Accident Analysis and Prevention 142 (2020) 105507

Contents lists available at ScienceDirect

Accident Analysis and Prevention

journal homepage: www.elsevier.com/locate/aap

Cognitive anticipation cellular automata model: An attempt to understand the relation between the traffic states and rear-end collisions

Kamal Jetto[a,b,*], Zineb Tahiri[b], Abdelilah Benyoussef[b,c], Abdallah El Kenz[b]

[a] Research Team Transport, Logistics and Road Safety, The Head of Department of Logistics and Transportation, Ecole Hassania des Travaux Publics km 7, route d'El Jadida, B.P 8108, Oasis Casablanca, Morocco

[b] Laboratoire de Matière Condensée et Sciences Interdisciplinaires, Département de Physique, B.P.1014, Faculté des Sciences, University Mohammed V, Rabat, Morocco

[c] Hassan II Academy of Science and Technology, Rabat, Morocco

ARTICLE INFO

Keywords:
Rear-end collisions
Cognitive psychology
Cellular automaton model
Anticipation
Conservative driver
Aggressive driver

ABSTRACT

We have investigated the accident's statistics of Europe and North America that are provided by the UN. This investigation has shown that accidents due to the traffic represent around 50 % of the total number of accidents every year. Among them, rear-end collisions hold a 20 % share. These numbers display the fact that the interaction between drivers can be held responsible of those incidents. In this respect, we have explored the reasons behind the conflict situations that may be responsible of the occurrence of rear-end collisions by the mean of a cognitive psychology based cellular automata model. Indeed, through field experiments performed by an embedded camera, we have extricated a psychological cognitive process of anticipation. We have defined the latter as the tendency of drivers to accelerate based on the history of their predecessor. Then, we have exploited the tools of the physics of traffic by which we have developed a CA-model that take into consideration this process. As a result, we were able to generate those incidents' situations. By considering two types of drivers: conservative who respect the learned information about the safe manoeuvres but make mistakes or aggressive who violate those secure processes, we have proved the complexity of the relationship between the states of the traffic flow and the drivers' behaviours. In fact, we have shown that rear-end collisions are a result of the anticipation as a response of the drivers to the traffic conditions: the congestion. Moreover, we have also highlighted an improvement of the flow in the congested state up to 11 % due to the anticipation, but that can only be achieved through vehicle-to-vehicle communication. Finally, we have investigated the hot spots. We have found that the traffic perturbations, that generate those hot spots and can be responsible of collisions, are more likely to be located away in the downstream direction. The distance between the two locations depends on the traffic density. This difference between the positions of the traffic perturbation and the hot spot has showcased the complexity, in time and space, of the transmission and the reception of deceleration information by the drivers.

1. Introduction

A strong transportation system is known for being one of the pillars of the economic growth of a country and of the prosperity of its citizens. The nature of the fleet, its cost and its speed affect significantly the capacity of a region to fully exploit its resources, thus directly influence its economic vitality. However, developing such efficient transportation system cannot be without any flaws. Indeed, encouraging more road using may put the drivers at the risk of having an accident, either consciously or unconsciously. Subsequently, every manager of a traffic network would face the dilemma of ameliorating the movement of people and goods while reducing, to the minimum, the risks of accidents that follow this amelioration.

Actually, the urge to improve the fluidity of the travelling facilities has represented a major concern of the USA since the 30′s. Since those early years, many researches have been produced in the aim to make the driving experience more comfortable (see Helbing, 2001; Kerner, 2004; Knospe et al., 2000; Transportation Research Board, 2001), and the references within).

Alongside that, the heavy consequences of traffic accidents have not gone unnoticed and have brought the need of investing in the interdisciplinary sciences. Then, the scientific world has known new

* Corresponding author at: Laboratoire de Matière Condensée et Sciences Interdisciplinaires, Département de physique, B.P.1014, Faculté des Sciences, University Mohammed V, Rabat, Morocco.

E-mail address: jettkam@gmail.com (K. Jetto).

https://doi.org/10.1016/j.aap.2020.105507
Received 20 November 2019; Received in revised form 21 January 2020; Accepted 12 March 2020
Available online 12 May 2020
0001-4575/

dynamics: after the 60′s, many paper and journals, focusing on this flea, have seen the day (Elvik et al., 2009; Evans, 2004; Jones and Joscelyn, 1976; Lamm et al., 1999; Voas and Lacey, 2011). Kazimierz Jamroz has described the evolution of the theories and the models on the road safety (Jamroz, 2008).

Unfortunately, despite all the efforts, the same dilemma persisted or kept reappearing in another form (Willumsen, 2011, p. 17) while no clear answer has been brought to the following question: ***how can we ameliorate the mobility on roads while assuring the highest level of security?***

Analysing the scientific literature has shown that there may be a disconnection when treating the relationship between the road safety and the traffic (Jamroz, 2008).

On the one hand, we can state that the security aspect of the road traffic has usually been studied all by itself, without, considering the fluidity. This may be due to two main reasons:

- The first reason is that the scientific demarche levies on only considering the relevant parameters of the problem: by nature, the major cause of the scientists is to save the human lives; the safety is then more relevant than the fluidity.
- The second reason is the fact that the amelioration of the fluidity and the reduction of the congestion are fully the responsibility of the traffic theory and engineering, those that we implicitly trust. This emerge from the belief that sciences and technologies are capable of resolving all of the humanity problems (Willumsen, 2011, p. 17).

On the other hand, the evaluation and the studying of the traffic performances as far as the fluidity is concerned, were established on safe basis, no accidents were predicted. Indeed, the researchers that have addressed the traffic theories aimed, primarily, to understand the mechanisms behind the formation and the propagation of jams, to define traffic states, to sort out the causes behind traffic congestion and finally to propose models of traffic that are inspired from the statistical physics and the fluid mechanics (Helbing, 2001; Kerner, 2004; Schadschneider et al., 2011). In the same frame, guides, technical reports and traffic manuals were developed so as to provide analysis methods (roads capacities, levels of service, delays caused by congestion…) and technical solutions (design, lanes number, optimum width…). The road safety was not considered in this type of work. For instance, the HCM does not include the safety when establishing service level (National Research Council (U.S.), 2000, p. 26).

In the attempt to remedy this situation, researchers have tapped into the traffic theory. In this respect, Oh et al. have studied the risks of accidents as a function of real-time traffic parameters. They have shown that the risk accidents occurrence depends on the standard deviation of velocity (Oh et al., 2001). Golob et al. have also proven that the road safety is mainly affected by the mean volume and the median speed as well as by the temporal variation of volume and velocity (Golob et al., 2004). Xu et al. have evaluated the security performances in relation with the traffic states; they have found that each traffic phase corresponds to a certain degree of unsafety (Xu et al., 2012). In this same context, Yeo et al. have shown that the accidents rate in the congestion state is five time higher than when in the free flow (Yeo et al., 2013). Many other papers have been produced in the field of real time crash risk models, for instance (Chatterjee, 2016; Lee et al., 2003; Retallack and Ostendorf, 2019; Theofilatos and Yannis, 2014; Wang et al., 2013; Xu et al., 2013a, 2013b, 2016).

Nevertheless, the majority of those studies were based on real data that are provided by the countries authorities. However, these data can present some shortcomings (Kraay, 1983; Mannering and Bhat, 2014; Migletz et al., 1985; Parker and Zegeer, 1989; Zajíc, 2012). Firstly, the row data is usually collected by human effort which can come with some observation errors (Yang and Ozbay, 2010; Zajíc, 2012).Moreover, the probability of accident is luckily very low (in space and in time), otherwise the consequences would be drastic, which makes the gathered data insignificant or insufficient in small period of time (Kalra and Paddock, 2016; Yang and Ozbay, 2010; Zajíc, 2012). Finally, the unexpected evolution of the dynamics of the traffic (Yang and Ozbay, 2010; Zajíc, 2012)makes it harder to fully relay on those data: accidents are usually the result of the complex evolution of the traffic through several phases that is not exhibited through statistical data (Archer et al., 2005; Zajíc, 2012).

Thus, relying solely on statistics may bring major difficulties when in the need of resolving current problems (Yang and Ozbay, 2010). For instance, the limits and the issues brought by this type of data brings a certain amount of uncertainty when defining the hot spots. Subsequently, the validity of the proposed resolutions to dissolve them becomes questionable and creates a dilemma between long term and short term solutions, knowing the costs that come along (Cheng and Washington, 2005; Elvik, 2014; Geurts et al., 2005).

To avoid these issues, researchers have exploited the potential of surrogate safety measures within the simulation models to treat the collisions between vehicles (Chai et al., 2017; Fan et al., 2013; Klunder et al., 2006; Ozbay et al., 2008; Tak et al., 2018; Yan et al., 2008; Young et al., 2014; Yang and Ozbay, 2010; Zajíc, 2012). They have developed simulation models that would help study predefined collisions situations, in order to evaluate the safeness of the adopted measures before their application by traffic managers. The strength of those models is their capability of estimating the probability of accidents occurrence based on microscopic traffic parameters such as vehicle speed, acceleration, time headway, and space headway (Tak et al., 2018).

It is worth to mention that all the previously cited works investigated the accidents occurrence as an automatic and intrinsic result of the traffic itself.

Nonetheless, there has been another tendency that claims that all the safety measures have already been considered in the engineering methods and that the road accidents are predominantly a consequence of the driver behaviour (errors, distraction, age, gender, disease…) (Bucchi et al., 2012; Chai et al., 2017; D'Addario and Donmez, 2019; Elander et al., 1993; Kontogiannis et al., 2002; Li et al., 2016). Thus, the traffic safety should, more likely, be considered in the light of the social sciences.

In this respect, a fundamental question arises: what does the human behaviour refer to?

To answer this question, it is primordial to know that a driver interacts with his/her environment. This interaction is the key to guaranteeing a movement without incidents. The environment can be for example:

- The road: width, number of lanes …
- The climate
- The pedestrians
- The traffic: type of drivers (aggressive, careful), composition of the fleet (heavy trucks, buses …)

The multiple components of a driver environment highlighted the fact that there are different behaviours that need to be elucidated. Studying those behaviours and responses should help in shelling the mechanisms of the interaction between the traffic and the driver, thus drawing near the causes of accidents.

Hence, we can conclude that treating the issues related to the road safety requires the intervention of both the traffic theory and the social sciences, such as psychology or psychosociology.

In this paper, we will present a modest attempt of combining the two fields by proposing a CA model that will consider the interaction between the driver and his/her environment. By analogy to the statistical mechanics, we will investigate the response of the driver to the traffic situations when going from the microscopic to the macroscopic scale, and we will examine the effects of one driver behaviour on the whole traffic, when taking the cross path.

For simplification aims, we will study the behaviour of a driver in a well-defined situation: the vehicles will be driven in one direction, under the different traffic states and parameters. Our object will be to define one mechanism responsible of the rear-end collisions between vehicles.

This article will be organised as follows: we will tackle our investigation by performing a statistical analysis. Then we will define the model on which we will base our discussion later. Finally, we will gather all the results into a conclusion.

2. Statistical analysis

Field observations differentiated three main types of accidents:

- Accidents between vehicles and pedestrians
- Single vehicle accidents
- Accidents between vehicles while travelling.

The latest type of accidents is the most concerned by the traffic theory since it is induced by the interaction between moving vehicles. The investigation of those accidents is then to be the most appropriate for us. Nevertheless, are those collisions more dangerous than the other types of accidents that they require a standout deep investigation?

To answer this question, we will conduct, in the following, a statistical analysis that defines the ratio of accidents between vehicles among all the types of accidents and quantifies the fatalities caused by them.

N.B: Unless stated otherwise, we mean by traffic accidents collisions between vehicles while traveling.

According to statistical data of traffic accidents in Europe and North America (Road Traffic Accidents Involving Personal Injury by Variable, Nature ofAccident, Country and Year, 2015), the numbers differ from a country to another being a function of demographical, political and socio-economic circumstances (Fig. 1) (Ansari et al., 2000; Skog, 2001; Bener and Crundall, 2005; Kopits and Cropper, 2005; Bishai et al., 2006).

Actually, if we define the ratio of accidents due to the traffic (RAT) as follows:

$$RAT = \frac{number\ of\ accidents\ due\ to\ the\ traffic}{total\ number\ of\ accidents}$$

We can observe that it represents a strong heterogeneity: it can reach very high values 80 %, for some countries when it cannot exceed 30 % for others (Fig. 1).

This observation leads us to compute the yearly mean *RAT* of this sample and follow its evolution from 1993 to 2015.

The Fig. 2 represents the yearly evolution of the mean values of *RAT* (black squares). We observe that despite the heterogeneity of the present sample the mean *RAT* does not show differences in behaviour from a year to another. Indeed, if we consider the first period (1993–2009) we observe that the mean *RAT* fluctuates slightly around 50 %. The same remark holds for second period i.e., from 2011 to 2015 where the mean RAT takes the value of 58 %.

Furthermore, traffic accidents are responsible for the biggest share of crushes, killed and injured people in comparison to the other types of accidents.

The urban investigation has shown the same patterns: the mean yearly ratio in the urban area maintains a value around 60 % over the years.

