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A crash prediction method based on bivariate extreme value theory and video-based vehicle trajectory data

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ABSTRACT

Traditional statistical crash prediction models oftentimes suffer from poor data quality and require large amount of historical data. In this paper, we propose a crash prediction method based on a bivariate extreme value theory (EVT) framework, considering both drivers' perception-reaction failure and failure to proper evasive actions. An unmanned aerial vehicle (UAV) was utilized to collect videos of ten intersections in Fengxian, China, at representative time periods. High-resolution vehicle trajectory data were extracted by a Kanade-Lucas-Tomasi (KLT) technique, based on four detailed metrics were derived including Time-to-accident (TA), Post-encroachment Time (PET), minimum Time-to-collision (mTTC), and Maximum Deceleration Rate (MaxD). TA was expected to capture the chance of perception-reaction failure, while other three metrics were used to measure the probability of failure to proper evasive actions. Univariate EVT models were applied to obtain marginal crash probability based on each metric. Bivariate EVT models were developed to obtain joint crash probability based on three pairs: TA and mTTC, TA and PET, and TA and MaxD. Thus, union crash probability within observation periods can be derived and the annual crash frequency of each intersection was predicted. The predictions were compared to actual annual crash frequencies, using multiple tests. The findings are three-folds: 1. The best conflict metrics for angle and rear-end crash predictions were different; 2. Bivariate EVT models were found to be superior to univariate models, regarding both angle and rear-end crash predictions; 3. TA appeared to be an important conflict metric that should be considered in a bivariate EVT model framework. In general, the proposed method can be considered as a promising tool for safety evaluation, when crash data are limited.

1. Introduction

Traffic safety Evaluation often relies on statistical crash prediction models. However, it is well known that crash models can suffer from poor quality of historical crash records. Not to mention that it normally takes years for crash data collection, in order to guarantee statistical power of those models. Thus, developing surrogate safety measures (SSM) and other safety evaluation methods are essential.

Traffic conflict, as a pre-crash event, has been studied for years. Conflicts are similar to traffic crashes in nature, but more frequent and easier to be observed in the real world (Tarko et al., 2009). Conflicts were traditionally collected based on manual observation, which are labor-intensive, time-consuming, and often inaccurate. Nowadays, automated video techniques (Saunier and Sayed, 2008) and traffic simulation (Wang et al., 2018) have been applied in this research field. Both simulation and video techniques allow detecting and continuously tracking moving objects (e.g. vehicles), based on which their trajectories can be extracted and conflict data can be easily collected.

Although new technologies have introduced to improve conflict data collection, there is still no benchmark conflict metric for safety evaluation. There are some fundamental conflict metrics, such as Time-to-Accident (TA), Time-to-Collision (TTC), Post-encroachment Time (PET), and maximum deceleration (MaxD). Based on these, some composite metrics were proposed, based on knowledge of physics, kinematics, and kinetics (Mahmud et al., 2018). In recent years, probabilistic metrics had also been developed. Saunier and Sayed (Saunier and Sayed, 2008; Mohamed and Saunier, 2013) estimated crash probability of conflicts, by projecting vehicles' trajectories using machine learning techniques. Davis et al., (2008) proposed a probabilistic framework for interpolating crash probability considering vehicles' emergency braking rates. Wang and Stamatiadis 2014, 2016) proposed probabilistic metrics considering drivers' reaction time and vehicles' breaking capabilities. Kuang et al., (2015) also proposed a probabilistic metric, considering both driver and vehicle failure. Promising results of

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probabilistic metrics had been reported. According to the research, perception-reaction failure and evasive action failure (e.g. braking limitation, improper actions and so on) were the two important aspects to consider for crash risk estimation.

Extreme value theory (EVT) has been utilized in many fields (e.g. finance, insurance, risk management), in order to predict the occurrence of extreme events that are less than normal observations or even non-observable (Racco, 2011; Dutfoy et al., 2014; Falk and Stupfler, 2017). Due to the similar nature of crash occurrence, EVT was also introduced for crash predictions of intersections and highway ramps (Songchitruska and Tarko, 2006; Zheng et al., 2014a, 2014b). PET was often used for EVT modeling, which indicates the consequence of a conflict (e.g. PET = 0 indicates a crash). Conflicts are normally observed in the field, based on any evasive actions. Thus, PET inherently reflect the consequence of drivers' evasive actions. However, as is known, there are other crash conditions when drivers are unable to perceive and react to potential conflicts. According to Davis's framework (2008), both perception-reaction failure and evasive action failure could result in a crash. PET = 0 could only indicate possible evasive action failure since conflicts are normally collected based on identification of evasive actions. In other words, PET values were collected from those conflicts, which have been reacted by drivers. This could raise a serious question that perception-reaction failure may not be fully considered by PET observations. Time-to-Accident (TA) can be a conflict metric indicating perception-reaction failure since it is defined as the initial condition of a conflict (i.e. initial TTC). Thus, EVT predictions based on TA may unveil the crash probability of driver' perception-reaction failure. In addition, other conflict metrics could also be considered for EVT modeling. For instance, when MaxD reaches the limitation of vehicles' braking performance, a crash can happen (i.e. evasive action failure). Plus, the minimum TTC (i.e. mTTC) during a conflict could also indicate possible evasive action failure. In general, conflict metrics associated with the two failure types (i.e. perception-reaction failure and evasive action failure) may need to be considered together, using a bivariate EVT framework. Recent literature also reported the superiority of such method to univariate EVT models in safety analysis (Zheng et al., 2018).

In light of these, we will propose a crash prediction method using a bivariate EVT model framework that considers both driver perception-reaction failure and evasive action failure. Four fundamental conflict metrics will be extracted from vehicle trajectories, including TA, PET, MaxD, and mTTC, using automated video techniques. TA will used as an indicator of driver perception-reaction failure, while PET, MaxD and minimum TTC will be considered as measures of evasive action failure. The bivariate EVT model performances based on these metrics will be examined and compared. The remainder of the paper is organized as follows: In section two, we present the methodology details, including data collection and the EVT model framework. Section three and four summarize EVT modeling results and comparisons among all EVT crash predictions. Section five concludes the paper and recommends future research directions.

2. Methodology

2.1. Video-based vehicle trajectory data and conflict data

Ten urban signalized intersections in Fengxian District in Shanghai were selected in this study. In previous research, video cameras were equipped at high-rise buildings to collect video data and manual conflict data collection were applied to get basic conflict data such as TTC and PET (Wang et al., 2018). In this study, since more detailed trajectory data need to be extracted, a different data collection strategy was applied. An unmanned aerial vehicle (UAV) were used to collect high-resolution videos from the very top of each intersection (mostly at 80–90 m height) to ensure clear and stable view. 4-hour video clips were collected for each intersection, during afternoon periods (i.e. 1 pm -4 pm). It should be noted that traffic conditions are time-varying due to many factors, such as traffic flow fluctuation, weather, time of day effects, and so on. Thus, observation periods need to be very carefully selected. By examining the 24-hour traffic volumes of those intersections at 10 different days, the time period between 1 pm and 4 pm was considered as representative. Previous literature also chose a similar time period (Zheng et al., 2018).

