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Calibration of crash risk models on freeways with limited real-time traffic data using Bayesian meta-analysis and Bayesian inference approach

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ABSTRACT

This study aimed to develop a real-time crash risk model with limited data in China by using Bayesian meta-analysis and Bayesian inference approach. A systematic review was first conducted by using three different Bayesian meta-analyses, including the fixed effect meta-analysis, the random effect meta-analysis, and the meta-regression. The meta-analyses provided a numerical summary of the effects of traffic variables on crash risks by quantitatively synthesizing results from previous studies. The random effect meta-analysis and the meta-regression produced a more conservative estimate for the effects of traffic variables compared with the fixed effect meta-analysis. Then, the meta-analyses results were used as informative priors for developing crash risk models with limited data. Three different meta-analyses significantly affect model fit and prediction accuracy. The model based on meta-regression can increase the prediction accuracy by about 15% as compared to the model that was directly developed with limited data. Finally, the Bayesian predictive densities analysis was used to identify the outliers in the limited data. It can further improve the prediction accuracy by 5.0%.

1. Introduction

Dynamic safety management systems provide a proactive way to improve traffic safety on freeway mainlines. The first and most important step in dynamic safety management is the identification of hazardous traffic conditions with high crash likelihood. To this end, numerous studies have developed crash risk models to investigate the relationship between crash risks and real-time traffic variables (Abdel-Aty et al., 2004, 2005, 2007, 2012a,b; Abdel-Aty and Pande, 2005, 2006; Abdel-Aty and Pemmanaboina, 2006; Lee and Abdel-Aty, 2008; Pande and Abdel-Aty, 2005; Hassan and Abdel-Aty, 2011, 2013; Ahmed and Abdel-Aty, 2011; Hossain and Muromachi, 2010; Zheng et al., 2010; Ahmed et al., 2012; Li et al., 2013, 2014; Xu et al., 2012a,b, 2013a,b, 2014, 2015; Yu and Abdel-Aty, 2013a). These models are used to predict the crash risks on freeway segments over a short time period, such as 5 min. The development of crash risk models usually requires high quality and quantity of real-time traffic data. The used traffic data are collected from freeway surveillance system for quite a long time, such as one or two years. The crash risk models that are directly developed with limited data cannot well capture the relationship between crash risks and traffic flow characteristics. Previous studies suggested that this kind of data can result in biased model estimation results and inadequate predictive performance (Lord, 2006; Lord and Miranda-Moreno, 2008; Xu et al., 2014).

When developing the crash risk models, the problem of limited real-time traffic data may occur. For example, the traffic surveillance equipment has just been placed on a freeway, or the local transportation agencies are not expected to store all the historic real-time traffic data. To develop a crash risk model for a freeway on which only limited data are available, one possible method is based on Bayesian inference approach (Lord and Miranda-Moreno, 2008; Khondakar et al., 2010; Xu et al., 2014; Yu and Abdel-Aty, 2013b). The central idea of Bayesian inference approach is to combine the information from observed data with the prior information from additional knowledge that does not depend on the observed data. The Bayesian inference approach achieves this by adding a prior distribution of the parameters in the model. Most of the existing crash risk models based on the Bayesian inference approach used the non-informative priors, as the dataset in these studies were

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enough to establish an accurate relationship between crash risks and real-time traffic variables. However, when the sample size of the dataset is small, the informative priors with plausible knowledge can increase the accuracy of model estimation and predictive performance (Lord and Miranda-Moreno, 2008; Khondakar et al., 2010; Hadayeghi et al., 2006; Xu et al., 2014).

Recently, several studies have been conducted to developed crash frequency model with limited data using the Bayesian inference method with informative priors (Lord and Miranda-Moreno, 2008; Khondakar et al., 2010; Hadayeghi et al., 2006). For example, Lord and Miranda-Moreno found that the Bayesian inference approach can greatly minimize the risk of inaccurate model estimation caused by limited data when an appropriate informative prior distribution is used (Lord and Miranda-Moreno, 2008). The study conducted by Hadayeghi et al. (2006) suggested that, when limited data are available, the accident prediction models based on the informative priors can produce better prediction accuracy than the model that was directly developed using the limited data. Most of these studies focused on developing the aggregate crash frequency models when only limited data are available. However, relatively fewer studies have considered how to develop real-time crash risk models with limited data. Besides, most of the previous studies developed the informative priors from a single model for another freeway (Hadayeghi et al., 2006; Xu et al., 2014). The disadvantage of this method is that the inaccurate informative priors from one single freeway may affect the accuracy of the model estimation.

This study fills the gap in developing the informative priors for the real-time crash risk models by using the Bayesian meta-analysis. The meta-analysis is a quantitative method that combines the results from numerous studies about real-time crash risk assessment and produces a more accurate estimate for the effects of traffic flow variables. The primary objective of this study is to develop a real-time crash risk model with limited data in China by using the Bayesian meta-analysis and the Bayesian inference method. More specifically, this study first conducted a systematic review of previous literature about real-time crash assessment using the Bayesian meta-analysis. Three different Bayesian meta-analyses, including the fixed effect meta-analysis, the random effect meta-analysis and the meta-regression, were used to formulate the informative priors. Then, the Bayesian inference method was used to develop the crash risk models based on the informative priors obtained from the three Bayesian meta-analyses. The effects of three Bayesian meta-analyses on the accuracy of model estimation were examined. Finally, the Bayesian predictive densities analysis for identifying the outliers in the limited data has been introduced and its effects on the accuracy of model estimation were examined.

2. Data sources

2.1. Data from systematic review of literature

A literature search for relevant studies published from 2001 to 2015 was conducted using several databases, including the transportation research international documentation (TRID) database, the Sciencedirect database, the Springer database, the Taylor and Francis database, and the Wiley database. In general, "real-time crash risk", "crash likelihood", "traffic flow", "traffic data", and "loop detector data" were used as the search terms.

The initial literature review indicated that 114 published papers used the high-resolution traffic flow data to conduct real-time crash risk assessment. These papers were selected for further examination in detail, of which 36 were included in the meta-analyses. The following criteria were used when selecting papers for the meta-analyses:

(1) The selected studies should focus on the general crashes instead of specific types of crashes. The studies focusing on specific crash types were excluded.
(2) The selected studies should report the effects of traffic variables as logarithms of odds ratios or odds ratios. The crash risks defined in the selected studies are consistent. Numerous studies using artificial intelligent (AI) or machine learning technique were excluded, because most of the AI models work as black-boxes. They cannot provide the estimated effects of traffic variables as logarithms of odds ratios or odds ratios.
(3) The traffic flow variables in the included studies should be aggregated during 5-min time interval. Because this study aimed to develop a real-time crash risk model based on the traffic data with 5-min aggregation interval on the freeway in China. Besides, most of the previous studies about real-time crash risk assessment aggregated traffic flow data at 5-min level. And they should also be observed in the time interval between 5 and 10 min prior to the crash occurrence time.
(4) The traffic flow variables in the included studies should be at the same location with respect to crash site (i.e., upstream or downstream).

To conduct meta-analyses, the following type of information was extracted from each selected study:

(1) Publication year. Since the real-time crash risk assessment is an emerging field in traffic safety analysis, the selected papers were published from 2001 to 2015.
(2) The freeway and country where the data were collected. Most of the studies were conducted in the United States. Only a few studies were conducted in other countries, including Korea, Japan, Belgium, and China.
(3) Considered traffic flow characteristics. Different traffic variables for measuring traffic conditions in previous studies were extracted, including the means and standard deviations of traffic flow measurements, the difference in traffic flow measurements between adjacent lanes, as well as the difference in traffic flow measurements between upstream and downstream stations.
(4) The estimator of the effects of traffic variables. The estimated coefficient and the standard error of the coefficient for each traffic flow variable were extracted from each study. Note that some studies only reported the odds ratio and the standard error of the odds ratio. In this case, the coefficient was calculated as the logarithm of the odds ratio; and the standard error of the coefficient is calculated using the following equation (Ntzoufras, 2009):

$$\hat{\sigma} = log\frac{U/L}{2\times 1.96} \tag{1}$$

where $\hat{\sigma}$ represents the standard error of the estimated coefficient of the traffic flow variable; U and L, respectively, represents the upper and the lower limits of the 95% confidence interval of the odds ratio.

