



# Performance measure for reliable travel time of emergency vehicles



Zhenhua Zhang<sup>a,1</sup>, Qing He<sup>a,b,\*</sup>, Jizhan Gou<sup>c,2</sup>, Xiaoling Li<sup>d,3</sup>

<sup>a</sup> Department of Civil, Structural and Environmental Engineering, The State University of New York, Buffalo, NY 14260, United States

<sup>b</sup> Department of Industrial and Systems Engineering, The State University of New York, Buffalo, NY 14260, United States

<sup>c</sup> Serco Inc., Reston, VA 20190, United States

<sup>d</sup> Northern Region Operations, Virginia Department of Transportation, Fairfax, VA 22030, United States

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## ABSTRACT

Travel time is very critical for emergency response and emergency vehicle (EV) operations. Compared to ordinary vehicles (OVs), EVs are permitted to break conventional road rules to reach the destination within shorter time. However, very few previous studies address the travel time performance of EVs. This study obtained nearly 4-year EV travel time data in Northern Virginia (NOVA) region using 76,000 preemption records at signalized intersections. First, the special characteristics of EV travel time are explored in mean, median, standard deviation and also the distribution, which display largely different characteristics from that of OVs in previous studies. Second, a utility-based model is proposed to quantify the travel time performance of EVs. Third, this paper further investigates two important components of the utility model: benchmark travel time and standardized travel time. The mode of the distribution is chosen as benchmark travel time, and its nonlinear decreasing relationship with the link length is revealed. At the same time, the distribution of standardized travel time is fitted with different candidate distributions and Inv. Gaussian distribution is proved to be the most suitable one. Finally, to validate the proposed model, we implement the model in case studies to estimate link and route travel time performance. The results of route comparisons also show that the proposed model can support EV route choice and eventually improve EV service and operations.

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## 1. Introduction

Emergency vehicles (EVs) are general terms for police cars, fire trucks or ambulances. When on a mission, they are designated and authorized to respond to firefighting, medical assistance, etc. which are highly pressed for time. To make a quick response, EVs expect and pursue possibly short and reliable on-route travel time to ensure the quality of emergency response.

\* Corresponding author at: Department of Civil, Structural and Environmental Engineering and Department of Industrial and Systems Engineering, University at Buffalo, The State University of New York, Buffalo, NY 14260, United States. Tel.: +1 (716)645 3470.

E-mail addresses: [zhenhuaz@buffalo.edu](mailto:zhenhuaz@buffalo.edu) (Z. Zhang), [qinghe@buffalo.edu](mailto:qinghe@buffalo.edu) (Q. He), [Jizhan.Gou@serco-na.com](mailto:Jizhan.Gou@serco-na.com) (J. Gou), [Ling.li@vdot.virginia.gov](mailto:Ling.li@vdot.virginia.gov) (X. Li).

<sup>1</sup> Tel.: +1 (716)939 8332.

<sup>2</sup> Tel.: +1 (703)259 3355.

<sup>3</sup> Tel.: +1 (571)350 2020.

Over the past few decades, travel time reliability has been upheld as an important research topic and attracted increasing attentions (Abdel-Aty et al., 1995; Asakura and Kashiwadani, 1991; Bates et al., 2001; Bogers and Van Zuylen, 2004; Richardson and Taylor, 1978; Senna, 1994; Tu et al., 2012). The concept of travel time variability (reliability) is accepted as one of the key indicators for the performance of transport systems (Tu et al., 2012). One can also say that travel time reliability is an important measure for assessing the operating efficiency of signalized arterials (Skabardonis and Geroliminis, 2005). Checking the probability features of travel time is one of the most common approaches to the study of vehicle routing and reliability.

The probability methods are very popular in the travel time study in ordinary vehicles (OVs). What they have in common is that they all relate to the properties of the (day-to-day or within-day) travel time distributions and particularly to the shape of the distribution (Tu et al., 2012). Measures of travel time reliability derived from distribution can be divided into percent variation (standard deviation over average travel time), misery index (length of delay of only the worst trips) and buffer Time (the difference between the average travel time and the 95th percentile travel time) (Lomax et al., 2003). Further approaches extensively studied the total delay (a portion of travel time composed of link delay and intersection delay) of OVs on an arterial, and argued that intersection delay is the main influential factor (Lin et al., 2004). Distribution based analysis proves to be valid for OV travel time and is also expected to be a good approach to reveal the characteristics of EV travel time. However, to our best knowledge, there are very few previous studies on EV travel time. The EV travel time may still follow a right-skewed distribution, but the travel time features of EVs and OVs are quite different because of their different priority levels. First, the EVs run for specific and urgent tasks, whereas most of the OVs run for everyday commuting; sometimes, EVs are allowed to run over the speed limit (Henchey et al., 2014) whereas OVs must not. Second, EVs siren when they are on duty, and other on-site OVs have to make way for them. These privileges possibly make EVs less sensitive to traffic congestions than OVs and also maintain a higher running speed. Third, traffic signals at intersections can be preempted to grant “green” for EVs despite the existing traffic conditions. This preemption process is called emergency vehicle preemption (EVP), which is the control logic providing priority for specific users. EVP is usually unconditional, providing higher priority consideration than transit signal priority (He et al., 2012). A case study on U.S.1 shows the average duration of EVP ranges from 16 to 26 s with no significant variability by time-of-day (Gkritza, 2003). These signal duration for EVP may exert an impact on the non-emergence vehicles travel time (McHale and Collura, 2003), and the reallocation of green time that results from preempting a traffic signal has the potential to affect the flow of traffic negatively (Obenberger and Collura, 2001). One can say that EVP offers much higher preferential treatments for EV movements than OVs as the signal-caused delay for EVs is substantially reduced. In this paper, the records of EVP process are employed to extract EV travel time.