Moreover, the rear end collisions hold, recently, a non-negligible share of fatalities compared to the other types of accidents due to traffic (Fig. 3). For this purpose, we narrow our investigation to only focus on that type of collisions. We denote their contribution by *RAC*.

$$RAC = \frac{number\ of\ rear-end\ collisions}{total\ number\ of\ accidents\ due\ to\ the\ traffic}$$

Indeed, after knowing an increase during the first years, we state that the RAC remains around 20 % since 1999 (Fig. 3- black squares).

Through the statistical analysis, we conclude that:

- The collisions between vehicles hold the biggest share between all the types of accidents (can reach 80 %) which makes them a crucial phenomenon that needs deep investigation.
- Despite the heterogeneity of the sample, and even though all the countries have necessarily known an evolution from 1993 to 2015, the mean *RAT* and *RAC* kept an almost constant value over this period of time. This made us believe in the existence of a universal

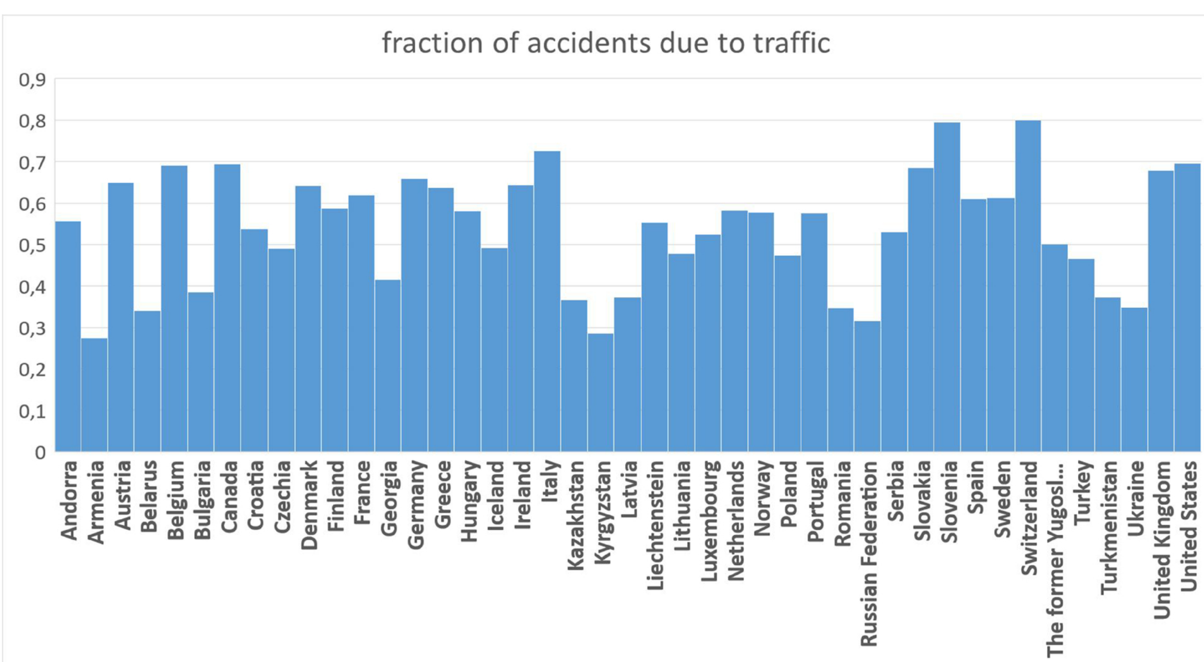

Fig. 1. the ratio of accidents due to traffic *RAT* for different countries.

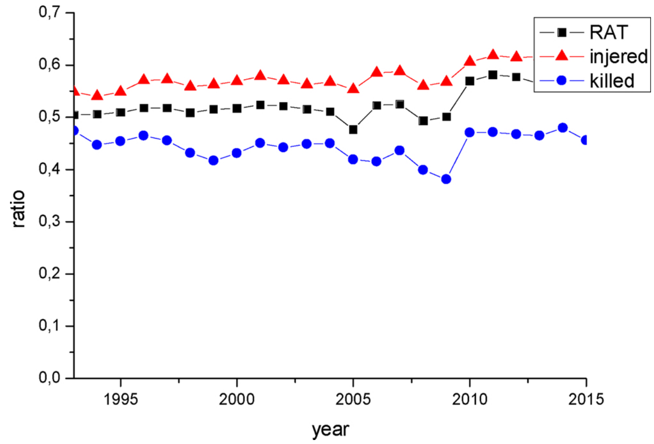

Fig. 2. the yearly mean ratio of accidents due to traffic *RAT* (black squares), the yearly mean ratio of fatalities caused by accidents due to traffic: injured (red triangles) and the killed (blue circles) versus the years from 1993 to 2015.

causality behind the accidents that, apparently, does not depend on the nature of the facilities nor on the fleet …

By analogy to the statistical physics that explain the macroscopic phenomena through microscopic details, we think that this causality is the result of mechanisms of interaction between the driver and the traffic.

The frame of this work cannot confirm, entirely this hypothesis. It will remain an open question for the scientists to approve or deny. In our opinion, the most relevant result is that we were able to highlight the important share of accidents due to the traffic. Thus, this finding confirms the importance of investigating the road security within the frame of the traffic theory.

Moreover, we will not discuss all the types of those accidents, we will only focus on the rear-end collisions.

It is worth to mention that this statistical analysis is not complete and would require an independent investigation that would justify the remarks stated previously.

3. Cognitive anticipation CA model

In order to quantify and reproduce accidents likelihood, scientists have opted for surrogates safety measures, for instance: time to collision, stopping distance index, vehicles speeds and headways… (see: Li et al., 2014 and the references within). Those measures were implemented into several simulation models, among them, cellular automata that we will adopt to carry on our study.

The traffic cellular automata "TCA" represent one of the most exploited methods of modelling the traffic due to their computational efficiency (for review see Schadschneider et al., 2011, pp. 213, 243–307), and the references within). Indeed, the TCA models are based on discretization of the time and space thus of: position, speed and acceleration of the vehicle.

Several TCA models have seen the lights during the last 3 decades. The NaSch model is one of the most popular (Nagel and Schreckenberg, 1992). Also known as the stochastic traffic cellular automaton, the NaSch model incorporate four basic steps necessary to reproduce most observed phenomena in the real traffic, namely the spontaneous Jam.

Nevertheless, this TCA model, among others, shows many limitations especially as far as the safety aspect of the traffic is concerned. In fact, this model can be described as intrinsically accident-free since it was based on secure bases and all collisions were prevented. Other models, on the other hand, are not intrinsically safe, but do not represent real cases of accidents when the simulations are run. Those TCA only displays dangerous situations instead of accidents (see Schadschneider et al., 2011, p. 332 and the references within).

In general, as their name suggests, TCA models quantify the human reactions in an automatic way, whereas, each driver is an independent

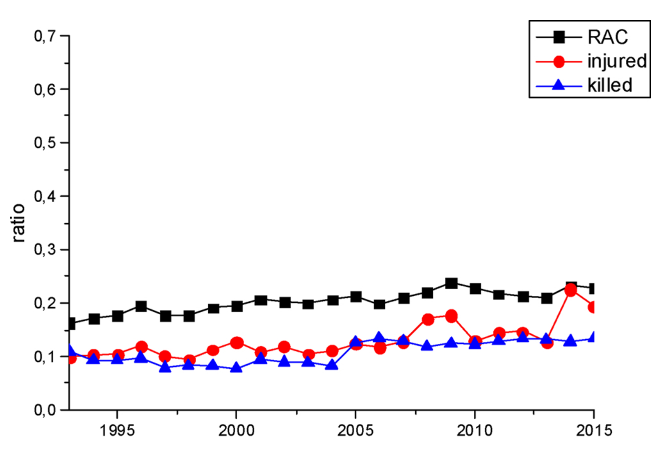

Fig. 3. the yearly mean ratio of rear-end collisions *RAC* (black squares), the yearly mean ratio of fatalities caused by rear-end collisions: injured (red triangles) and the killed (blue circles) versus the years from 1993 to 2015.

intelligent agent who thinks, learns and decides differently (Chowdhury, 2000, p. 5). Therefore, the human behaviour should be taken into account when trying to reproduce real events such as collisions.

3.1. Anticipation process in the frame work of cellular automata models

In the attempt to simulate the human behaviour, researchers have defined different parameters or moving steps such as randomization or more importantly anticipation.

The anticipation can theoretically be defined by the susceptibility of drivers to predict the behaviour of their predecessors (velocity, deceleration, headway…).

This behaviour has largely been studied in the scope of the car following models (to name a few: Jun-Fang et al., 2010; Peng and Cheng, 2013; Zheng et al., 2012; Saifuzzaman and Zheng, 2014). In the frame TCA, many models have incorporated the anticipation during deceleration as a tool to avoid collisions (Knospe et al., 2000; Lárraga et al., 2004; Lee et al., 2002; Nishinari et al., 2004; Nishinari and Takahashi, 1999).

On the other hand, fewer introduced the anticipation as a cause of accidents during acceleration. Hamdar et al. stated that drivers, while observing the behaviour of their predecessors, may take the risk of making an accident in order to increase their velocity (Hamdar et al., 2008).

Nevertheless, the previous models incorporated the anticipation as an automatic step, i.e, all the drivers anticipate in the same way, whereas, in fact, no one can assure that two drivers would have the same respond to the same situation. Additionally, this anticipation was performed as a step among others, and did not result from a certain process.

Our field experiments have highlighted (see next Subsection 3.2) that the anticipation during acceleration is an intricate reaction of drivers to the strong interaction between them.

Actually, due to the anisotropy of the traffic, drivers mainly manoeuvre with respect to their predecessors (Daganzo, 1995; van Wageningen-Kessels et al., 2013). A ***transfer of information*** between successive vehicles is then present. Indeed, this communication is guaranteed by the strong correlation of their velocities as proven empirically by Appert-Ronald (Appert-Rolland, 2009) and through simulations by Lakouari et al. (Lakouari et al., 2014).

Thus, shelling the drivers' response to those interactions, we were able to state that anticipation is a complex human driving progression that may lead to speed improvement but could also be responsible of generating rear-end collisions.

In this paper, we define anticipation as a driver's tendency to accelerate on the basis of their expectation of the next move of the leading driver. This expectation is preceded by a learning process on the previous actions of their leader.

We will show in the next subsection that the learning process results from cognitive considerations and differs from one driver to another.

3.2. Cognitive process

In order to deeply understand the cognitive process followed by a driver, we have opted for the moving observer method (experimental approach). Indeed, by the mean of an embedded camera, we recorded the behaviour of a driver while travelling. We were mainly focused on:

- Detecting the situations that may lead to a rear-end collision.
- Measuring the frequency of the occurrence of those situations.
- Defining the causes behind the existence of those situations.

N.B: the results of this field experiment are beyond the scope of this article and will be discussed in a future paper.

By synthesizing all of that information, we were able to state that the anticipation process, that we believe to be strongly responsible of the potential rear end collision, is performed in four steps:

- Step 1: Attention: during this first step, the follower driver monitors the leading driver and records its accelerations and decelerations history.
- Step 2: Perception: at this stage, the follower driver analyses the behaviour of the leading driver based on:
  - The historic of his/her leader's acceleration (deceleration): is he/she a carful driver? How does he/she perform in front of obstacles or sudden incidents?
  - The evolution of the empty space between the two vehicles
- Step 3: Learning: The recurrence of this analysis allows the follower driver to understand and learn about the behaviour of his/her leading driver.
- Step 4: Decision-making: based on the results of his/her analysis and learning about his/her predecessor, the driver may anticipate, if he/she is urged to synchronize his/her speed with his/her leading vehicle.

To incorporate the anticipation process, we were brought to translate the learning process (step 1–3) into measurable quantities and Boolean conditions. To do so, we will define new pillar parameters:

- Historic parameter H: this variable registers the accelerations and the decelerations of each single vehicle: if a driver accelerates, his/her H is incremented by 1, if not, it is decremented by 1.
- Threshold learning parameter Hs: this variable determines the minimum number of accelerations that should be realized by a driver so as his/ her follower can predict his/ her next move. It is an attempt to quantify the level of the caution of a driver. It varies from 1 to Hsmax to represent different levels of caution. If a driver succeeds an anticipation attempt, he/ she becomes more confident in his /her judgment and Hs is decremented by 1. If he/she fails, Hs is brought to Hsmax. Each driver would be attributed initially, with an Hs chosen randomly from 1 to Hsmax, to reproduce the differences between drivers.

It is worth to mention that modelling the human behaviour is not a trivial exercise (Hamdar, 2012) and far beyond the scope of this article. To our knowledge, no one has measured or found the unity of learning. Nevertheless, the values added, subtracted, compared and taken by H and Hs are only a mean to quantify those two parameters.

Furthermore, step 4 cannot be executed automatically by all the drivers. Each driver reacts differently, i.e. might or not anticipate, even if the learning process is performed.

As a consequence, it is crucial to consider the differences of the psychology of the drivers. In other words, we should know and try to capture the behaviour of each driver while driving so as to understand the way he/she anticipates. In the next subsection, we will present some finding regarding this point of view.

