, including all the 10 input variables. Time-of-day, headway, arrival time deviation, service reliability, and load	all	92.44
Sce1	Arrival time deviation is not included.	$T, h_{i+1,k}, h_{i,j-1}, \Phi_i^{\text{mean}}, \psi_i^{\text{mean}}, L_{i-1,j-1}, L_{i,j-1}, L_{i+1,k}, v_{l,j}$	91.12
Sce2	Headway of vehicles is not included.	$T, \Delta t_{i,j-1}, \Phi_i^{\text{mean}}, \psi_i^{\text{mean}}, L_{i-1,j-1}, L_{i,j-1}, L_{i+1,k}, v_{l,j}$	82.19
Sce3	Service reliability of the impact set is not included	$T, h_{i+1,k}, h_{i,j-1}, \Delta t_{i,j-1}, L_{i-1,j-1}, L_{i,j-1}, L_{i+1,k}, v_{l,j}$	85.26
Sce4	Load is not included.	$T, h_{i+1,k}, h_{i,j-1}, \Delta t_{i,j-1}, \Phi_i^{\text{mean}}, \psi_i^{\text{mean}}, v_{l,j}$	86.67
Sce5	Cruising speed is not included	$T, h_{i+1,k}, h_{i,j-1}, \Delta t_{i,j-1}, \Phi_i^{\text{mean}}, \psi_i^{\text{mean}}, L_{i-1,j-1}, L_{i,j-1}, L_{i+1,k}$	90.56

each individual tree increases but the correlation among trees also increases. When  $mtry = 3$ , the correlation among trees in our RF model is low while the strength of each tree is maintained, thus the highest accuracy rate appears. As  $mtry$  continues to increase ( $mtry = 4$  or 5), the correlation increases and the accuracy decreases. The impact of the value of  $ntree$  on classification accuracy has no obvious regularity. Overall, the RF model has the highest accuracy rate when  $ntree = 1000$  and  $mtry = 3$  (approximately one third of the total 10 input variables). Therefore, the parameter set  $\{ntree = 1000, mtry = 3\}$  is adopted for the classification of the control category in this study.

#### 5.3.4. Effectiveness of explanatory variables

The data-driven control module with the RF model is the core of the DDHC framework. As long as the data-driven approach is used to accurately estimate whether the upcoming vehicle needs to be controlled and its control category, the specific control actions in the optimization module can be any feasible control strategy. This paper selects controlling dispatching time, service stations and number of boarding passengers. Therefore, the use of effective explanatory variables to improve the accuracy of the RF model in the data-driven control module is significant. To verify the validity of the selected explanatory variables, this paper set five contrast scenarios, and the scenario settings and their corresponding accuracy rates are shown in Table 3.

The accuracy rate of the classification with the RF model under the six scenarios in Table 3 is calculated with the parameters:  $ntree = 1000$ ,  $mtry = 1/3$  of the total number of explanatory variables. The accuracy rate of the basic scenario Sce0 is the highest. The accuracy rate of Sce1 is slightly lower than that of Sce0 and higher than that of other scenarios, which indicates that the input variable arrival time deviation has little effect on the accuracy of the classification results. This is because the major purpose of control is to improve the service level of the public transport system when the headway between vehicles is irregular and the service reliability is low. That is, the priority of headway-based and service reliability based control is higher than arrival time deviation based control. However, we cannot ignore the effect of this explanatory variable. The accuracy rate of Sce2, Sce3 and Sce4 is much lower than that of Sce0, which illustrates these explanatory variables – the headway between the vehicle under control and its preceding and subsequent vehicles, service reliability and load - have significant impacts on the classification accuracy of the RF model. Although a control category is judged by a combination of various factors, it is undeniable that the headway between vehicles is the most intuitive factor in the judgement of control decisions. Service reliability, which is not included in Sce3, combines the operation state of all the stations in the impact set and passenger demand at each station. It is the only indicator that describes the overall operating state of the system and has an important influence on the judgment of control strategies. Fully loaded vehicles may be classified to provide deceleration service in Sce4, which is invalid to improve transit performance and will induce negative utility (extra passenger in-vehicle time). Therefore, the accuracy of Sce4 is low. Sce5 indicates that the historical cruising speed truly affects the classification accuracy. If the cruising speed is slow, then there may be congestion along the road segment ahead within the next 10 min, and the vehicle needs to be controlled at the current moment to reduce the possibility of deterioration of the service level. In contrast, if the road ahead is unblocked, then the system can be expected to self-repair, and no interference with the operation of the line is required. In summary, the input variables selected in this paper are reasonable and effective, and can accurately estimate the control category for most upcoming vehicles at control stations.

#### 5.3.5. Computation time

The proposed DDHC framework and algorithms were run on a computer with Intel Core i7-3930K processor running at 3.20 GHz. For the problem DDHC-TT, the average time to obtain a control decision with Algorithm 1 is 4.84 s, in which the data-driven control module needs 0.67 s and the optimization method needs 4.17 s. The 90th percentile computation time for the framework was 5.03 s, and the maximum was 5.26 s. For the problem DDHC-AA, the average time to obtain a control decision with Algorithm 2 is 5.44 s, in which the data-driven control module needs 0.67 s and the optimization module needs 4.77 s. The 90th percentile computation time for the framework was 5.72 s, and the maximum was 5.91 s. For the OBHC strategy, the average time to obtain a control decision is 5.49 s. The 90th percentile computation time for the strategy OBHC is 5.76 s, and the maximum was 5.98 s. When a vehicle departs from the previous station of a control station, it begins to optimize its control action at the control station. In general, the running time of a vehicle between adjacent stations is more than 1 min. Therefore, it is feasible to apply the DDHC framework and the proposed algorithm in real-time applications.

