Comprehensive Examination
Algorithms and Theory

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17 August 2007

1 Short Questions: Answer any 3 of 4.

[S_1] Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of Θ-function, prove that

$$max\ (f(n), g(n)) = \Theta(f(n) + g(n)).$$

[S_2] From the following recurrence determine the growth rate of $T(n)$:

$$\begin{cases} T(n) = & 2T(\frac{n}{2}) + n^3 \\ T(1) = 1 & \cancel{T(2) = 4} \end{cases}.$$

[S_3] Give a regular expression or a finite automaton for each language:

1. $\{x \in \{a,b\}^* : abb$ occurs at least twice in $x\}$
2. $\{x \in \{a,b\}^* : abb$ occurs exactly once in $x\}$

[S_4] Give a context free grammar for each language:

1. $\{a^n b^{n+m} c^m : n, m > 0\}$
2. $\left\{xc^n d^n x^R : x \in \{a,b\}^*, n > 0\right\}$

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2 Long Questions: Answer any 3 of 4.

[L_1] Suppose we have an instance of TSP given by the cost matrix:

$$\begin{bmatrix} \infty & 3 & 5 & 8 & 1 & 2 \\ 3 & \infty & 6 & 4 & 5 & 9 \\ 5 & 6 & \infty & 2 & 4 & 1 \\ 8 & 4 & 2 & \infty & 7 & 5 \\ 1 & 5 & 4 & 7 & \infty & 6 \\ 2 & 9 & 1 & 5 & 6 & \infty \end{bmatrix}$$

a) Give the partial solution $X = (5, 2, -, -, -)$, calculate $B(X)$ using the reducing technique on the matrix.
b) For X as in a), use backtracking with branch-and-bound to find the best solution which is an extension of the given partiala solution. Draw the portion of the state space tree you are investigating.

[L_2] Solve the instance of minimum tardy task weight with 6 objects, all of length 1, having deadlines 3, 2, 1, 2, 4, 3; and weight 7, 5, 4, 3, 2, 1 (resp.).

[L_3] Classify each of the following languages as regular, context free but not regular, or in P but not context free. Prove your answers.

1. $\left\{a^n b^{n^2} : n > 0\right\}$
2. $\{a^{2n} b^{2n+1} : n > 0\}$
3. $\{a^{2n} b^{2m+1} : n, m > 0\}$

[L_4] Briefly prove or disprove:

1. The intersection of two context free languages is context free.
2. The intersection of two Turing acceptable languages is Turing acceptable.
3. The intersection of two NP languages is NP.