Then, a Kanade-Lucas-Tomasi (KLT) method was introduced to extract vehicle trajectories from videos. To be specific, Shi-Tomasi features were used to detect vehicles and the optical flow method (i.e. Kanade-Lucas algorithm) was used to track vehicles (Cao et al., 2011; Buddubariki et al., 2015). Notably, in an initial experiment, the detection rate of black vehicles was relatively low due to their similar color to the background (i.e. roadway surface). Thus, a multi-color space information fusion strategy (Shao and Duan, 2012) was introduced to transfer each frame into several different color spaces, where black vehicles were more easily detected. Then, the detection information from multiple color spaces was merged, allowing the detection and tracking of most vehicles. The total detection rate was over 91.2%. The process was applied with 10 frames per seconds, resulting in vehicle trajectory data with .1 s interval. Cubic spline curves were utilized to smooth vehicle trajectories. Georeferencing was conducted for each intersection, based on reference markings (i.e. lane width) on roadways (Ke et al., 2017).

As such, original vehicles' trajectory data can be automatically extracted and Fig. 1 shows the important steps of this process, using an intersection (No.10) as an illustration. Note that this intersection will be also used to demonstrate EVT modeling details in Section 3.

In previous literature, conflicts were normally identified by trained observers based on traffic conflict techniques (TCT). Some automatic methods were also introduced, including semi-supervised learning, supervised learning, and unsupervised learning (Saunier and Sayed, 2007). However, in essence, such efforts still rely on human judgment. Supervised learning requires positive and negative samples, which need to be labeled by observers. An unsupervised learning model also requires 'labeled' samples to validate its reliability, though it does not need such for training. Thus, in our study, conflicts were identified based on traditional TCT methods, by four trained observers. Those observers were early examined for their intra-reliability and inter-reliability, by watching other video clips (Wang et al., 2018). A clear sign of an evasive action (e.g. braking, swerve) was used as the sign of a conflict. Conflict metrics were calculated and collected based on vehicle trajectory data, such as Time-to-Collision (TTC), Post-encroachment Time (PET), and maximum deceleration rate (MaxD). The detailed calculation of these metrics can be found in previous literature (Federal Highway Administration (FHWA), 2008). Fig. 2 shows the deceleration rates of a vehicle and TTC during a rear-end conflict process.

2.2. Bivariate EVT model

Previous literature utilized univariate EVT models to predict crash frequency. For a univariate case (Coles, 2001), suppose there is a variable X with certain probability distribution. $x_1, x_2 \ldots x_i$, are independent random observations from X. Let $M = max\ (x_1, x_2 \ldots x_n)$. When $n \to \infty$, the M will converge to a General Extreme Value (GEV) distribution:

$$F(X) = exp\{-[1 + \varepsilon(\frac{x-\mu}{\sigma})]^{-1/\varepsilon}\} \tag{1}$$

Where μ is the location parameter, σ is the scale parameter, and ε is the shape parameter.

Regarding the Peak-Over-Threshold (POT) EVT approach, an observation is considered as an extreme if a measurement exceeds a certain threshold that is determined beforehand. When the threshold (u) is large enough, the distribution $F_u(x)$ is approximately a General Prato Distribution (GPD) which can be written as follow:

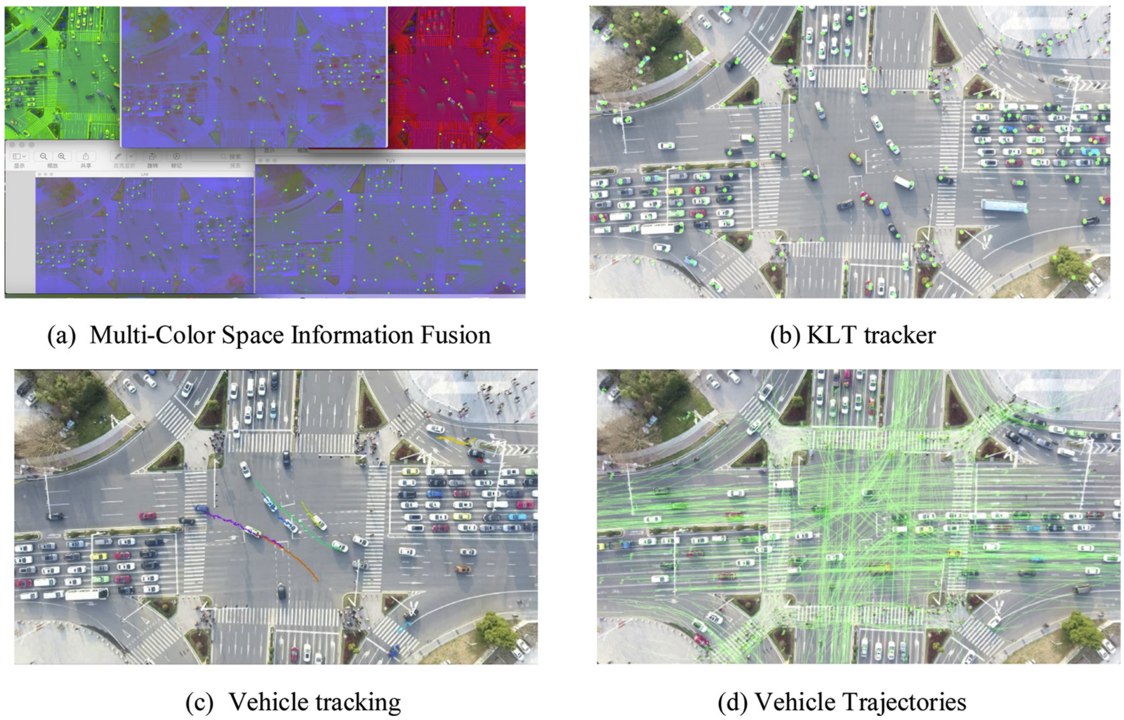


Fig. 1. UAV Video Trajectory Processing of Intersection #10.