Table 1 gives the studies included in the meta-analysis. The main reasons for excluding studies were as follows: (1) the studies did not meet the above selection criteria; (2) the studies cannot provide the sufficient information, i.e., the above four type information; (3) the studies were review, i.e., a secondary source based on existing literature.

Table 1
Systematic review of literature.

Study no	Authors	Year	Country	Freeway	Average spacing between stations	Considered Traffic Flow Variable
1	Abdel-Aty	2004	America	I-4	0.49 miles	● Coefficient of variation of speed at downstream station ● Average of detector occupancy at upstream station
2	Abdel-Aty and Pande	2005	America	I-4	About 0.5 miles	● Average of detector occupancy at upstream station ● Average of detector occupancy at downstream station
3	Abdel-Aty et al.	2005	America	I-4	0.53 miles	● Average of detector occupancy at upstream station ● Average traffic volume at upstream station ● Coefficient of variation of speed at the nearest station
4	Pande and Abdel-Aty	2005	America	I-4	0.53 miles	● Coefficient of variation of speed at the nearest station ● Std. dev. of volume at the nearest station ● Average detector occupancy at the nearest station
5	Abdel-Aty and Abdalla	2004	America	I-4	0.49 miles	● Average speed at the nearest station ● Average traffic volume at the nearest station ● Std. dev. of speed at upstream station ● Std. dev. of speed at downstream station
6	Abdel-Aty and Pemmanaboina	2006	America	I-4	0.53 miles	● Average of detector occupancy at downstream station ● Coefficient of variation of speed at upstream station
7	Abdel-Aty and Pande	2006	America	I-4	0.52 miles	● Coefficient of variation of speed at the nearest station ● Average of detector occupancy at downstream station
8	Abdel-Aty et al.	2007	America	I-4	About 0.5 miles	● Standard deviation of traffic volume at the nearest station ● Average traffic volume at upstream station ● Coefficient of variation of speed at the nearest station ● Average detector occupancy at upstream station ● Traffic volume at upstream station
9	Park and Oh	2008	Korea	Seohaean freeway	Not mentioned	● Average detector occupancy at upstream station ● Average traffic volume at upstream station ● Std. dev. of speed at downstream station ● Std. dev. of speed at nearest station
10	Lee and Abdel-Aty	2008	America	I-4	Not mentioned	● Speed difference between upstream and downstream
11	Abdel-Aty and Pande	2008	America	I-4	About 0.5 miles	● Average traffic volume at upstream station ● Std. dev. of volume at downstream station ● Coefficient of variation of speed at the nearest station ● Average detector occupancy at upstream station
12	Park and Oh	2009	Korea	Seohaean freeway	Not mentioned	● Average detector occupancy at upstream station ● Average traffic volume at upstream station ● Average traffic volume at downstream station ● Std. dev. of speed at upstream station
13	Hossain and Muromachi	2010	Japan	Shinjuku 4	0.17 miles	● Average traffic volume at upstream station ● Average vehicle speed at downstream station ● Average traffic volume at upstream station
14	Zheng et al.	2010	America	I-5	1.06 miles	● Average of detector occupancy at the nearest station ● Standard deviation of speed at the nearest station
15	Abdel-Aty et al.	2010	America	I-4	About 0.5 miles	● Coefficient of variation of speed at the nearest station ● Average detector occupancy at downstream station ● Std. dev. of occupancy at downstream station
16	Hassan and Abdel-Aty	2011	America	I-4 & I-95	About 0.5 miles	● Average vehicle speed at upstream station ● Average vehicle speed at downstream station ● Average of detector occupancy at downstream station
17	Ahmed and Abdel-Aty	2011	America	SR408, SR417, & SR528	SR408: 0.90 miles SR408: 1.47 miles SR528: 2.88 miles	● Coefficient of variation of speed at downstream station ● Standard deviation of speed at the nearest station ● Average vehicle speed at downstream station
18	Abdel-Aty et al.	2012	America	I-4 & I-95	0.24 miles	● Average vehicle speed at downstream station ● Coefficient of variation of speed at upstream station
19	Xu et al.	2012	America	I-880	0.48 miles	● Average detector occupancy at upstream station ● Standard deviation of speed at downstream station
20	Hossain and Muromachi	2012	Japan	Shibuya & Shinjuku	0.17 miles	● Occupancy difference between up- and downstream station ● Speed difference between up- and downstream station
21	Xu et al.	2012	America	I-880	0.48 miles	● Std. dev. of speed at upstream station Std. dev. of speed at downstream station ● Occupancy difference between adjacent lanes
22	Abdel-Aty et al.	2012	America	I-70	Not mentioned	● Average speed at the nearest station ● Coefficient of variation of speed at the nearest station
23	Ahmed et al.	2012	America	SR417	1.47 miles	● Standard deviation of speed at the nearest station ● Average vehicle speed at downstream station
24	Abdel-Aty et al.	2012	America	I-4 & I-95	Not mentioned	● Average vehicle speed at downstream station ● Coefficient of variation of speed at upstream station
25	Xu et al.	2013	America	I-880	0.53 miles	● Average detector occupancy at upstream station ● Std. dev. of speed at downstream station ● Average detector occupancy at downstream station
26	Yu and Abdel-Aty	2013	America	I-70	0.50 miles	● Occupancy difference between lanes at the nearest station ● Std. dev. of occupancy at downstream station ● Average vehicle speed at downstream station
27	Xu et al.	2013	America	I-880	0.51 miles	● Std. dev. of detector occupancy at upstream station ● Std. dev. of speed at downstream station ● Occupancy difference between lanes at downstream station

Table 1 (*Continued*)

Study no	Authors	Year	Country	Freeway	Average spacing between stations	Considered Traffic Flow Variable
28	Yu et al.	2013	America	I-70	0.50 miles	● Average speed at the nearest station ● Average detector occupancy at upstream station
29	Xu et al.	2013	America	I-880	0.50 miles	● Average detector occupancy at upstream station ● Average vehicle speed at downstream station ● Speed difference between upstream and downstream
30	Hassan and Abdel-Aty	2013	America	I-4 & I-95	About 0.5 miles	● Average vehicle speed at upstream station ● Average detector occupancy at downstream station
31	Li et al.	2014	America	I-880	0.43 miles	● Average vehicle speed ● Standard deviation of speed ● Coefficient of variation of speed at the nearest station
32	Xu et al.	2014	America	I-880	0.54 miles	● Average detector occupancy at upstream station ● Occupancy difference between lanes at the nearest station ● Average traffic volume at downstream station ● Coefficient of variation of speed at downstream station
33	Pirdavani	2014	Belgium	European route E313	Not mentioned	● Std. dev. of speed at upstream station ● Average detector occupancy at upstream station ● Average speed at the upstream station
34	Xu et al.	2014	America	I-880 & I-5	I-880: 0.49 miles I-5: 0.63 miles	● Average traffic volume at the nearest station ● Average vehicle speed at the nearest station ● Coefficient of variation of speed at the nearest station ● Occupancy difference between lanes at the nearest station
35	Yu et al.	2015	China	Shanghai urban expressway	Not mentioned	● Average speed at upstream station ● Average speed at downstream station
36	Xu et al.	2015	America	I-880	0.49 miles	● Average traffic volume at the nearest station ● Std. dev. of speed at the nearest station

2.2. Data from study site

Data were obtained from a twelve-kilometer section on the Hangyong freeway in the Zhejiang Province, China. There are eight remote traffic microwave sensors (RTMSs) and one weather station along the selected freeway section. These RTMSs were implemented for the variable speed limit system on the twelve-kilometer freeway section. Since the local transportation agencies only stored the real-time traffic data for two months before the operation of the variable speed limit system, we can only collect the crash, weather and traffic data for these two months. A total of 41 crashes were identified and used. For each crash, the authors extracted traffic data in the time interval between 5 and 10 min prior to crash occurrence. The purpose was to compensate for the possible inaccuracies in the reported crash time (Abdel-Aty et al., 2012b; Ahmed and Abdel-Aty, 2011; Ahmed et al., 2012). Doing so can also help identify hazardous traffic conditions ahead of the crash occurrence time to make preemptive measures possible (Abdel-Aty et al., 2012b; Ahmed and Abdel-Aty, 2011; Ahmed et al., 2012). The traffic data were collected from the nearest upstream and downstream RTMSs to each crash location.