3.3. Conservative and aggressive drivers

As stated previously, we should increment the Human behaviour at certain level of our model. Before doing this we should be able to understand why two drivers in the same situation may (or not) react in the same way while driving.

In this respect, a driver behaviour questionnaire "DBQ" was proposed by Reason et al. (Reason et al., 1990).

The DBQ aims to define the types of actions lead by a driver when in a certain situation. Indeed, those actions, that may or not cause an accident, are correlated to demographical specificities (age, sexes…) (Elander et al., 1993; Kontogiannis et al., 2002) and more strongly to the state of mind of the driver (Stephens et al., 2018).

Accordingly, different categorizations of drivers have been proposed:

- Drivers who commits errors/violations (Reason et al., 1990).
- Mindful/angry drivers (Stephens et al., 2018).

In the same perspective, and so as to consider all those types of drivers in the anticipation process, we propose the following classes:

- Conservative drivers: who are mindful but can make unintentional errors. Such driver may anticipate after a careful reading on his/her predecessor. Indeed, even if the learning process is performed, this kind of driver is indecisive. Thus its anticipation takes time and not always smooth.
- Aggressive drivers: who are angry and deliberately commits violations. This Kind of drivers will not take seriously the learning step and may anticipated after a short amount of time.

3.4. Our cognitive cellular automata model

In order to perform simulations, we will write the psychological cognitive process presented above in CA language. In this respect, we will consider the NaSch Cellular Automaton Model with periodic boundaries in which we will implement the cognitive anticipation process.

3.4.1. Standard NaSch model

The strength of the NaSch model is its ability to be extended to study more complicated situations. It is based on specific but simple steps that govern the movement of vehicles (Nagel and Schreckenberg, 1992). In the following, we will recall and comment on these steps. Let us focus on one vehicle which we denote by ***I*** and its leader by ***I+1***.

X_i^t: is the position of the vehicle "***I***" at a given instant of time t.

X_{i+1}^t: is the position of the vehicle "***I+1***" at the time t.

V_i^t : is the velocity of the vehicle ***I*** at the time t.

V_{i+1}^t : is the velocity of the vehicle ***I+1*** at the time t.

According to the parallel dynamics approach, the position and the velocity of the vehicle ***I*** are updated (from the time t to t+1) as follow:

- Step 1 is about acceleration: $V_i^{t+1} = Min[(V_i^t + 1), V_{max}]$, where V_{max} is the maximal velocity of vehicles. This rule reflects the human innate to move ahead.
- Step2, deceleration: $V_i^{t+1} = Min(V_i^{t+1}, D_i^t)$ where $D_i^t = [(X_{i+1}^t - 1) - X_i^t]$ is the gap between the leading vehicle (***I+1***) and the following one ***I***. This step limits the previous one for safety purpose.
- Step 3, randomization: $V_i^{t+1} = Max[(V_i^{t+1} - 1), 0]$ with a probability ***Pr***. It is well known that this probability is what makes NaSch model able to reproduce the formation of the phantom Jam. Indeed, ***Pr*** quantifies the driver's response to the variation of the external conditions. In the following, this deceleration parameter would only reflect the probability that a driver may face a perturbation namely a sudden event in the road that would force him/her to decelerate, for instance, drivers that join the flow from ramps and may cause unexpected braking.
- Vehicle's motion: $X_i^{t+1} = X_i^t + V_i^{t+1}$

3.4.2. Anticipation process in CA language

At low density, the gap D_i^t is large enough that vehicles do not interact. As long as the density of cars increases, vehicles are more constrained to "cooperate" in order to avoid rear-end collisions. The definition of $D_i^t = [(X_{i+1}^t - 1) - X_i^t]$ ensures that no rear-end collision will occur between vehicles ***I*** and (***I+1***). Indeed, the dynamics of NaSch model are a compromise between speed improvement and collision avoidance (steps 1 & 2).

However, rear-end collisions take place in real traffic. According to our previous discussion (Sections 3.1 and 3.2), the anticipation process allows speed improvement to the detriment of the safety. In the CA language, anticipation means that:

$$X_i^{t+1} = X_{i+1}^t \tag{1}$$

It is based mainly on two conditions:

$$\mathbf{C}1:\ D_{c_i}^t \leq R_i^{t+1} \tag{2}$$

where,

$D_{c_i}^t = [X_{i+1}^t - X_i^t]$: the critical gap between vehicle ***I*** and his leading ***I+1.***

and

$R_i^{t+1} = Min[(V_i^t + 1), V_{max}]$: the risk velocity. If the driver maintains this velocity he/she will surely have a collision with the leading car.

As a consequence, ***C1*** is a spatial condition. It translates the situation when the vehicles' gap makes it impossible for the drivers to travel at their desired speed R_i^{t+1} that they feel the urge to predict and anticipate the next move (next time steps t+1) of their predecessors.

If ***C1*** is not verified, it means that the vehicles ***I*** and ***I+1*** are far enough that there is no need for anticipation.

$$\mathbf{C}2:\ H_{i+1}^t \geq H_{s_i}^t \tag{3}$$

This condition translates the learning process: the fact that the driver of vehicle ***I*** has, in his/her personal opinion, enough information about his/her predecessor that will able him/her to anticipate. It is a conciliation between the level of caution $H_{s_i}^t$ of the following driver **(*I*)**, and the recorded behaviour (acceleration/deceleration) H_{i+1}^t of the leading **(*I+1*).**

It is worth to mention that the respect of condition ***C2*** will depend on the psychology of the driver ***I***. In the previous Section 3.3, we distinguished two types of behaviour.

For an aggressive driver, if both condition ***C1*** and ***C2*** are fulfilled, then he/she surely anticipates i.e. try to $X_i^{t+1} = X_{i+1}^t$ (1).

However, if ***C1*** is verified whereas ***C2*** is not, the driver may anticipate with a probability ***Pe***. For such drivers, the learning process limits the anticipation so they "Gamble: deliberately commits violations " in order to improve their speed and reduce their journey time.

For a conservative driver, both conditions ***C1*** and ***C2*** must be verified. After that, he /she may anticipate with a probability ***Pe***.

3.4.3. Rear-end collision

According to the anticipation definition (Section 3.1), the driver of vehicle ***I*** expects that ***I+1*** will move in the next time steps (t+1). The success or the failure of the anticipation surely depend on the state of X_{i+1}^{t+1}.

If $X_{i+1}^{t+1} = 0$. i.e. the leading vehicle has moved forward, it follows that the anticipation was a successful maneuver from the following driver ***I***. Fortunately, the rear-end collision was avoided. In this case:

$$X_i^{t+1} = X_{i+1}^t \tag{1}$$

and

$$H_{s_i}^{t+1} = Max[(H_{s_i}^t - 1), 1] \tag{4}$$

the driver **I** becomes more self-confident.

If $X_{i+1}^{t+1} \neq 0$. i.e. the leading vehicle has not moved forward, then the anticipation was not a successful maneuver from the following driver ***I.*** In this case, the driver ***I*** decelerates and his/her velocity is reduced to none so as to try to avoid a collision.

$$V_i^{t+1} = 0 \tag{5}$$

This brutal brake may or not generate rear-end collision. In this model, we are not concerned by the occurrence of a real rear-end collisions. In other words, this situation only represents a potential collision that will not solicit the elimination of the concerned vehicles.

After that the driver ***I*** becomes aware

$$H_{s_i}^{t+1} = H_{smax} \tag{6}$$

and stays in the safe position

$$X_i^{t+1} = X_{i+1}^{t} - 1 \quad (7)$$

obtained by the execution of NaSch rules.

N.B: In the following, rear-end collisions, collisions or accidents refer to potential rear-end collisions.

3.4.4. Algorithm of the cognitive anticipation model

This algorithm sums up the main stages of our cognitive anticipation CA model. For simplicity, let us consider the case of a conservative driver. The movement of this driver will be performed according to the following steps:

Step 1: **acceleration and deceleration**

$$V_i^{t+1} = Min[(V_i^t + 1), D_i^t, V_{max}]$$

Step 2: **randomization**

Yes, with a probability ***Pr***

No, with ***1-Pr***

Step 3: Following the standard NaSch rules

Update the velocity

$$V_i^{t+1} = Max[(V_i^{t+1} - 1), 0]$$

Update position

$$X_i^{t+1} = X_i^t + V_i^{t+1}$$

Record acceleration/deceleration

If $V_i^{t+1} \geq V_i^t$ ***(Acceleration)***

Then $H_i^{t+1} = H_i^t + 1$

Else (deceleration) $H_i^{t+1} = H_i^t - 1$

Step 4: Anticipation

1) Spatial condition C1:

$$D_{c_i}^t \leq R_i^{t+1}$$

2) Learning condition C2:

$$H_{i+1}^t \geq H_{s_i}^t$$

3) Attempt to anticipate with a probability Pe:

$$X_i^{t+1} = X_{i+1}^t$$

Step 5: **Anticipation and accidents detection**

If the vehicle ***I*** has fulfilled all the conditions in step 4, then two cases are possible:

$X_{i+1}^{t+1} \neq 0$ Rear-end collision

$$V_i^{t+1} = 0$$

$$H_{s_i}^{t+1} = H_{smax}$$

$X_{i+1}^{t+1} = 0$ Successful anticipation

$$X_i^{t+1} = X_{i+1}^t$$

$$V_i^{t+1} = D_{c_i}^t$$

$$H_{s_i}^{t+1} = Max[(H_{s_i}^t - 1), 1]$$

3.4.5. Monte Carlo simulations

In order to run the simulations and reproduce rear-end collisions, we translate the algorithm above into a program that would be compiled by programming machine.

To guarantee the realization of the stochastic process of the model, we resort to the Monte Carlo simulation. Indeed, each MC simulation generates one specific realization of the stochastic process: by generating a random number r from [0,1] and comparing it to a fixed probability (in our case: probability of randomization Pr, of anticipation Pe). For instance, if $r \leq Pe$, the process of anticipation will happen, if $r > Pe$, it will not (Schadschneider et al., 2011, p. 58, 59).

In this respect, we execute 50,000 Monte Carlo iteration while we average 40 initial configurations to eliminate statistical fluctuations.

4. Results and discussion

As defined in the previous Section 3, the anticipation process, which is believed to be among the generators of rear end collisions, strongly depends on the gap between vehicles. Therefore, in this section, we will follow the behaviour of this phenomenon (anticipation) in relation with the traffic states and track how it leads to rear end collisions.

It is worth to mention that we will quantify the traffic states by considering the variation of the density for simplification purposes. This can also be performed using the traffic volume or the vehicles' mean velocity.

4.1. Conservative drivers

We start our investigation by taking into account one type of drivers, for instance, conservative drivers. If not stated otherwise, Vmax = 5, Pe = 0.8, Pr = 0.005 and Hsmax = 5.

4.1.1. Anticipation, rear end collision and traffic states

We define the rate of anticipation “Ra” by the mean value of the ratio of the number of drivers that attempted anticipating to the total number of drivers.

We plot this rate as a function of the density ρ of vehicles for different values of Pe (Fig. 4) so as to show how the anticipation evolves according to the state of the traffic flow.

The Fig. 4 represents the variation of Ra for two values of Pe. It highlights that the Ra, like other natural phenomena, follows a Gaussian law. Indeed, drivers mainly anticipate for a specific range of densities $[\rho_{s1}, \rho_{s2}]$, where ρ_{s1} corresponds to the density at which the anticipation starts to occur ($Ra \geq 0$) while ρ_{s2} represents the density at which the driver's manoeuvre fades away ($Ra \approx 0$).

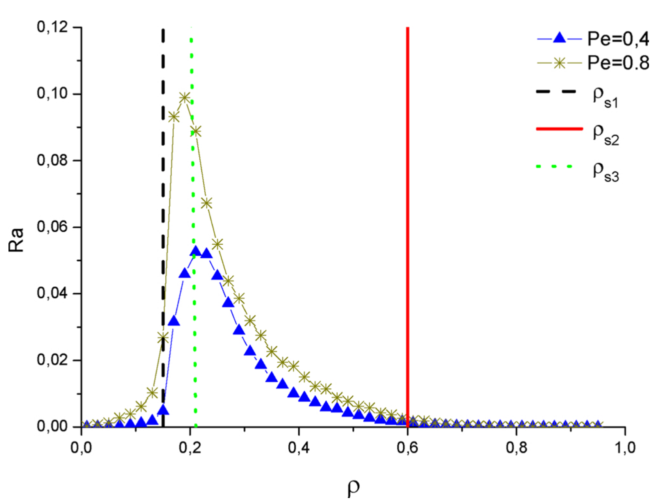

Fig. 4. The variation of the rate of anticipation Ra versus the density ρ for different values of Pe.

To understand the Ra's Gaussian behaviour, we investigate (according to traffic density) the accomplishment of the two main conditions: the spatial condition (1) “ $R_i^{t+1} \geq Dc_i^t$ “and the learning condition (2) “ $H_{i+1}^t \geq Hs_i^t$” . We recall that these conditions must be fulfilled simultaneously to perform anticipation (see the model Section 3.4.2 for more details).

On the one hand, Fig. 5 displays that when $\rho < \rho_{s1}$, the spatial condition is mostly not verified since $Dc_i^t > R_i^{t+1}$. Thus, the anticipation does usually not occur.