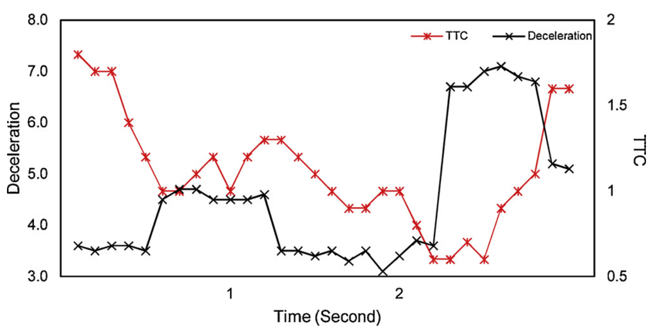


Fig. 2. Deceleration Rates of a Vehicle and TTC during a Rear-End Conflict Process.

$$F_u(X) = \begin{cases} \left(1 + \frac{\varepsilon(X-u)}{\sigma}\right)^{-\frac{1}{\varepsilon}} \varepsilon \neq 0 \\ e^{\left(\frac{-X+u}{\sigma}\right)} \varepsilon = 0 \end{cases} \tag{2}$$

Assume that a number of random observations $\{x_1, x_2, x_n\}$ are independently and identically distributed, the conditional distribution function of exceedances X over a threshold u can be given as:

$$\Pr(X > u) = 1 - F_u(X) \tag{3}$$

In previous literature, PET was mostly utilized in univariate EVT modeling to predict crash probability (i.e. PET = 0). However, PET may not be able to reflect all crash conditions. For example, drivers are unaware of an upcoming conflict or cannot be able to react during very short time (e.g. TA = 0). Thus, more aspects may need to be considered and a multivariate EVT framework needs to be applied.

As to a multivariate case, suppose y is the D-dimensional vector of variables $\boldsymbol{Y} = \{Y_1, Y_2, Y_j, j = 1, d\}$. Let $\boldsymbol{u}$ be a multivariate threshold such that $F_j(u_j) = exp(-\lambda_j)$ for some small positive λ_j. For the region $[\boldsymbol{u},\infty]$, the joint distribution can be written as:

$$F_{\boldsymbol{u}}(\boldsymbol{Y}) \approx \exp\{-l(\boldsymbol{v}; \theta)\}, \ \boldsymbol{Y} \geq \boldsymbol{u} \tag{4}$$

$$v_j = \lambda_j \left(1 + \gamma_j \frac{Y_j - u_j}{\sigma_j}\right)_+^{1/\gamma_j}, j = 1, \ldots, d \tag{5}$$

Where $l(\boldsymbol{v}; \theta)$ is called the table tail dependence function. In this case, for a small λ_j, the margin distribution F_{u_j} on the region $[u_j, \ \infty]$ approximates to a Generalized Pareto model for univariate extremes:

$$F_{u_j}(Y_j) \approx \exp\left\{-\lambda_j (1 + \gamma_j \frac{Y_j - u_j}{\sigma_j})_+^{-1/\gamma_j}\right\}, \ Y_j \geq u_j \tag{6}$$

Since the joint distribution (i.e. Eq. (4)) is defined on the region $[\boldsymbol{u},\infty]$, only observations within this region should be considered. In other words, for any dimension Y_j, observations smaller than its corresponding threshold u_j should be censored from below at u_j. Thus, the 'censored' likelihood of the parameters given a sample y_1, y_2, y_n can be written as:

$$L\{(y_i)_{i=1}^n; (\lambda_j, \gamma_j, \sigma_j)_{j=1}^d, \theta\} = \prod_{i=1}^{n} L(y_i) \tag{7}$$

The marginal parameters $(\lambda_j, \gamma_j, \sigma_j)$ and the dependence parameters θ can be estimated jointly by maximum likelihood. By maximizing likelihood, multivariate EVT model parameters can be estimated and the full probability of multivariate can be estimated.

$$Pr(\boldsymbol{Y} > \boldsymbol{u}) = 1 - F_{\boldsymbol{u}}(\boldsymbol{Y}), \ \boldsymbol{Y} \geq \boldsymbol{u} \tag{8}$$

As for dependence function $l(\boldsymbol{v}; \theta)$, there are various non-parametric or parametric estimations. For instance, a multivariate asymmetric logistic distribution can be written as:

$$l(\boldsymbol{v}, \theta) = \sum_{c \in C_d} \left\{\sum_{j \in c} (\psi_{c,j} v_j)^{1/a_c}\right\}^{a_c} \tag{9}$$

Where C_d is the collection of non-empty subsets c of $\{1, \ldots, d\}$; $\theta = \{a_c, \psi_{c,j}\}$ are dependence parameters, and $0 < a_c < 1$, $\psi_{c,j} \geq 0$, and $\sum_{j \in c} \psi_{c,j} = 1$ for $j = 1, \ldots, d$.

In this study, since we consider two aspects (i.e. perception-reaction failure and evasive action failure), bivariate EVT models were developed, which can be considered as a special case of multivariate EVT models. For instance, a logistic bivariate EVT model has the stable tail dependence function as:

$$l(\boldsymbol{v}, \theta) = \left(v_1^{\frac{1}{a}} + v_2^{\frac{1}{a}}\right)^a, \ 0 <= a < 1 \tag{10}$$

where $\theta = \{a\}$ is the measure of the strength of dependence. In particular, $a = 1$ indicates independence and $a \downarrow 0$ implies complete dependence.

For an asymmetric logistic bivariate model:

$$l(\boldsymbol{v}, \theta) = (1 - \psi_1)v_1 + (1 - \psi_2)v_2 + \{(\psi_1 v_1)^{\frac{1}{a}} + (\psi_2 v_2)^{\frac{1}{a}}\}^a \tag{11}$$

where $\theta = \{\psi_1 , \psi_2, \alpha\}$ are dependence parameters, and $0 < \psi_j < 1$ and $0 < a < 1$. Independence arise when $\psi_j = 0$ or $a = 1$. Other types of bivariate model details can be found in (Zheng et al., 2018).

Depending on whether each variable exceeds its corresponding threshold, the likelihood contributions of bivariate EVT models can be written as:

$$L(Y_1, Y_2) \propto \begin{cases} F(u_1, u_2)\ if Y_1 \le u_1, Y_2 \le u_2 \\ \frac{\partial F}{\partial Y_1}(Y_1, u_2)\ if Y_1 > u_1, Y_2 \le u_2 \\ \frac{\partial F}{\partial Y_2}(u_1, Y_2)\ if Y_1 \le u_1, Y_2 > u_2 \\ \frac{\partial^2 F}{\partial Y_1 \partial Y_2}(Y_1, Y_2)\ if Y_1 > u_1, Y_2 > u_2 \end{cases} \tag{12}$$

As to conflict metrics ($\boldsymbol{Y}$) and thresholds ($\boldsymbol{u}$) selections, TA was used as the measure for initial perception-reaction failureTA = 0 s was considered as the threshold. Other three conflict metrics were used for measuring the failure to proper evasive action. PET = 0 s indicates a crash with the failure of proper evasive actions. Minimum TTC (mTTC) = 0 s also implies a failure to evasive action during a conflict. For Maximum deceleration (i.e. MaxD), there are two thresholds examined, a radical one and a conservative one. Based on Cunto (2008), 12.7 m/s^2 was considered as the upper limit of vehicles' braking performance. Thus, if MaxD is larger than this value, a crash can be considered as unavoidable. According to Ceunynck (2017), 8 m/s^2 can be considered as a conservative value for maximum deceleration rate that all vehicles can achieve. Thus, if MaxD is larger than this value, a crash may also be considered as nearly unavoidable. In general, both values indicate extreme difficult conditions for braking evasive actions.