In this study, the unmatched case–control design was used to develop crash risk models. Previous studies in epidemiology suggested that both matched and unmatched case–control study design can control for the impacts of confounding variables (Rothman and Greenland, 1998; Bruce et al., 2008). The major difference is that the matched case–control study account for the impacts of confounding factors at the stage of selecting controls; while the unmatched case–control study takes account of the impacts of confounding factors at the stage of data analysis. In the unmatched case–control design, four non-crash cases were randomly selected from crash-free days for each crash case. Two decisions need to be made, including the time and the location of each non-crash case. More specifically, each non-crash case was randomly selected from 17,568 5-min time intervals in the two months. Two neighboring RTMSs were also randomly selected from the eight RTMSs along the selected freeway segment. The sampling universe contains about $17{,}568 \times 7 = 1.229 \times 10^5$ potential time–space points. The non-crash cases were randomly selected from the sampling universe. In addition, it was also ensured that there were no crashes observed at the location of each non-crash case during the whole day. The traffic flow data for each non-crash case were then extracted based on the randomly generated time and RTMSs. The weather data for each crash and non-crash case was extracted based on the time of them. The geometric data for the crash and non-crash cases were obtained from the Google earth software.

3. Methodology

Since only limited data were available for developing a crash risk model in this study, the Bayesian meta-analysis was first conducted to produce the informative priors of several traffic variables by combing the results from numerous previous studies. Then, the Bayesian inference method was used to develop the crash risk model with the limited data based on the extract informative priors. Finally, the Bayesian predictive densities analysis was conducted to identify the outliers in the limited data.

3.1. Bayesian meta-analysis

The meta-analysis is a quantitative method to conduct systematic review of literature. It can produce a more accurate estimate for the effects of traffic flow variables on crash risks by synthesizing results from numerous previous studies. This method has been applied in different fields of transportation engineering, such as travel time value analysis (Abrantes and Wardman, 2011; Dimitropoulos et al., 2013), public transport demand forecast (Holmgren, 2007), and traffic safety evaluation (Elvik, 1996, 1999, 2001, 2013; Cairda et al., 2014). Compared with the commonly used meta-analysis by the frequentist formulation in previous studies, the Bayesian meta-analysis used in this study has several advantages, including accounting for the uncertainty of parameters in the meta-analysis, extending the meta-analyses to accommodate more complex scenarios, and full allowance of the probability statement regarding the quantities of interest (Sutton and Abrams, 2001).

3.1.1. Bayesian fixed effect meta-analysis

The fixed effect and random effect models are the two commonly used meta-analyses. In the fixed effect meta-analysis, the result of each study estimates a common unknown overall pooled effect. While in the random effect meta-analysis, the results of the studies estimate their own unknown underlying effects, which are estimating a common population mean. Assuming that the parameter of the traffic variable in crash risk models is normally distributed, and then the fixed effect meta-analysis can be expressed as:

$$R_i \sim Normal(\mu, \sigma_i^2), \quad i = 1, \ldots, k$$
$$\mu \sim Normal(a, b) \tag{2}$$

where R_i represents the regression parameter of the traffic flow variables in previous studies; μ represents the underling effect size obtained in the meta-analysis; ${\sigma_i}^2$ represents the observed within-study variances. A non-informative prior normal distribution is used for μ; the hyper-parameters a and b are set to be 0 and 10^6, respectively.

3.1.2. Bayesian random effect meta-analysis

In the random effect meta-analysis, the true effects in different studies are assumed to follow a distribution, such as a normal distribution with mean μ and variance τ^2. The Bayesian random effect meta-analysis can be expressed as:

$$R_i \sim Normal(\theta_i, \delta_i^2), \quad i = 1, \ldots, k$$
$$\theta_i \sim Normal(\mu, \tau^2), \quad \mu \sim Normal(c, d), \quad \tau^2 \sim Inverse \quad Gamma(e, f) \tag{3}$$

where R_i represents the regression parameter of the traffic flow variables in previous studies; θ_i represents the true effect in each observed study; ${\delta_i}^2$ represents the observed within-study variances in each study; μ denotes the overall pooled effect; τ^2 represents the between-study variances. Thus, the random effect meta-analysis allows for the existence of both between-study heterogeneity and within-study variability.

A non-informative prior normal distribution is used for the overall pooled effect μ, and the hyper-parameters c and d are set to be 0 and 10^6, respectively. The inverse gamma distribution was used as the non-informative prior for the between-study variances τ^2, and the hyper-parameters e and f are set to be 0.001 and 0.001, respectively. Including the between-study heterogeneity can reduce the relative weighting given to the more precise studies. This can help produce a more accurate and conservative confidence interval for the overall pooled effect estimate (Sutton and Abrams, 2001).

3.1.3. Bayesian meta-regression

As shown in Eq. (3), the random effect meta-analysis allows the true effect in each observed study θ_i to be different, but it cannot explain why study results vary systematically. To overcome this shortcoming, meta-regression uses study-level variables, such as the studied freeway in the publication, to explain the difference in true effect θ_i across studies. This can help get a more accurate estimate of the overall pooled effect estimate. The Bayesian meta-regression can be expressed as:

$$R_i \sim Normal(\theta_i, \delta_i^2), \quad i = 1, \ldots, k$$
$$\theta_i = \mu + \beta x_i + \varepsilon, \quad \varepsilon \sim Normal(0, \tau^2) \tag{4}$$
$$\mu \sim Normal(c, d), \quad \tau^2 \sim Inverse \quad Gamma(e, f), \quad \beta \sim Normal(g, h)$$

where R_i represents the regression parameter of the traffic flow variables in previous studies; x_i is the study-level variable represents the characteristics of each study; β is coefficient of the explanation variable x_i; βx_i can answer whether the study-level variables contribute to the difference across studies, i.e., the heterogeneity between studies. Therefore, the linear function of theta in meta-regression means that true effect in each observed study is different, and that this heterogeneity between studies can be explained by the characteristics of studies; A non-informative prior normal distribution is used for the coefficient β; the hyper-parameters g and h are set to be 0 and 10^6, respectively; the other parameters in Eq. (4) are the same as those in the Eq. (3).

3.2. Bayesian inference approach

A Markov chain Monte Carlo (MCMC) simulation-based Bayesian inference approach is used to develop the crash risk models using the limited data. Based on the Bayes' theorem, the posterior joint distribution of parameters $\mathbf{\Theta}$ in the crash risk model can be estimated as follows:

$$f(\mathbf{\Theta}|Y) = \frac{f(Y, \mathbf{\Theta})}{f(Y)} = \frac{f(Y|\mathbf{\Theta})\pi(\mathbf{\Theta})}{\int f(Y, \mathbf{\Theta})d\mathbf{\Theta}} \propto f(Y|\mathbf{\Theta})\pi(\mathbf{\Theta}) \tag{5}$$

where $f(\mathbf{\Theta} \mid \boldsymbol{Y})$ denotes the posterior joint distribution of parameters $\mathbf{\Theta}$ conditional on the limited data $\boldsymbol{Y}$; π $(\mathbf{\Theta})$ is the prior distribution of parameters $\mathbf{\Theta}$ in the model. In previous studies, the non-informative priors were used, as the dataset in these studies were enough to establish an accurate relationship between crash risks and traffic flow variables. In this study, crash risk models were developed by the informative prior π $(\mathbf{\Theta})$ that was obtained from the meta-analyses.

