Ph.D. Comprehensive Examination

Design and Analysis of Algorithms

Fall 2011

Short Questions

Answer 3 of 4 questions.

[S_1] Let $h(n) = \sum_{i=1}^{n} \frac{1}{i}$, prove $h(n) = \Theta(\log_2 n)$.

[S_2] From the following recurrence determine the growth rate of $T(n)$:

$$\begin{cases} T(n) = 4\,T(n-1) - 4\,T(n-2) \\ T(1) = 1, \quad T(2) = 4 \end{cases}.$$

[S_3] (a) Order by asymptotic growth rate from slowest to fastest:

$$4^{n^{1.005}},\ (1200n+1)^2,\ n^3,\ \lg^2 n,\ e^n,\ \ln\ln n,\ 5^{n^{1.004}}.$$

(b) Calculate $\sum_{i=1}^{n} i(i+1)(i+2)(i+3)(i+4)$.

[S_4] Construct

[a] a finite automaton or a regular expression for the language

$\{ x \in \{0,1\}^* :$ the first two characters of x are identical to the last two$\}$.

[b] a context free grammar or pushdown automaton for the language

$\{ a^{2n} b^{2n+1} : n > 0 \}$.

Long Questions

Answer 3 of 4 questions.

[L_1] Suppose we have an instance of TSP given by the cost matrix:

$$\begin{bmatrix} \infty & 3 & 5 & 8 & 1 & 2 \\ 3 & \infty & 6 & 4 & 5 & 9 \\ 5 & 6 & \infty & 2 & 4 & 1 \\ 8 & 4 & 2 & \infty & 7 & 5 \\ 1 & 5 & 4 & 7 & \infty & 6 \\ 2 & 9 & 1 & 5 & 6 & \infty \end{bmatrix}$$

a) Given the partial solution $X = (5, 2, -, -, -)$, calculate $B(X)$ using the reducing technique on the matrix.

b) For X as in a), use backtracking with branch-and-bound to find the best solution which is an extension of the given partial solution. Draw the portion of the state space tree you are investigating.

[L_2] Solve the instance of minimum tardy task weight with 6 objects, all of length 1, having deadlines 3, 2, 1, 2, 4, 3; and weight 7, 5, 4, 3, 2, 1 (resp.).

[L_3] Keys a, b, c, d, e have frequencies 10, 3, 4, 7, 15, respectively. Find OBST using Dynamic programming algorithm.

[L_4] Classify each of the following languages as regular, context free but not regular, or decidable but not context free. Prove your answers.

a) $\{a^{n+1}b^{n-1}c^m : \ n, m > 0\}$

b) $\{a^{2n}b^{2m+1} \ : \ n, m \geq 0\}$

c) $\{a^{n+1}b^{n-1}c^n : \ n > 0\}$