On the other hand, for higher densities $\rho > \rho_{s2}$, Fig. 6 shows that the mean Historical parameter $<H> \approx 0$. Thus, the learning conditions is not satisfied and the anticipation rate nearly approaches none for this range of density.

When $\rho_{s1} \leq \rho \leq \rho_{s3}$, the number of vehicles for which both conditions are verified increases as well as Ra according to the density ρ. For $\rho = \rho_{s3}$, the spatial and learning conditions are fulfilled for all drivers and the anticipation is very permeant, it takes its maximal value (Fig. 4).

When $\rho_{s3} < \rho \leq \rho_{s2}$ (Fig. 4), the learning conditions is, in the average, not always verified (Fig. 6), which brings the decrease if the anticipation rate Ra.

N.B: According to the Fig. 4 (and other figures omitted here for space management), the density thresholds ρ_{s1}, ρ_{s2} and ρ_{s3} do not depends of Pe nor Pr. However, those thresholds depend on Vmax which since they also correspond to the traffic phase transitions (as will be presented in the following).

The question that arises then is: how do the traffic states contribute in the appearance and the disappearance of the anticipation?

We observe that the drivers mainly anticipate for medium densities $[\rho_{s1}, \rho_{s2}]$ (Fig. 4). According to the fundamental diagram, those densities correspond to the congested traffic state (Fig. 7).

The free flow state ($\rho < \rho_{s1}$): Drivers run at their desired speed. There is no interaction between the drivers and the gap between the vehicles is very large (low densities): the spatial condition is mostly not verified (Fig. 5) and very few successful anticipations occur.

The jamming state ($\rho > \rho_{s2}$): Drivers are obliged to decelerate which significantly affects their historical parameter H and prohibits the verification of the learning condition (Fig. 6): there is no anticipation.

The congested traffic state ($\rho_{s1} \leq \rho \leq \rho_{s2}$), drivers start approaching from each other. The gap between the leader and the follower forced the latter to drive at a velocity lower than what he/ she wants. Such situation may urge some drivers to anticipate.

Indeed, the spatial condition is verified by the effect of the density (Fig. 5) while the learning condition is still checked (Fig. 6). Major anticipations are then present as an attempt to maintain the velocity of the free flow state. At $\rho = \rho_{s3}$, the anticipation becomes a collective behaviour of drivers.

It is worth to mention that the obtained fundamental diagram corresponding to this case of anticipation where only conservative drivers are considered does not differ from the one obtained by the standard NaSch model (Fig. 7). However, this model is able to shed light on the congested traffic state and marks off its limit $[\rho_{s1}, \rho_{s2}]$.

Nevertheless, the anticipation is not a safe manoeuvre: the appearance of this process brings accidents along. Indeed, at $\rho = \rho_{s1}$, potential rear end collisions appear (Fig. 8). Moreover, the ratio of those probable accidents Pcr follows the tendency of the variations of Ra (Gaussian law): it increases with the increase of Ra and decreases with its decrease. At the density ρ_{s2}, potential accidents also disappear. Therefore, potential rear-end collisions strongly result from the anticipation, they are then more likely to occur in the congested traffic state.

4.1.2. Drivers' behaviour and the traffic states

In the previous Subsection 4.1.1, we have explained that the anticipation is a collective response of the drivers to the congested traffic state. It is important now to understand how this collective behaviour

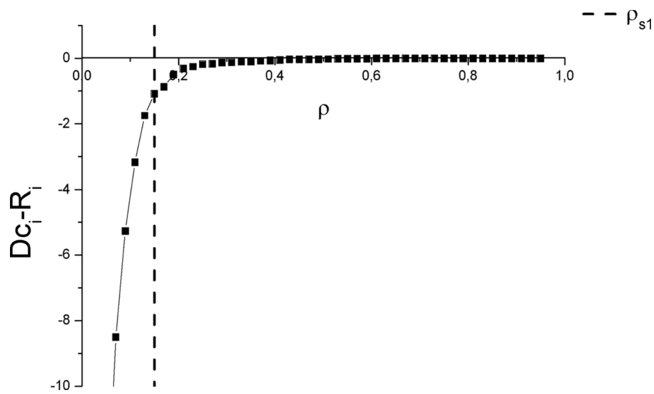

Fig. 5. Accomplishment or not of the spatial condition for Pe = 0,8. We plot here the mean difference between the speed of anticipation R_i and the critical gap Dc_i versus traffic density ρ.

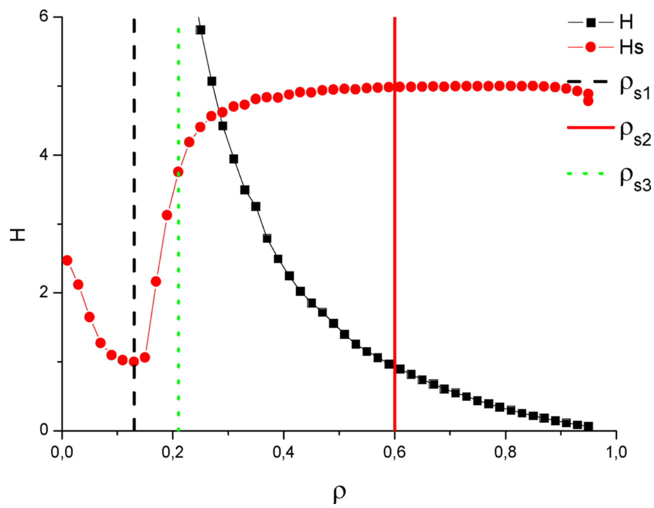

Fig. 6. the learning condition versus the density ρ for Pe = 0.8: the historic H (black scare) and the mean learning Hs thresholds (red circles).

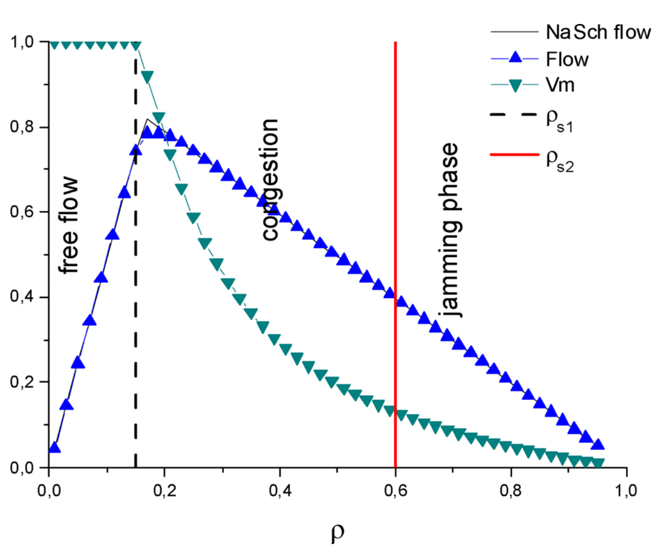

Fig. 7. fundamental diagram (flux versus density) and normalized velocity (V/Vmax) versus the density ρ for Pe = 0.8.

emerges according to traffic states.

The Fig. 6 shows the variation of the mean learning threshold parameter Hs that represents the degree of caution of drivers, versus traffic density.

For ($\rho < \rho_{s1}$), the traffic is in the free flow state (no anticipation). The mean Hs is around 2.5, which shows the heterogeneity between the drivers as far as their caution is concerned.

When the density increases, by the effect of the traffic, few successful anticipations take place, those drivers reduce their degree of awareness, which brings a decrease of the mean Hs observed in Fig. 6. At the density $\rho = \rho_{s1}$, all the drivers have an Hs that is equal to 1.

Note that $\rho = \rho_{s1}$ is the density at which the traffic undergoes a phase transition: It switches from free flow to the state of congestion. Thus, this traffic flow transition has forced the drivers' community to self-organized in homogeneous but hazardous situation (Hs = 1). Following that, the lack of caution combined with the effect of the shortage of space due to the density generate the first potential accidents. This results in the augmentation of Hs for the concerned drivers, from 1 to 5 (Hs = Hsmax and V = 0), while the remaining drivers are not affected (Fig. 6) since the mean value of Hs is lower than 5.

This event starts a chain reaction of rear end collisions. Certainly, for $\rho_{s1} < \rho < \rho_{s3,}$ the mixture of drivers' Hs combined with the

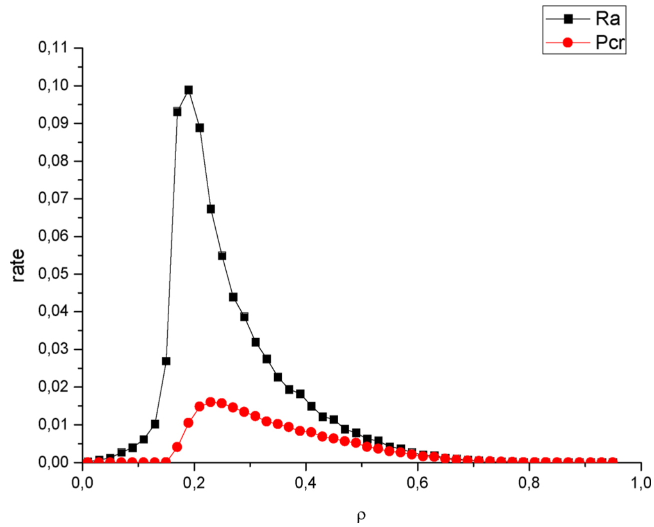

Fig. 8. rate of potential collisions Pcr compared to the rate of anticipation Pa versus the density ρ, for Pe = 0.8.

congested state of traffic produce more potential accidents that affect again the Hs and go on. When the mean learning threshold reaches Hsmax at $\rho = \rho_{s3}$, the human behaviour is homogenous; the level of awareness is maximum for all the drivers and the number of potential accidents starts decreasing.

In the range of density $\rho_{s3} < \rho < \rho_{s2}$, the interaction between vehicles is still strong (congested state) but the drivers are more aware.

The safe state is restored at $\rho \geq \rho_{s2}$ (the traffic is in the Jamming state, so no anticipation is needed).

4.1.3. *Summary*

To sum up, the analysis of the traffic states, the anticipation and the accidents probabilities shades lights on the existence of a complex causality relation between the traffic flow states, the drivers' behaviours states and consequently accidents' occurrence.

Indeed, the traffic states (free flow, congestion and jamming) are correlated to the driver's behaviour states (anticipation/no anticipation). The phase transitions occur for the same transitions' densities ρ_{s1} and ρ_{s2}.

At ($\rho = \rho_{s1}$), the traffic flow undergoes a phase transition from free flow to the congested state. In the same way, the drivers' behaviour switches from the state of no anticipation to the anticipation state.

At ($\rho = \rho_{s2}$), the traffic flow changes from the congested state to the jamming state. Also, at the same density, the drivers' behaviour undergoes a phase transition from the state of anticipation to the no anticipation state.

Moreover, when the first transition ($\rho = \rho_{s1}$) takes place, the successful anticipations lead the drivers to self-organize but into a hazardous situation ($<\mathbf{Hs}> = \mathbf{1}$). This situation generates a potential rear-end collision. This first rear end collision is what brings back the heterogeneity in the human awareness ($\mathbf{1} < \mathbf{Hs} < \mathbf{5}$) which leads to more and more accidents. Surprisingly, this situation provokes another self-organization i.e. the drivers become more aware ($<\mathbf{Hs}> = \mathbf{Hsmax}$) and the number of accidents decreases progressively.

The Fig. 9 sums up the different traffic and security phases when only conservative drivers are considered.

4.2. *Aggressive drivers*

In the following, we consider Pr = 0.005 and Hsmax = 5.

After initiating the anticipation process in the previous section, we will now investigate the impact of only considering the second type of drivers "the aggressive drivers" on the results stated above (4.1).

On the one hand, the Fig. 10a represents a comparison of the ratio of anticipation Ra between the conservative and the aggressive drivers. We observe that Ra corresponding to the aggressive drivers is very important compared to the other type of drivers. Firstly, the ratio of anticipation starts increasing much earlier in the low densities at a density ($\rho'_{s1} < \rho_{s1}$). Indeed, in this range of densities, the spatial condition controls the anticipation process, as the learning condition is usually verified (Fig. 6). In such situation, conservative drivers anticipate with a probability Pe while aggressive drivers surely anticipate despite the value of Pe (see Section 3.4.2). After that, Ra keeps growing even in the higher densities ($\rho > \rho_{s3}$), so as it reaches very high values whereas conservative drivers start ceasing anticipation. This is due to the fact that those aggressive drivers do not always respect (effect of Pe) the learning condition of anticipation (condition 2) that is responsible of the decrease of the Ra as in the Section 4.1.1.

On the other hand, the Fig. 10b compares the probabilities of the potential rear-end collisions for both conservative and aggressive drivers. Those probabilities start increasing at the same density $\rho_{=}\rho_{s1}$ for both types of drivers. This shows the existence of a secure and more fluid interval of densities [ρ'_{s1}, ρ_{s1}] (*This finding will be discussed later in the paper*). However, no safe state is restored in the higher densities ($\rho > \rho_{s3}$).