3.3. Bayes factor

The Bayes factor can be used to compare two competing model M_1 and model M_2 (Kass and Raftery, 1995). Assuming equal preferences of these two models, the prior probability of model m_1 is equal to model m_2. In this case, the ratio of the models' posterior probabilities can be expressed as:

$$PO_{21} = \frac{f(M_2|Y)}{f(M_1|Y)} = \frac{f(M_2, Y)/f(Y)}{f(M_1, Y)/f(Y)} = \frac{f(Y|M_2)(M_2)}{f(Y|M_1)(M_1)} = \frac{f(Y|M_2)}{f(Y|M_1)} = B_{2,1} \tag{6}$$

where PO_{21} is termed the posterior model odds of model M_2 versus model M_1; B_{21} is the Bayes factor of model M_2 versus model M_1; $f(M \mid Y)$ is the posterior probability for model M; $f(Y \mid M)$ is the marginal likelihood of the data under model M, which can estimated by the harmonic mean estimator (Newton and Raftery, 1994) as follows:

$$f(Y|M)^{-1} = \frac{1}{R}\sum_{r=1}^{R}\{f(Y|\Theta^{(r)}, M)\}^{-1} \tag{7}$$

where $r = 1, 2, \ldots, R$, and R represents the total number of iterations. Previous study in Bayesian statistics suggested that significant difference exists between two models if the log of Bayes factor for these two models is larger than 3 (Kass and Raftery, 1995).

3.4. Bayesian predictive densities for model checking

The Bayesian predictive densities can be used to identify the outliers in the data. The leave-one-out cross validation predictive density was used to estimate the posterior predictive ordinate $f(y_i|\boldsymbol{y}_{\backslash i})$ using the following equation:

$$f(y_i|\boldsymbol{y}_{\backslash i}) = \int f(y_i|\mathbf{\Theta})f(\mathbf{\Theta}|\boldsymbol{y}_{\backslash i})d\mathbf{\Theta} \tag{8}$$

where y_i represents the ith observation; $\boldsymbol{y}_{\backslash i}$ represents the data $\boldsymbol{y}$ after omitting y_i. This quantity is known as the conditional predictive ordinate (CPO) (Ntzoufras, 2009). This quantity estimates the probability of y_i after having observed $\boldsymbol{y}_{\backslash i}$. Low value of $f(y_i|\boldsymbol{y}_{\backslash i})$ indicates that observation y_i comes from the tail areas of the assumed distribution, while extremely low values indicate the possible outliers.

The CPO for each observation can be estimated by using the MCMC output generated from the full posterior density as follows (Ntzoufras, 2009):

$$[f(y_i|\boldsymbol{y}_{\setminus i})]^{-1} = \frac{f(\boldsymbol{y}_{\setminus i})}{f(\boldsymbol{y})} = \int \frac{f(\boldsymbol{y}_{\setminus i}|\boldsymbol{\Theta})f(\boldsymbol{\Theta})}{f(\boldsymbol{y})} d\Theta$$

$$= \int \frac{1}{f(y_i|\boldsymbol{\Theta})} \frac{f(\boldsymbol{y}|\boldsymbol{\Theta})f(\boldsymbol{\Theta})}{f(\boldsymbol{y})} d\boldsymbol{\Theta} = \int \frac{1}{f(y_i|\boldsymbol{\Theta})} f(\boldsymbol{\Theta}|\boldsymbol{y}) d\boldsymbol{\Theta} \quad (9)$$

Therefore, the inverse CPO_i (ICPO) can be estimated as the sample mean of the inverse density function evaluated at y_i for each $\boldsymbol{\Theta}^{(t)}$ generated from the full posterior distribution. The CPO_i is then given by:

$$\mathrm{CPO}_i = \left(\frac{1}{T} \sum_{t=1}^{T} \frac{1}{f(y_i|\boldsymbol{\Theta}^{(t)})} \right)^{-1} \quad (10)$$

where T = 1, 2, . . ., T, and T represents the total number of iterations. Thus, CPO can be obtained based on the harmonic mean of the probability distribution function evaluated at y_i in each step of an MCMC algorithm.

4. Results of data analysis

4.1. Results of meta-analyses

Table 2 summarizes the number of extracted estimates for different traffic variables presented in Table 1. In total, 105 estimates were extracted from the studies listed in Table 1. The fixed-effect meta-analysis, the random-effect meta-analysis and the meta-regressions were conducted for different traffic variables in Table 2, expected for the variables with number of estimates lower than 2. Different combinations of traffic flows variables were used to develop the crash risk models with the informative priors from meta-analyses. It was found that the crash risk model has the best predictive performance when the following combinations of traffic flow variables were used: (1) the average detector occupancy at the upstream station ($\mathrm{AvgOcc_u}$); (2) the standard deviation of speed at the downstream station ($\mathrm{DevSpd_d}$); (3) the average traffic volume at the upstream station ($\mathrm{AvgCnt_u}$); and (4) the coefficient of variation of speed at the nearest station ($\mathrm{CovSpd_c}$). To present this paper in a concise way, Table 3 only gives the results of the Bayesian fixed effect meta-analysis, the Bayesian random effect meta-analysis and the Bayesian meta-regression for these four traffic flow variables used for the following crash risk model development. The meta-regression used the freeway as the explanatory variables to explain the between-study heterogeneity.

Fig. 1 illustrates the main results of the three meta-analyses for the traffic flow variable $\mathrm{DevSpd_d}$, and Table 3 provides the details of the parameter estimates. In Fig. 1, the solid line represents the 95% confidence interval of the effects of DevSpd on crash risks in each individual study. The dashed line directly below each solid line represents the shrunken estimate for each study, based on the posterior distributions of θ_i obtained from the Bayesian random effect meta-analysis (see Eq. (3)). As expected, the 95% confidence intervals obtained from the random effect meta-analysis are generally narrower than the observed estimates in these studies (see Fig. 1), as these estimates are produced by combining information from other studies.

Below the individual studies are three diamonds, which represent the overall pooled effects for $\mathrm{DevSpd_d}$, estimated by the Bayesian fixed effect meta-analysis, the Bayesian random effect meta-analysis, and the Bayesian meta-regression, respectively. The overall pooled point estimates are 0.015, 0.092, and 0.095, for the fixed effect meta-analysis, the random effect meta-analysis, and the meta-regression, respectively. Their confidence intervals do not include zero, indicating the DevSpd has significant impacts on crash risks. Crash risks on freeway increase with an increase in the speed variations at the downstream station.

Table 2
Number of extracted estimates for different traffic variables.

Variable category	Variable	Number of estimates
Occupancy	Average occupancy at upstream station	16
	Average occupancy at downstream station	7
	Average occupancy at nearest station	2
	Std. dev. of detector occupancy at upstream station	1
	Std. dev. of detector occupancy at downstream station	2
	Occupancy difference between lanes at downstream station	1
	Occupancy difference between lanes at the nearest station	4
	Occupancy difference between up- and downstream station	1
Speed	Average speed at upstream station	4
	Average speed at downstream station	9
	Average speed at nearest station	5
	Coefficient of variation of speed at upstream station	3
	Coefficient of variation of speed at downstream station	3
	Coefficient of variation of speed at nearest station	9
	Std. dev. of speed at upstream station	4
	Std. dev. of speed at downstream station	8
	Std. dev. of speed at nearest station	6
	Speed difference between up- and downstream station	3
Volume	Average traffic volume at upstream station	9
	Average traffic volume at downstream station	2
	Average traffic volume at nearest station	3
	Std. dev. of volume at the nearest station	2
	Std. dev. of volume at downstream station	1

However, the standard errors, and thus confidence intervals, are quite different. The 95% confidence interval of the pooled effect in the random effect meta-analysis is wider than that in the fixed effect meta-analysis. The wider confidence interval of the pooled effect for the random effect meta-analysis does not necessarily indicate the disadvantage of the random effect meta-analysis. In fact, one of the appeals of the random effect meta-analysis is that it can produce a more conservative estimate for the overall pooled effect as compared to the fixed effect meta-analysis, since it includes a component of variation to account for the heterogeneity between studies. Similarly, the meta-regression produced an even more conservative estimate for the overall pooled effect than the random effect meta-analysis, since it includes explanatory variables to control the between-study heterogeneity.