Thus, the union probability of perception-reaction failure and proper evasive action failure can be derived by the following:

$$\begin{aligned} Pr(Y_1 > u_1 \vee Y_2 > u_2) &= Pr(Y_1 > u_1) + Pr(Y_2 > u_2) \\ &- Pr(Y_1 > u_1 \bigwedge Y_2 > u_2) \end{aligned} \tag{13}$$

Where Y_1, Y_2 can be two selected conflict measures (e.g. TA and PET); u_1 and u_2 are their extreme thresholds(e.g. 0 s for TA and 0 s for PET). Given a representative observation period (e.g. O hours), the annual crash frequency of an intersection can be predicted as:

$$N = \frac{Pr(Y_1 > u_1 \ \vee \ Y_2 > u_2)^* 365 * 24}{O} \tag{14}$$

3. EVT modeling results

To apply a bivariate EVT model, the existence of asymptotic dependency between two variables needs to be examined. The reason is that only asymptotically dependent extreme upper tails can be properly described by EVT models. To examine asymptotic dependency, a Chi-statistic was early adopted (Coles, 2001). Let Z_1, Z_2 be a bivariate random vector, with a joint distribution function F and marginal distribution functions F_1 and F_2. Assuming F_1 and F_2 are identical, the dependence between Y_1 and Y_2 at extreme levels can be written as:

$$\chi = \lim_{y \uparrow y^*} P[Y_2 > y | \ Y_1 > y] \tag{15}$$

where y^* denotes the right end-point of the common marginal distribution.

When the marginal distribution function F_1 and F_2 are non-identical, the Chi-statistic can be generalized as:

$$\chi = \lim_{q \uparrow 1} P[Q_2 > q | \ Q_1 > q] \tag{16}$$

Where $Q_j = F_j(Z_j)(j = 1{,}2)$ are uniformly distributed on (0, 1). The number χ can be interpreted as the tendency for one variable to be extreme given that the other is extreme. When $\chi = 0$, the variables are considered as asymptotically independent. Whereras if $0 < \chi < 1$, they can be considered as asymptotically dependent.

However, the Chi-statistic fails to discriminate between the degrees of relative strength of dependence for asymptotically independent variables ($\chi = 0$). Thus, a modified measure Chi-bar statistic $\bar{\chi}$ was developed to tackle this issue:

$$\bar{\chi} = \lim_{q \to 1^-} \bar{\chi}(q), \ 0 < q < 1 \tag{17}$$

$$\begin{aligned} Wh\bar{e}re\chi(q) &= \frac{2logP[Q1 > q]}{logP[Q1 > q, Q2 > q]} - 1 \\ &= \frac{2logP[F_{u1}(Y_1) > q]}{logP[F_{u1}(Y_1) > q, F_{u2}(Y_2) > q]} - 1 \end{aligned} \tag{18}$$

$\bar{\chi}$ provides a limiting measure that increases with relative dependence strength within this class. If chi-bar values are close 1 at very high quantiles, two variables can be considered as asymptotically dependent. More details can be found in Beirlant et al. (2004). Otherwise, asymptotic independence can be determined and other methods, such as conditional EVT methods and copula methods (Doyon, 2013), need to be used instead. According to Fig. 3, it can be found that all three pairs are asymptotically dependent for both crossing and rear-end conflicts for intersection No.10, including TA and PET, TA and mTTC, and TA and MaxD. Such asymptotic independence holds for all 10 intersections. This could be reasonable that when a conflict occurs in a very immense situation (i.e. TA is very low), mTTC, PET could be expected to be low as well. In addition, drivers' reaction tends to be more intense, resulting in larger deceleration rate (i.e. MaxD).

Since all three pairs were found to be asymptotically dependent, bivariate EVT models were applied. POT-based EVT methods were often perfered since they had shown advantages over the Block Maxima (BM) approach under limited data samples (Zheng et al., 2014a). For univariate POT-based modeling, mean excess plots and parameter stability plots were often used to determine threholds. In this study, univariate EVT modeling was also conducted on each conflict metric for crash predictions. These predictions were compared to bivariate predictions and acutal crash frequencies. Based on univarite EVT modeling, the marginal probability of each conflict metric exceeding its threhold was estimated. For PET, TA and TTC, thresholds were set as 0. For MaxD, two threshold values were used for crash predictions, including 8 m/s^2 and 12.7 m/^{s}. Note that the details of univariate EVT model development were not discussed here, since the major intent of this research is to discuss bivariate EVT models. Table 2 presents crash probability (i.e. marginal probability) predicted by univariate EVT models.

For bivariate POT-based modeling, the threshold for each conflict metric in a pair needs to be determined. An approximation of spectral measure was largely used to determine thresholds for multivarite EVT models. Theoretically, at properly largest thresholds, the empirical spectral value should be close to the theoratical spectral measure. In particular for a bivariate extreme case, the theoretical spectral value is 2. Fig. 4 presents the optimal threhold selections based on empirical spectral values. For example, at k = 30, the emperical spcturm value was observed to be close to 2 for the pair of TA and mTTC. Thus, observations with the first 30 largest radial coordinates will be selected as extreme cases. Then, the threshold of TA and mTTC for bivariate EVT modeling can be determined, respectively. More details can be found in Beirlant et al. (2004).