Moreover, the Fig. 11 shows the behaviour of the mean learning threshold $<$Hs$>$ versus traffic density for both types of drivers. In the low densities $\rho < \rho_{s1}$, the mean Hs decreases until it reaches 1 despite the type of the drivers. When $\rho > \rho_{s1}$, the mean learning threshold starts increasing as a result of the potential accidents that bring some drivers to the highest level of caution, i.e., Hs = Hsmax. However, unlike the previous case, this mean Hs does not go back to Hsmax unless the density is very important, $\rho \approx \mathbf{1}$. Therefore, the system needs more traffic density to restore a homogenous state (Hs = Hsmax for all drivers).

This highlights the resistance of aggressive drivers to the traffic conditions. Consequently, this resistance translates into a higher risk of potential collisions. Actually, their rate increases exponentially when increasing the density (Fig. 10b).

Furthermore, unlike the case of conservative drivers where the first transitions (no anticipation/anticipation; free flow/congestion) happen at the same density, the Fig. 12 shows that the aggressive drivers generate two different transitions: a first transition at $\rho = \rho'_{s1}$ where anticipations appear and a second transition at $\rho = \rho_{s1}$ where the system switches from a free flow safe state to a congested unsafe one.

These findings rise the question about whether this type of drivers would have any impact on the fluidity of the traffic.

In this respect, we plot the flux corresponding to Pe = 0.8 and

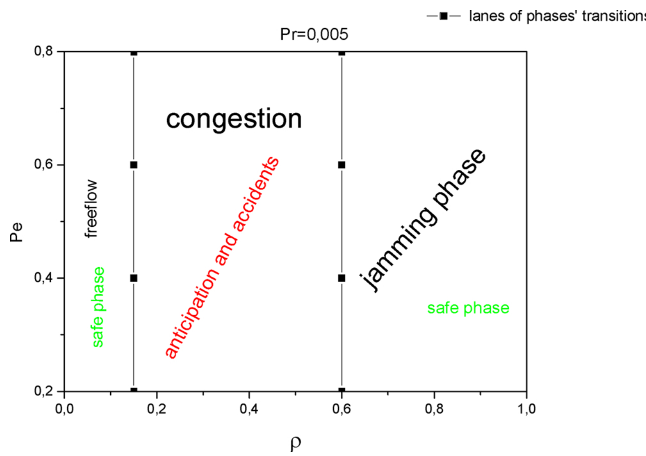

Fig. 9. traffic and security phase diagram: define the three traffic phases (free flow, congestion and jamming phase) along with the corresponding security phases for different densities and anticipation probabilities, when Pr = 0.005.

Pe = 1.

According to the Fig. 13, we can identify three different flow behaviours:

- When $\rho \leq \rho_{s1}$: the system is in the free flow state.
- When $\rho_{s1} \leq \rho \leq \rho_{s2}$: the system is in the congested state (Fig. 14)
- When $\rho = \rho_{s1}$: the system undergoes a transition from the free flow state to the congestion state (Fig. 15)
- When $\rho > \rho_{s2}$: the system is in the jamming state.
- When $\rho = \rho_{s2}$: the system undergoes a transition from the congestion to the jamming state (Fig. 16)

4.2.1. In the vicinity of the traffic flow transition (free flow/congestion)

Based on the Fig. 13, the system is characterized by the free flow when $\rho \leq \rho_{s1}$.

Indeed, when $\rho \leq \rho'_{s1}$, the gap is sufficiently big that all drivers drive at their desired speed (Fig. 12). No anticipation is needed.

When $\rho > \rho'_{s1}$, few anticipations take place. However, due to their low number, the flow is not affected.

Nevertheless, when ρ approaches ρ_{s1}, the effects of the congestion appear and the gap between the vehicles decreases in comparison to the free flow state. At this point, the anticipation intervenes so as the drivers maintain their wanted velocity (Fig. 12). This anticipation leads to an increase of the mean velocity which results into an amelioration of the flow (Fig. 13). This amelioration is more important when Pe = 1. Indeed, by using an intelligent transportation system (for instance vehicle to vehicle communication "V2V" (jean.yoder.ctr@dot.gov, 2016)) that allows the drivers to anticipate, we can improve, in a very secure way, the flow by 11 % up to a density of 0,21. The challenge that rises at this step is how to implement this finding in the V2V concepts.

4.2.2. The effects of the aggressive behaviour on the congestion

When $\rho > \rho_{s1}$, the flow q starts decreasing with the density ρ but its slope $\frac{dq}{d\rho}$ remains very low. Its values are higher than the case of conservative drivers and of the standard NaSch model too (Fig. 13). The anticipation is however more important while more potential accidents appear: it is the congestion.

In fact, even though the anticipation is higher, the drivers can no longer operate at their desired speed (Fig. 12). The impact of the density is more present: the average gap is smaller in comparison to the free flow state. A competition between the anticipation that symbolizes the drivers' will to maintain their wanted speed and the density that represents the space appears.

To understand this competition, we will perform a microscopic spatio-temporal investigation.

In the following, we will consider Pe = 1 in order to make the anticipation a sure event that only depends on the spatial condition (the density) (see Section 3.4.2 for more details), in the aim to showcase their competition.

We consider ρ = 0.3 and we draw the corresponding space-time

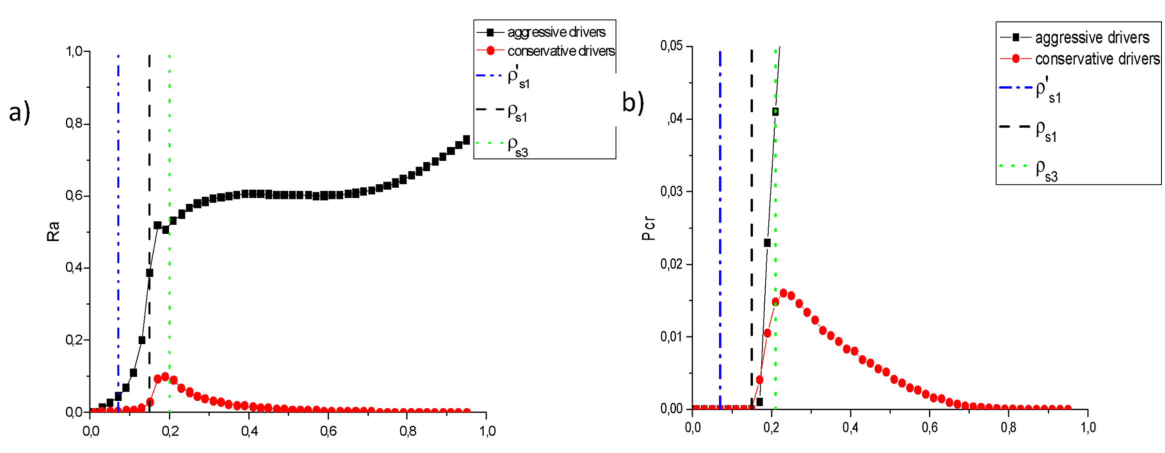

Fig. 10. a-ratio of anticipation Ra versus the density ρ of both aggressive and conservative drivers when Pe = 0.8, b-probability of rear-end collisions versus the density ρ of both aggressive and conservative drivers when Pe = 0.8.

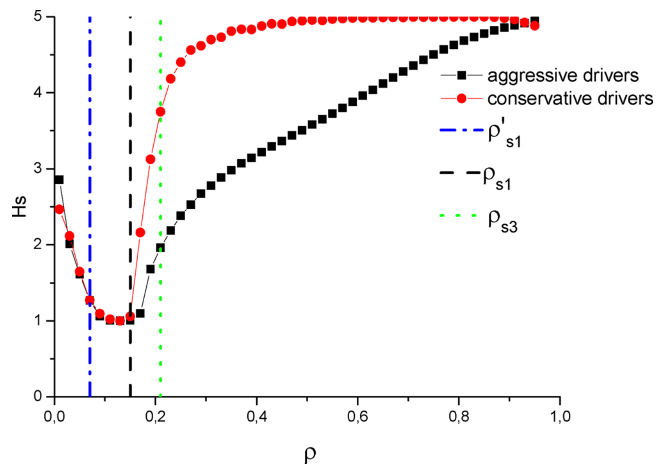

Fig. 11. mean learning threshold <Hs> of both aggressive and conservative drivers for Pe = 0.8 versus the density ρ.

diagram (Fig. 14).

The figure displays two types of lines:

- Vertical lines representing anticipating drivers: by the effect of the anticipation, when a driver empties a cell, the follower occupies it instantly.
- Inclined lines representing platoons.

As stated in the previous Sections 3.4.3 and 4.1, an anticipating driver may face a potential accident which will force him/her to brutally brake. This incident, as shown in Fig. 14 creates a small jam or platoon that propagates in time and space in the opposite direction of the traffic and of which the slope varies in time (nonlinear behaviour). At an undefined instant, this mini jam, that is usually observed in empirical time space diagrams (Treiterer, 1975), dissociates and numerous and successive anticipations occur (vertical lines in Fig. 14). Those same anticipations may lead to more potential accidents and so on. This is a retroactive chain of reaction (anticipation -> accident -> platoon -> anticipation).

This phenomenon is characterized by the competition between the anticipation and the platoons' creation persists until $\rho=\rho_{s2}\approx 0.6$.

4.2.3. The jamming phase

When $\rho > \rho_{s2}$, the average gap tends to none and the anticipation does not improve significantly the velocity of the drivers (Fig. 12b). The density wins the competition over the anticipation, and the flow starts decreasing rapidly.

However, this situation brings major probable accidents since the anticipation is still existent because the learning condition is not respected, while no space is available.

4.2.4. The transition points

The Points $\rho=\rho_{s1}\approx 0.15$ and $\rho=\rho_{s2}\approx 0.6$ correspond to the transition points between the three state: free flow, congestion, and Jamming.

The space-time diagrams at those critical densities show the coexistence of two phases at a time:

$\rho\approx\ \rho_{s1}$: the coexistence of the free flow and the congestion: the appearance of two slopes, negative and positive (Fig. 15).

$\rho\approx\ \rho_{s2}$: the coexistence of the congestion that has nonlinear slope and the Jamming state that is defined by jams of which the slope is linear (Fig. 16).

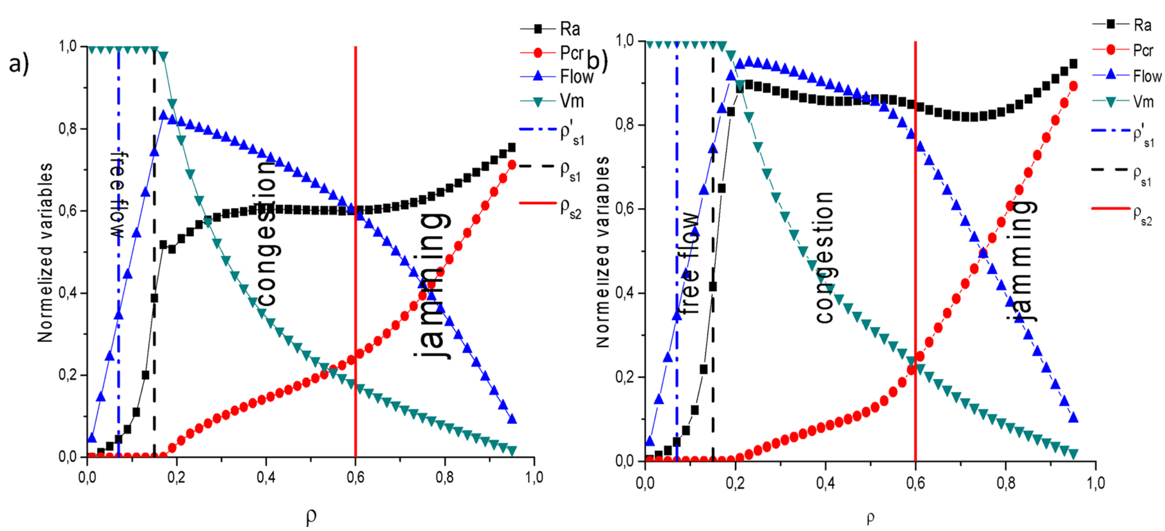

Fig. 12. different observables for a-Pe = 0.8 and b- Pe = 1: rate of anticipation Ra, probability of collisions Pcr, flow and normalized velocity (V/Vmax) versus the density ρ.

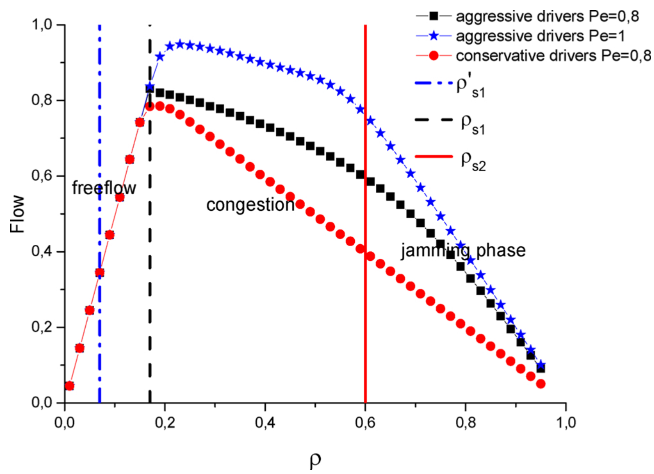

Fig. 13. the fundamental diagram of both types of drivers for Pe = 0.8 and Pe = 1 (flux versus density).

