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An online updating method for time-varying preference learning

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ABSTRACT

The rapid proliferation of smart, personal technologies has given birth to smart Transportation Demand Management (TDM) systems that can give personalized incentives to users. This personalization capacity builds on accurate modeling of user behaviors; however, in practice, a user's behavior data is often limited, and his preferences in the discrete choice-making process may change or evolve. In this paper, we propose a new online-updating model that can accurately and efficiently estimate an individual's preferences from his discrete choices. Our model is built on the concept of canonical structure, where a set of canonical models are identified as the common preference patterns shared by the whole population, and a membership vector is also identified for each individual to capture the degrees of the resemblance of his preferences to those common preference patterns. To allow preference to vary in the choice-making process, a time-varying model can be integrated with the canonical structure. In the current study, we use a simple cubic polynomial model with a single variant and show the detailed formulation of the integrated model. An online-updating strategy is also proposed, such that it is possible to update the parameters partially in practice. The proposed model is suitable for modeling a heterogeneous population with insufficient data from each individual. Both simulation studies and a real-world application are taken in the current study. The results show that comparing with other frequently used models, the model we proposed has the highest accuracy in preference learning and behavior prediction.

1. Introduction

A new approach in Transportation Demand Management (TDM) has emerged in recent years, along with the rapid proliferation of smart, personal technologies. As mediums of individual interactions between individuals and devices (Andersson et al., 2018), personal technologies make it possible to modify an individual's travel behavior by offering personalized incentives that suit his demands, constraints, and preferences. Several recent studies have shown that providing personalized incentives rather than generic incentives holds great promise in TDM (Song et al., 2018; Azevedo et al., 2018; Xiong et al., 2019; Zhu et al., 2020). With personalized incentives, individuals are more likely to accept the promoted alternatives, the congestion would be mitigated, and the energy could be used more efficiently.

To provide personalized incentives, a model that could learn an individual's preferences in discrete choices from his behavior data is required. In the discrete choice analysis, an individual's preferences refer to a set of parameters evaluating how the individual values those influential attributes in his choice-making process. With individual preferences, it is possible to predict whether the individual is more likely to accept an alternative, or to provide an appropriate amount of incentives such that the individual would accept a proposed travel choice with a probability larger than a threshold (Zhu et al., 2020), according to the theory of Random

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Utility Maximization (RUM) (Ben-Akiva et al., 1985; Hensher, 1994; Hess et al., 2018; McFadden et al., 1975). In our previous work of Zhu et al. (2020), we designed a framework of a personalized system which could trigger desired behavioral change by iteratively learning the individual's preferences and providing personalized incentives: (1) when an individual is to conduct a trip, the system presents an incentive for the promoted alternative relying on its previously learned preferences along with other trip information; (2) the individual responds by choosing either the promoted alternative or the default choice; (3) his decision is captured by the system so that the preference estimates are updated with the preference learning model embedded in the system.

However, individual behavior data is usually limited, since a single individual can only generate a limited number of observations (Zhu et al., 2020). This problem prevents the use of common preference learning methods, such as regressions, econometric models, or machine learning techniques, in individual preference learning process (O'Mahony et al., 2009; Park et al., 2012; Lops et al., 2011). Since an individual's preferences might not be constants but change along with choice scenarios or evolve in the choice-making process (Bettman et al., 1998; Kivetz et al., 2008; Hess and Giergiczny, 2015; Krueger et al., 2019), this problem may further be the obstacle to update and capture varying preferences.

How to learn and update an individual's preferences when only a few observations are available from him? In Zhu et al. (2020), we dealt with the problem by proposing a particle filter approach (Doucet and Johansen, 2009) with domain knowledge and divide-and-conquer strategy, and soliciting additional information on utility ratio of the two alternatives based on the theory of RUM. While each individual's preferences can be estimated with only one observation with the bayesian theory-based methodology, some limitations still exist. For the first, soliciting information on the utility ratio introduces measurement noises into the data collection process. This is because it is always easier and more accurate for an individual to make binary choices rather than evaluating the levels of attractiveness with a scale. For the second, the individual preference learning algorithm proposed in Zhu et al. (2020) only utilizes the information in each individual's data (individual-level information), while group-level information such as the similarities and commonalities among individuals in the whole population is not considered. It might help improve the accuracy of individual preference learning if knowledge from the massive individuals' data is also extracted in the preference learning process.

In the present study, we propose a Logistic Collaborative Model with Time-varying Parameters (LCM-T), which is a model integrating a time-varying model with a collaborative learning framework, to address the issues mentioned above in individual preference learning. The time-varying model integrated replaces the original constant preference parameter β to a time-dependent function $\beta(t)$, allowing an individual's preferences to fluctuate in his discrete choice-making process. The basic idea of the collaborative learning structure is to exploit the underlying canonical structure in individuals' preference variation of a given population with heterogeneity. It splits the learned preferences into two parts: (1) canonical models in the format of the time-dependent model $\beta(t)$, which represent the common patterns/types of preference variation identified from all the individuals in the population, and (2) a membership vector,[1] representing the degrees of the resemblance of the individual's preferences to those canonical models. It can be seen that with the structure, each individual's preferences integrate the group-level similarities (represented by canonical models) and individual-level personality (represented by his membership vector). From the perspective of a single individual, more information is introduced in his preference learning process.

The proposed LCM-T can be solved with a parameter estimation algorithm similar to the parameter estimation algorithm presented in Lin et al. (2015, 2018a,b). With the algorithm, all the parameters in the canonical models and membership vectors could be estimated. When new observations come into the system and the parameters need to be updated, the estimation algorithm will be re-run with all the data, and all the parameters will be re-estimated together. Apparently, considering the computational cost, the algorithm can be very inefficient when the population is large but only several parameters might change and need to be updated.

Thus, we propose a two-stage (online and offline stages) parameter updating strategy, in which the online-updating algorithm can be used to just update the individual-specific parameters without the need to re-estimate the whole model once a new choice is obtained. The assumption of the two-stage updating method is that the common patterns/types of individual preferences (i.e., the parameters in canonical models) shared by all individuals in the population would not change significantly in a short time, thus do not need to be updated frequently. We call the updating process "online updating" when only part of the parameters, i.e., an individual's membership vector, are revised while the canonical models are fixed. Accordingly, "offline updating" refers to the process when all parameters are updated together with the iterative updating method used by the original collaborative learning model. In practice, the online updating process can take place several times each day for an individual, while the offline-updating may only happen periodically, e.g., once a week or so.

In the current study, we propose an individual preference learning algorithm which can successfully tackle the problem that only a few observations are available in the learning process, and is also able to allow preference variations in an individual's choice-making process. The online parameter estimation method presents as a real-time parameter updating strategy such that the parameters of the whole model can be partially updated, such that an individual's preferences can be estimated in an accurate and efficient way without the need to re-run the algorithm and estimate all the parameters for the whole population. With the learned preferences, an individual's choice-making behavior is expected to be predicted, and personalized incentives might be provided. The current study further presents simulations and a real-world case study of the proposed LCM-T with the online-updating algorithm to learn and update individual preferences in discrete choice data. The results show that both the parameter estimation accuracy

[1] There could be several membership vectors for each individual if the preferences have multiple dimensions, i.e., if there are multiple attributes that matter in the decision-making process, as shown in this paper. In general, the number of the membership vectors or the formulation of the membership vector is decided by the formulation of the canonical models.

and the choice-making prediction accuracy of the proposed model are better than those of the traditional logit model, mixed effect model, latent class choice model, and the particle filter method we use in Zhu et al. (2020).

The remainder of the paper is organized as follows. In Section 2, we present the details of the proposed OLCM-T, which is the online stage of the two-stage Logistic Collaborative Model with Time-varying Parameters (LCM-T). This section includes the mathematical formulation of time-varying parameters, the framework and the mathematical formulation of the two-stage LCM-T, the OLCM-T, and the relationship between two stages — the online and offline updating. In Section 3, we provide the parameter estimation algorithm for the proposed OLCM-T. In Section 4, some simulation studies of the proposed model are presented. In Section 5, our model is applied to a real-world dataset and the performances are presented. Finally, we give a conclusion of the present study and some directions for future work in Section 6.

2. Logistic collaborative model with time-varying parameters (LCM-T)

2.1. Comparison with latent class model and some intra-/inter-individual preference models

Before describing details of the proposed model, we briefly discuss two other types of models that are also used to model individual preferences in his discrete choice-making process in transportation.

The latent class choice model (LCCM) posits that an individual's discrete choice behavior depends not only on the attributes in a choice scenario that can be observed, but also on the latent preference heterogeneity that varies with factors that cannot be directly observed by the analyst (Greene and Hensher, 2003). Thus, LCCM assumes that there are group-level preference classes, and each individual belongs to one of them. In the process of identifying the classes with the choice data, LCCM also assigns a membership vector to each individual, representing the probabilities of the individual belonging to those preference classes. Though LCCM looks similar to the collaborative learning structure in the model we propose, our model is different from the latent class model of discrete choice in three aspects:

For the first, the assumptions are different. The latent class model of discrete choice assumes that there exist several classes or groups of individuals, and the individuals in the same class have the same preferences (Breffle et al., 2011). An individual's membership vector in the latent class model represents the probabilities for the individual to be in each class (Gopinath, 1996; Campbell et al., 2011). In other words, the latent class model assumes that each individual can be seen as belonging to one class identified by the model, and his preference is the same as other members in the class. For the collaborative model proposed in the manuscript, we assume that there are several unique underlying preference patterns that can be identified from the data. An individual's membership vector, however, represents the degree of resemblance of the individual's preferences to each preference pattern. His preference is a linear combination of all the preference patterns identified by the model, where the weights are his membership vector.

For the second, the questions to be solved by the two models are different. LCM tries to answer the question of how many classes/groups there are among all individuals. The functional relationship between the identified classes and some covariates are explored such that it is possible to predict which class/group an individual may belong to, or what response the individual would give, with probabilities. The proposed model aims to learn each individual's preferences on all the attributes in the choice-making process and to be able to update the preferences efficiently.

For the third, the estimates (i.e., the preferences) obtained are different. Though the individual's class may be predicted, his preferences conditional on a certain class are not personal preferences but estimated at an aggregated level, assuming that the individual has the same preferences as other people in the same class. The proposed model, however, aims to learn each individual's personal preferences with the canonical models and the individual's membership vector identified from the data.