Similar results were found for the other three traffic variables. As shown in Table 3, the meta-regressions have the largest variance for the overall pooled effects, while the fixed effect meta-analyses have the lowest variance for the overall pooled effects, indicating that the estimated pooled effects are not robust to the type of used model. The results of the meta-analyses can be used to establish the informative priors for the traffic flow variables. These informative priors can help develop a crash risk model using the limited data in China. Fig. 2 illustrates the developed informative priors based

Table 3
Results of the Bayesian fixed effect meta-analysis, Bayesian random effect meta-analysis, and Bayesian meta-regression.

	Bayesian fixed effect				Bayesian random effect				Bayesian meta-regression			
	$AvgOcc_u$	$CovSpd_c$	$DevSpd_d$	$AvgCnt_u$	$AvgOcc_u$	$CovSpd_c$	$DevSpd_d$	$AvgCnt_u$	$AvgOcc_u$	$CovSpd_c$	$DevSpd_d$	$AvgCnt_u$
Pooled effect (μ)												
Prior distribution	$N(0,10^6)$	$N(0,10^6)$	$N(0,10^6)$	$N(0,10^6)$	$N(0,10^6)$[a]	$N(0,106)$	$N(0,10^6)$	$N(0,10^6)$	$N(0,10^6)$	$N(0,10^6)$	$N(0,10^6)$	$N(0,10^6)$
Posterior mean of μ	0.050	0.990	0.015	0.053	0.094	1.018	0.092	0.065	0.072	3.903	0.095	0.091
Std. dev. of μ	0.004	0.080	0.004	0.016	0.033	0.260	0.035	0.032	0.041	1.048	0.050	0.054
MC error of μ	0.000	0.001	0.000	0.000	0.001	0.014	0.001	0.001	0.001	0.118	0.001	0.003
2.5% of μ	0.042	0.836	0.008	0.021	0.042	0.672	0.031	0.009	0.007	1.742	0.029	0.012
97.5% of μ	0.058	1.148	0.023	0.085	0.166	1.669	0.173	0.136	0.136	5.379	0.181	0.183
Probability ($\mu>0$)	0.999	0.999	0.999	0.999	0.999	0.999	0.997	0.994	0.965	0.999	0.992	0.972
Between-study variance (τ^2)												
Prior distribution	–[c]	–	–	–	IG(c,d)[b]	IG(c,d)	IG(c,d)	IG(c,d)	IG(c,d)	IG(c,d)	IG(c,d)	IG(c,d)
Posterior Mean of τ^2	–	–	–	–	0.008	0.199	0.008	0.004	0.094	0.094	0.007	0.102
Std. dev. of τ^2	–	–	–	–	0.025	0.694	0.028	0.005	3.358	1.331	0.014	2.660
MC error of τ^2	–	–	–	–	0.001	0.043	0.000	0.000	0.085	0.026	0.000	0.097
2.5% of τ^2	–	–	–	–	0.002	0.001	0.001	0.000	0.002	0.001	0.001	0.001
97.5% of τ^2	–	–	–	–	0.026	1.829	0.028	0.017	0.034	0.567	0.033	0.037

[a] Normal distribution with mean.
[b] Inverse gamma distribution with $c=0.001$ and $d=0.001$.
[c] Not applicable.

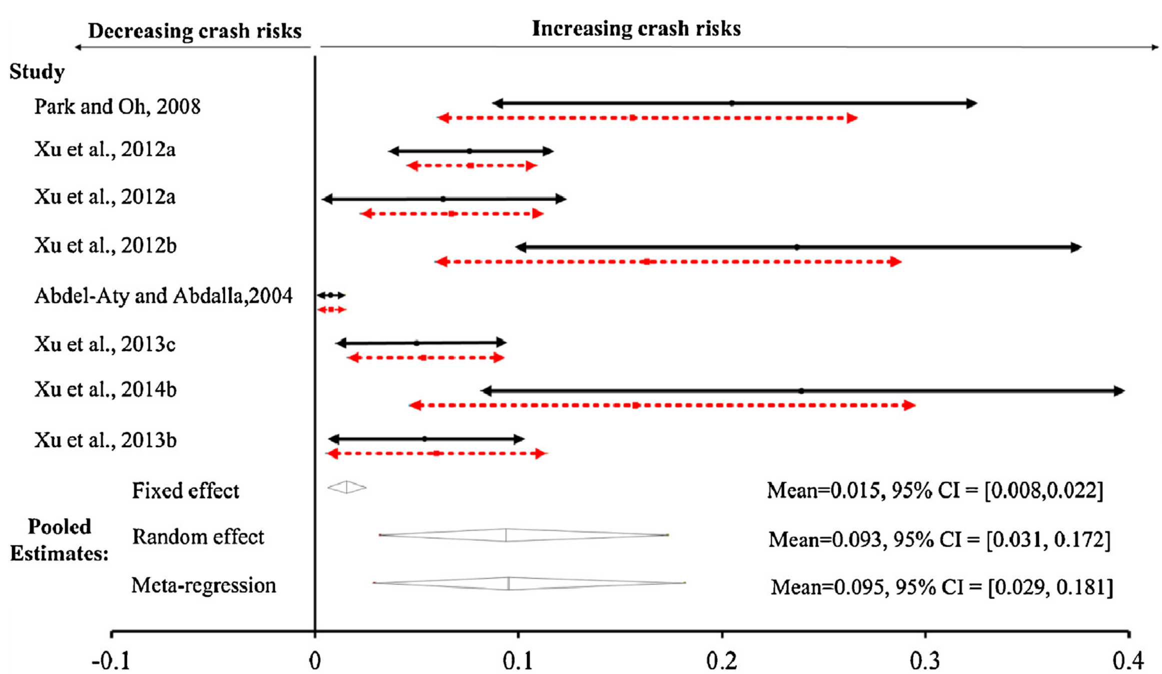

Fig. 1. Forest plot for the different meta-analyses of the effects of speed variations.

on the posterior distributions of the overall pooled effects obtained from the three meta-analyses.

4.2. Crash risk model development

To determine whether the type of used meta-analysis affects the inferences of the crash risk models, the informative priors based on the three meta-analysis models were all used to develop the crash risk model. The MCMC simulation-based Bayesian logistic regressions were used to develop the crash risk models. Three parallel MCMC chains were constructed for Bayesian inference. Each MCMC chain consisted of 10,000 iterations, including an initial "burn-in" of 5000 iterations. The Gelman-Rubin potential scale reduction (PSR) of 1.1 was used to check the convergence of the MCMC simulations (Gelman et al., 2004).

Table 4 gives the estimation results of the crash risk models based on the informative priors obtained from the three meta-analyses. Models 1, 2 and 3 represent the crash risk models developed based on the informative priors obtained from the fixed effect meta-analysis, the random effect meta-analysis, and the meta-regression, respectively. For comparison, we have also developed a logistic regression using non-informative priors, i.e., model 0. The logarithm of the marginal likelihoods of the model by the non-informative priors is −104.746. As shown in Table 4, the three models using the informative priors provide improvements in the logarithm of the marginal likelihood (ranging from 4.12 to 5.56) as compared to the model using the non-informative priors, indicating that the informative priors extracted from the previous studies by the meta-analyses can significantly increase the fitness of the models (Ntzoufras, 2009).