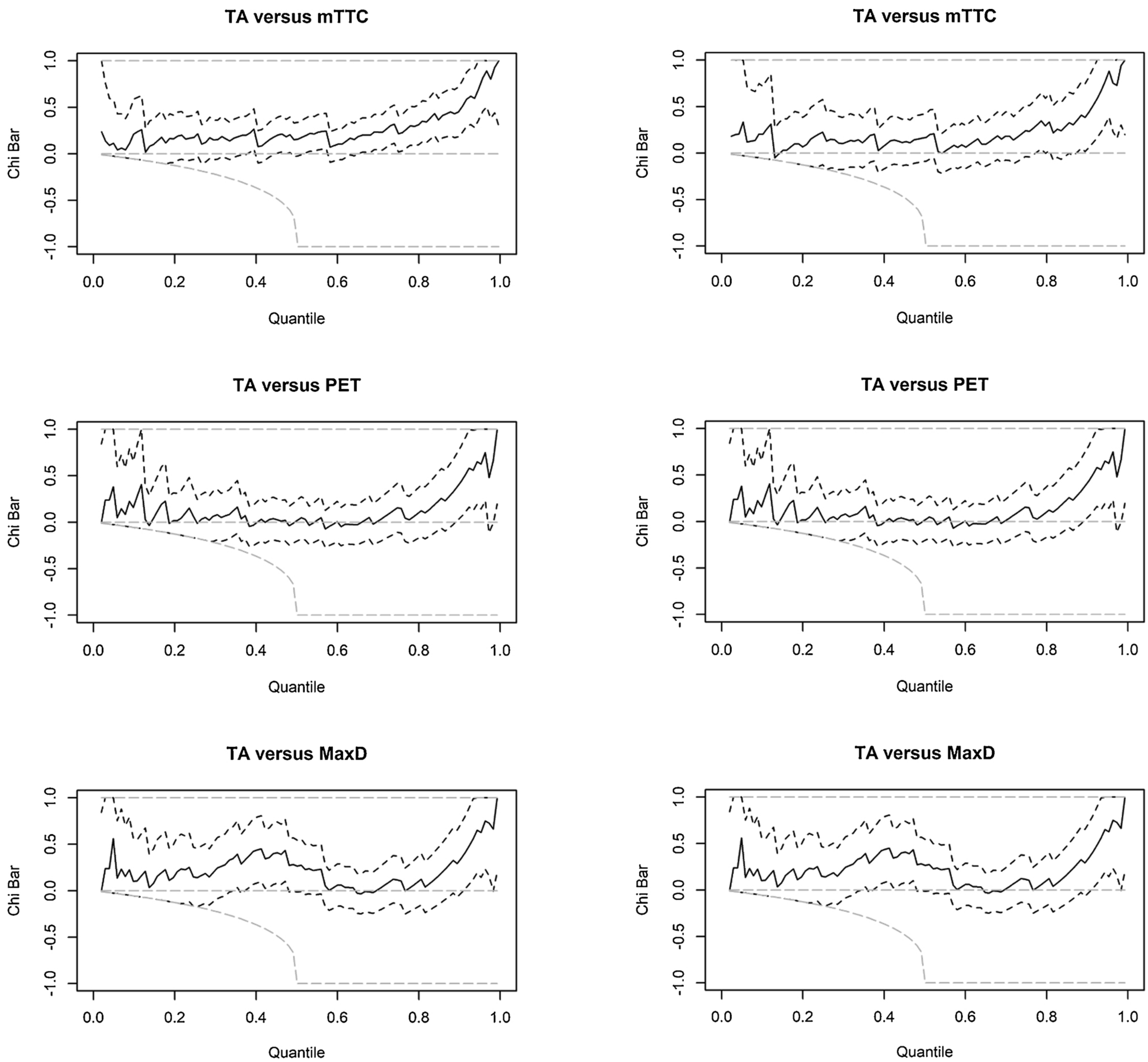


Fig. 3. asymptotic independence of crossing and rear-end conflicts at Intersection No.10.

After determining thresholds for each pair, a set of parametric bivariate EVT models were fitted based on maximum-likelihood methods, including logistic, asymmetric logistic, negative logistic, bi-logistic, Coles-Tawn, and asymmetric mixed models (Beirlant et al., 2004). The best model for each pair was finally selected due to AIC values.

$$AIC = 2p - 2ln(\tilde{L}) \quad (19)$$

Where p is the number of parameters in the model; L is the maximum value of the likelihood function of the model. A model with the lowest AIC can be considered as the best one. Table 1 shows the details of bivariate EVT models for angle and rear-end crash predictions, including their AIC, scale parameters, shape parameters, and thresholds. Note that in this study, crossing conflict data was used for angle crash predictions, while rear-end conflict data was used for rear-end crash predictions. Since rear-end conflicts were more than crossing conflicts across ten intersections, rear-end crash prediction models generally had larger AICs than angle crash prediction models.

Based on best fitted models, the bivariate excess probability density of each pair can be calculated. Thus, the joint crash probability for each pair can be derived. Fig. 5 presents the density plots and thresholds of all three pairs.

Based on joint probability estimated from bivariate EVT models and marginal probability calculated from univariate EVT models, the union probability of each pair can be determined. This estimator considers two extreme conditions, including both perception-reaction failure and evasive action failure. To be more specific, the union crash probability of TA and PET include crash conditions when either crash is not reacted (TA = 0 means a failure of reaction to the conflict) or not avoided (i.e. PET = 0). The union probability of TA and MaxD indicates conditions when either a crash is not reacted or the deceleration rate is unrealistically high (i.e. the crash is unavoidable). The union probability of TA and minTTC suggests conditions when either a crash was not reacted at first or crash occurred during the evasive actions. Table 3 presents the results.

4. Results comparison

Five-year crash data of ten intersections was acquired from the Fengxian Police Department. For each intersection, annual crash frequencies were predicted based on univariate and bivariate EVT models. These values were compared to the actual annual crash frequency (AACF) in multiple tests, regarding correlation, rank correlation, and