Different from LCCM which are used to cluster individuals into different preference classes, mixed logit model and related augments are proposed to capture preference heterogeneity, which is believed to be a major concern in the discrete choice analysis that can significantly impact the accuracy of preference estimation and behavioral prediction (Revelt and Train, 1998; Krueger et al., 2019; Ben-Akiva et al., 2019). By accommodating the preference variation with the standard logit model, the mixed logit model accounts for unobserved preference heterogeneity among individuals, assuming that the preferences vary randomly across individuals but are unique and stable for each individual (Revelt and Train, 1998; McFadden and Train, 2000). Along with the improved performance of the mixed logit model in various applications, some researchers further hold that an individual's preference may not be stable but vary according to the choice scenarios or evolve gradually with personal experiences, and both "intra- and inter-individual preference heterogeneity" need to be accounted for in the discrete choice analysis (Bettman et al., 1998; Kivetz et al., 2008; Hess and Giergiczny, 2015; Krueger et al., 2019). Motivated by this, an augmented mixed logit model has been developed, in which scenario-specific preferences are also random variables and generated as perturbations around the individual-specific preference parameters with a certain type of multivariant mixing distribution (Bhat and Sardesai, 2006; Hess and Train, 2011; Becker et al., 2018; Danaf et al., 2019; Krueger et al., 2019). Given the complexity of the model, various estimation methods have also been proposed and applied, including maximum simulated likelihood (MSL) estimation methods (Hess and Train, 2011), MCMC (Becker et al., 2018), and Variational Bayesian Inference (Krueger et al., 2019).

However, besides the limitations of the augmented mixed logit model such as long computation times, serial correlation, high storage costs for the posterior draws, and difficulties in assessing convergence (Depraetere and Vandebroek, 2017; Krueger et al., 2019; Bansal et al., 2020), a major problem for the model is that the values of each individual's preferences at each time step are still not estimated, thus could not be used to estimate the utility of an alternative for an individual, or make behavioral predictions or recommendations. Also, both inter- and intra-individual preferences are assumed to be in normal distributions in the mixed logit model, which may not be the case in reality.

The present work aims to account for the inter- and intra-individual preference heterogeneity in discrete choice with a different model structure other than the logit models. The proposed model is able to capture the inter-individual heterogeneity with the collaborative learning structure and the intra-individual heterogeneity with the time-dependent model, which allows variations or perturbations in an individual's preferences during his choice-making process. Most importantly, the model is able to estimate each individual's preferences at each time step, and the estimation is free from the form of distributions given the data-driven nature of the model.

2.2. Time-varying preference $\beta(t)$ in Logistic Collaborative Model

The proposed Logistic Collaborative Model with Time-varying Parameters (LCM-T) aims to allow an individual's preferences to vary over time in the individualized modeling. In other words, while in the traditional logistic model for binary choices, the probability of an alternative being chosen by individual i at tth time step ($y_{it} = 1$) is

$$Pr(y_{it} = 1|\boldsymbol{x}_{it}) = \frac{\exp(\boldsymbol{\beta}_i^T \boldsymbol{x}_{it})}{1 + \exp(\boldsymbol{\beta}_i^T \boldsymbol{x}_{it})} \tag{1}$$

where $\boldsymbol{x}_{it}$ is the differences of the factor variables (attributes) between two alternatives at time step t. In this paper individual i's preference vector $\boldsymbol{\beta}_i$ is converted to $\boldsymbol{\beta}_i(t)$, which means that the vector $\boldsymbol{\beta}_i$ may be different when time t is different.

An individual's preferences may vary according to specific scenarios/attributes, or evolve gradually with personal experiences (Bettman et al., 1998; Kivetz et al., 2008; Hess and Giergiczny, 2015; Krueger et al., 2019). Considering the possible influential factors, there can be various time-dependent models for $\boldsymbol{\beta}(t)$. In the current study, we use polynomial models as the formulation of the time-dependent model $\boldsymbol{\beta}(t)$. Since the focus of the current work is to elaborate an online preference learning and updating algorithm whose basic structure is integrating the canonical structure and a time-dependent preference model, to illustrate the algorithm, we would like to start from a simple case where the preference model is only influenced by one variable. The polynomial models allow us to express time-dependent variations, while the only influential variate is t. This simple model is effective and can capture patterns in a range of applications. More delicate dynamic models will be explored in future research.

To further prove that the polynomial model could capture preference changes in individual choice-making process, we have also run some regressions with individual preference data. In our previous study Zhu et al. (2020), we have collected responses for a sequence of 13 travel choice scenarios from 826 individuals and learned their preferences during the process with the particle filter approach. We ran polynomial regression models with the learned preferences. The results show that either the linear, quadratic or cubic polynomial model can describe the preference variation statistically significantly. Hereafter from Section 2 though Section 4, we use a formulation of cubic model $\beta(t) = q_0 + q_1 t + q_2 t^2 + q_3 t^3$ as an example to construct our model and introduce our method.

2.3. The model formulation of logistic collaborative learning with time-varying parameters (LCM-T)

2.3.1. The collaborative learning framework with time-dependent preference: a simple example

We will first describe the idea of the collaborative learning structure and the proposed model with the following example.

Assume that a group of people has made their travel choices for morning commuting for several days. When deciding whether to bike or not, they all concern four factors: total commuting time, total commuting cost, their personal preferences on biking, and the weather. Different individuals may value each factor differently when making their commuting choices. We could say that an individual's preference on each factor forms a dimension of an individual's preferences, i.e., the preferences have 4 dimensions: preference on commuting time, preference on commuting cost, preference on biking, and preference on the weather. It can be imagined that when the attributes of these factors in choice scenarios are different, people's preferences may change. For example, given that the weather is cloudy, some people's preference on biking may fall due to a concern of rain. Some other people, however, may feel it a good temperature and humidity to bike to work. Thus, when the weather changes every day, an individual's preference on biking may also vary accordingly, e.g., the preference rises from 0.5 to 0.8 on the second day which is cloudy, falls to −0.3 on the third day which is a rainy day, and so on. We define the curve that describes how the preference value changes over time or in a sequence of scenarios as "preference changing pattern". It can also be imagined that the changing patterns of different factors may not be the same: on a cloudy day, when an individual's preference on biking rises, his preference on commuting time or commuting cost may not rise correspondingly. Thus we could see that the changing patterns of each preference dimension may be discussed separately.

To simplify in the explanation, let us only focus on one preference dimension (i.e., the preference on biking) as an example, and assume that the changes of the preference on biking will only be impacted by weather. Also, assume that we have only two types of common preference changing patterns in the group: for preference changing pattern $\beta_A(t)$, an individual's preference on biking will be 1 on a sunny day, be 1 on a cloudy day, and be 0 on a rainy day; for preference changing pattern $\beta_B(t)$, an individual's preference on biking will be 0 on a sunny day, be −1 on a cloudy day, and be −1 on a rainy day. We can see that these two types are significantly different from each other: $\beta_A(t)$ represents a bike-lover since the values of preference on biking in the choice-making process are always non-negative, while $\beta_B(t)$ may represent someone who never bikes as the preference curve keeps staying in the non-positive zone. These people exist, yet most people, as we could imagine, are likely to be in between: in some choice scenarios, their preference on biking is positive, and in some others, the preference is negative.

According to the collaborative learning framework, the two common preference changing patterns $\beta_A(t)$ and $\beta_B(t)$ identified from this group of people are called "canonical models", which form the basic references for all the individuals' preferences on biking. Specifically, each individual's preference on biking is assumed to be obtained with a combination of $\beta_A(t)$ and $\beta_B(t)$. For example, an individual's preference on biking can resemble $\beta_A(t)$ at a degree of α. Since we assume that there are only two types of preference changing patterns here in the group, this individual's preference changing pattern would then resemble $\beta_B(t)$ at a percentage of $1-\alpha$. The individual's preference on biking would be $\alpha\beta_A(t)+(1-\alpha)\beta_B(t)$. The vector of the parameters $(\alpha, 1-\alpha)$ is called this individual's "membership vector" of his preference on biking. Since each individual may have different values of α, the preferences of each individual would be different.

With the example, we briefly describe the collaborative learning framework and the preference changing patterns we mentioned in previous sections. We interpret the identified "canonical models" as a set of "common types of preference changing patterns", meaning that these changing patterns are shared by all the individuals in the group, as each individual's preferences are composed by these canonical models with weights. Now, we can move to the general description and mathematical formulation of the proposed LCM-T.

2.3.2. Mathematical formulation of LCM-T

As we state in the introduction section, the collaborative learning framework exploits the underlying canonical structure of the preference changing patterns in the population with heterogeneity. With the framework, several common preference changing patterns are identified from all individuals' data, which are assumed to be the basic structural elements of the individual preferences for all individuals. Specifically, each individual's preferences are generated by linearly combining all the common preference changing patterns, in which the weights towards these common preference changing patterns are identified from his own choices. For each individual, the weights form his "membership vector", which describes to what degree his own preference changing pattern resembles each of the common patterns.

Now we write this model in a mathematical way. Assume that an individual's preferences have R dimensions, meaning that there are R attributes in each choice scenario, and $\boldsymbol{\beta}_i$ in Eq. (1) is an R-dimensional vector. Also assume that for individual i, the changing pattern of the rth dimension of his preferences can be described by a cubic polynomial function $\beta_{ir}(t)=q_{ir,0}+q_{ir,1}t+q_{ir,2}t^2+q_{ir,3}t^3$ ($r=1,2,\dots,R$; $i=1,2,\dots,N$), as discussed in the previous section. For the whole population, we assume that there are K_r canonical models $(f_{1,r}(t), f_{2,r}(t),\dots,f_{K_r,r}(t))$ for rth preference dimension $\beta_r(t)$, representing K_r common changing patterns identified from all N individuals. Thus, formulation of the canonical models should be the same as the changing pattern of the preference dimension, which means the cubic polynomial equation, i.e., $f_{k,r}(t)=q_{kr,0}+q_{kr,1}t+q_{kr,2}t^2+q_{kr,3}t^3$. Assigning each individual i with a membership vector $\boldsymbol{c}_{i,r}=[c_{1,ir},c_{2,ir},\dots,c_{K_r,ir}]^T$, $\sum_k c_{k,ir}=1$, representing the degree of resemblance of the individual's rth dimension of his preferences $\beta_{ir}(t)$, to the canonical models. Now, with the canonical models and the individual's membership vector, the individual i's preference $\beta_{ir}(t)$ could be expressed with a linear combination of the membership vector and the canonical models: $\beta_{ir}(t)=\sum_k c_{k,ir}f_{k,r}(t)$, $k=1,2,\dots,K$. Similarly, we could assume that other preference dimensions for individual i can also be expressed with the corresponding membership vectors and the canonical model sets. In total, there will be R different sets of canonical models for all R dimensions of the preferences, and each individual has R membership vectors correspondingly. With the canonical models and the membership vectors, all the individuals' preferences can be expressed.

