Ph.D. Comprehensive Examination
Algorithms and Theory of Computation

Fall, 2008

Short Questions

Answer 3 of 4 questions.

[S1]

(a) Briefly explain the difference between *determinism* and *nondeterminism* for any computing device.

(b) What does this mean specifically for (1) a finite automaton, (2) a pushdown automaton, and (3) a polynomial time decision algorithm?

(c) What do we know, and not know, about the equivalence of determinism and nondeterminism in each of the three cases of (b).

[S2] Construct

(a) a finite automaton or a regular expression for the language

$\{ x \in \{0,1\}^* :$ "11" can only occur immediately after "00" $\}$.

(b) a context free grammar or pushdown automaton for the language

$\{ a^n b^m c^{n+m} : n, m > 0 \}$

[S3] Let $h(n) = \sum_{i=1}^{n} \frac{1}{i}$, prove

$$h(n) = \Theta(log_2\, n).$$

[S4] From the following recurrence determine the exact formula of $T(n)$ in terms of n and the growth rate of $T(n)$:

$$\begin{cases} T(n) = & 6T(n-1) - 9T(n-2) + 4n \\ T(0) = 4, & T(1) = 10 \end{cases}$$

Long Questions

Answer 3 of 4 questions.

[L1] Briefly prove (if true) or disprove (if false):

(a) The complement of a regular language is regular.

(b) The complement of a context free language is context free.

(c) The complement of a Turing acceptable language is Turing accepatble.

[L2] Classify each of the following languages as regular, context free but not regular, or decidable but not context free. Prove your answers.

(a) $\{ a^n b^m c^p : n > m > p > 0 \}$

(b) $\{ a^n b^m c^p : n = 2p > 0 \}$

[L_3] Suppose we have an instance of TSP given by the cost matrix:

$$\begin{bmatrix} \infty & 8 & 3 & 6 & 7 \\ 8 & \infty & 5 & 7 & 4 \\ 3 & 5 & \infty & 9 & 8 \\ 6 & 7 & 9 & \infty & 6 \\ 7 & 4 & 8 & 6 & \infty \end{bmatrix}.$$

For this instance use backtracking with branch-and-bound to find the best solution and draw the state space tree you are investigating.

[L_4] Using Dynamic Programming to calculate the best product $ABCD$ of four matrices, where A is 13×5, B is 5×89, C is 89×3, D is 3×34.