Due to these differences in operations between OVs and EVs, the study on EV travel time may also pose certain differences in several aspects. A good reliable index for EV travel time may have not only theoretical value but also practical significance. To do this, the first problem to be addressed is what the probability distribution of EV travel time follows. Previous literature for OVs applied several distributions on travel time in the past few decades including lognormal distribution (Herman and Lam, 1974), Gamma distribution (Polus, 1979), Weibull distribution (Emam and Ai-Deek, 2006), etc. Most of the distributions are right-skewed and have a complicated logarithmic calculation. To make the computation of distribution fitting less expensive, Taylor and Susilawati (2012) suggested the Burr distribution be a better statistical model than the lognormal distribution because of its relative computational advantages and sufficient “bent” to capture the spread of observations in the upper tail. Compared to OVs, EV travel time distributions are also important because they can unveil the mean and standard deviation of travel time, and distributions over different time periods can even reveal how EV travel time varies over time-of-day.

The second problem is how to quantify the EV travel time performance in different road links. Previous studies conducted questionnaire survey for OVs and travelers are asked to choose between alternatives that differ in terms of cost, average travel time, variability of travel times and departure time (Asensio and Matas, 2008). These considerations are certainly unrealistic for EVs. In contrast, theoretical analysis based on historical data of EV travel time may be applicable and give more meaningful results. Link travel time percentiles still play an important role in evaluating the travel time reliability of road links, suggested by van Lint et al. (2005). Besides the single statistical indicators derived from the travel time distribution such as mean or median, the unexpected delay must be incorporated into the model because the risk of encountering a travel time in the right-tail of that distribution should be considered (Tu et al., 2012). A statistical model is a good choice to quantify the travel time reliability, and provides a good indicator for evaluating the EV travel time performance.

This paper aims to close the gaps in travel time study between OVs and EVs, as the latter draws relatively less attention in the past few years (Westgate et al., 2013). Preemption records are employed to extract the travel times of EVs in this study with a couple of reasonable assumptions. The extracted travel time data is fully examined to reveal the different characteristics between EVs and OVs. The major contribution of this paper is to build a utility model to evaluate EV travel time performance with consideration of reliability. This utility-based model consists of three major parts: benchmark travel time, delay late and delay early. In the model, EVs that arrive earlier than benchmark are rewarded, and those arriving later are penalized. The model gives a straightforward indicator of whether the given road link or route is reliable or not and which is the best route for a given origin–destination pair.

The rest of this paper is organized as follows. Section 2 describes the raw data and the methods of collecting travel time. Section 3 examines the characteristics of EV travel time. Section 4 proposes a model to measure EV travel time performance. Several assumptions are validated through numerical studies in selected areas, and the characteristics of robust benchmark travel time and standardized travel time are revealed. In Section 5, case studies of the road network are conducted to present the travel time performance both in different time-of-day and different routes.

## 2. Data description

### 2.1. The study area

The study area, shown in Fig. 1, is located in the network of Northern Virginia (NOVA). With 2.8 million residents (about a third of the state), NOVA is the most populous region of Virginia and the Washington D.C. Metropolitan Area. NOVA has long been known for its heavy traffic (Cervero, 1994) which could potentially affect the travel time performance of EVs. We focus on arterials and connectors within several census-designated places, including Hoadly, Minnieville, Montclair, Dumfries, Dale City, and Agnewville. The study area contains more than 200 signalized intersections, which form a large road network connecting different land properties including residential land, commercial land, industrial land, public land, etc. There are more than 15 fire departments and 20 medical service centers in this area. Therefore, this is a proper place for studying the characteristics of EV travel time.

### 2.2. Emergency vehicle preemption process

All the signalized intersections in our study area can provide emergency vehicle preemption service (EVP) to EVs. Emergency vehicle preemption is a type of system that allows the normal operation of traffic signals to be preempted at the signalized intersections. EVP has two major objectives: reduce the travel time and improve operator safety (Gifford et al., 2001). EVs are equipped with strobe-light based preemption devices, which interact with signal-side receivers. Once the priority request is recognized and validated by EVP system, an electrical relay in the traffic signal cabinet is closed which allows the controller to service the call (Nelson et al., 2000). A control strategy immediately switches from the current phase to a pre-selected phase for the first request received (He et al., 2014). After the EVP ends, an EVP recovery strategy is actuated, and traffic signals switch back to normal. With the implementation of EVP, emergency vehicles can receive green indications along their routes to reach their destinations more quickly (Park et al., 2008). The EVP process has been proved to be very effective in shortening the trip travel times for EVs in several field tests.