Now we rewrite some of the notations such that we could use them in the formula expressions. The parameters of a canonical model $f_{k,r}(t)=q_{kr,0}+q_{kr,1}t+q_{kr,2}t^2+q_{kr,3}t^3$ could be denoted by a R-dimensional parametric vector: $\boldsymbol{q}_{k,r}=[q_{kr,0},q_{kr,1},\dots,q_{kr,R}]^T$. Then, $\beta_{ir}(t)$ could be written as a product of two vectors, i.e., $\beta_{ir}(t)=\sum_k c_{k,ir}\boldsymbol{q}_{k,r}{}^T\boldsymbol{v}_r(t)$, where $\boldsymbol{v}_r(t)$ is the time-dependent vector which gives the form of the canonical models. For example, if the canonical model is in a linear form $f_{k,r}(t)=q_0+q_1t$, it could be seen as $f_{k,r}=\boldsymbol{q}_{k,r}{}^T\boldsymbol{v}_r(t)$, where the parameter vector $\boldsymbol{q}_{k,r}=[q_0,q_1]^T$ and the time-dependent vector $\boldsymbol{v}_r(t)=[1,t]^T$.

For the binary logistic model as shown in Eq. (1), we can have:

$$
\begin{aligned}
Pr(y_{it}=1|\boldsymbol{x}_{it}) &= \frac{\exp(\boldsymbol{\beta}_i^T\boldsymbol{x}_{it})}{1+\exp(\boldsymbol{\beta}_i^T\boldsymbol{x}_{it})} = \frac{\exp(\boldsymbol{x}_{it}^T(\boldsymbol{Q}\boldsymbol{C}_i)^T\boldsymbol{V}(t))}{1+\exp(\boldsymbol{x}_{it}^T(\boldsymbol{Q}\boldsymbol{C}_i)^T\boldsymbol{V}(t))}\\
Pr(y_{it}=0|\boldsymbol{x}_{it}) &= \frac{1}{1+\exp(\boldsymbol{\beta}_i^T\boldsymbol{x}_{it})} = \frac{1}{1+\exp(\boldsymbol{x}_{it}^T(\boldsymbol{Q}\boldsymbol{C}_i)^T\boldsymbol{V}(t))}
\end{aligned}
\tag{2}
$$

The log-likelihood function of the binary logistic model then can be written as:

$$
l=-\log(1+\exp(\boldsymbol{\beta}_i^T\boldsymbol{x}_{it}))+y_{it}(\boldsymbol{\beta}_i^T\boldsymbol{x}_{it})=-\log(1+\exp(\boldsymbol{x}_{it}^T(\boldsymbol{Q}\boldsymbol{C}_i)^T\boldsymbol{V}(t)))+y_{it}(\boldsymbol{x}_{it}^T(\boldsymbol{Q}\boldsymbol{C}_i)^T\boldsymbol{V}(t))
\tag{3}
$$

To formulate the log-likelihood function in the collaborative learning framework for parameter estimation, we further write up the parameters of the K_r canonical models for β_r as a matrix: $\boldsymbol{Q}_r\equiv[\boldsymbol{q}_{1,r},\boldsymbol{q}_{2,r},\dots,\boldsymbol{q}_{K_r,r}]\in\mathbb{R}^{R\times K_r}$. Then, we could rewrite $\beta_{ir}(t)=(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_r(t)$. Given that each individual's preferences have R dimensions, and each dimension has a set of canonical models, an individual may have R different membership vectors towards R dimensions of his preferences. The individual i's preference vector is:

$$
\boldsymbol{\beta}_i=[\beta_{i1}(t),\beta_{i2}(t),\dots,\beta_{iR}(t)]^T=[(\boldsymbol{Q}_1\boldsymbol{c}_{i1})^T\boldsymbol{v}_1(t),(\boldsymbol{Q}_2\boldsymbol{c}_{i2})^T\boldsymbol{v}_2(t),\dots,(\boldsymbol{Q}_R\boldsymbol{c}_{iR})^T\boldsymbol{v}_R(t)]^T
$$

We also need to formulate $\boldsymbol{\beta}_i^T\boldsymbol{x}_{it}$. To do this, we combine the membership vectors for all dimensions of the preferences of individual i into a diagonal matrix $\boldsymbol{C}_i=diag(\boldsymbol{c}_{i1},\boldsymbol{c}_{i2},\dots,\boldsymbol{c}_{iR})$. We further combine the canonical models for each dimension into a big matrix $\boldsymbol{Q}=diag(\boldsymbol{Q}_1,\boldsymbol{Q}_2,\dots,\boldsymbol{Q}_R)$ and combine the time-dependent vectors $\boldsymbol{v}_r(t)$ for the R forms of the canonical models into

$\boldsymbol{V}(t) = [\boldsymbol{v}_1(t), \boldsymbol{v}_2(t), \ldots, \boldsymbol{v}_R(t)]^T$. Then $\boldsymbol{\beta}_i^T \boldsymbol{x}_{it}$ could be written as $\boldsymbol{\beta}_i^T \boldsymbol{x}_{it} = \boldsymbol{x}_{it}^T (\boldsymbol{Q}\boldsymbol{C}_i)^T \boldsymbol{V}(t)$. At time step t, the result of $(\boldsymbol{Q}\boldsymbol{C}_i)^T \boldsymbol{V}(t)$ is a $R \times 1$ vector $[\beta_{i1}(t), \beta_{i2}(t), \ldots, \beta_{iR}(t)]^T$.

To learn all the parameters in $\boldsymbol{C}$ and $\boldsymbol{Q}$, we pool all the data from all individuals together, which leads to the following formulation:

$$\begin{aligned} \min_{\mathbf{C_1},\mathbf{C_2},\ldots,\mathbf{C_N},\mathbf{Q}} \quad & \sum_{i=1}^{N} \frac{1}{n_i} \sum_{t=1}^{n_i} \left\{ \log(1 + \exp(\boldsymbol{x}_{it}^T \cdot (\boldsymbol{Q}\boldsymbol{C}_i)^T \boldsymbol{V}(t))) - y_{it}(\boldsymbol{x}_{it}^T \cdot (\boldsymbol{Q}\boldsymbol{C}_i)^T \boldsymbol{V}(t)) \right\}, \\ \text{s.t.} \quad & \boldsymbol{c}_{ir} \geq 0,\ \boldsymbol{c}_{ir}^T \mathbf{1} = 1 \quad i = 1, \ldots, N; r = 1, \ldots, R. \end{aligned} \tag{4}$$

Here, n_i is the number of measurements/observations from individual i. We name the model of the optimization framework in Eq. (4) as the Logistic Collaborative Model with Time-varying Preferences (LCM-T). In LCM-T, while t changes, the attributes $\boldsymbol{x}_{it}$ changes accordingly, representing the specific attributes for the scenario at time step t. The time-dependent vector $V(t)$ also changes, leading to the changes of each dimension of individual i's preferences. It could be noticed that now in our LCM-T, the sequences of the choice scenarios play a role in determining the decision variables. This is different from the previous optimization works done with the collaborative learning framework in (Lin et al., 2018b,a, 2015).

As stated in the previous section, we assume that all the preference dimensions for an individual could be described with cubic polynomial models, i.e., $\boldsymbol{v}_1(t) = \boldsymbol{v}_2(t) = \cdots = \boldsymbol{v}_R(t) = [1, t, t^2, t^3]^T = \boldsymbol{v}_t$. Thus, the formation of all the canonical models ($k = 1, 2, \ldots, K_r$) for any dimension β_r could be written as $f_{k,r}(t) = \boldsymbol{q}_{k,r}{}^T \boldsymbol{v}_t$, where the parameter vector $\boldsymbol{q}_{k,r} = [q_{1,kr}, q_{2,kr}, q_{3,kr}, q_{4,kr}]^T$, i.e., $R = 4, \forall r \in \{1, 2, \ldots, R\}$.

3. Online logistic collaborative model with time-varying parameters (OLCM-T)

3.1. OLCM-T and parameter estimation algorithm

With the optimization problem shown in Eq. (4), we can estimate the canonical parameters matrix $\boldsymbol{Q}$ and the membership matrix $\boldsymbol{C}$ iteratively with a parameter estimation algorithm similar to the one in (Lin et al., 2018a). However, the computation process may take time. Thus, this wholesome update of all parameters of all individuals based on only a few observations from one individual is not efficient. To update an individual's preferences when a new observation is available, we separate the process of canonical model updating and membership vector updating of the LCM-T into two stages — "offline updating" and "online updating". The proposed Online Logistic Collaborative Model with Time-varying Parameters (OLCM-T) we are focusing on in this paper is the algorithm of the online stage.

In the online updating stage, an individual's membership vectors will be updated given his own data, while the canonical models are not updated. The online updating could also be used to learn the membership vectors when a new user comes to the system.

Assume that for individual i, there are n_i observations $y_{it}, t = 1, 2, \ldots, n_i$ available. In online updating process, the optimization problem in Eq. (4) becomes:

$$\begin{aligned} \min_{c_{ir}, r=1,2,\ldots,R} \quad & \sum_{t=1}^{n_i} \left\{ \log\left(1 + \exp\left(\sum_{r=1}^{R} x_{r,it} (\boldsymbol{Q}_r \boldsymbol{c}_{ir})^T \boldsymbol{v}_t\right)\right) - y_{it}\left(\sum_{r=1}^{R} x_{r,it} (\boldsymbol{Q}_r \boldsymbol{c}_{ir})^T \boldsymbol{v}_t\right) \right\}, \\ \text{s.t.} \quad & \boldsymbol{c}_{ir} \geq 0,\ \boldsymbol{c}_{ir}{}^T \mathbf{1} = 1 \quad r = 1, \ldots, R. \end{aligned} \tag{5}$$

Here, $\boldsymbol{V}_t = [\boldsymbol{v}_t, \boldsymbol{v}_t, \ldots, \boldsymbol{v}_t]^T$, where $\boldsymbol{v}_t = [1, t, t^2, t^3]^T$. $\boldsymbol{Q} = diag(\boldsymbol{Q}_1, \boldsymbol{Q}_2, \ldots, \boldsymbol{Q}_R)$ is the diagonal matrix of the canonical models of each dimension, where $\boldsymbol{Q}_r = [\boldsymbol{q}_{1,r}, \boldsymbol{q}_{2,r}, \ldots, \boldsymbol{q}_{K_r,r}]$, $r = 1, 2, \ldots, R$ is the parameter matrix of K_r canonical models for preference dimension r. $x_{r,it}$ and y_{it} ($t = 1, 2, \ldots, n_i$, $r = 1, 2, \ldots, R$) are the attributes and choices in the t's scenario for individual i, which are also known. The decision variables are individual i's $\boldsymbol{R}$ membership vectors towards each dimension of the preferences. For each dimension r, his membership vector $\boldsymbol{c}_{ir} \geq 0$, and the $\sum_{k=1}^{K_r} c_{k,ir} = 1$. Since each dimension of the individual's preferences is a linear combination of the corresponding canonical models, each preference dimension is also time-dependent and change in a form of the cubic polynomial.