As shown in Table 4, all the four traffic flow variables have significant impacts on crash risks in these three models. However, the parameters of the same traffic flow variables are quite different in these models. A series of t-tests indicate that the differences in these parameters between various models are statistically significant. Therefore, the type of used meta-analysis significant significantly

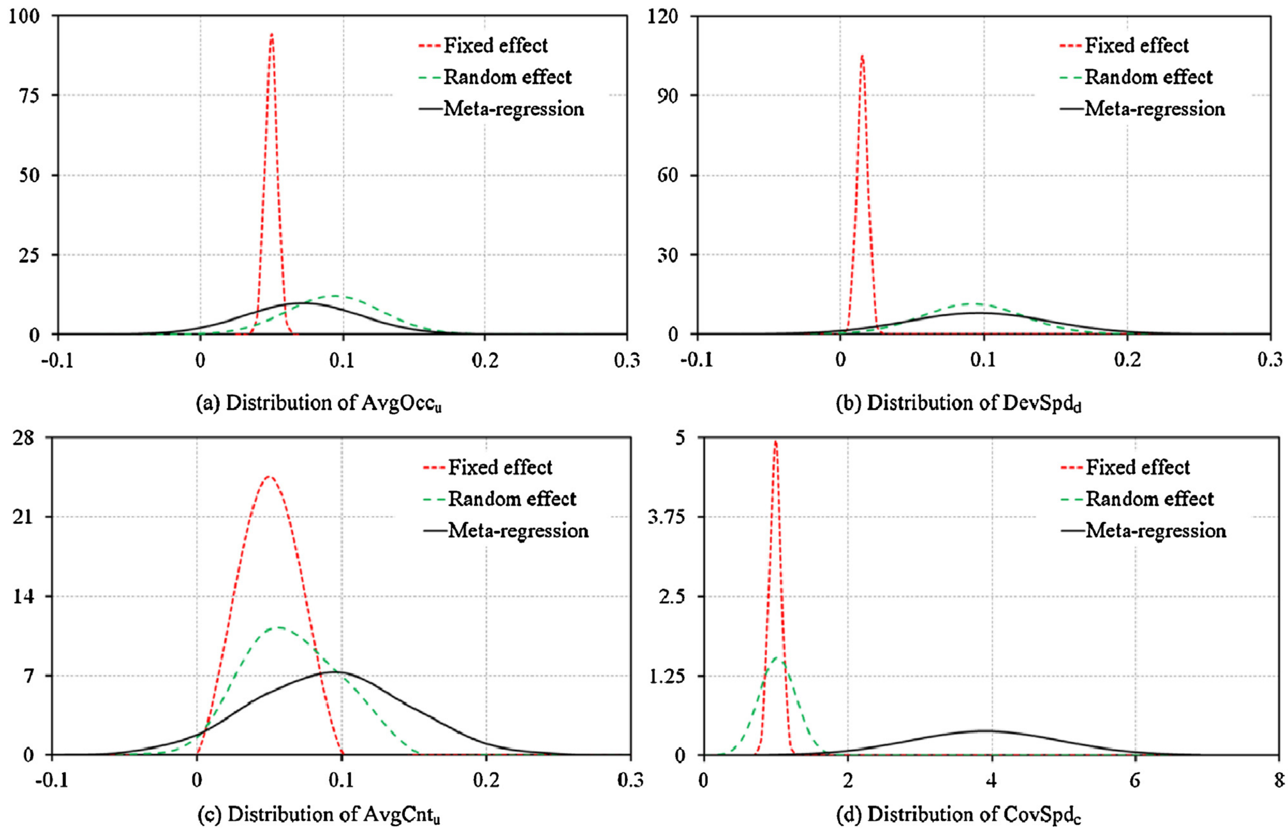

Fig. 2. Distributions of the effects of traffic variables obtained from the meta-analyses.

Table 4
Results of the crash risk models using different meta-analyses results.

Variables	Crash risk model 1[a]		Crash risk model 2[b]		Crash risk model 3[c]		Crash risk model 0[d]	
	Mean	95% CI[e]	Mean	95% CI	Mean	95% CI	Mean	95% CI
Constant	−2.786	(−3.405,−2.229)	−4.066	(−5.335,−2.758)	−5.251	(−7.327,−3.322)	−2.475	(−3.756,−1.364)
$AvgOcc_u$	0.05	(0.042,0.058)	0.104	(0.043,0.164)	0.135	(0.011,0.263)	0.139	(−0.031,0.328)[g]
$DevSpd_d$	0.016	(0.009,0.023)	0.057	(0.007,0.104)	0.054	(0.002,0.106)	–	–
$AvgCnt_u$	0.055	(0.027,0.084)	0.074	(0.001,0.147)	0.165	(0.037, 0.290)	–	–
$CovSpd_c$	0.995	(0.841,1.148)	1.039	(0.539,1.542)	3.859	(1.856,5.781)	–	–
Lane	−0.302	(−0.548,−0.068)	−0.309	(−0.556,−0.07)	−0.308	(−0.552,−0.061)	−0.298	(−0.546,−0.055)
Ramp	0.771	(0.057,1.493)	0.805	(0.106,1.537)	0.841	(0.134,1.597)	0.772	(0.062,1.486)
DIC	199.423		199.228		199.214		203.705	
Ln[f(Y|M)][f]	−100.626		−99.715		−99.187		−104.746	

[a] Model developed with the informative priors obtained from fixed effect meta-analysis.
[b] Model developed with the informative priors obtained from random effect meta-analysis.
[c] Model developed with the informative priors obtained from meta-regression.
[d] Model developed with non-informative priors.
[e] 95% confidence interval.
[f] Logarithm of marginal likelihood of data.
[g] This variable is significant at the 0.1 level, and the 90% CI of the parameter is (0.002, 0.297).

affects the inferences of the crash risk models. In addition to the four traffic variables, two geometric variables Lane and Ramp were also significant in the different models. The variable Lane represents the number of lanes, and the variable Ramp represents whether there is a ramp between upstream and downstream stations. The adverse weather conditions were not significant in the model because of the limited sample.

The authors further investigate whether the type of used meta-analysis affects the predictive performance of the crash risk models. To estimate the predictive performance of the models with informative priors based on the limited Hangyong freeway data sample, the 20-fold cross-validation method was used. The Hangyong freeway data sample was randomly partitioned into 20 mutually exclusive sub-samples of approximately equal size. Among the 20 datasets, each single sub-sample was used as a validation sample and the other 19 sub-samples were combined as a training sample. The crash risk models with informative and non-informative priors were developed for different training samples. Note that the explanatory variables included in each sample were the same variables shown in Table 4. The estimated models for each training data sample were then used to calculate the crash likelihood in each corresponding validation sample.