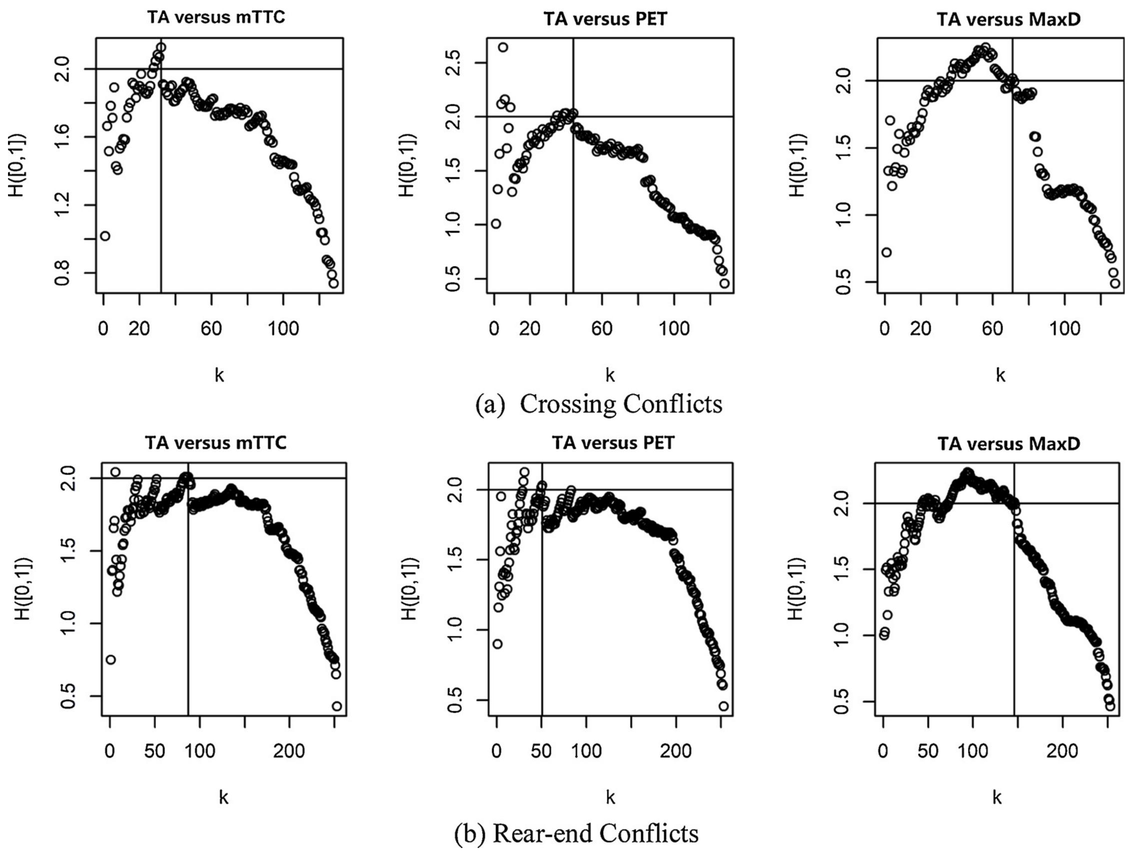


Fig. 4. Threshold Selection for Bivariate EVT models based on Emperical Spectrum Value.

prediction accuracy.

Table 4a and b displays AACF, the Poisson interval estimators of AACF and the predicted annual crash frequencies based on multiple univariate and bivariate EVT models. The detail of Poisson interval estimators will be discussed in 5.2.

4.1. Pearson correlation and rank coefficient

Fig. 6a presents the results of Pearson correlation tests. For univariate angle crash predictions, only PET had an acceptable coefficient of 0.67, while other metrics had lower or even negative correlation coefficients. For rear-end crash predictions, PET, mTTC, and MaxD12 had acceptable performances. mTTC appeared to be slightly better than PET, with a higher correlation coefficient. TA appeared to be unable to link to actual crash records. This could be reasonable, as TA may only indicate perception-reaction failure accounting for a small portion of crash occurrence. MaxD did not perform well, either. This could be related to the selection of a single threshold.

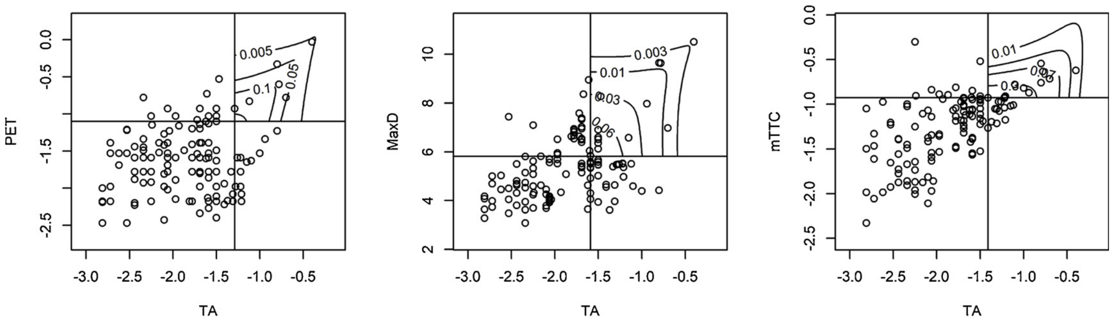
As for bivariate EVT models, the pair of TA and PET showed the best performance in angle crash predictions, with a correlation coefficient of 0.76. More importantly, it had a better performance than the univariate predictions based on PET. This could be reasonable, as the bivariate

Table 2
Marginal Probability based on univariate EVT and Each Conflict Metric.

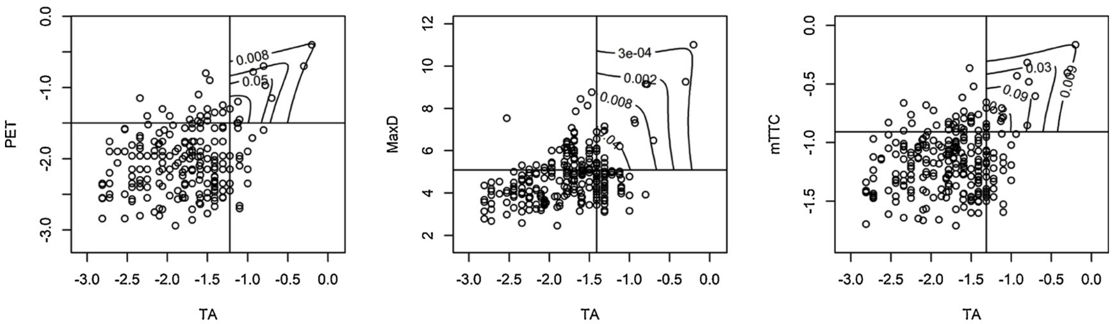
Probability	TA < 0	PET < 0	mTTC < 0	MaxD > 8	MaxD > 12
Angle Crash	0.00016	0.00091	0.00058	0.00057	0.00008
Rear-end Crash	0.00021	0.00032	0.00073	0.00254	0.00003

Table 1
Bivariate EVT Model Parameters.

Model Parameters		Angle Crash			Rear-End Crash		
		TA versus PET	TA versus MaxD	TA versus TTC	TA versus PET	TA versus MaxD	TA versus TTC
TA	Scale	0.4755	0.4227	0.3631	0.3412	0.2838	0.2944
	Shape	−0.4451	−0.2684	−0.2981	0.1014	−0.1590	−0.2163
	Threshold	−1.22	−1.59	−1.27	−1.22	−1.41	−1.31
PET	Scale	0.1935	–	–	0.4052	–	–
	Shape	−0.6378	–	–	−0.3156	–	–
	Threshold	−1.27	–	–	−1.50	–	–
MaxD	Scale	–	1.7888	–	–	1.3063	–
	Shape	–	−0.3588	–	–	−0.1521	–
	Threshold	–	5.81	–	–	5.09	–
mTTC	Scale	–	–	0.3299	–	–	0.2475
	Shape	–	–	−0.2967	–	–	−0.1477
	Threshold	–	–	−1.11	–	–	−0.91
Model AIC		143.3525	186.609	77.68	227.1889	699.4667	164.7



(a) Three Bivariate Excess ProbabilityDensities for crossing conflicts



(b) Three Bivariate Excess Probability Densities for rear-end conflicts

Fig. 5. Bivariate Excess Probability Densities (TA and PET, TA and MaxD, TA and mTTC).