An iterative parameter estimation algorithm can be derived on the basis of Karush–Kuhn–Tucker (KKT) conditions. The detailed derivation process of the updating rule can be found in Appendix A of this paper. Similar to Lin et al. (2015, 2018b,a), the final updating rule for $\boldsymbol{c}_{ir}$ is:

$$\begin{aligned} c_{k,ir}^{m+1} = c_{k,ir}^{m} \times & \left\{ \sum_{t=1}^{n_i} \left[-\delta_-\left(\frac{\exp(\sum_{r=1}^{R} x_{r,it} (\boldsymbol{Q}_r \boldsymbol{c}_{ir}^m)^T \boldsymbol{v}_t)}{1 + \exp(\sum_{r=1}^{R} x_{r,it} (\boldsymbol{Q}_r \boldsymbol{c}_{ir}^m)^T \boldsymbol{v}_t)} \times x_{r,it} (\boldsymbol{Q}_r{}^T \boldsymbol{v}_t)_k \right) + \delta_+\left(y_{it} \times x_{r,it} (\boldsymbol{Q}_r{}^T \boldsymbol{v}_t)_k \right) \right.\right. \\ & \left.\left. -\delta_-\left(y_{it} \times x_{r,it} (\boldsymbol{Q}_r{}^T \boldsymbol{v}_t)^T \boldsymbol{c}_{ir}^m \right) + \delta_+\left(\frac{\exp(\sum_{r=1}^{R} x_{r,it} (\boldsymbol{Q}_r \boldsymbol{c}_{ir}^m)^T \boldsymbol{v}_t)}{1 + \exp(\sum_{r=1}^{R} x_{r,it} (\boldsymbol{Q}_r \boldsymbol{c}_{ir}^m)^T \boldsymbol{v}_t)} \times x_{r,it} (\boldsymbol{Q}_r{}^T \boldsymbol{v}_t)^T \boldsymbol{c}_{ir}^m \right) \right] \right\} \\ / & \left\{ \sum_{t=1}^{n_i} \left[\delta_+\left(\frac{\exp(\sum_{r=1}^{R} x_{r,it} (\boldsymbol{Q}_r \boldsymbol{c}_{ir}^m)^T \boldsymbol{v}_t)}{1 + \exp(\sum_{r=1}^{R} x_{r,it} (\boldsymbol{Q}_r \boldsymbol{c}_{ir}^m)^T \boldsymbol{v}_t)} \times x_{r,it} (\boldsymbol{Q}_r{}^T \boldsymbol{v}_t)_k \right) - \delta_-\left(y_{it} \times x_{r,it} (\boldsymbol{Q}_r{}^T \boldsymbol{v}_t)_k \right) \right.\right. \\ & \left.\left. +\delta_+\left(y_{it} \times x_{r,it} (\boldsymbol{Q}_r{}^T \boldsymbol{v}_t)^T \boldsymbol{c}_{ir}^m \right) - \delta_-\left(\frac{\exp(\sum_{r=1}^{R} x_{r,it} (\boldsymbol{Q}_r \boldsymbol{c}_{ir}^m)^T \boldsymbol{v}_t)}{1 + \exp(\sum_{r=1}^{R} x_{r,it} (\boldsymbol{Q}_r \boldsymbol{c}_{ir}^m)^T \boldsymbol{v}_t)} \times x_{r,it} (\boldsymbol{Q}_r{}^T \boldsymbol{v}_t)^T \boldsymbol{c}_{ir}^m \right) \right] \right\} \end{aligned} \tag{6}$$

where the superscript m refers to the order of iteration.

In summary, the membership vectors could be learned and updated iteratively with the following steps:

1. Input: $\boldsymbol{Q}$, $\boldsymbol{v}_t = [1, t, t^2, t^3]^T$, $x_{r,it}$, y_{it}, initial value $\boldsymbol{c}_{ir}^0$, for all $r = 1, 2, \dots, R$; $t = 1, 2, \dots, n_i$;
2. For $m = 1, 2, \dots$, iteratively update each dimension of each membership vector $c_{k,ir}^{m+1}$ with Eq. (6), given $\boldsymbol{c}_{ir}^m$ calculated in the previous iteration step;
3. Give a pre-determined criteria ϵ. When $\gamma^m = \sum_{r=1}^R \|\boldsymbol{c}_{ir}^{m+1} - \boldsymbol{c}_{ir}^m\|_2^2 \le \epsilon$, stop the iteration.

3.2. Relationship between online updating and offline updating

In the work of Zhu et al. (2020), an algorithm based on Random Utility Maximization theory utilizing a particle filtering method is used to learn and update an individual's preferences, such that tracking preference changes is possible. The proposed LCM-T can capture the preferences changes by estimating $\boldsymbol{Q}$ and $\boldsymbol{C}$ iteratively. However, compared with the particle filter method, the updating process of the proposed LCM-T would be cumbersome, especially when we are interested in updating only several individual's preferences (assume that the whole population is large). In other words, the whole computation process of the optimization problem needs to be taken, even when only a few individuals have new data available and their preferences are to be updated. Because of this, the real-time preference updating for each individual may be inefficient to implement with the proposed LCM-T.

The two-stage updating structure of the proposed LCM-T could solve the problem: by having an online and an offline updating stage, we could apply the proposed online updating method to the new data obtained by an individual and only update his membership vectors with much shorter computation time. Since the canonical models are vital in the preference learning process but they are not updated in the online updating stage, the offline updating stage needs to be applied once after a time, during which both canonical models and individuals' membership vectors are updated using much more data in history. In other words, the online updating stage is taken more frequently than the offline updating stage.

Since the online and offline updating stages are taken iteratively in the LCM-T model, a linkage between the two stages should be built, and several questions need to be answered. For example, one question is related to the updating rule of the membership vectors. As the membership vectors are updated both in online and offline stages, the updates from the two stages may be different from each other. How to update the membership vectors in the iterative process such that little information is lost may worth exploration. One possible method is to add friction factors in the offline-updating process, such that the information from both online and offline stages could be maintained in the membership vectors. Another question could be related to the time interval between two offline updates. If offline updating is taken rarely, the model may have low accuracy, while if the offline updating is taken very frequently, the model may have low efficiency. This may need more explorations with real-world data and the conclusions may differ significantly when having different applications. These questions are beyond the scope of the current paper, where we primarily focus on the online updating process when the canonical models are assumed known from previous offline updating step. In the following sections, we are only considering the online updating stage.

4. Simulation studies

4.1. General design of the simulations

The purpose of the simulation is to test the performance of the proposed model using online-updating strategy. Given the online-updating strategy is a real-time updating method, the model is expected to estimate and update an individual's preferences at each time step when new data points are available.

The performance of the model is evaluated with two metrics, Average Absolute Error, and Prediction Accuracy. Average Absolute Error measures the difference between learned coefficients and the true ones (Eq. (15)):

$$\text{Average Absolute Error} = \frac{1}{N}\sum_{i=1}^{N} |\beta_i - \hat{\beta}_i|. \tag{7}$$

Prediction Accuracy measures how the model performs in behavioral predictions, as shown in Eq. (8):

$$\text{Prediction Accuracy} = \frac{N_{\text{Number of predictions that are consistent with the actual choices made}}}{N_{\text{Number of predictions made in total}}} \tag{8}$$

The simulations in this section test the performance of the proposed OCLM-T in scenarios with different numbers of influential factors $R \in (2, 3, 4)$ and when having different numbers of canonical models $K \in (2, 3, 4, 5)$. Besides, the simulation also explores how different levels of noisy responses $(10\%, 20\%, 30\%)$ may affect the learning and prediction accuracy of the model. In the simulations, we (1) generate the true values of the preferences for each individual for several successive time steps, (2) generate attributes for the choice scenario at each time step, and (3) generate the responses to the scenario based on his the true preferences at each time step. Notice that when generating the true preferences for each individual at step (1), we need to generate the canonical models (polynomial models in the current study) and membership vectors for all individuals, such that we can obtain each individual's preference changing model with the method we propose in Section 2, and estimate his preferences at each time step. The details of how the models and parameters are generated can be found in Appendix B of this paper.

The performance of the proposed OLCM-T is compared with several benchmark methods including (1) the independent logistic regression model (ILM) that learns the regression coefficients of each individual solely based on his own data; (2) the mixed-effect logistic regression model (logistic MEM) that considers the coefficients of individuals are extracted from a certain distribution; (3) the one-size-fits-all logistic regression model (LR) that treats all individuals homogeneously, combines all individuals' data and

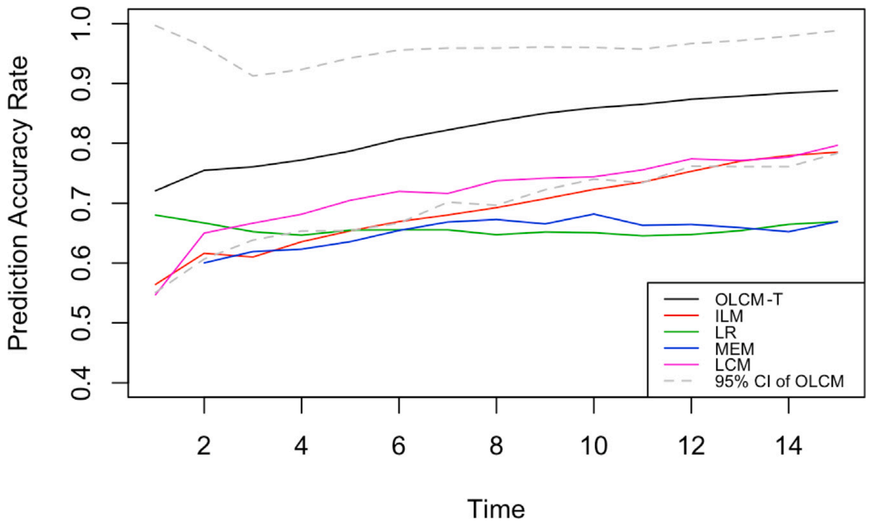

Fig. 1. The prediction accuracy of each model over time in online updating process. (Each curve is the average of 100 independent runs.)

estimates one set of preferences; (4) the original Logistic Collaborative Learning (LCM) model, where individual preferences are constant values β_r rather than time-varying variables $\beta_r(t)$. In LCM, the canonical models represent different utility models in a format of $U = \sum_r \beta_r x_r$, and different canonical models have different β_r.