4.3. Heterogeneous traffic

The differences between the flow of the standard NaSch model and the flow of this cognitive anticipation model, in the congestion phase, shown in the Figs. 7 and 12, approach the scattering observed in the empirical fundamental diagram. This brings as to speculate that the heterogeneity of the traffic as far as the drivers' behaviours are concerned may be partly responsible, as advanced in (Ez-Zahraouy et al., 2006; Jetto et al., 2010).

In this respect, we propose this third variant of the algorithm in which the two types of drivers are present in the road that will offer a more realistic representation of the traffic. We define a new probability Pa that designates the expected behaviour of a driver: if the random number is less than Pa, the driver will anticipate as an aggressive driver. If not, he/she will be a conservative driver.

According to the Fig. 17, we observe that the evolution of the flow varies with respect to Pa. Indeed, for low densities, the flow plot evolves from a parabolic shape to a linear shape.

To understand this behaviour, we plot the flow, the accident's and the anticipation's rates for $\rho = 0.2$ (Fig. 18a).

In fact, when Pa is very low, the appearance of potential accidents combined with very little anticipation leads to a decrease of the flow. When Pa reaches a threshold Pa_{th}, the probability of accidents decreases while the anticipation keeps increasing. This brings an amelioration of the flow. These same observations hold for $\rho = 0.4$ with a difference in Pa_{th} (Fig. 18b).

The Fig. 19a displays the variation of Pa_{th} in function of the density. This variation of Pa_{th} shows that there is no standard dangerous mixture of drivers that can be held responsible of the most potential accidents. The density of vehicles is the one that determines degree of risks that can face a mixed fleet.

Therefore, to guarantee a safe and fluid circulation, a combination of strong anticipation (aggressive drivers that do not respect the learning condition) and a relatively low density should be respected (Fig. 18a). However, this cannot be achieved without the contribution of the V2V technologies.

To generalize those observations on the behaviour of the flow, we plot the flow for a random Pa and a random Pr.

The Fig. 19b confirms that the heterogeneity of the drivers contributes in the wide scattering of the fundamental diagram in the

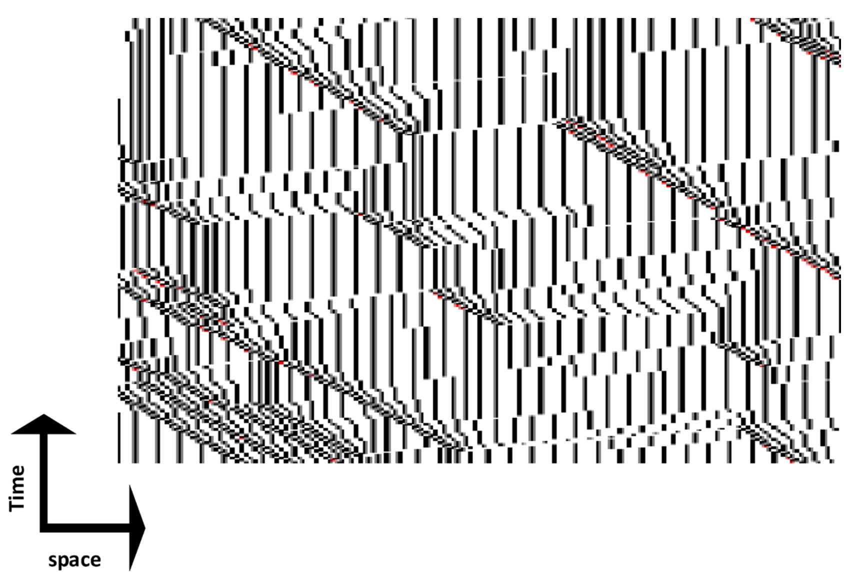

Fig. 14. space-time diagram for $\rho = 0.3$: red cells represent potential rear-end collisions, vertical lanes represent anticipations, the black for full cell (vehicle) and the white for an empty cell.

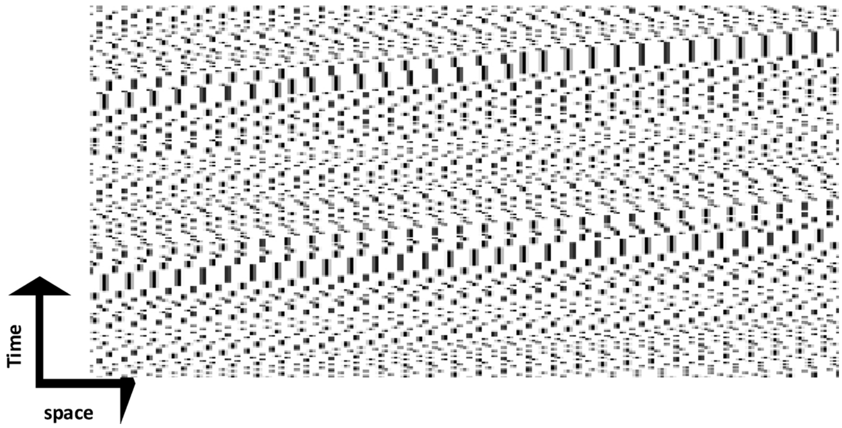

Fig. 15. Space-time diagram for ρ = 0.15: vertical lanes represent anticipations, the black for full cell (vehicle) and the white for an empty cell.

congestion phase.

A similar finding was developed by Sakai et al. when they have found that varying deceleration and acceleration probabilities generates a variation in the magnitude of the scattered area in a fundamental phase diagram (Sakai et al., 2006).

4.4. Hot spots

Behind knowing the causes of accidents, defining the region where potential accidents may take place is also crucial.

In this respect, we adopt a road of 500 cells and we insert a perturbation zone at 400–405 with a deceleration rate Pr = 0.5. We run the simulation and we observe the spatial distribution of accidents for different densities.

We only consider conservative drivers with Pe = 1.

We can observe that the spatial distribution of accidents follows the Gaussian law. The centre of the plot moves when we increase the density (Figs. 20 and 21).

The perturbation zone creates a deceleration wave that generates a high density (Fig. 22). The dissipation of this wave generates an interface between the low and high density. This causes numerous potential accidents.

It would then more beneficial to the zone of the perturbation than the zone of the accidents.

5. Conclusion

Modern societies are increasingly facing complex problems. Improving the mobility with zero accident is one of these problems.

It is well known that complex systems constitute a big challenge for scientists since they display emergent behaviour that cannot be predicted from knowing microscopic details of interactions of the constituent parts (Knowing the behaviour of a single neuron or how neurons interact does not lead to understand the behaviour of the brain).

Without doubt, traffic flow is one of the most studied complex system. The plenitude of interactions (driver to driver, driver with his/her environment) has made modelling insufficient for forecasting and control. Thus, it is of paramount importance to develop new approaches and theories. In this paper, we have tried to combine the tools of the physics of traffic (statistical mechanics) with cognitive psychology to understand how a driver's behaviour influences the traffic and vice versa.

Indeed, to analyse the safety of traffic facilities, scientists have opted for the surrogate safety measures founded models as an alternative approach to the statistical based methods. Through numerical simulations, they were able to reproduce different types of accidents by defining new parameters or incorporating additional movement steps that may be behind those incidents. However, few of these models, if not none, have considered the human behaviour when modelling collisions.

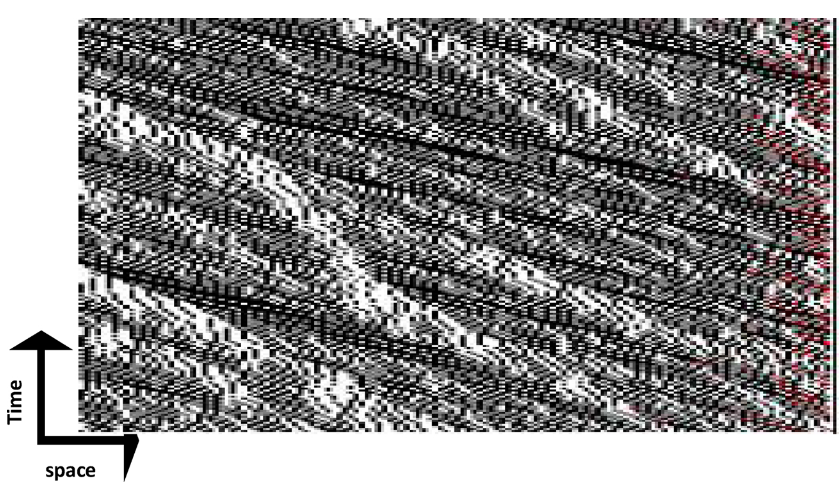

Fig. 16. Space-time diagram for ρ = 0.6: red cells represent potential rear-end collisions, vertical lanes represent anticipations, the black for full cell (vehicle) and the white for an empty cell.

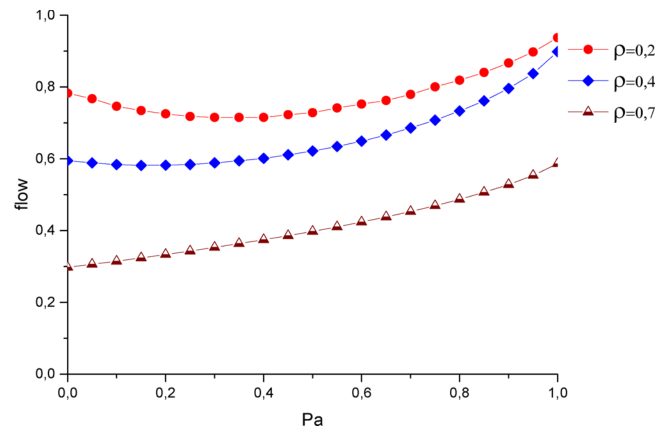

Fig. 17. variation of the flow as a function of Pa that represents the fraction of aggressive drivers for different densities ρ.

For this purpose, through the means proposed by the statistical mechanics, we developed a new cellular automata model, inspired by field experiments, in which we implemented, in addition to the regular rules of movement, the anticipation. Indeed, based on a cognitive psychological model that we named "cognitive anticipation model", we defined the anticipation by the tendency of drivers, based on historical data of the leading vehicles, to or not to accelerate. This dilemma sectioned our article so as two study two types of drivers:

- Conservative drivers: those drivers take their decision on anticipation by judging the behaviour of their predecessor. Considering only this type of chauffeurs represents a realistic schema as far as the security aspect is concerned, since the probability of accidents is very weak and follows the Gaussian law. Moreover, those potential rear-end collisions are strongly correlated to the anticipation. Indeed, they only occur in a limited interval of densities in which the anticipation is present. This range of densities happens to correspond to the congestion state. Thus, this model's case is able to delimit the scope of the congestion state that stimulates the anticipation and may cause collisions. But more importantly, this configuration highlights the complexity of the relationship between the state of traffic and the driver response as shown by the variation of the mean learning threshold plot. Indeed, the transitions of the traffic states generate a self-organization of the drivers; either into a hazardous state ($<Hs> = 1$) that may generate accidents or into a safe state ($<Hs> = 5$).
Then, the anticipation represents the response of the driver, as an individual to a general situation that forced him/her to go against his/her will. This response can have major consequences: in this case, the congestion obliged him/her to decelerate and to not adopt his / her desired speed, which urged him/her to anticipate and probably cause a collision.
- Aggressive drivers: who choose to overlook the historic of the leading vehicle and accelerate despite their analysis. Even though representing a greater risk in the higher densities, the investigation of this range of drivers has shown that allowing the existence of a small risk of anticipation would allow a significant amelioration of the flow in the congested state (up to 11 % compared to the standard case of NaSch). The problem that prevails at this stage is the fact that those drivers may compromise the security of the traffic.
The dilemma of the fluidity/ security appears again but can be settled through the vehicle-to-vehicle communication technologies (V2V).

In addition, in the attempt to reproduce a real traffic situation, we studied next, a traffic flow with a mixture of both conservative and aggressive drivers. This investigation displays a scattering in the fundamental diagram. This statement joins previous statements that

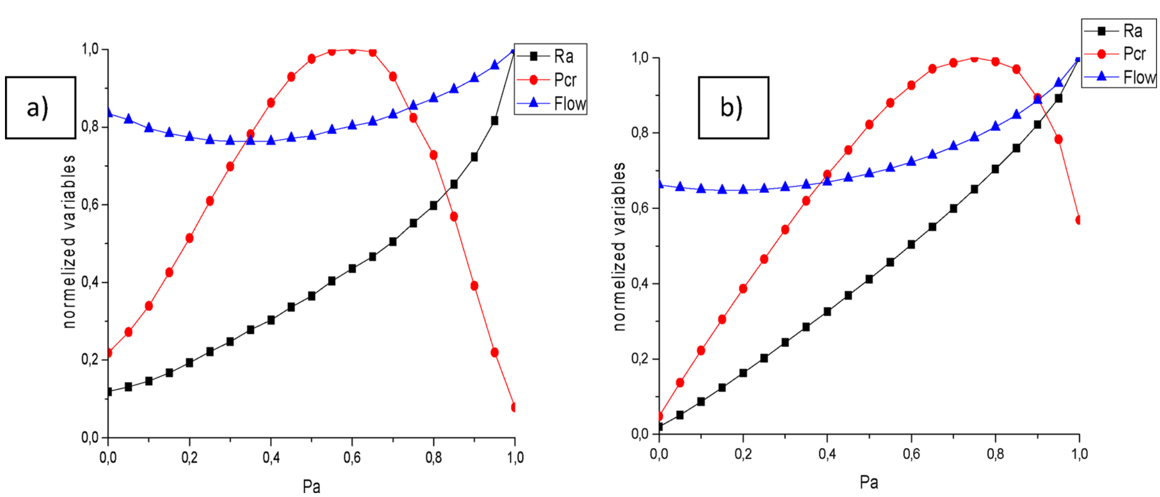

Fig. 18. variation of the flow, probability of rear-end collisions Pcr/Pcrmax and rate of anticipation Ra/Ramax as a function of Pa for a) ρ = 0.2, b) ρ = 0.4.