The entire EVP process is illustrated in Fig. 2. When an EV approaches the intersection, the strobe device on EVs is sensed, and EVP is activated at the intersection when an “Alert” event is recorded. The signal phase is then preempted to be green for EVs. Once the EV passes the intersection and the strobe signal is no longer received, the EVP is deactivated when an “OK” event is recorded.

### 2.3. Travel time extraction and assumptions

To collect the travel time of EVs, there were attempts using GPS information to acquire the travel time information of ambulances (Westgate et al., 2013). In this paper, we employ the EVP data collected by the event log system in signal controllers instead. Recorded by EVP, both “Alert” and “OK” events contain useful information including vehicle types, the timestamps, event types, intersection ID. The time difference between the two “OK” events from two adjacent intersections can be regarded as the link travel time. More than 3 million valid preemption records were collected for nearly 4 years, from July 2010 to June 2014.

In addition, each intersection along selected arterials is labeled with three types of information: the name of the arterials that crossed at the intersection, geographic coordinates of the intersections in the arterial and a unique intersection ID. According to the geographic coordinates, the locations of the intersection are revealed, and the distance between two adjacent intersections can be obtained accordingly. The preemption records together with geographic information constitute the raw data required to extract EV travel time data.

The existing challenge in this dataset is that no vehicle ID is recorded during EVP process. Therefore, we need to pair two “OK” events in order to obtain the travel time between two intersections. As EVs do not appear as frequently as OV, the situation that two or more EVs appear in the same location at the same time interval is rare. We can guarantee the quality of extracted travel time by making the following assumptions:

- **Assumption 1:** In urban streets, EVs travel within a range of speed because the speed of the EVs should obey certain rules (Henchey et al., 2014). In this study, we assume the highest speed is  $V_{max}$  and lowest is  $V_{min}$ . Given the distance between two consecutive intersections, we can obtain the reasonable travel time range. For an “OK” event at a given intersection, the matched “OK” event in adjacent intersections should occur within the reasonable travel time range.
- **Assumption 2:** For each “OK” event recorded by an intersection, there is only one reasonable paired “OK” event in the total preemption records of its nearest four intersections. If there are multiple paired “OK” events, the assumption is violated and we remove the record of this “OK” event.
- **Assumption 3:** Of all “OK” events recorded in one intersection, there is only one “OK” event that can be paired with an “OK” event in the other intersection. If there are more than one “OK” events in one intersection that are paired with the same “OK” event in the other intersection, the assumption is violated and we abandon these “OK” events in this intersection.

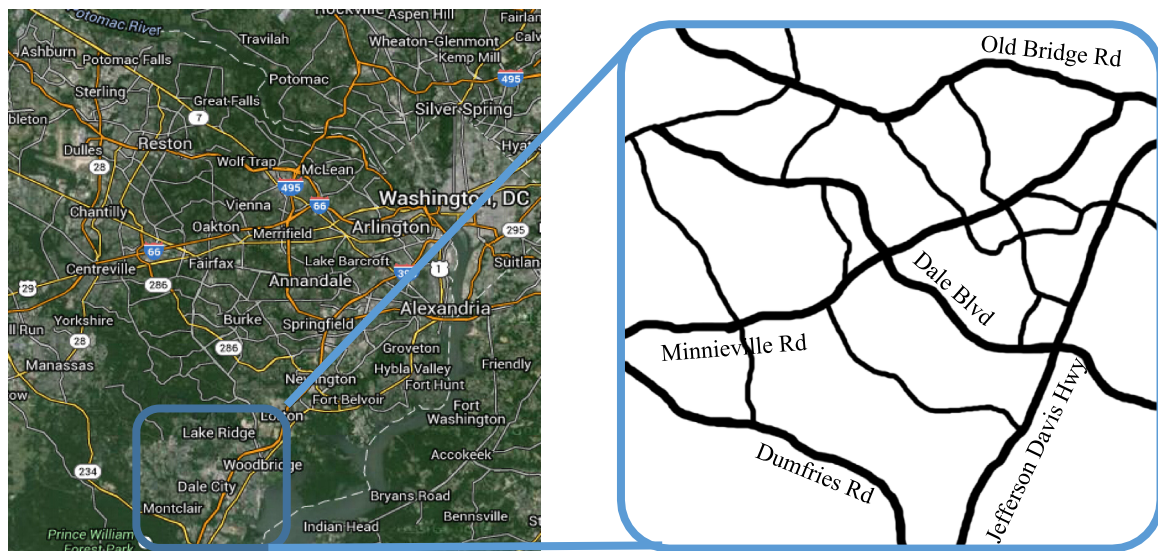


Fig. 1. The road networks and selected road facilities.