The simulations in this section test each model's performance when available data points increase over time. In other words, at the beginning of the updating period, the data points obtained from each individual may be limited (i.e., the so-called sparse sampling condition), and as time goes by, the sampling condition may become denser when more and more available data points could be collected. This happens commonly in various realistic circumstances, and we will show that the proposed method has advantages when dealing with it.

4.2. The online-updating process in simulation

To mimic the online updating process over time, we assume that at each time point, one data point could be obtained from each individual. For a given time step, the data that could be used to learn the preferences of the individual is the set of the data points obtained from all previous time steps and the new data point obtained at the current time step. Another 10 data points are also generated at each time step for testing. In other words, at time step t, we generate $1 + 10 = 11$ data points with the performances of the time step t. Among all the 11 data points, one is the new observation at this time step, and the other 10 data points consist of the testing set.

In our simulation, we start from a basic setting of (1) the total number of individuals $N = 120$, (2) the number of preference parameters in utility function $R = 4$ (i.e., the individual preferences have four dimensions), and (3) the number of canonical models for each preference dimension $K = 2$. With this basic setting, each individual has four membership vectors towards the four dimensions of his preferences, and each dimension of his preferences could be described by the two canonical models of that preference dimension and his membership vector towards that dimension. Tests regarding the performances of the model with different numbers of canonical models and different numbers of dimensions (i.e., the number of variables in the utility function) are also conducted accordingly.

We present the results of the prediction accuracy, computation time, and the Average Absolute Error of an estimate for each model (OLCM-T, ILM, LR, MEM, and LCM) in online updating over time, with the OLCM-T having four parameters in the utility model and two canonical models in each preference dimension. Notice that though the scenarios and responses are the same when testing all the five models, the division of training sets and testing sets are only used in simulations of OLCM-T model. We then show the differences in prediction accuracy rates when changing the number of canonical models in each preference dimension, and when changing the variable numbers for our proposed OLCM-T model.

4.3. Simulation results

The results of the prediction accuracy of each model at each time step are presented in Fig. 1. In the experiment, at time $t = 1$, each individual has 1 data point available for estimation. As time goes from $t = 1$ to $t = 15$, the data points collected from each individual increase from 1 to 15. In general, the prediction accuracy of all the five models increases over time when more data are available in preference learning and updating process, among which the proposed OLCM-T considering time-varying preferences has better performances in prediction accuracy comparing with all other models. Since the best number of canonical models of

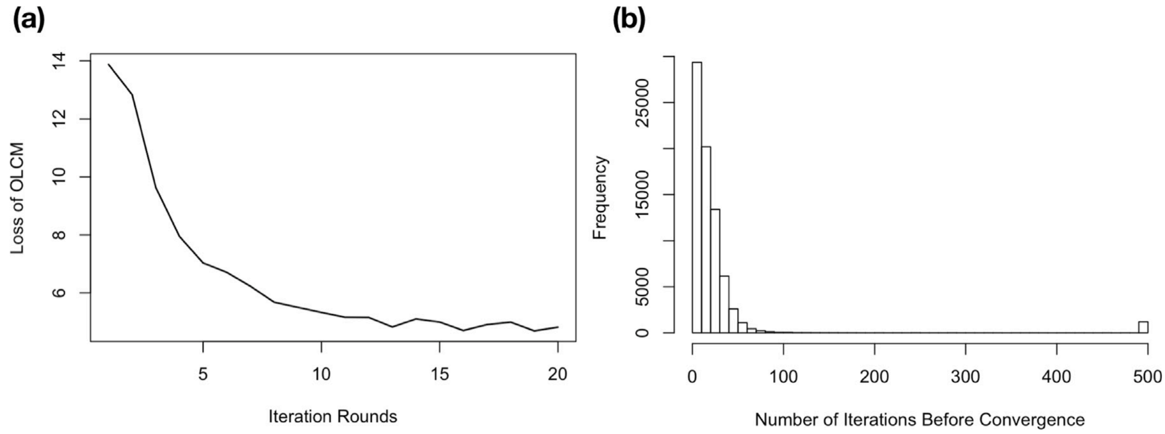

Fig. 2. Convergence performance of the computational algorithm in the proposed OLCM-T.

Table 1
Results model comparison.

Model	OLCM-T	LCM	ILM	LR	MEM
Average computation time (s)	7788.792	713.321	17.123	0.644	835.062
Average absolute error	0.101	0.945	5.175	1.100	0.445

LCM is not known in the simulation, cross-validation is applied to identify optimal K before evaluating the performance of LCM. The average absolute error of the estimates obtained by each model is presented in Table 1. The proposed OLCM-T has the lowest absolute error for each preference parameter when the available data is very limited.

The loss function of our model could be the sum of the minus log-likelihood of the logistic function. Fig. 2a shows the average loss after each iteration of updating with the updating rule shown in Eq. (6), and Fig. 2b presents the histogram of the number of iterations before reaching a pre-specified convergence criterion, i.e., $|Loss(t) - Loss(t-1)| < 0.1$. Besides the criterion, we also set the maximum number of iterations to be 500. It could be seen that more than two-thirds of the total simulations converge within 20 iterations, and only about 1.5% of the simulations do not converge within 500 iterations.

The computation time for the proposed OLCM-T is significantly longer (7788.792 s) than the LCM method (713.321 s) where the preference changing is not considered. This could be reasonable because each individual now has multiple membership vectors (consistent with the number of preference dimensions), such that more parameters in membership vectors need to be estimated. Because of this, the current method may require more iterations before convergence, which may significantly extend the computation time. Notice that the computation time counted here in Table 1 is the total computation time for 15 time steps and 120 individuals. As the proposed online algorithm OLCM-T is designed to be applied to each individual at each time step, the individual updating time at each time step would be much shorter (about 4.33 s).

We further test the performance of the proposed OLCM-T when the number of dimensions and the number of canonical models change. From Fig. 3a we could see that when the number of canonical models increases (i.e., the parameters that need to be estimated increase), the performance of the model decreases. Meanwhile, the computation time does not show significant differences in the process. In Fig. 3b we could see that for the first few time steps, the prediction accuracy would be higher when there are fewer numbers of dimensions (fewer variables in the model). As more data are obtained in the learning process, the performances would become similar to models with different numbers of dimensions. Fig. 4 also shows that in general, the proposed OLCM-T has higher prediction accuracy compared with LCM when the responses fed to the model have some noises.

5. Real-world case study

To see the performance of OLCM-T with real-world data, we apply our model to the dataset we collected in an online experiment we conducted in Zhu et al. (2020). Similar to the simulation section, in the real-world case study, we again compare the performance of the proposed OLCM-T with independent logit model (ILM), traditional logit model (LM), mixed-effect model (MEM), and the original logistic collaborative model where the parameters are constant rather than time-varying (LCM). Moreover, we also compare the performance of OLCM-T with that of the latent class choice model (LCCM) and the particle filter method we used in Zhu et al. (2020).

5.1. Dataset of commuting departure time choices

The dataset includes responses from 826 individuals recruited from AMT (Amazon Mechanical Turk) platform.

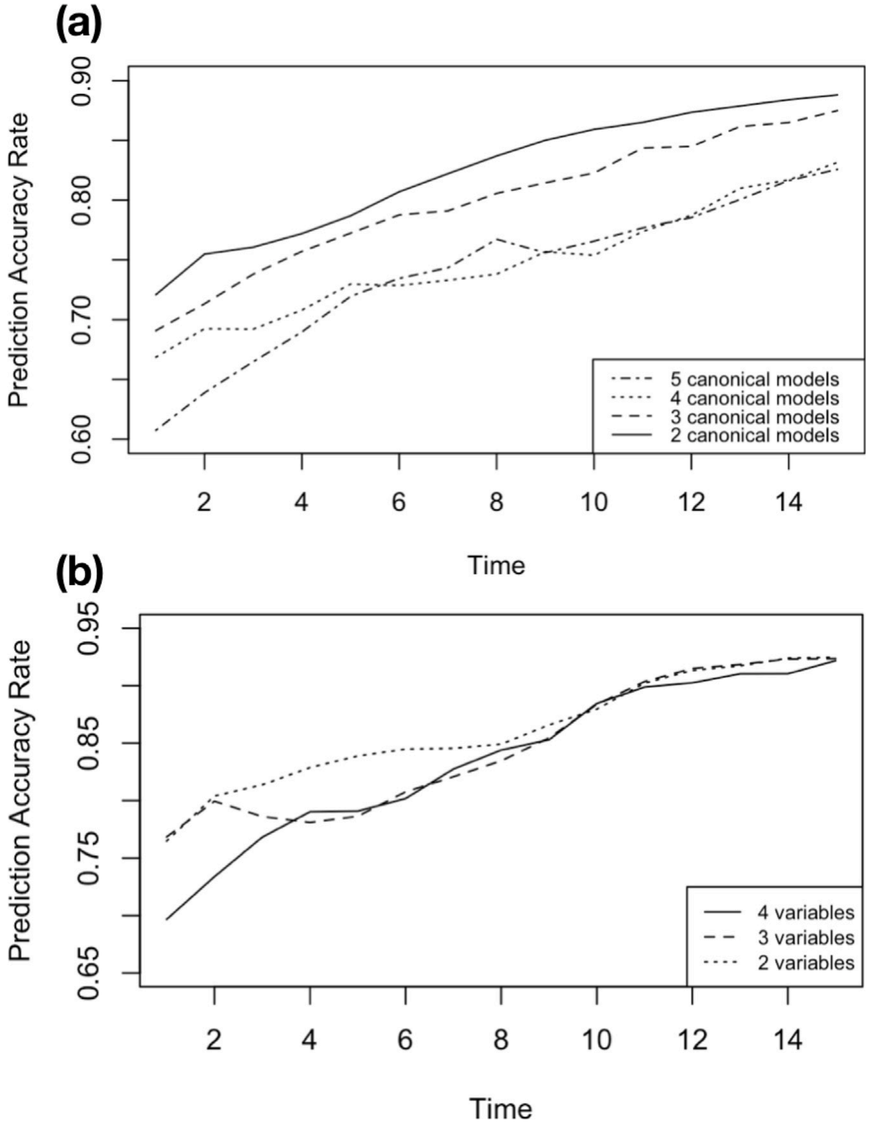

Fig. 3. The prediction accuracy of (a) OLCM-T with different number of canonical models for each preference dimension, and (b) OLCM-T with different number of dimensions. (Each curve is the average of 20 independent runs.)