## 6. Conclusions

Our study has been motivated by the practical requirements of dynamic control, where it is difficult to determine “control timing” because of the time-varying feature of the traffic environment. This paper proposed a data-driven hybrid control framework to study when and how to dispatch vehicles. The DDHC framework was tested with the data of a real transit route in Urumqi, China and its performance was compared with two other hybrid control methods. The performance measure adopted to evaluate the DDHC framework was total passenger travel time, which reflects passenger waiting time, in-vehicle time and penalty of extra travel time induced by acceleration strategy. The findings are summarized as follows.

- i. The DDHC framework proposed in this paper is a competitive method for real-time control problems. The proposed method consists of two phases. In the first phase, the RF model is used to judge the control timing and control categories. In the second phase, the optimization model is applied to determine the magnitude of each control action. The consideration of the human experience and historical data can estimate the future evolution trend of bus systems. Thus, the DDHC framework is suitable for real-time control problems with randomness. We also tested the impact of subsequent control decisions on control effects, respectively during peak and off-peak periods. The results showed that, under the operational environment like Urumqi where the traffic conditions changes obviously during peak periods, the inclusion of the control decisions of vehicles and stations far away performed well during off-peak periods, but did not achieve better results during peak periods.
- ii. The control percentage during peak hours is higher than that during off-peak hours. Within peak hours, the passenger demand is higher and the service regularity of the bus system is easily disturbed. Since the traffic status is relatively stable during off-peak hours, the performance of the complex DDHC framework is not much better than that of the simple headway-based approach. The effect of the simple control strategy, HCBH, is also acceptable during off-peak hours.
- iii. In the control category of the acceleration strategy, the control action “Skip next station” is more popular across all types of stations. If the passengers whose destinations are the next station of the control station get off at the control station, they only need to endure the extra waiting time of the interval between two vehicles. However, if the control station is skipped, the passengers with the destination of the control station must endure double running time between the two stations and extra waiting time.
- iv. The features related to the operational state of the public transport system, headway, arrival time deviation, service reliability of the impact set, load, and historical cruising speed all affect the accuracy of the classification by the RF model.

In this study, only the control timing of the first vehicle at the first control station is determined by the RF model. Further study will use the RF model or other classification methods to judge the control timing for more vehicles at more control stations, in order to consider more random factors for real-time control problems. Thus, the performance of the proposed method can be enhanced.

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## Appendix A.: Stochastic running times

See [Tables 4 and 5](#).

**Table 4**

The mean and coefficient of variation of running times for each section during peak periods.

Section	Mean (s)	Coefficient of variation	Section	Mean (s)	Coefficient of variation	Section	Mean (s)	Coefficient of variation
1-2	162	0.3	15-16	181	0.1	29-30	145	0.2
2-3	158	0.4	16-17	174	0.3	30-31	105	0.4
3-4	78	0.3	17-18	295	0.2	31-32	158	0.4
4-5	172	0.4	18-19	167	0.2	32-33	74	0.2
5-6	124	0.3	19-20	302	0.3	33-34	152	0.3
6-7	204	0.4	20-21	295	0.3	34-35	92	0.2
7-8	354	0.4	21-22	–	–	35-36	336	0.2
8-9	92	0.2	22-23	396	0.2	36-37	342	0.3
9-10	132	0.3	23-24	215	0.2	37-38	104	0.1
10-11	135	0.3	24-25	175	0.1	38-39	165	0.1
11-12	142	0.2	25-26	154	0.2	39-40	65	0.2
12-13	322	0.4	26-27	106	0.2	40-41	183	0.4
13-14	173	0.2	27-28	174	0.3	41-42	110	0.1
14-15	125	0.4	28-29	150	0.3			

**Table 5**

The mean and coefficient of variation of running times for each section during off-peak periods.

Section	Mean (s)	Coefficient of variation	Section	Mean (s)	Coefficient of variation	Section	Mean (s)	Coefficient of variation
1-2	147	0.2	15-16	176	0.1	29-30	132	0.2
2-3	142	0.3	16-17	158	0.2	30-31	92	0.3
3-4	72	0.3	17-18	276	0.2	31-32	139	0.3
4-5	155	0.3	18-19	154	0.2	32-33	69	0.2
5-6	116	0.2	19-20	282	0.3	33-34	135	0.3
6-7	178	0.3	20-21	271	0.2	34-35	87	0.2
7-8	304	0.3	21-22	–	–	35-36	312	0.2
8-9	83	0.2	22-23	357	0.1	36-37	316	0.2
9-10	118	0.2	23-24	194	0.2	37-38	95	0.1
10-11	129	0.3	24-25	163	0.1	38-39	157	0.1
11-12	127	0.2	25-26	146	0.2	39-40	61	0.2
12-13	284	0.3	26-27	88	0.2	40-41	168	0.3
13-14	161	0.2	27-28	158	0.2	41-42	103	0.1
14-15	121	0.3	28-29	136	0.2			

**Appendix B. Supplementary data**Supplementary data to this article can be found online at <https://doi.org/10.1016/j.trc.2019.08.017>.**References**

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