Table 3
Joint and Union Crash Probability for Each Bivariate Combination of Conflict Metrics.

Probabilit	Angle Crash				Rear-End Crash			
y	TA and PET	TA and MaxD8	TA and MaxD12	TA and mTTC	TA and PET	TA and MaxD8	TA and MaxD12	TA and mTTC
Joint Probability	0.00011	0.00016	0.00005	0.00040	0.00008	0.00009	0.00004	0.00009
Union Probability	0.00096	0.00057	0.00019	0.00034	0.00041	0.00192	0.00021	0.00064

Based on the estimated union crash probability (of representative periods), the annual crash frequency of each intersection can be predicted. In next section, we will present the prediction results for all intersections and compared them to actual annual crash frequency.

Table 4a
EVT Predictions, Actual Annual Frequencies, and Poisson Interval Estimators (Angle Crash).

Intersection No.	TA	MaxD8	MaxD12	mTTC	PET	TA + PET	TA + mTTC	TA + MaxD8	TA + MaxD12	AACF	Poisson Estimator	
											lower	Upper
1	1.1	4.4	0.5	0.5	0.0	0.5	1.4	5.2	1.5	0.4	0.05	1.44
2	0.6	2.8	0.3	0.6	2.9	3.2	1.0	3.1	0.8	2.0	0.96	3.68
3	1.1	2.6	0.3	0.4	0	0.8	1.3	2.9	1.2	1.2	0.44	2.61
4	0.4	3.1	0.6	0.7	1.9	2.1	0.9	3.4	0.7	1.4	0.56	2.88
5	0.7	2.6	0.4	0.9	1.8	2.4	1.2	3	1.0	2.6	1.38	4.55
6	0.6	2.9	0.5	1.0	0.0	0.4	0.9	3.1	0.9	0.8	0.22	2.05
7	0.4	4.1	0.8	0.8	3.1	3.2	1.0	4.2	1.0	1.6	0.69	3.15
8	0.8	3.7	0.7	1.1	2.1	2.6	1.5	4.1	1.4	2.2	1.1	3.94
9	0.7	2.2	0.4	0.9	0.9	1.4	1.4	2.6	0.8	1.6	0.82	3.42
10	0.3	1.2	0.2	1.3	2.0	2.1	0.7	1.2	0.4	1.4	0.56	2.88

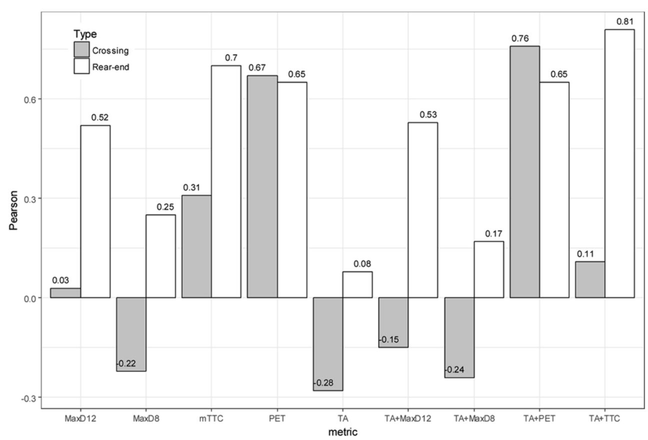
modeling considered two critical aspects (i.e. perception-reaction failure and evasive action failure). Regarding rear-end crash predictions, the pair of TA and mTTC had the best performance, followed by the pair of TA and PET. It could be due to the fact that a number of rear-end conflicts were observed as car-following conditions. Thus, mTTC may reveal more accurate information than PET in a car-following process, since it was derived based on a series of observations (i.e. TTC).

Fig. 6b presents the results of Spearman Rank tests, which indicate the relative safety among ten intersections. Bivariate EVT model predictions had better performances in identifying relative safety, regarding angle and rear-end crashes. The pair of TA and PET achieved the best rank coefficient of 0.81 for angle crash predictions, while the pair of TA and mTTC had the highest coefficient of 0.64 for rear-end crash predictions.

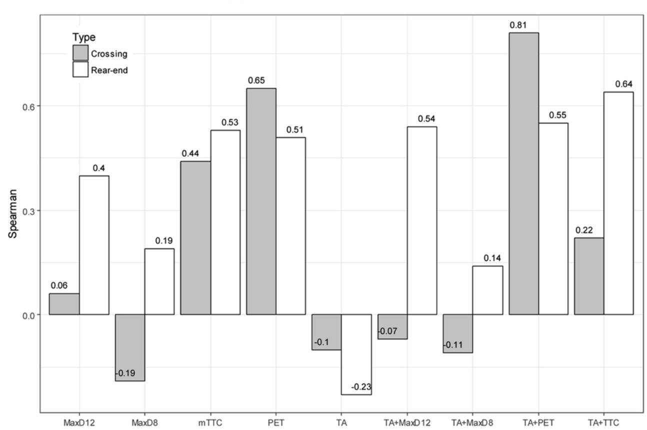
In general, by incorporating a bivariate EVT framework and TA as the measure of initial perception-reaction failure, the crash prediction performances of EVT models had been improved. In addition, for each different crash type, the best pair of conflict metric was found to be

Table 4b
EVT Predictions, Actual Annual Frequencies, and Poisson Interval Estimators (Rear-end Crash).

Intersection No.	TA	MaxD8	MaxD12	mTTC	PET	TA + PET	TA + mTTC	TA + MaxD8	TA + MaxD12	AACF	Poisson Estimator	
											lower	upper
1	0.5	5.2	1.5	1.0	3.6	4.0	1.2	5.5	1.7	1.4	0.56	2.88
2	0.3	4.5	1.8	3.2	8.8	8.9	3.3	4.6	2	3.8	2.29	5.93
3	0.4	4.1	1.6	1.8	3.1	3.3	2.1	4.3	1.7	1.8	0.82	3.42
4	0.8	4.8	0.7	0.6	2.9	3.2	1.2	5.2	1.1	2.0	0.96	3.68
5	0.4	4.6	1.7	1.6	2.4	2.7	1.8	4.9	1.9	3.6	2.13	5.69
6	0.4	4.0	0.8	0.9	4.4	4.5	1.2	4.1	1.0	2.0	0.96	4.68
7	0.3	4.6	1.3	1.4	5.5	5.7	1.6	4.8	1.4	2.4	1.24	4.19
8	0.3	5.1	1.1	1.7	6.9	7.0	1.9	5.1	1.2	3.0	1.68	4.95
9	0.6	3.6	0.6	1.3	2.7	3.0	1.7	3.9	0.9	2.0	0.96	3.68
10	0.4	4.0	0.9	1.2	0.7	0.9	1.4	4.2	0.5	1.2	0.44	2.61