In the experiment, we randomly assign each participant into one of 9 experimental groups, each with a specific hypothetical background setting on his original commuting departure time, arrival time, typical commuting time, and the level of the flexibility in his working starting time. Then the participant is required to make binary choices on commuting departure time in 13 scenarios. In each scenario, he could select to depart at an earlier or later time point suggested in the experiment, or to stay unchanged and depart at his original departure time. Information provided in each alternative includes departure time, arrival time, total commuting time, and possible reward points that are used to encourage the individual to accept the suggested alternative. Thus, we have four variables in the utility model: changes in arrival time (including both scheduled delay early *SDE* and scheduled delay late *SDL*), the minutes of travel time savings if the suggested departure time is accepted *TTS*, and the reward points *RP*.

$$U = \beta_{SDE}SDE + \beta_{SDL}SDL + \beta_{TTS}TTS + \beta_{RP}RP \tag{9}$$

For each background setting of an experimental group, there are corresponding levels for the attributes of a proposed alternative in a scenario. Specifically, there are three levels for *SDE* and *SDE* (10 min, 25 min, and 60 min), two levels for *TTS* (10% for slight congestion, and 60 for severe congestion), and *RP* is no larger than 100. The experimental groups guarantee that there is preference heterogeneity in the population.

The first two scenarios are the same for all respondents in the experiment. Start from the third scenario, the proposed alternative presented to a participant's in the next time step will be decided by the participant's choice at the current time step and his

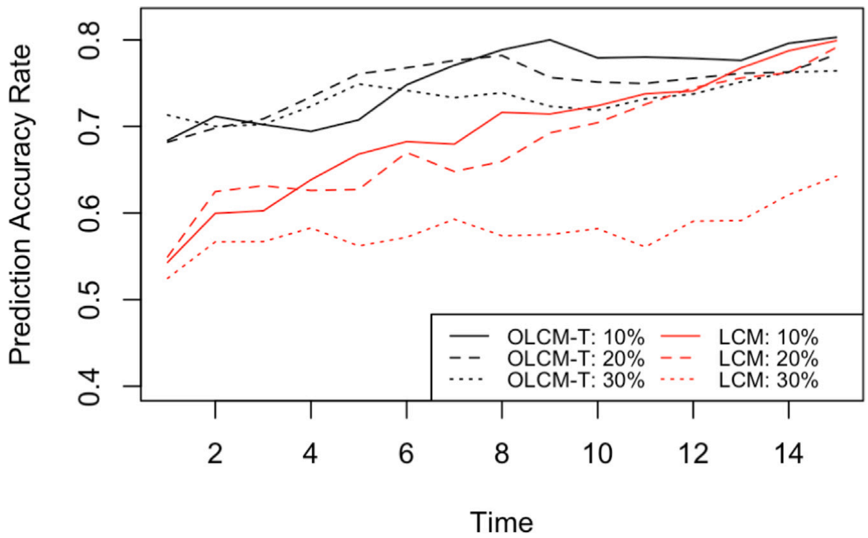

Fig. 4. The prediction accuracy of OLCM-T and LCM with different level of noise in response. (Each curve is the average of 20 independent runs.)

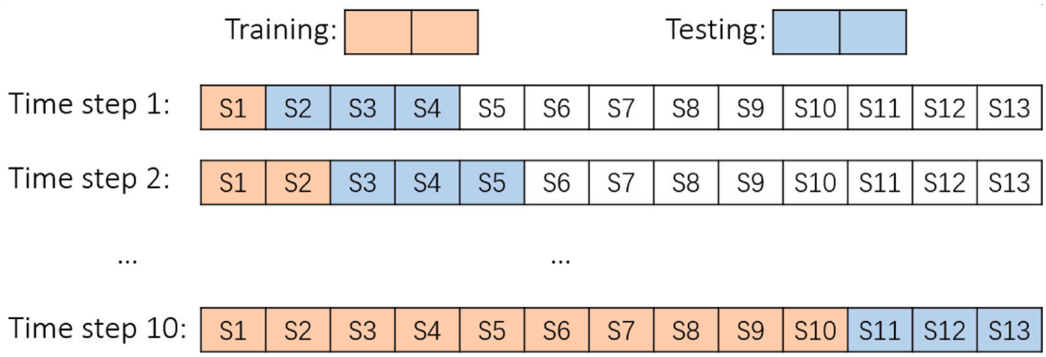

Fig. 5. The data points used for model training and testing with real-world dataset at each time step.

preferences learned by the individual preference learning algorithm proposed in Zhu et al. (2020). With the updated preferences at each time step, the background algorithm further prepares a proposed alternative for the following scenario that is likely to be accepted by the participant. Thus, the amount of *RP* needed is calculated according to RUM such that the proposed alternative is attractive enough for the participant. If the required *RP* exceeds 100 points, the algorithm will change the alternative by adjusting the levels of other attributes.

5.2. Application procedures of different models

Similar to the simulation section, we assume that at each time point, one data point could be obtained from each individual. At each time step, the new data points obtained at the current time step, together with all the data points obtained in previous steps, could be used to learn the individual's preferences. Also, at each time step, the following three scenarios and choices are used as the test dataset to test the prediction accuracy given the preferences learned at the current time step (as shown in Fig. 5).

For the independent logit model (ILM), we run logit regressions for each individual with his own data points at each time step. Since there are four parameters to be estimated, we could not obtain estimates from time step 1 to time step 3 with ILM. For the traditional logit model (LM), we run logit regression with data points from all individuals at each time step, and make predictions assuming that all the individuals have the same preferences. Since the scenarios at the first two time steps are the same for all respondents, we can only obtain estimates starting from the third time step. Similar problems also exist in the mixed-effect model (MEM) and the original logistic collaborative model (LCM).

For the particle filter method used in Zhu et al. (2020), we simply use the percentages of the acceptance at each time step in the dataset as the prediction accuracy. This is because that the preference learning algorithm utilizing particle filter approach itself is embedded in the experiment and each individual's preferences are updated at each time step, and the proposed alternative presented at each time step (starting from the third time step) to each respondent is expected to be accepted by the respondent. Thus, we are

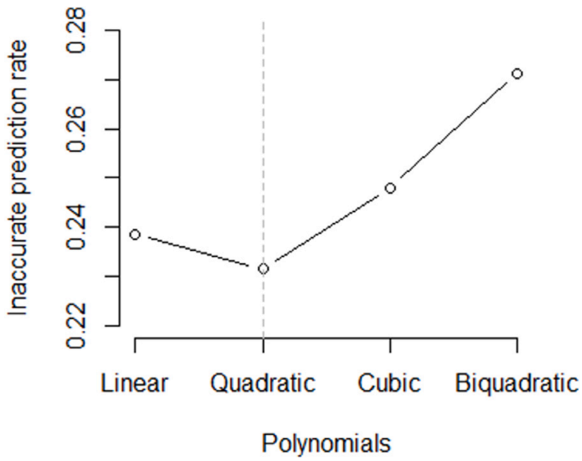

Fig. 6. Inaccurate prediction rates with different degrees of polynomial models.

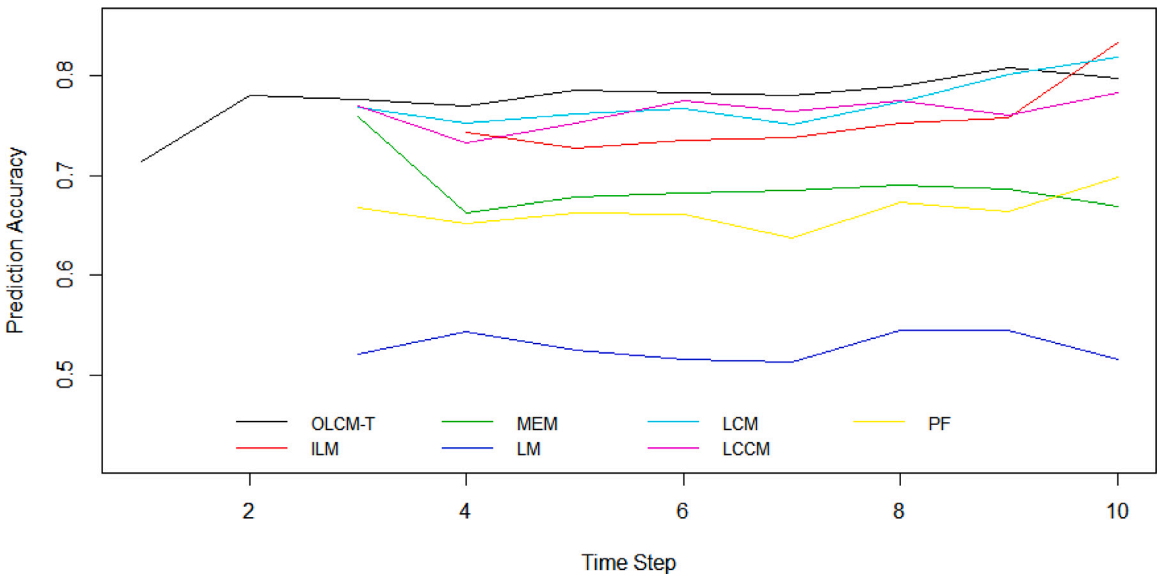

Fig. 7. The prediction accuracy of different models.

able to obtain the prediction accuracy of the particle filter algorithm proposed in Zhu et al. (2020) by calculating the acceptance ratio at each time step in the experiment.

For the proposed OLCM-T, we first decide the suitable degree of the polynomial models applied to this dataset using cross-validation. The results shown in Fig. 6 suggest that the best model suitable for our dataset is the quadratic polynomial model. We then explore how many canonical models are needed for each dimension of the preferences. Similar to the conclusion we obtained from simulations (Fig. 3a), the results also show that having two canonical models may lead to higher performance than having more canonical models. Thus in this real-world case study, we use two canonical modes for each preference dimension for OLCM-T. Cross-validation is also used to decide the number of canonical models for each preference dimension for the original LCM, and we let it be 5.