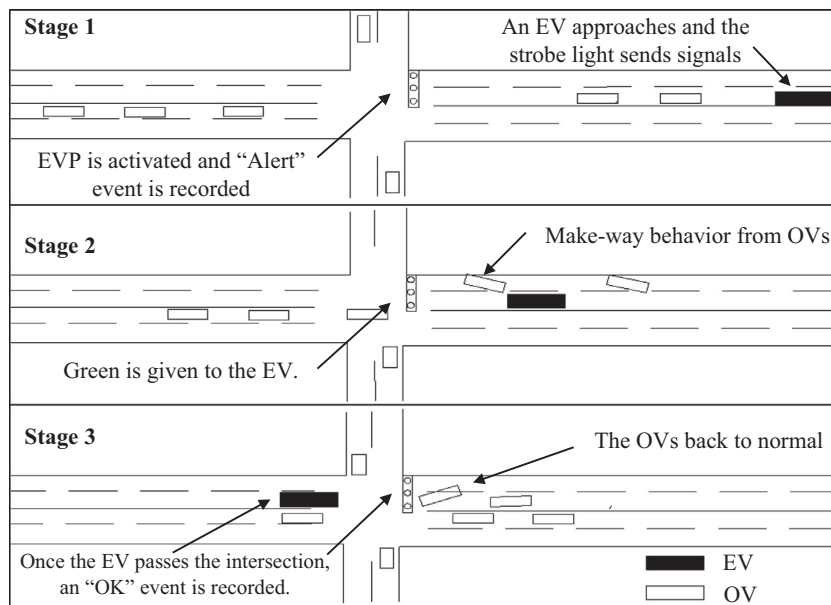


Fig. 2. Preemption process for EVs with strobe lights (EVP based on other technologies contains similar logic).

Assumption 1 helps calculate the bounds of the travel time between the two consecutive signalized intersections and filter unreasonable travel times. Assumption 2 ensures that there are no multiple pairs of “OK” events in adjacent intersections that match the same “OK” event in a given intersection. Assumption 3 ensures that there are no multiple pairs of “OK” events in a given intersection that matches an “OK” event in any adjacent intersection. Benefiting from a large sample size, the study finally obtains more than 500,000 link travel time records.

### 3. Characteristics of EV travel time

In this section, an empirical analysis of EV travel time is conducted. To normalize link travel time, travel time obtained in Section 2 is converted into unit travel time, defined as follows:

$$S = \text{link travel time} / \text{link length} \quad (1)$$

The unit travel time, which is the inverse of speed, is an indirect manifestation of average running speed on a road link. The unit travel time has been widely used in OV's and can unveil the travel time characteristics within a certain scale of the road network.

For travel time study, both the probability distribution and the temporal relations are critical to evaluate the travel time reliability. Fig. 3(a) shows the probability density plot of unit travel time for the entire study area. Similar to OV's, the distribution displays a right-skewed characteristic with a very long tail (Taylor and Susilawati, 2012). The long tail of EV travel time distribution indicates that it is still possible for EV's to experience unexpected delay, although EV's are endowed privileges such as make-way behavior and EVP. This is rare but its consequences of unexpected delay could be very severe. The figure also shows the mean unit travel time is around 0.084 s/m, indicating an average speed of 45 km/h (28.1 mph). The mean of EV unit travel time is much smaller than that of OV's (0.145 s/m), reported in the previous study (Susilawati et al., 2013). Lower than the mean, the median is around 0.067 s/m, with an average speed of 60 km/h (37.5 mph). The median of EV unit travel time is very low and much smaller than the one of public transit unit travel time (Taylor, 1982) and the OV's (Chien and Kuchipudi, 2003) in previous studies.

Fig. 3(b) shows the boxplots of EV travel time over time-of-day on weekdays. The time interval of each boxplot is 15 min. One can see that the mean, median and standard deviation do not fluctuate greatly over time-of-day, although a small rise of travel time is observed in the afternoon. The “flat” time-of-day pattern can be explained by three aspects: First, the siren of EV's can notice the nearby OV's, and OV's make way for EV's, so that EV's can maintain a high speed. Second, the routing of EV's is already optimized by experienced drivers, and they may try to avoid the links with low capacity, bad pavement, etc. Third, EV's benefits from EVP system, and EV's are less influenced by intersection delay.

The mean and standard deviation are useful to identify important information concerning the performance of travel time. Given a specified origin and destination, the ideal reliable route is the one with both the least mean and the least standard deviation. However, in practice, from Fig. 3(b), one can see that conditions of higher mean and lower standard deviation coexist with that of lower mean and higher standard deviation. Thus, it is difficult for EV operators to make choices under such circumstances. What's more, a reliable road link should also refrain from the unexpectedly extreme delay. Simple statistical measures, such as mean, median or standard deviation, are obviously not sufficient.