(a) Pearson Correlation



(b) Spearman Rank Coefficient

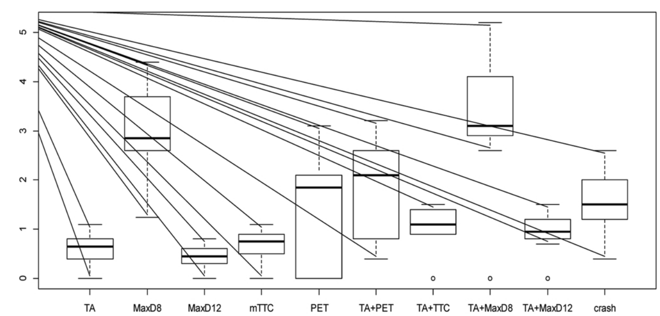
Fig. 6. Pearson Correlation and Spearman Rank Test Results.

different.

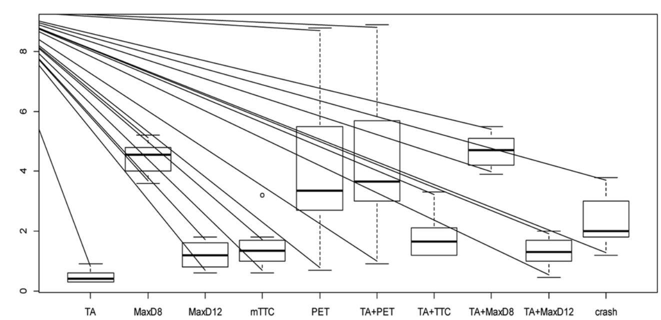
4.2. Prediction accuracy and mean difference

Poisson interval estimators (95% confidence interval) were calculated to examine prediction accuracy of all EVT models. Let $K_1, \ldots, K_n$ be random crash counts in n years for location i from a Poisson population with the true mean annual crash occurrences equal to τ_i. It can be shown that $1\text{-}a$ confidence interval for τ_i can be obtained as (Casella and Berger, 2001):

$$\left\{\frac{1}{2n}x^2_{2k_i,1-\alpha/2} \leq \tau_i \leq \frac{1}{2n}x^2_{2(k_i+1),\alpha/2}\right\} \tag{20}$$



(a) Angle Crash Predictions



(b) Rear-End Crash Predictions

Fig. 7. Boxplots for Annual Crash Prediction based on univariate and bivariate EVT models.

Where k_i is the total number of crashes for the i[th] location in n years.

For angle crash predictions, the bivariate EVT model based on the pair of TA and PET achieved the best performance that 9 of 10 predictions fell into the corresponding Poisson estimator intervals. The best univariate EVT predictions based on PET successfully predicted 7 of 10 intersections. For rear-end crash, the bivariate EVT model based on the pair of TA and mTTC successfully predicted 9 of 10 intersections. The univariate EVT model based on mTTC had 7 of 10 successful predictions. Thus, bivariate EVT models had the best prediction accuracies for both crash types.

Fig. 7 shows the box-plots of EVT crash predictions and actual crash frequencies. For angle crash, it appears that the bivariate EVT predictions based on TA and PET has the similar range with that of actual observations. For rear-end crash, the most similar predictions to actual observations is the bivariate EVT prediction based on TA and mTTC. Then, paired Wilcoxon signed rank tests were also applied to compare the mean difference among these EVT predictions and actual crash frequencies. The reason of using non-parametric tests is due to the non-gaussian nature of crash distribution. For angle crash, the univariate prediction based on PET ($w = 51.5$, $p = 0.44$) and the bivariate predictions based on TA and PET (w = 61.5, p = 0.40) were found with p-value lager than 0.05. For rear-end crash, only the bivariate predictions based on TA and mTTC (w = 30.5, p = 0.1485) was found with p-value larger than 0.05. Thus, under 95% confidence, the identification assumption between predictions and observations cannot be rejected for these cases. In other words, those predictions can be considered as identical as actual observations.

5. Conclusion

This paper presents a crash prediction method based on a bivariate extreme value theory (EVT) framework and UAV trajectory data processing. Ten intersections in Fengxian District, Shanghai were selected, where videos of representative traffic periods were captured using an unmanned aerial vehicle (UAV). High-resolution vehicle trajectories were automatically extracted from UAV videos, based on which conflicts were identified and detailed metrics were further calculated. To obtain vehicle trajectories, a KLT-based detection and tracking method was applied and a multi-color space information fusion strategy was introduced to improve detection accuracy.

Four fundamental conflict metrics were used for EVT modeling, including TA, mTTC, PET, and MaxD. TA was considered as a measure of perception-reaction failure and the other three were used for measuring the failure to proper evasive actions. Each metric was modelled by a univariate EVT model and each pair was modelled by a bivariate EVT model. A set of tests were applied to examine the performances of those models based on actual crash observations, in terms of correlation, rank correlation, and prediction accuracy.

Among univariate EVT models, those based on PET appeared to be the best for predicting angle crash frequency, while those based on mTTC outperformed others in terms of estimating rear-end crash frequency. The results indicated that the best conflict metrics for different crash types could vary. Better bivariate EVT model predictions were found for both angle and rear-end crash, compared to univariate models. To be more specific, bivariate EVT crash models based on PET and TA outperformed univariate PET models, in angle crash predictions. Those based on mTTC and TA were better than univariate EVT models. This indicated the importance of considering both perception-reaction failure and evasive action failure for crash predictions based on an EVT modeling framework. TA could improve bivariate EVT model performances, as a measure of initial perception-reaction failure. Moreover, for different crash types, the best pair of conflict metrics could also vary.

In general, the crash prediction method proposed in this paper appeared to be a promising tool for safety evaluation on signalized intersections. To be admitted, there are still some issues that need to be addressed. First, only 10 intersections were examined in this study, although a lot of efforts had been made. Future research can incorporate more intersections. Second, the automated video techniques utilized in this study may not be the optimal choice, although a fairly high detection rate was obtained. Other state-of-the-art computer vision techniques may be used in the future, such as deep learning methods. However, it will take additional efforts to collect samples for model training. Moreover, there is no benchmark video technique recommended for video-based conflict studies. Third, only bivariate EVT models were examined. A multivariate EVT model framework could also be attempted, incorporating multiple conflict metrics. However, before applying such models, additional aspects other than reaction and evasive action failure need to be considered. Last, although day-time non-peak hours were considered as representative in previous literature (Zheng et al., 2018), it would be better to collect more data of representative hours for EVT modeling. We recommended that future research could be focused on these topics.

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