Since we are testing the performance of the online-updating strategy for the proposed LCM-T, at each time step, we will fix the canonical models and only update each individual's membership vector. Thus, we randomly select 80% of the 826 respondents (661 individuals) as a subset to learn the canonical models for this population, assuming that the canonical models are also applicable for the rest 20% respondents (165 individuals). Given the canonical models, the dataset of those 165 individuals is used to test the algorithm of OLCM-T presented in Section 3. As the 661 individuals in the canonical learning group are randomly selected from the population, we take independent random selection for 10 times.

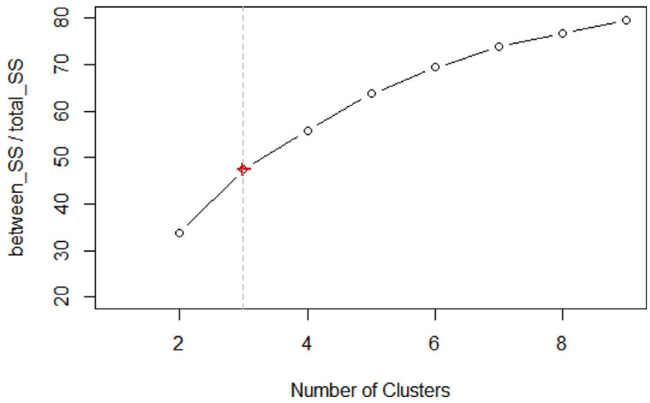

Fig. 8. The ratio of between-cluster-sum-of-squares and total-sum-of-squares when dividing the individuals in to different number of clusters.

5.3. Model application results

The performance of OLCM-T shown in Fig. 7 is the average of the 10 trials. From Fig. 7 we could see that the proposed OLCM-T has higher prediction accuracy than other models from the first time step until the 9th time step, at which the prediction accuracy of LCM exceeds that of OLCM-T. This might because while the canonical models of OLCM-T are fixed in the whole process, the original LCM updates both canonical models and membership vectors at each time step, which may impact the performance of the model in later time steps. Another observation one may notice is that at the 10th time step (i.e., when 10 data points are available for each individual), the performance of ILM has a sudden increase and becomes higher than the performance of the model we propose. This is possibly due to some properties of the dataset we have in the real-world case study and the characteristics of the individuals in the experiment. Besides, our model is shown to be able to estimate a large number of unknown variables, where ILM may require many more data points to get estimates. From Fig. 7, we could also notice that the performance of OCLM-T is just slightly better than other models in early time steps. One reason may be that the polynomial model used here to represent an individual's changing preference is just a tentative example. Better models that could capture the dynamic of the changing preferences may lead to better performance and would require further explorations in our future work.

We would like to present some results showing the changing preferences we learned in the choice-making process. The results are obtained in one of the 10 times of independent algorithm running. Same as other 9 times, in the beginning, 661 individuals' data is randomly selected to learn canonical models for the population. For each preference dimension, two canonical models are identified. The dataset from the other 165 individuals is used to test OLCM-T and learn their changing preferences.

To illustrate the changing preferences, we present the individual preferences learned by OLCM-T in an aggregated way: we cluster the 165 individuals into clusters using k-means according to their membership vectors (see Fig. 8), and show each individual's preference changing curves in Fig. 9. It can be seen that each individual has his own preferences learned by the model, and each individual's preferences are changing in his choice-making process. As we expected, most individuals have negative β_{SDE} and β_{SDL}, and positive β_{TTS} and β_{RP}, which is consistent with the common knowledge we have in transportation behaviors. The changing pattern of each individual is different from each other, possibly due to the different scenarios presented to him.

The results of the real-world case study show that our proposed model contributes to the prediction of the behaviors, which further proves that people's preferences may vary over time when making sequential choices.

6. Conclusion

In this paper, we propose a model that could learn and capture personalized preferences from their behavior data. With simulations and an application to a real-world dataset collected in a recent study (Zhu et al., 2020), the proposed model shows promising outcomes in individual preference learning and behavior prediction, which may be a potential method used to provide personalized incentives and trigger behavioral changes in transportation demand management.

Given the promising results of the OLCM-T, there are at least two directions for our future studies. The first one is to build a suitable mathematical model to describe individuals' time-varying preferences. We will present a more detailed exploration regarding this issue in our future work. The second one would be integrating the online and offline stages in the preference learning process. As we point out earlier, several questions may need further explorations in the integration. For instance, a linkage should be built between the online and offline updating stages such that little information is lost in the process. Besides, the best frequency ratio of online and offline updating should be identified such that the performance of the model could be the best. Given that the proposed model is data-driven, it is possible that the offline-updating frequency may depend on the property and nature of the dataset. With the algorithm of the whole two-stage Logistic Collaborative Model with Time-varying Parameters (LCM-T), some further studies on the property of the model could be explored.

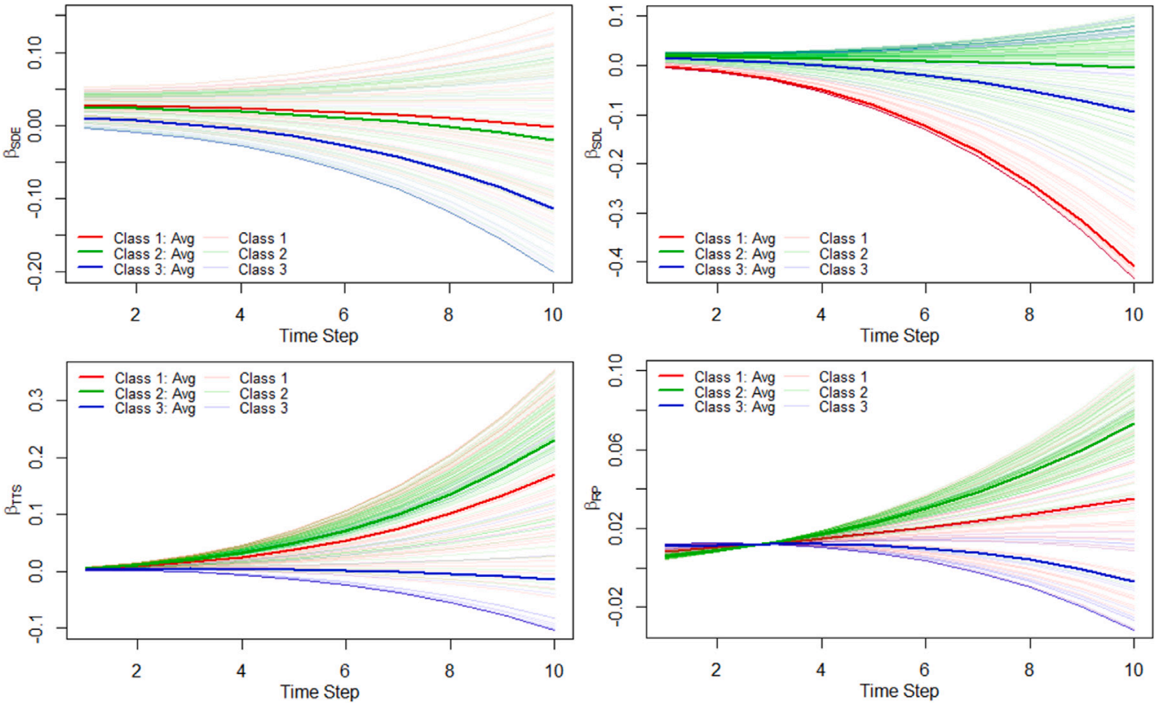

Fig. 9. The curves of each individual's changing preferences (in light red, light green, and light blue), and the average preference changing curves in each clusters (in red, green, and blue). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

CRediT authorship contribution statement

Xi Zhu: Formal analysis, Investigation, Methodology, Writing - original draft, Writing - review & editing. **Jingshuo Feng:** Methodology, Writing - original draft. **Shuai Huang:** Conceptualization, Funding acquisition, Methodology, Supervision, Writing - review & editing. **Cynthia Chen:** Conceptualization, Funding acquisition, Supervision.

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Appendix A. Derivation process of the parameter estimation algorithm for OLCM-T in Section 4

In Eq. (5), the decision variables are the membership vectors of individual i. Given $\boldsymbol{Q}_1, \boldsymbol{Q}_2, \ldots, \boldsymbol{Q}_R$ and $\boldsymbol{v}_t$, the Lagrangian function of the formulation shown in Eq. (5) could be written as:

$$L = \sum_{t=1}^{n_i} \left\{ \log\left(1 + \exp\Big(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r \boldsymbol{c}_{ir})^T \boldsymbol{v}_t\Big)\right) - y_{it}\Big(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r \boldsymbol{c}_{ir})^T \boldsymbol{v}_t\Big) \right\} + \sum_{r=1}^{R} \mu_r(\boldsymbol{c}_{ir}^T \mathbf{1} - 1) \tag{10}$$

In Eq. (10), we introduce the Lagrangian multiplier μ_r for constraint $\boldsymbol{c}_{ir}^T \mathbf{1} = 1$. To get the optimal $\boldsymbol{c}_{ir}, r = 1, 2, \ldots, R$, we could derive the gradient of the objective function regarding $\boldsymbol{c}_{ir}$:

$$\frac{\partial L}{\partial \boldsymbol{c}_{ir}} = \sum_{t=1}^{n_i} \left\{ \left(\frac{\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r \boldsymbol{c}_{ir})^T \boldsymbol{v}_t)}{1 + \exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r \boldsymbol{c}_{ir})^T \boldsymbol{v}_t)} - y_{it} \right) \times x_{r,it} \boldsymbol{Q}_r^T \boldsymbol{v}_t \right\} + \mu_r \mathbf{1} = 0 \tag{11}$$

According to Kuhn–Tucker Lagrangian, we also have $\frac{\partial L}{\partial c_{k,ir}} c_{k,ir} = 0$ ($k = 1, 2, \ldots, K_r; r = 1, 2, \ldots, R$), which could lead to the following equation:

$$\frac{\partial L}{\partial c_{k,ir}} c_{k,ir} = \sum_{t=1}^{n_i} \left\{ \left(\frac{\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r \boldsymbol{c}_{ir})^T \boldsymbol{v}_t)}{1 + \exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r \boldsymbol{c}_{ir})^T \boldsymbol{v}_t)} - y_{it} \right) \times x_{r,it} (\boldsymbol{Q}_r{}^T \boldsymbol{v}_t)_k c_{k,ir} \right\} + \mu_r c_{k,ir} = 0 \tag{12}$$