The predictive performance of a model with a binary outcome can be measured with two complementary indicators, including the proportion of crash cases classified as a crash (sensitivity), and the proportion of non-crash cases misclassified as a crash (false alarm

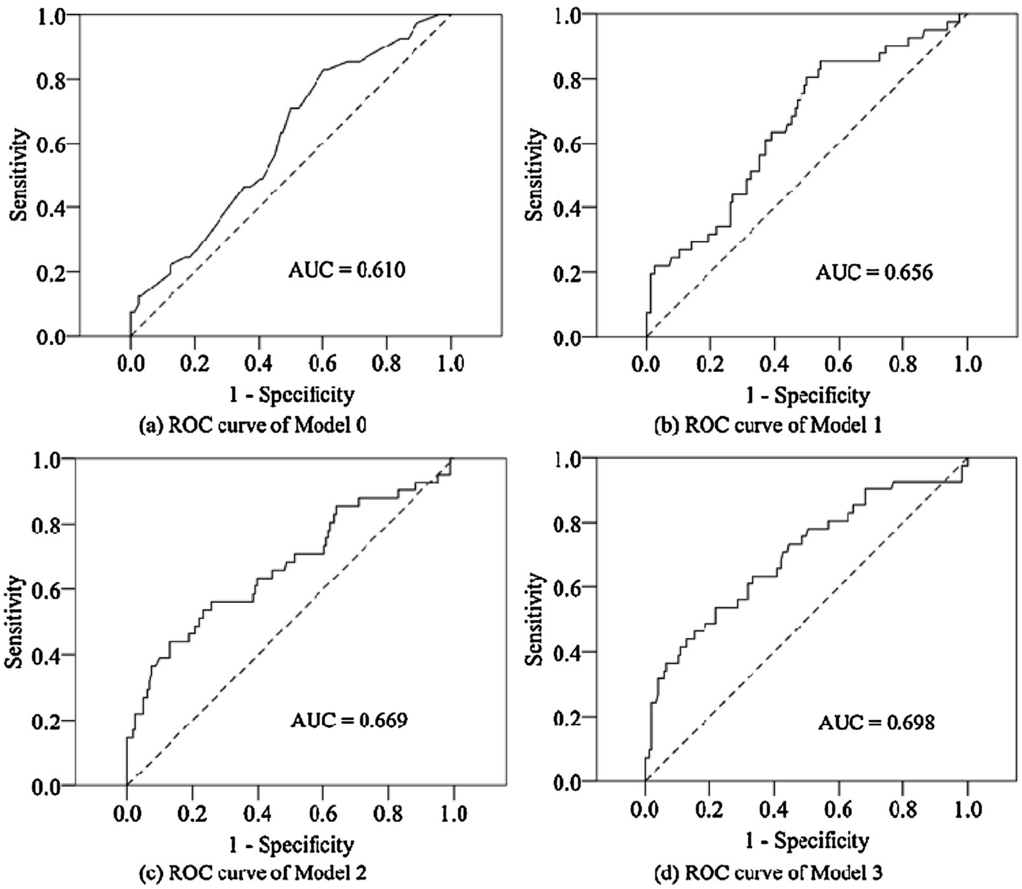

Fig. 3. Results of the crash risk models using different meta-analyses results.

Table 5
Predictive performance of different models.

False alarm rates	Sensitivity				Difference		
	Model 1[a]	Model 2[b]	Model 3[c]	Model 0[d]	Model 1 vs Model 0	Model 2 vs Model 0	Model 3 vs Model 0
0.1	0.268	0.390	0.390	0.195	0.073	0.195	0.195
0.2	0.317	0.488	0.512	0.268	0.049	0.220	0.244
0.3	0.512	0.561	0.585	0.463	0.049	0.098	0.122
0.4	0.634	0.634	0.634	0.488	0.146	0.146	0.146
0.5	0.756	0.707	0.756	0.707	0.049	0	0.049

[a] Model developed using the informative priors obtained from fixed effect meta-analysis.
[b] Model developed using the informative priors obtained from random effect meta-analysis.
[c] Model developed using the informative priors obtained from meta-regression.
[d] Model developed using the non-informative priors.

rate). The receiver operating characteristic (ROC) curves were used to compare the sensitivities of the three models at different false alarm rates. The ROC curves were developed by using all validation samples. Fig. 3 illustrates the ROC curve of the model 0, i.e., the model developed by the non-informative priors. The area under the curve (AUC) of the model 0 is as low as 0.61. Table 5 gives the crash prediction accuracy of the model 0 at different false alarm rates. As expected, the predictive performance of the model 0 is very limited, indicating that the crash risk model that is directly developed with the limited data cannot produce adequate predictive performance.

For comparison, the ROC curves of the models 1, 2, and 3 were also illustrated in Fig. 3. The AUCs of these three models are also given in Fig. 3. The predictive performance of these three models are noticeably better than that of the model 0, indicating that, when only limited data are available, the informative priors extracted from previous studies by meta-analysis can significantly increase the predictive performance of the developed crash risk models.

However, the predictive performances of the three models are different. As shown in Fig. 3, among the three models developed by the informative priors, the model 1 has the worst predictive performance, while the model 3 has the best predictive performance. Table 5 also gives the crash prediction accuracy of these three models at different false alarm rates. The average difference in crash prediction accuracy between models 1 and 0 is 7.3%, indicating that the informative priors obtained from the fixed effect meta-analysis can increase the crash prediction accuracy by an average of 7.3%. The average difference in crash prediction accuracy between models 3 and 4 is 15.1%, indicating that the informative priors obtained from meta-regression can increase the crash prediction accuracy by an average of 15.1%. Therefore, the informative priors obtained from the meta-regression are more appropriate for developing the crash risk models with limited data.

The fixed effect meta-analysis provides a lower standard error for the pooled effect than the meta-regression (see Table 3), but

Table 6
Estimation results of model 3 before and after CPO check.

Variable	Model 3[b] before checking		Model 3 after checking		t-Test	
	Mean	95% CI[a]	Mean	95% CI[a]	t Value	p Value
Constant	−5.251	(−7.327,−3.322)	−6.517	(−8.751,−4.373)	−54.706	<0.001
$AvgOcc_u$	0.135	(0.011,0.263)	0.164	(0.013, 0.327)	13.3304	<0.001
$DevSpd_d$	0.054	(0.002,0.106)	0.080	(0.028,0.136)	44.7686	<0.001
$AvgCnt_u$	0.165	(0.037, 0.290)	0.197	(0.025, 0.358)	14.106	<0.001
$CovSpd_c$	3.859	(1.856,5.781)	4.332	(2.418,6.294)	21.2427	<0.001
Lane	−0.308	(−0.552,−0.061)	−0.301	(−0.547,−0.061)	2.55459	0.011
Ramp	0.841	(0.134,1.597)	1.133	(0.385,1.96)	33.7456	<0.001

[a] 95% confidence interval.
[b] Model developed using the informative priors obtained from meta-regression.

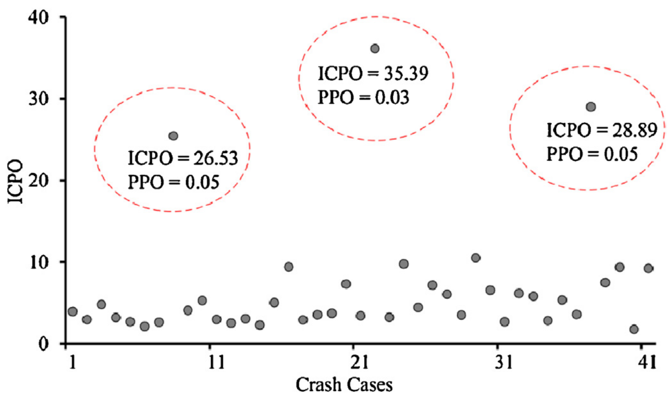

Fig. 4. Bayesian predictive densities for model checking.

Table 7
Predictive performance of model 3 before and after CPO check.

False alarm rates	Sensitivity		Difference
	Model 3 before checking	Model 3 after checking	
0.05	0.317	0.368	0.051
0.1	0.390	0.474	0.084
0.2	0.512	0.526	0.014
0.3	0.585	0.632	0.047
0.4	0.634	0.711	0.077
0.5	0.756	0.789	0.033

a lower improvement in the predictive performance of the crash risk model than the meta-regression. A possible explanation is that the fixed effect meta-analysis underestimates the standard error, as it assigns exaggerated weighting to the more precise studies. On the other hand, the meta-regression takes into account the within-study heterogeneity, resulting in more conservative and accurate confidence interval of the overall pooled effects.