## 4. Modeling travel time performance of EV's

### 4.1. Utility-based models

The travel time performance can be simply formulated as a utility value that consists of three major components: benchmark unit travel time, arriving early and arriving, which is proposed by (Fosgerau and Karlström, 2010).

$$U_D = BL + \beta L(S_D - B)^+ + \gamma L(S_D - B)^- \quad (2)$$

$U_D$  is the utility of travel time of the road link over a certain time period at departure time  $D$ . The time period is usually set to be one hour. In the first component,  $B$  is the benchmark unit travel time, and  $BL$  measures the reasonable travel time without unexpected delay.  $L$  is the link length. We assume that EV operators are aware of all possible routes to the emergency site, and they have a rough estimation of possible link travel time  $BL$  to reach the emergency site. The second component measures the delay late,  $S_D$  represents the dataset of real unit travel time of a road link over a certain time period from departure time  $D$ .  $(S_D - B)^+$  is defined as  $\max(S_D - B, 0)$ . If  $S_D$  is larger than  $B$ ,  $\beta L(S_D - B)^+$  increases the utility value. The third component measures the delay early,  $(S_D - B)^-$  is defined as  $\min(S_D - B, 0)$ . If  $S_D$  is smaller than  $B$ ,  $\gamma L(S_D - B)^-$  reduces the utility value.  $\beta$  and  $\gamma$  are non-negative preference ratio parameters of the delay late and delay early over benchmark unit travel time.

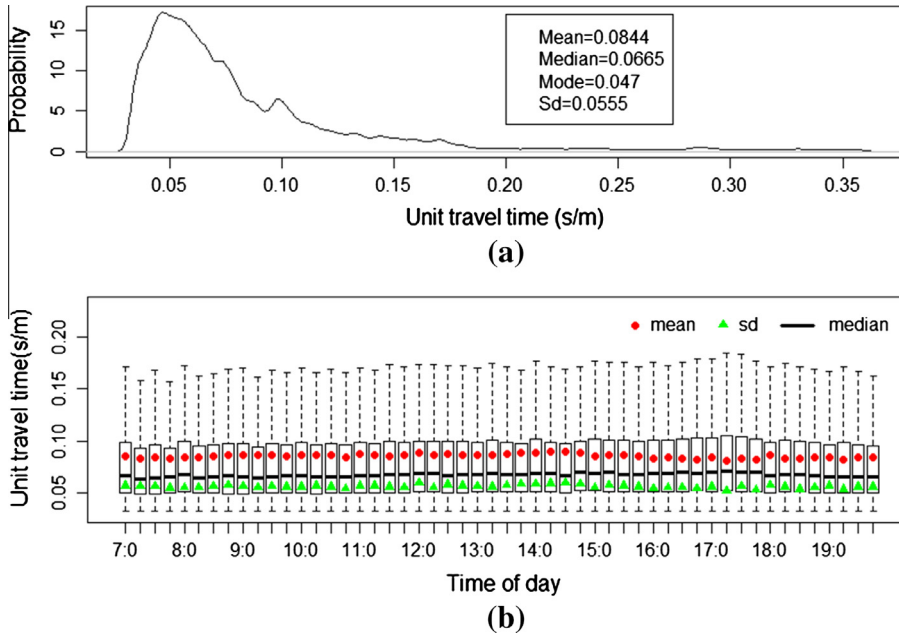
The benchmark unit travel time, which is the inverse of travel speed, can be directly perceived by the emergency vehicle drivers. Estimation of benchmark travel time of the road link directs to a reasonable travel time value that is neither late nor early. Usually, this estimation may vary with road links because the travel speed may be influenced by the unique features of the road link such as geometry features, slopes, cross sections, and time of day traffic conditions. Also, the weight of each influential factors is difficult to determine because of the emergency vehicle's privileges mentioned in Section 1. We then turn to the empirical results of historical data. In this paper, the benchmark travel time is set to be the mode of unit travel time distribution, which most EV operators have experienced.

$$B = \text{mode}(S_D) \quad (3)$$

It is worth mentioning that we do not choose mean as the benchmark because mean travel time is especially sensitive to the right tail of the distribution of travel times (Small et al., 2005). Also, compared to mean or median, mode is less sensitive to the right-skewed feature of travel time and range around mode represents the travel time owns the highest probability.

Besides benchmark travel time, in daily emergency operations, EV's are expected to arrive as early as possible in response to emergencies. Travel time that is higher than the benchmark travel time is regarded as unreliable and should be penalized accordingly. On the contrary, travel time that is lower than the benchmark travel time should be reliable and rewarded because this wins more time for rescuing operations. The utility function of a certain link describes the travel time perfor-





**Fig. 3.** EV unit travel time analysis: (a) Probability density distribution of EV unit travel time; (b) boxplots of EV unit travel time over of time-of-day on weekdays.

mance of a road link. The larger the utility is, the worse the performance should be, and the less likely the road link would be chosen by EV operators. In this sense, one can also take it as a disutility function.

Also, we can further decompose the unit travel time on a road link over a certain period as follows:

$$S_D = \mu_D + X\sigma_D \quad (4)$$

where  $\mu_D$  is the mean of unit travel time and  $\sigma_D$  is the standard deviation.  $X$  is defined as the standardized travel time with probability density distribution  $f()$  and cumulative distribution  $F()$ .  $X$  plays an important role in evaluating the travel time performance. The reason we write  $X$  instead of  $X_D$  is that standardized travel time for EVs may not be influenced by departure time. For OV, Fosgerau and Karlström (2010) assumes the distribution of  $X$  is independent from both departure time and links. In this paper, we also assume this is valid for EVs and the assumption is verified in Section 4.3.