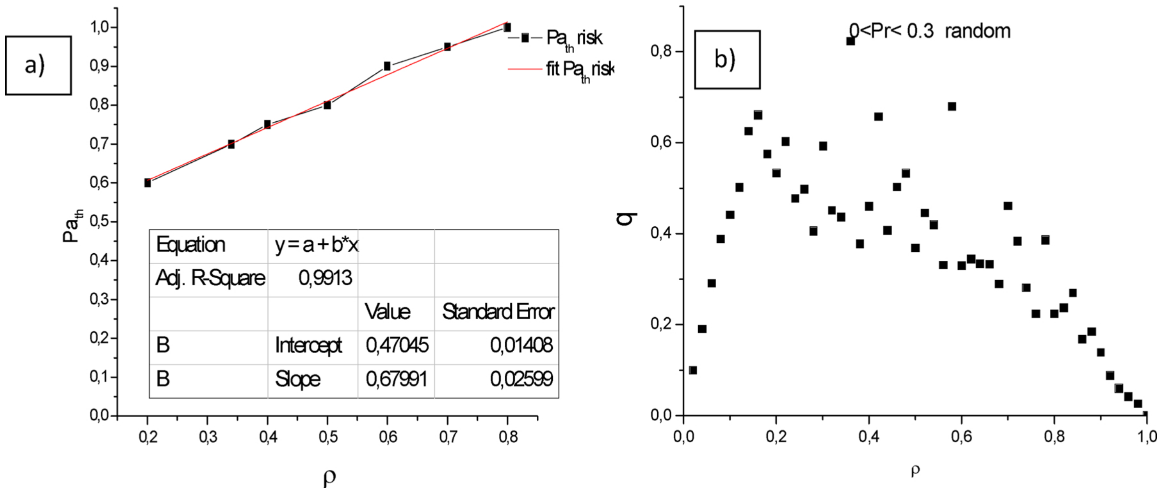

Fig. 19. a) variation of Pa_{th} in function of the density ρ; b) fundamental diagram for random Pa: ratio of aggressive drivers, and random Pr: probability of random braking ($0 < Pr < 0.3$).

proposed that the heterogeneity of the traffic flow might be among of the causes behind the scattering observed in the experimental fundamental diagram.

Furthermore, we inspected the features of hot spots. We found that to resolve this problem, measures should be taken at the perturbation zones instead of the accidents areas, since the latter strongly depend of the density. Moreover, this finding has highlighted the complexity of the information transmission and reception while driving.

Actually, the anticipation can be reduced to the simple fact that a driver would expect the leading driver to free his/her position at the following instant. This prediction is generally the result of a long process of learning and is expected to always be successful. However, sometimes, anticipation leads to collisions. In fact, each vehicle $\boldsymbol{I}$ only reacts with respect to the vehicle $\boldsymbol{I+1}$ in front of him/her, the vehicle $\boldsymbol{I+1}$ with $\boldsymbol{I+2}$ and ongoing to $\boldsymbol{I+n}$ with $\boldsymbol{I+n+1}$. Thus, if a perturbation occurs at the position of $\boldsymbol{I+n+1}$ at an instant $\boldsymbol{t}$, a shock wave of which the speed is unknown for the drivers, would form at this same position $\boldsymbol{I+n+1}$ and the same instant $\boldsymbol{t}$. It is then impossible to predict when and where this information (deceleration) wave would reach the vehicle $\boldsymbol{I}$. Subsequently, this uncertainty makes the learning process limited in space and time, which generates rear-end collisions.

It is worth to mention that we have based our investigation on only one type of anticipation, which is the anticipation by acceleration that we have considered as a major cause of rear-end collisions. However, even if this investigation has brought into the evidence interesting information about the mechanism behind the accidents occurrence, conducting extensive empirical studies to confirm our assumption seem to be an interesting field of research as far as the security and the fluidity aspects of the traffic are concerned.

CRediT authorship contribution statement

Kamal Jetto: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Writing - original draft, Writing - review & editing, Visualization, Supervision. **Zineb Tahiri:** Software, Validation, Formal analysis, Investigation, Writing - original draft, Writing - review & editing, Visualization. **Abdelilah Benyoussef:** Supervision. **Abdallah El Kenz:** Supervision, Project administration.

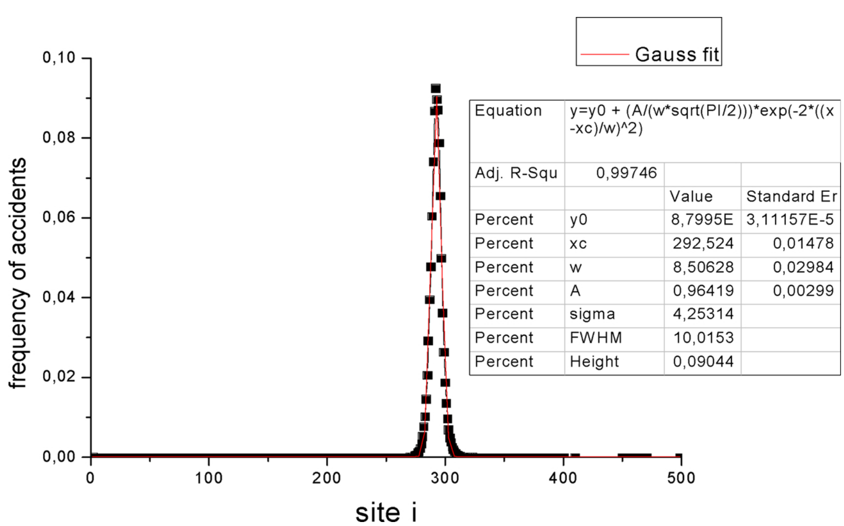

Fig. 20. spatial distribution of accidents for ρ = 0.2.

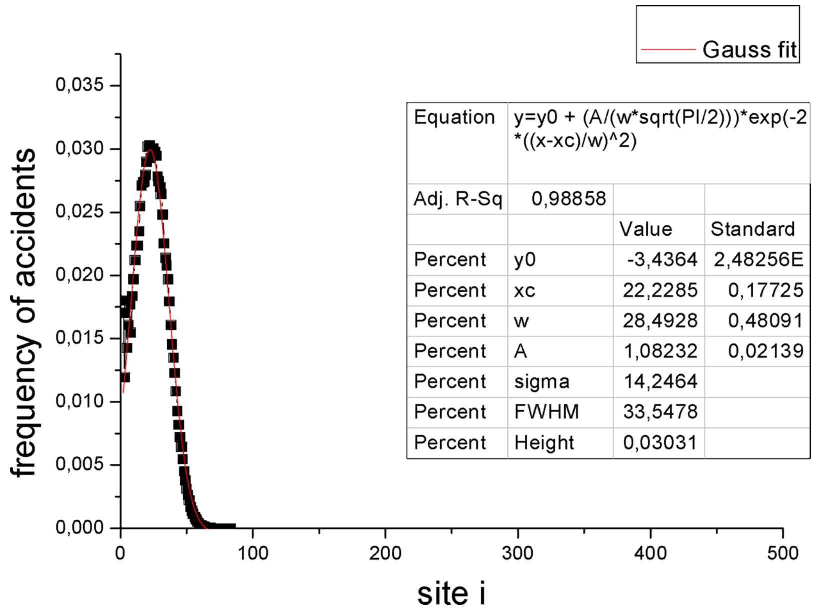

Fig. 21. spatial distribution of accidents for ρ = 0.5.

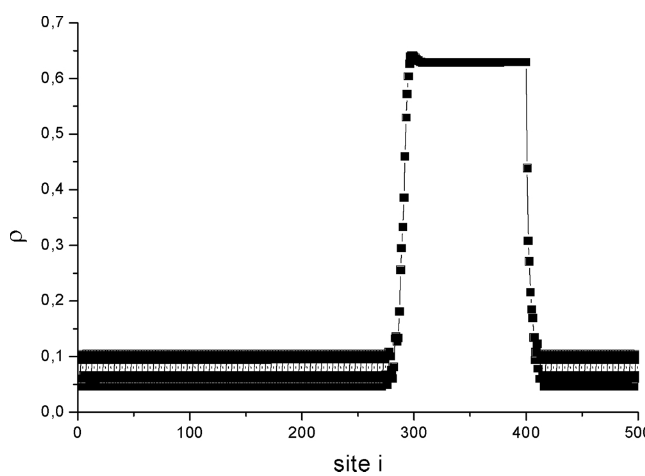

Fig. 22. density profile for ρ = 0.2.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary material related to this article can be found, in the online version, at doi:https://doi.org/10.1016/j.aap.2020.105507.

References

Ansari, S., Akhdar, F., Mandoorah, M., Moutaery, K., 2000. Causes and Effects of Road Traffic Accidents in Saudi Arabia. pp. 3.

Appert-Rolland, C., 2009. Experimental study of short-range interactions in vehicular traffic. Phys. Rev. E 80. https://doi.org/10.1103/PhysRevE.80.036102.

Archer, J., Kungl. Tekniska högskolan, Institutionen för infrastruktur, 2005. Indicators for Traffic Safety Assessment and Prediction and Their Application in Micro-Simulation Modelling: a Study of Urban and Suburban Intersections. Stockholm. .

Bener, A., Crundall, D., 2005. Road Traffic Accidents in the United Arab Emirates Compared to Western Countries. pp. 9.

Bishai, D., Quresh, A., James, P., Ghaffar, A., 2006. National road casualties and economic development. Health Econ. 15, 65–81. https://doi.org/10.1002/hec.1020.

Bucchi, A., Sangiorgi, C., Vignali, V., 2012. Traffic psychology and driver behavior. Procedia - Soc. Behav. Sci. 53, 972–979. https://doi.org/10.1016/j.sbspro.2012.09.946.

Chai, C., Wong, Y.D., Wang, X., 2017. Safety evaluation of driver cognitive failures and driving errors on right-turn filtering movement at signalized road intersections based on Fuzzy Cellular Automata (FCA) model. Accid. Anal. Prev. 104, 156–164. https://doi.org/10.1016/j.aap.2017.04.026.

Chatterjee, I., 2016. Understanding Driver Contributions to Rear-End Crashes on Congested Freeways and Their Implications for Future Safety Measures. A Thesis Submitted to the Faculty of University of Minnesota. pp. 149.

Cheng, W., Washington, S.P., 2005. Experimental evaluation of hotspot identification methods. Accid. Anal. Prev. 37, 870–881. https://doi.org/10.1016/j.aap.2005.04.015.

Chowdhury, D., 2000. Statistical physics of vehicular traffic and some related systems.