4.3. Model checking using Bayesian predictive densities

The Bayesian predictive densities analysis was further conducted to identify the outliers in the limited data and to evaluate the effects of outliers on the inferences of the crash risk model. The Eq. (10) was used to estimate the CPO for each observation in the limited data. Since the scaling of CPO depends on the structure of the assumed data distribution, there is no general rule for identifying outliers using CPOs. Previous studies suggested that each CPO value must be monitored relative to other CPO values such as, the average and the maximum values (Ntzoufras, 2009). Fig. 4 illustrates the CPOs for the each crash case. Three observations can be considered the potential outliers.

The research team further checked the raw data for the three potential outliers, and found that the raw data were invalid traffic data which might be caused by the random errors or hardware problems of RTMSs. Thus, it is reasonable to treat these three samples as potential outliers. To evaluate the impacts of the potential outliers on the model development, the crash risk model was further developed by the informative priors obtained from meta-regression based on the data after omitting the three outliers. Table 6 gives the estimation results of the model 3 before and after omitting the three outliers. The parameters of the traffic flow variables are quite different between these two models. A series of *t*-tests indicate that all the parameters are statistically different between these two models (see Table 6), indicating that, when limited data are available, even a small number of outliers can significant affect the inferences of the crash risk model. Therefore, appropriate techniques, such as the Bayesian predictive densities analysis, are needed for model checking when only limited data are available.

As shown in Table 6, the coefficient of traffic variable $AvgOcc_u$ is positive, indicating that the crash risks increases with an increase in the upstream detector occupancy. The odds ratio of $AvgOcc_u$ can be calculated as exp(0.164) = 1.178, indicating that a unit increase in the upstream detector occupancy is associated with 17.8% increases in crash odds. The positive parameter of the traffic variable $DevSpd_d$ indicates that a crash is more likely to occur in the traffic conditions with high speed variations. The odds ratio of $DevSpd_d$ is 1.083, indicating that the crash odds increases by 8.3% when speed variation increase by one unit. Similarly, both traffic variables $AvgCnt_u$ and $CovSpd_c$ have positive parameters, indicating that freeway crash risks tend to be high when traffic volume at upstream station and coefficient of variation of speed at nearest station are high.

The authors further investigate the effects of the outliers on the predictive performance of the crash risk model. The 20-fold cross-validation method was further used to evaluate the predictive performance of the model after CPO check. Fig. 5 illustrates the ROC curves of the model 3 before and after the Bayesian predictive densities analysis. The predictive densities analysis can increase the AUC of the model 3 from 0.698 to 0.746. Table 7 compares the crash prediction accuracy of the model 3 at different false alarm rates before and after the Bayesian predictive densities analysis. The average difference in crash prediction accuracy between two models is 5.10%, indicating that the Bayesian predictive densities analysis for identifying outliers can further improve the crash prediction accuracy of the crash risk model by an average of 5.10%.

As shown in Fig. 5, there is a strong trade-off between crash prediction accuracy and false alarm rate. The crash prediction accuracy increases as the false alarm rate increases. This trade-off must be carefully considered when using the crash risk model for mitigating crash risks. When the developed models are used to calculate the crash likelihood and to warn drivers about the high-risk traffic conditions, a reasonably low false alarm rate should be selected to reduce the danger of losing drivers' responsiveness to the alerts. The developed model can achieve 36.8% crash prediction accuracy at a low false alarm rate of 0.05, indicating that the crash prediction

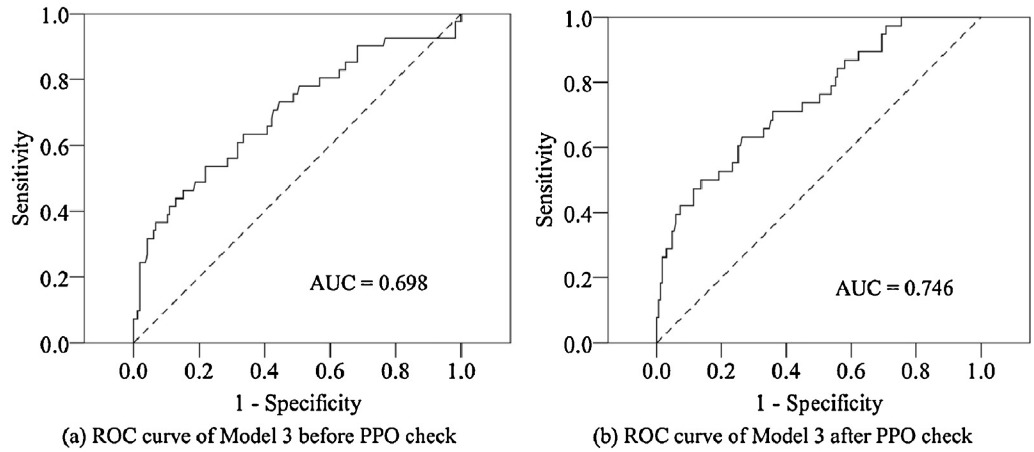

Fig. 5. ROC curves of the models before and after checking.

accuracy and false alarm rate of the developed model are acceptable for practical implementation. The developed model can also be used in dynamic safety management systems, in which, the false alarm rate is not as critical as it is for warning-based solutions, since road users are not aware of the reason for a change in speed limit or a ramp metering rate. A relative high false alarm rate can be selected to increase crash prediction accuracy.

5. Summary and conclusions

This paper proposed a method to develop a real-time crash risk model with the limited data in China by using the Bayesian meta-analysis and Bayesian inference approach. This study has made two contributions to the field of real-time crash risk modeling. First, a systematic review of previous studies about real-time crash risk assessment was conducted by using the Bayesian meta-analysis. The meta-analyses provided a numerical summary of the effects of traffic flow variables on crash risks by quantitatively synthesizing results from previous studies. The second contribution is to propose a method of developing crash risk models on the basis of the Bayesian meta-analysis and the Bayesian inference when only limited data are available. This method is expected to reduce the requirement of the sample size for developing crash risk models.

The Bayesian meta-analyses were used to extract knowledge about the effects of traffic variables on crash risks by synthesizing results from 36 published studies from 2001 to 2015. Three different Bayesian meta-analyses have been utilized, including the fixed effect meta-analysis, the random effect meta-analysis, and the meta-regression. The estimation results showed that all the three models can increase the accurate of the effects of the traffic flow variables on crash risks. The random effect meta-analysis and the meta-regression produce more conservative estimates for the overall pooled effect compared with the fixed effect meta-regression, since they account for the heterogeneity between studies.

The results of the three meta-analyses were used as the informative priors for developing the real-time crash risk model with the limited data in China. The results indicated that the three different meta-analyses significantly affect the inference results and the predictive performance of the crash risk model. The models based on the random-effect meta-analysis and meta-regression can produce better predictive performance. The possible reason is that the unobserved factors in different countries, such as the traffic regulation and driver behavior, might lead to different estimate for the effects of traffic variables on crash risks. The random effect meta-analysis and meta-regression can partly account for the unobserved heterogeneity caused by these unobserved factors. According, the informative priors extracted by them are more accurate than those extracted by the fixed effect meta-analysis. They can better capture the effects of traffic variables in China. The model based on meta-regression can increase the crash prediction accuracy by 15% as compared to the model that was directly developed with limited data.

Finally, the Bayesian predictive densities analysis was further conducted to identify the outliers in the limited data and to evaluate the effects of outliers on the inferences of the crash risk model. The results showed that the outliers significantly affect the estimation results and the predictive performance of the crash risk model. The Bayesian predictive densities analysis for identifying outliers can further improve the crash prediction accuracy by 5%.

The results of this study can help transportation agency to develop real-time crash risk models, when only limited data are available, such as the traffic surveillance equipment has just been placed on a freeway, or the local transportation agencies are not expected to store all the historic real-time traffic data. However, before these results are used in practical engineering applications, research is still needed to test the validity of the proposed method using freeway data collected from other countries. Besides, some more advanced meta-analyses, such as the three-level meta-analysis, can be used to further improve the accuracy of the model estimation. The authors recommend that future studies may focus on these issues.

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