According to Assumption 1 in Section 2.3, the highest and lowest speed for EVs are  $V_{max}$  and  $V_{min}$  which indicate largest (labeled as  $B_2$ ) and the smallest (labeled as  $B_1$ ) unit travel time, respectively. This defines the upper bound and lower bound of distribution  $X$ .

Given a road link or a route, the expected utility of EVs can be derived as follows,

$$E(U_D) = L \cdot \left[ B + \beta \int_{\frac{B_1 - \mu_D}{\sigma}}^{\frac{B_2 - \mu_D}{\sigma}} (\mu_D + X\sigma_D - B)f(X)dX + \gamma \int_{\frac{B_1 - \mu_D}{\sigma}}^{\frac{B_2 - \mu_D}{\sigma}} (\mu_D + X\sigma_D - B)f(X)dX \right] \quad (5)$$

One can see that besides the effect of link length, the preference parameters  $\beta$  and  $\gamma$  are also very important in evaluating the performance of the travel time. These parameters can be decided by EV operators. Also, to strengthen the proposed model, we proceed to examine the robust benchmark unit travel time and the distribution of standardized travel time.

#### 4.2. Robust benchmark unit travel time

According to the last subsection, one assume that different links possess different benchmark unit travel time. In order to uniform benchmark unit travel time, we further derive a robust benchmark unit travel time directly from link length. Also, EV travel time datasets are not evenly distributed in the network. Minor arterials may lack sufficient EVP records to obtain a good estimation of link travel time distribution. In this case, the robust benchmark unit travel time provides a good estimation of benchmark unit travel time without collecting lots of EV historical data.

Fig. 4(a) and (b) shows how link travel speed and benchmark unit travel time change over link length, respectively. It is found that the upper bound of the scatter points in Fig. 4(b) displays a power relationship between the benchmark unit travel time and the link length. This agrees with common sense that longer link travel distance usually leads to a lower portion of intersection delay, higher average speed, and thus lower unit travel time. Therefore, we define the maximum of benchmark unit travel times over a certain range of link length, as robust benchmark unit travel time. Fig. 4(c) shows the relation-

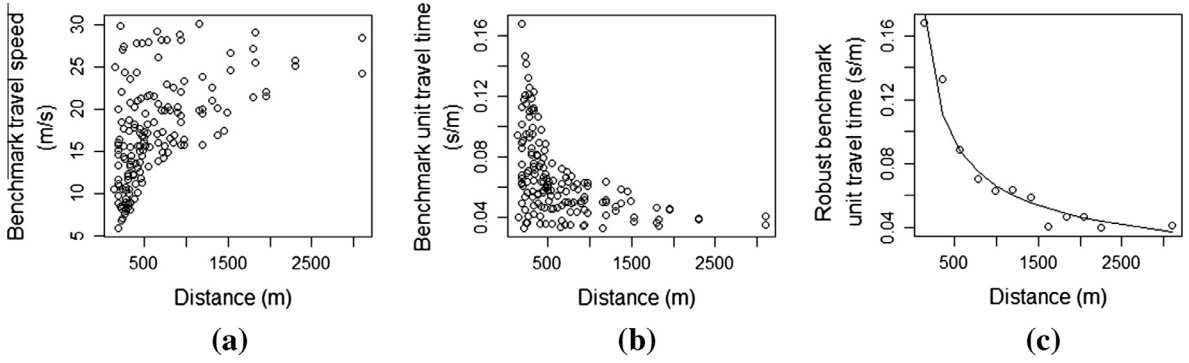


Fig. 4. Relationship between link lengths and benchmark travel speed (a), benchmark unit travel time (b), robust benchmark unit travel time (c).

ship between the robust benchmark unit travel time and link length in every 200 meters. One can see that there is a clear decreasing trend when the link length increases. Power transformation is adopted to fit the relationship between robust benchmark unit travel time and link length as follows.

$$\bar{B} = a * (L + b)^{(-\frac{1}{c})} \quad (6)$$

where  $\bar{B}$  is the robust benchmark unit travel time; Parameter  $a$ ,  $b$ , and  $c$  represent the scale parameter, the location parameter and the power parameter, respectively.

#### 4.3. Standardized travel time estimation

According to Section 4.1, we assume that the standardized travel time (STT) is independent of both time and space (links) for EVs. As one can see, Eq. (4) decomposes the unit travel time as the summation of mean and standard deviation multiplied by a coefficient of standardized travel time  $X$ . One can say that  $X$  follows a distribution with mean equal to 0 and standard deviation equal to 1. The standardization process potentially eliminates the probability distribution differences among travel time datasets from different road links.