Phys. Rep. 329, 199–329. https://doi.org/10.1016/S0370-1573(99)00117-9.
D'Addario, P., Donmez, B., 2019. The effect of cognitive distraction on perception-response time to unexpected abrupt and gradually onset roadway hazards. Accid. Anal. Prev. 127, 177–185. https://doi.org/10.1016/j.aap.2019.03.003.
Daganzo, C.F., 1995. The cell transmission model, part II: network traffic. Transp. Res. Part B Methodol. 29, 79–93. https://doi.org/10.1016/0191-2615(94)00022-R.
Elander, J., West, R., French, D., 1993. Behavioral Correlates of Individual Differences in Road-Traffic Crash Risk: An Examination of Methods and Findings. pp. 16.
Elvik, R., 2014. Problems in determining the optimal use of road safety measures. Res. Transp. Econ. 47, 27–36. https://doi.org/10.1016/j.retrec.2014.09.016.
Elvik, R., Sorensen, M., Vaa, T., Hoye, A., 2009. The Handbook of Road Safety Measures [WWW Document]. URL https://www.bookdepository.com/Handbook-Road-Safety-Measures-Rune-Elvik/9781848552500 (Accessed 11.3.18). .
Evans, L., 2004. Traffic Safety.
Ez-Zahraouy, H., Jetto, K., Benyoussef, A., 2006. New topologies in the fundamental diagram of a cellular automata traffic flow model. Chin. J. Phys. 44.
Fan, R., Wang, W., Liu, P., Yu, H., 2013. Using VISSIM simulation model and Surrogate Safety Assessment Model for estimating field measured traffic conflicts at freeway merge areas. IET Intell. Transp. Syst. 7, 68–77. https://doi.org/10.1049/iet-its.2011.0232.
Geurts, K., Thomas, I., Wets, G., 2005. Understanding spatial concentrations of road accidents using frequent item sets. Accid. Anal. Prev. 37, 787–799. https://doi.org/10.1016/j.aap.2005.03.023.
Golob, T.F., Recker, W.W., Alvarez, V.M., 2004. Freeway safety as a function of traffic flow. Accid. Anal. Prev. 36, 933–946. https://doi.org/10.1016/j.aap.2003.09.006.
Hamdar, S., 2012. Driver behavior modeling. In: Eskandarian, A. (Ed.), Handbook of Intelligent Vehicles. Springer, London, London, pp. 537–558. https://doi.org/10.1007/978-0-85729-085-4_20.
Hamdar, S.H., Treiber, M., Mahmassani, H.S., Kesting, A., 2008. Modeling driver behavior as sequential risk-taking task. Trans. Res. Rec.: J. Trans. Res. Board 2088, 208–217. https://doi.org/10.3141/2088-22.
Helbing, D., 2001. Traffic and Related Self-driven Many-particle Systems. American Physical Society.
Jamroz, K., 2008. Review of road safety theories and models. J. Konbin 4, 89–108. https://doi.org/10.2478/v10040-008-0012-z.
jean.yoder.ctr@dot.gov, 2016. Vehicle-to-Vehicle Communication [WWW Document]. NHTSA. URL https://www.nhtsa.gov/technology-innovation/vehicle-vehicle-communication (Accessed 1.30.19).
Jetto, K., Ez-Zahraouy, H., Benyoussef, A., 2010. The effect of the heterogeneity on the traffic flow behavior. Int. J. Mod. Phys. 21, 1311–1327. https://doi.org/10.1142/S0129183110015865.
Jones, R.K., Joscelyn, K.B., 1976. Alcohoi, and Highway Safety 1978: a Review of the State of Knowledge. pp. 226.
Jun-Fang, T., Bin, J., Xin-Gang, L., Zi-You, G., 2010. A new car-following model considering velocity anticipation. Chin. Phys. B 19, 010511–010517. https://doi.org/10.1088/1674-1056/19/1/010511.
Kalra, N., Paddock, S.M., 2016. Driving to Safety: How Many Miles of Driving Would It Take to Demonstrate Autonomous Vehicle Reliability? pp. 15.
Kerner, B.S., 2004. The Physics of Traffic: Empirical Freeway Pattern Features, Engineering Applications, and Theory, Understanding Complex Systems. Springer-Verlag, Berlin Heidelberg.
Klunder, G., Abdoelbasier, A., Immers, B., 2006. Development of a Micro-Simulation Model to Predict Road Traffic Safety on Intersections with Surrogate Safety Measures. pp. 8.
Knospe, W., Santen, L., Schadschneider, A., Schreckenberg, M., 2000. Towards a realistic microscopic description of highway traffic. J. Phys. A Math. Gen. 33, L477–L485. https://doi.org/10.1088/0305-4470/33/48/103.
Kontogiannis, T., Kossiavelou, Z., Marmaras, N., 2002. Self-reports of aberrant behaviour on the roads: errors and violations in a sample of Greek drivers. Accid. Anal. Prev. 34, 381–399. https://doi.org/10.1016/S0001-4575(01)00035-5.
Kopits, E., Cropper, M., 2005. Traffic fatalities and economic growth. Accid. Anal. Prev. 37, 169–178. https://doi.org/10.1016/j.aap.2004.04.006.
Kraay, J.H., 1983. REVIEW-of-traffic-flow-conflict.
Lakouari, N., Jetto, K., Ez-Zahraouy, H., Benyoussef, A., 2014. Velocity correlations in the Nagel–Schreckenberg model. Int. J. Mod. Phys. C 25, 1350089. https://doi.org/10.1142/S0129183113500897.
Lamm, R., Psarianos, B., Mailaender, T., 1999. Highway Design and Traffic Safety Engineering Handbook.
Lárraga, M.E., del Río, J.A., Schadschneider, A., 2004. New kind of phase separation in a CA traffic model with anticipation. J. Phys. A Math. Gen. 37, 3769–3781. https://doi.org/10.1088/0305-4470/37/12/004.
Lee, K., Hui, P.M., Mao, D., Wang, B.-H., Wu, Q.-S., 2002. Fukui–Ishibashi traffic flow models with anticipation of movement of the car ahead. J. Phys. Soc. Jpn. 71, 1651–1654. https://doi.org/10.1143/JPSJ.71.1651.
Lee, C., Hellinga, B., Saccomanno, F., 2003. Real-time crash prediction model for application to crash prevention in freeway traffic. Trans. Res. Rec.: J. Trans. Res. Board 1840, 67–77. https://doi.org/10.3141/1840-08.
Li, Z., Ahn, S., Chung, K., Ragland, D.R., Wang, W., Yu, J.W., 2014. Surrogate safety measure for evaluating rear-end collision risk related to kinematic waves near freeway recurrent bottlenecks. Accid. Anal. Prev. 64, 52–61. https://doi.org/10.1016/j.aap.2013.11.003.
Li, X., Yan, X., Wu, J., Radwan, E., Zhang, Y., 2016. A rear-end collision risk assessment model based on drivers' collision avoidance process under influences of cell phone use and gender—a driving simulator based study. Accid. Anal. Prev. 97, 1–18. https://doi.org/10.1016/j.aap.2016.08.021.
Mannering, F.L., Bhat, C.R., 2014. Analytic methods in accident research: methodological frontier and future directions. Anal. Methods Accid. Res. 1, 1–22. https://doi.org/10.1016/j.amar.2013.09.001.
Migletz, D.J., Glauz, W.D., Bauer, K.M., 1985. Relationships between Traffic Conflicts and Accidents.
Nagel, K., Schreckenberg, M., 1992. A cellular automaton model for freeway traffic. J. Phys. I 2, 2221–2229. https://doi.org/10.1051/jp1:1992277.
National Research Council (U.S.) (Ed.), 2000. Highway Capacity Manual. Transportation Research Board, National Research Council, Washington, D.C.
Nishinari, K., Takahashi, D., 1999. A new deterministic CA model for traffic flow with multiple states. J. Phys. A Math. Gen. 32, 93–104. https://doi.org/10.1088/0305-4470/32/1/010.
Nishinari, K., Fukui, M., Schadschneider, A., 2004. A stochastic cellular automaton model for traffic flow with multiple metastable states. J. Phys. A Math. Gen. 37, 3101–3110. https://doi.org/10.1088/0305-4470/37/9/003.
Oh, C., Oh, J.-S., Ritchie, S.G., Chang, M., 2001. Real-Time Estimation of Freeway Accident Likelihood.
Ozbay, K., Yang, H., Bartin, B., Mudigonda, S., 2008. Derivation and validation of new simulation-based surrogate safety measure. Trans. Res. Rec.: J. Trans. Res. Board 2083, 105–113. https://doi.org/10.3141/2083-12.
Parker, M.R., Zegeer, C.V., 1989. Traffic Conflict Techniques for Safety and Operations–Observers Manual.
Peng, G., Cheng, R., 2013. A new car-following model with the consideration of anticipation optimal velocity. Phys. A Stat. Mech. Appl. 392, 3563–3569. https://doi.org/10.1016/j.physa.2013.04.011.
Reason, J., Manstead, A., Stradling, S., Baxter, J., Campbell, K., 1990. Errors and violations on the roads: a real distinction? Ergonomics 33, 1315–1332. https://doi.org/10.1080/00140139008925335.
Retallack, A.E., Ostendorf, B., 2019. Current understanding of the effects of congestion on traffic accidents. Int. J. Environ. Res. Public Health 16, 3400. https://doi.org/10.3390/ijerph16183400.
Road Traffic Accidents Involving Personal Injury by Variable, Nature ofAccident, Country and Year, 2015. Road Traffic Accidents Involving Personal Injury by Variable, Nature ofAccident, Country and Year [WWW Document]. United Nations Economic Commission for Europe. URL http://w3.unece.org/PXWeb2015PXWeb2015/pxweb/en/STAT/STAT__40-TRTRANS__01-TRACCIDENTS/07_en_TRAccNature_r.px/ (Accessed 11.10.18).
Saifuzzaman, M., Zheng, Z., 2014. Incorporating human-factors in car-following models: a review of recent developments and research needs. Transp. Res. Part C Emerg. Technol. 48, 379–403. https://doi.org/10.1016/j.trc.2014.09.008.
Sakai, S., Nishinari, K., Iida, S., 2006. A new stochastic cellular automaton model on traffic flow and its jamming phase transition. J. Phys. A Math. Gen. 39, 15327–15339. https://doi.org/10.1088/0305-4470/39/50/002.
Schadschneider, A., Chowdhury, D., Nishinari, K., 2011. Stochastic Transport in Complex Systems: From Molecules to Vehicles. Elsevier, Amsterdam.
Skog, O.-J., 2001. Alcohol consumption and mortality rates from traffic accidents, accidental falls, and other accidents in 14 European countries. Addiction 96, 49–58. https://doi.org/10.1080/09652140020021170.
Stephens, A.N., Koppel, S., Young, K.L., Chambers, R., Hassed, C., 2018. Associations between self-reported mindfulness, driving anger and aggressive driving. Transp. Res. Part F Traffic Psychol. Behav. 56, 149–155. https://doi.org/10.1016/j.trf.2018.04.011.
Tak, S., Kim, S., Lee, D., Yeo, H., 2018. A comparison analysis of surrogate safety measures with car-following perspectives for advanced driver assistance system. J. Adv. Transp. 2018, 1–14. https://doi.org/10.1155/2018/8040815.
Theofilatos, A., Yannis, G., 2014. A review of the effect of traffic and weather characteristics on road safety. Accid. Anal. Prev. 72, 244–256. https://doi.org/10.1016/j.aap.2014.06.017.
75 Years of the Fundamental Diagram for Traffic Flow Theory: Greenshields Symposium. In: Transportation Research Board (Ed.), pp. 246.
Treiterer, J., 1975. Investigation of Traffic Dynamics by Aerial Photogrammetry Techniques. Final Report EES278. Transportation Research Center, Department of Civil Engineering, Ohio State University.
van Wageningen-Kessels, F., van't Hof, B., Hoogendoorn, S.P., van Lint, H., Vuik, K., 2013. Anisotropy in generic multi-class traffic flow models. Transp. A Transp. Sci. 9, 451–472. https://doi.org/10.1080/18128602.2011.596289.
Voas, R.B., Lacey, J.C., 2011. Alcohol and Highway Safety 2006: A Review of the State of Knowledge.
Wang, C., Quddus, M.A., Ison, S.G., 2013. The effect of traffic and road characteristics on road safety: a review and future research direction. Saf. Sci. 57, 264–275. https://doi.org/10.1016/j.ssci.2013.02.012.
Willumsen, L.G., 2011. Department of Transport Engineering and Logistics Pontificia Universidad Católica de Chile Santiago Chile. SPH P 588. .
Xu, C., Liu, P., Wang, W., Li, Z., 2012. Evaluation of the impacts of traffic states on crash risks on freeways. Accid. Anal. Prev. 47, 162–171. https://doi.org/10.1016/j.aap.2012.01.020.
Xu, C., Liu, P., Wang, W., Jiang, X., 2013a. Development of a crash risk index to identify real time crash risks on freeways. KSCE J. Civ. Eng. 17, 1788–1797. https://doi.org/10.1007/s12205-013-0353-6.

Xu, C., Tarko, A.P., Wang, W., Liu, P., 2013b. Predicting crash likelihood and severity on freeways with real-time loop detector data. Accid. Anal. Prev. 57, 30–39. https://doi.org/10.1016/j.aap.2013.03.035.

Xu, C., Liu, P., Yang, B., Wang, W., 2016. Real-time estimation of secondary crash likelihood on freeways using high-resolution loop detector data. Transp. Res. Part C Emerg. Technol. 71, 406–418. https://doi.org/10.1016/j.trc.2016.08.015.

Yan, X., Abdel-Aty, M., Radwan, E., Wang, X., Chilakapati, P., 2008. Validating a driving simulator using surrogate safety measures. Accid. Anal. Prev. 40, 274–288. https://doi.org/10.1016/j.aap.2007.06.007.

Yang, H., Ozbay, K., 2010. Application of Simulation-Based Traffic Conflict Analysis for Highway Safety Evaluation. pp. 14.

Yeo, H., Jang, K., Skabardonis, A., Kang, S., 2013. Impact of traffic states on freeway crash involvement rates. Accid. Anal. Prev. 50, 713–723. https://doi.org/10.1016/j.aap.2012.06.023.

Young, W., Sobhani, A., Lenné, M.G., Sarvi, M., 2014. Simulation of safety: a review of the state of the art in road safety simulation modelling. Accid. Anal. Prev. 66, 89–103. https://doi.org/10.1016/j.aap.2014.01.008.

Zajíc, P., 2012. Traffic Conflicts and Road Transport Safety New Development. pp. 10.

Zheng, L.-J., Tian, C., Sun, D.-H., Liu, W.-N., 2012. A new car-following model with consideration of anticipation driving behavior. Nonlinear Dyn. 70, 1205–1211. https://doi.org/10.1007/s11071-012-0524-x.