Given $\boldsymbol{c}_{ir}\mathbf{1} = 1$, i.e., $\sum_{k=1}^{K_r} c_{k,ir} = 1$, Eq. (12) could further be modified as:

$$\sum_{t=1}^{n_i}\left\{\left(\frac{\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_t)}{1+\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_t)} - y_{it}\right)\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)^T\boldsymbol{c}_{ir}\right\} + \mu_r = 0 \tag{13}$$

With Eq. (13), we could write μ_r in the following way:

$$\mu_r = \sum_{t=1}^{n_i} y_{it}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)^T\boldsymbol{c}_{ir} - \sum_{t=1}^{n_i}\frac{\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_t)}{1+\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_t)}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)^T\boldsymbol{c}_{ir} \tag{14}$$

Replace μ_r in Eq. (12) with Eq. (14):

$$\begin{aligned} c_{k,ir}\times\Bigg\{&\sum_{t=1}^{n_i}\left(\frac{\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_t)}{1+\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_t)} - y_{it}\right)\times x_{r,it}(\boldsymbol{Q}_r{}^T v(t))_k \\ &+\sum_{t=1}^{n_i}\left(y_{it} - \frac{\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_t)}{1+\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_t)}\right)\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)^T\boldsymbol{c}_{ir}\Bigg\} = 0 \end{aligned} \tag{15}$$

Since $c_{k,ir}$ should be non-negative, we define $\delta_+(x) \equiv max(x,0)$ and $\delta_-(x) \equiv min(x,0)$, with which Eq. (15) could be separated into a positive part and a negative part:

$$\begin{aligned} &c_{k,ir}\times\Bigg\{\sum_{t=1}^{n_i}\Bigg[\delta_+\left(\frac{\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_t)}{1+\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_t)}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)_k\right) - \delta_-\left(y_{it}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)_k\right) \\ &\quad+\delta_+\left(y_{it}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)^T\boldsymbol{c}_{ir}\right) - \delta_-\left(\frac{\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_t)}{1+\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_t)}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)^T\boldsymbol{c}_{ir}\right)\Bigg]\Bigg\} \\ &-c_{k,ir}\times\Bigg\{\sum_{t=1}^{n_i}\Bigg[-\delta_-\left(\frac{\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_t)}{1+\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_t)}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)_k\right) + \delta_+\left(y_{it}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)_k\right) \\ &\quad-\delta_-\left(y_{it}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)^T\boldsymbol{c}_{ir}\right) + \delta_+\left(\frac{\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_t)}{1+\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir})^T\boldsymbol{v}_t)}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)^T\boldsymbol{c}_{ir}\right)\Bigg]\Bigg\} = 0 \end{aligned} \tag{16}$$

With Eq. (16), we could derive an iteratively updating rule for $\boldsymbol{c}_{ir}$ similarly as Lin et al. (2015, 2018b,a):

$$\begin{aligned} c_{k,ir}^{m+1} = c_{k,ir}^{m}\times\Bigg\{&\sum_{t=1}^{n_i}\Bigg[-\delta_-\left(\frac{\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir}^m)^T\boldsymbol{v}_t)}{1+\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir}^m)^T\boldsymbol{v}_t)}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)_k\right) + \delta_+\left(y_{it}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)_k\right) \\ &-\delta_-\left(y_{it}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)^T\boldsymbol{c}_{ir}^m\right) + \delta_+\left(\frac{\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir}^m)^T\boldsymbol{v}_t)}{1+\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir}^m)^T\boldsymbol{v}_t)}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)^T\boldsymbol{c}_{ir}^m\right)\Bigg]\Bigg\} \\ \Big/\Bigg\{&\sum_{t=1}^{n_i}\Bigg[\delta_+\left(\frac{\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir}^m)^T\boldsymbol{v}_t)}{1+\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir}^m)^T\boldsymbol{v}_t)}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)_k\right) - \delta_-\left(y_{it}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)_k\right) \\ &+\delta_+\left(y_{it}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)^T\boldsymbol{c}_{ir}^m\right) - \delta_-\left(\frac{\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir}^m)^T\boldsymbol{v}_t)}{1+\exp(\sum_{r=1}^{R} x_{r,it}(\boldsymbol{Q}_r\boldsymbol{c}_{ir}^m)^T\boldsymbol{v}_t)}\times x_{r,it}(\boldsymbol{Q}_r{}^T\boldsymbol{v}_t)^T\boldsymbol{c}_{ir}^m\right)\Bigg]\Bigg\} \end{aligned} \tag{17}$$

In Eq. (17), the superscript m refers to the order of iteration. By introducing δ-functions, we ensure that the numerator and the denominator are both non-negative. Therefore, given any positive initial membership vectors $\boldsymbol{c}_{ir}, r = 1, 2, \ldots, R$, the non-negativity requirement of the membership vectors is guaranteed.

Appendix B. Simulation experiment data generation

Two coefficients are varying in the simulation, such that we could see the performance of the proposed algorithm and make comparisons: (1) the number of attributes in the utility model R, i.e., the number of factors in a choice scenario, or the number of the preference dimensions; (2) the number of canonical models for each preference dimension K (here in the simulation we are assuming that different preference dimension has the same number of canonical models, though the models are different). With R and K assigned, we could set the true preferences for the group of individuals with preference heterogeneity.

Generate suitable formulation for different canonical models

A given number of canonical models are generated by randomly setting the parameters of the cubic polynomial models. For each cubic polynomial model $q_0 + q_1 t + q_2 t^2 + q_3 t^2$, we need to set 4 parameters. Since the canonical models represent different preference changing patterns, they need to be different enough from each other. To guarantee this, for each preference dimension, we can let the canonical models be polynomial models of different degrees. For example, we may let one canonical model be a cubic polynomial, and other canonical models be a quadratic model, a linear model, and a constant model. We can also let different canonical models have opposite signs. For example, in canonical model $q_0^1 + q_1^1 t + q_2^1 t^2 + q_3^1 t^3$ and canonical model $q_0^2 + q_1^2 t + q_2^2 t^2 + q_3^2 t^3$, q_3^1 and q_3^2 could have opposite signs such that the two canonical models are significantly different from each other.

Generate appropriate parameters for canonical models

We obtain the magnitudes/ranges of the 4 parameters in the cubic polynomial model by running regressions with the learned preferences we obtained in our previous study (Zhu et al., 2020). As we stated in Section 2.2, we have individual preferences for 826 respondents in 13 binary choice scenarios. We run regressions for each individual, and obtain the magnitudes for q_0, q_1, q_2, and q_3 in the cubic polynomial model. Then, the parameters of each canonical model are selected randomly from corresponding ranges:

$$q_0 \in [-0.25, 0.25];\; q_1 \in [-0.05, 0.05];\; q_2 \in [-0.008, 0.008];\; q_3 \in [-0.0005, 0.0005]$$

Since we would like to guarantee that the canonical models are different from each other, some parameters are set to be 0 intentionally. For example, to obtain a quadratic polynomial model, we will let $q_3 = 0$ and $q_2 \neq 0$.

Generate membership vectors

With canonical models, each individual's membership vectors need to be decided such that his preferences $\boldsymbol{\beta}_i$ could be obtained via $\boldsymbol{\beta}_i = (\boldsymbol{QC}_i)^T \boldsymbol{V}_t$. Given the requirements that $\boldsymbol{c}_{ir}\mathbf{1} = 1$ and $c_{k,ir} \geq 0$, we use Dirichlet distribution to model the membership vectors (the commend is *rdirichlet* in R). In the simulation, we let each individual has his dominant preference changing model for each dimension of his preferences. This is achieved by setting one large value (e.g., 10) in the p-length vector $\boldsymbol{\alpha}$ and let other values of the vector be small (e.g., 1). With the given number of canonical models K, we equally split the total population into K sub-groups, and individuals in one sub-group will have the same "dominant model", which refers to the canonical model to which the preference changing patterns of these individuals will resemble more than others.

For example, assume that we are generating true preferences for a condition where the individual preferences are 3-dimensional and there are 2 canonical models for each dimension. The whole population is split into 2 sub-groups, each with $120/2 = 60$ individuals. Each individual has 3 membership vectors corresponding to the 3-dimensional preferences. Each membership vector has 2 values in response to the 2 canonical models. For the first sub-group, we may let the first canonical models be the "dominant models" for all preference dimensions. The corresponding value for the "dominant model" in each membership vector will be significantly larger than the other, e.g., $(0.9, 0.1)$. Meanwhile, for the second sub-group, we need to set the second canonical models be the "dominant models" for all preference dimensions. An example of the membership vector for one preference dimension of an individual in the second sub-group could be $(0.1, 0.9)$.

Generate attributes in choice scenarios

We then generate choice scenario attributes $\boldsymbol{x}_{it}$ for each individual i at each time step. With the generated attributes at each time step t, an individual's choice in the scenario can be predicted using binary logit model:

$$p_1 = Pr(y_{it}|\boldsymbol{x}_{it}) = \frac{\exp(\boldsymbol{x}_{it}^T\boldsymbol{\beta}_{it})}{1 + \exp(\boldsymbol{x}_{it}^T\boldsymbol{\beta}_{it})} = \begin{cases} > 0.5 & y_{it} = 1 \\ \leq 0.5 & y_{it} = 0 \end{cases} \tag{18}$$

We randomly select the values of the attributes from a uniform distribution $U(0, 100)$, which is consistent with the magnitudes of the attributes in the online experiment in Zhu et al. (2020). The consistency in the magnitudes of the attributes and the preferences helps ensure the reliability of the simulations in the current study. Given the number of attributes R, we first randomly select $R - 1$ values from a uniform distribution $U(0, 100)$ for each individual at each time step. We then randomly select p_1 from a uniform distribution $U(0, 1)$. With p_1, we could calculate the last attribute required using the discrete choice-making model such that his probability to select the promoted choice is the pre-determined value p_1. By generating attributes in this way, we avoid the system from aborting due to large $\exp(\boldsymbol{x}_{it}^T\boldsymbol{\beta}_{it})$, and make it possible for us to control the balance of the responses (i.e., the percentages of acceptance and rejections in responses).

Levels of noise in responses

We also test the performance of the model given different levels of noise in responses, i.e., 10% (0.1), 20% (0.2), 30% (0.3), which represent the percentages of the incorrect responses for each individual in all his choices. To achieve this, a random number between 0 and 1 is generated after generating a scenario and the corresponding true response from the true preferences. If the number is smaller than the noise level (e.g., 0.1), we turn the response to the opposite (from acceptance to rejection, or from rejection to acceptance), making the response in this scenario an incorrect response.

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