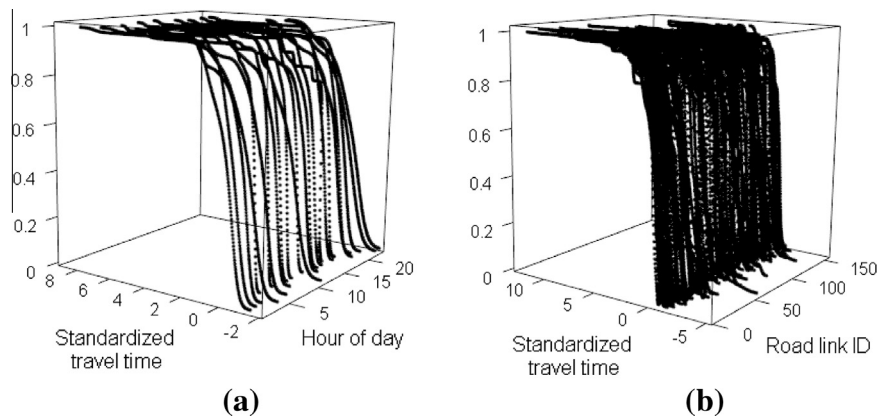
To verify that STT does not vary over different time-of-day on a certain link, one link that contains more than 1000 observations is selected for this study. The collected travel time data are divided according to hourly time periods. The cumulative density distribution of STT for each hour is shown in Fig. 5(a). To verify that the STT does not vary on different road links, more than 50 links in the study area are examined, and the collected travel time data are separated by links. The cumulative density distributions of STT in different links are depicted in Fig. 5(b).

For all distributions in Fig. 5(a) and (b), the maximum probability difference between any two distributions is less than 0.07. Although minor differences exist in both cumulative density distributions, we can assume that the distribution of STT is not influenced by time-of-day and different links. On the assumption that the STT does not vary over time and links, we can estimate its empirical probability density distribution  $f()$  and cumulative distribution  $F()$ . To compensate deficiencies of different sample sizes in different road links, we choose links with sample size larger than 60 and conduct equal sampling in each link.

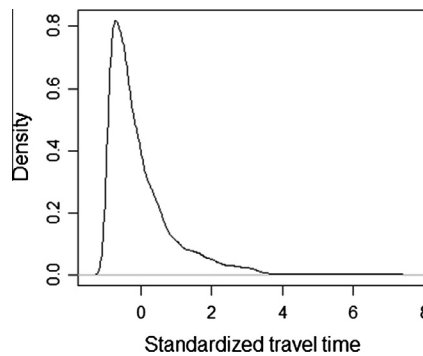
As shown in Fig. 6, the STT of total samples presents a long-tailed and right skewed distribution. Thus the candidate density functions to model STT should possess the same characteristics. Previous studies suggest that Burr distributions and lognormal distributions be suitable to model the travel time distribution while none of them has been validated for the STT of EVs. It also seems desirable to test stable distributions families including Gaussian distributions, Levy distributions, etc. which can be described by four parameters as  $X \sim f(\alpha, \beta, \gamma, \delta)$ , where  $\alpha$  is a stability parameter;  $\beta$  is the skewness parameter;  $\gamma$  is the scale parameter; and  $\delta$  is the location parameter (Zolotarev, 1986). Here we use maximum likelihood estimation (MLE) to estimate the parameters of the distributions and the Kolmogorov–Smirnov (KS) test to determine the goodness of fit for a certain distribution. KS test can derive a KS statistic which measures the maximum probability differences between the fitted distribution and the original data distribution. The less the KS statistic is, the better the fitness should be. More than 50 kinds of statistical distributions are tested and preliminary results tell that some of the stable distributions, such as the Levy distribution, are not suitable to model standardized travel time of EVs. It is also found that adding a location parameter  $\delta$  in the distribution functions improves the overall fit in some other distributions. For example, lognormal distribution is not likely to be a candidate distribution, but three-parameter lognormal distribution with one additional location parameter performs well in our study.

To identify the best distribution model, we focus on some candidate distributions listed in Table 1. Through KS test, we obtain fitted results for each distribution, shown in Table 2. One can see that Inv. Gaussian (3P) is the best distribution to model STT of EVs.

The identified probability density function in Eq. (7) and cumulative probability function in Eq. (8) are listed as below:



**Fig. 5.** Cumulative density distributions of STT, (a) for travel time in a certain link over time-of-day, (b) for travel time in different road links over a certain period.



**Fig. 6.** Distribution of standardized travel time of EVs.

**Table 1**  
Details of the candidate distributions.

Type	Density function	Parameters
Four-parameter Burr distribution	$f(x) = \frac{\alpha k}{\beta} \cdot \left(\frac{x-\gamma}{\beta}\right)^{\alpha-1} \cdot \left(1 + \left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^{-k-1}$	$k, \alpha$ : shape parameter $\beta$ : scale parameter $\gamma$ : location parameter
Three-parameter lognormal distribution	$f(x) = \frac{1}{(x-\gamma)\sqrt{2\pi}\sigma} \cdot \exp\left(-\frac{1}{2} \cdot \left(\frac{\ln(x-\gamma)-\mu}{\sigma}\right)^2\right)$	$\mu, \sigma$ : shape parameter $\gamma$ : location parameter
Three-parameter Inv. Gaussian distribution	$f(x) = \sqrt{\frac{\lambda}{2\pi(x-\gamma)^3}} \cdot \exp\left(-\frac{\lambda(x-\gamma-\mu)^2}{2\mu^2(x-\gamma)}\right)$	$\lambda$ : shape parameter $\mu$ : shape parameter, also the mean $\gamma$ : location parameter
Three-parameter fatigue life distribution	$f(x) = \frac{\sqrt{(x-\gamma)/\beta + \sqrt{\beta/(x-\gamma)}}}{2\alpha(x-\gamma)} \cdot \mathcal{Q}\left(\frac{1}{\alpha}(\sqrt{(x-\gamma)/\beta} - \sqrt{\beta/(x-\gamma)})\right)$	$k, \alpha$ : shape parameter $\beta$ : scale parameter $\gamma$ : location parameter
Three-parameter Frechet distribution	$f(x) = \frac{\alpha}{s} \left(\frac{x-\gamma}{s}\right)^{-1-\alpha} \cdot \exp\left(-\left(\frac{x-\gamma}{s}\right)^{-\alpha}\right)$	$\alpha$ : shape parameter $s$ : scale parameter $\gamma$ : location parameter
Three-parameter PearsonV distribution	$f(x) = \frac{\exp(-\beta/(x-\gamma))}{\beta \cdot \Gamma(\alpha) \cdot ((x-\gamma)/\beta)^{\alpha-1}}$	$\alpha$ : shape parameter $\beta$ : scale parameter $\gamma$ : location parameter



**Table 2**  
Results of goodness of fit for the candidate distributions.

Type	Kolmogorov–Smirnov test	Result	Rank
Inv. Gaussian (3P <sup>a</sup> )	0.00982	Accepted	1
Lognormal (3P)	0.01287	Accepted	2
Burr (4P <sup>b</sup> )	0.01307	Accepted	3
PearsonV (3P)	0.01863	Accepted	4
Fatigue Life (3P)	0.01998	Accepted	5
Frechet (3P)	0.02255	Accepted	6

<sup>a</sup> Three-parameter.

<sup>b</sup> Four-parameter.

$$f(x) = \sqrt{\frac{\lambda}{2\pi(x-\gamma)^3}} \exp\left(-\frac{\lambda(x-\gamma-\mu)^2}{2\mu^2(x-\gamma)}\right) \quad (7)$$

$$F(x) = \Phi\left[\sqrt{\frac{\lambda}{x}}\left(\frac{x-\gamma}{\mu}-1\right)\right] + \exp\left(\frac{2\lambda}{\mu}\right)\Phi\left[-\sqrt{\frac{\lambda}{x}}\left(\frac{x-\gamma}{\mu}+1\right)\right] \quad (8)$$

where  $\lambda = 1.8584$ ;  $\mu = 1.1959$ ;  $\gamma = -1.1959$ .  $\Phi()$  is the cumulative probability function of standard normal distribution.

#### 4.4. Updating the utility function

Utility function of Eq. (5) defines the preference of selecting a road link for EVs. We can update the function by implementing the findings in Sections 4.2 and 4.3. So the utility function can be further derived and expressed as:

$$E(U_D) = L \cdot \left[ \bar{B} + \beta(\mu_D - \bar{B})F(X)\left|\frac{B_2 - \mu_D}{\sigma_D}\right| + \gamma(\mu - \bar{B})F(X)\left|\frac{\bar{B} - \mu_D}{\sigma_D}\right| + \beta\sigma_D \int_{F\left(\frac{\bar{B} - \mu_D}{\sigma_D}\right)}^{F\left(\frac{B_2 - \mu_D}{\sigma_D}\right)} F^{-1}(X)dX + \gamma\sigma_D \int_{F\left(\frac{B_1 - \mu_D}{\sigma_D}\right)}^{F\left(\frac{\bar{B} - \mu_D}{\sigma_D}\right)} F^{-1}(X)dX \right] \quad (9)$$

where  $L$  is the length of the road link;  $\bar{B}$  is the robust benchmark unit travel time;  $\mu_D$  and  $\sigma_D$  are the mean and standard deviation of the road link travel time;  $B_1$  and  $B_2$  is the upper bound and lower bound of the unit travel time;  $F(X)$  is the cumulative probability function of Inv. Gaussian distribution;  $\beta, \gamma$  are preference parameters for delay late and delay early. One may refer to Appendix for deriving Eq. (9).

In Eq. (9), in order to obtain  $E(U_D)$ , we need to estimate values of two parameters:  $\bar{B}$  and  $\mu_D$ . We also need to consider two important things: first, the departure time should be predefined both for link utility estimation and route utility estimation; second, when estimating  $\mu_D$ , an accordant time interval should be predefined during which the travel time records be utilized to calculate the average travel time. If not specified, we usually set the time interval which starts at the departure time and ends in 1 h, given the assumption that the link travel time of an emergency vehicle is less than 1 h.

#### 4.5. Route utility estimation

Route choice is one of the most important decisions for EV operators. Compared to link utility estimation, one more challenge exists in estimation of departure times between contiguous road links. As is mentioned in Section 4.4, the average travel time estimation depends on the departure time and the time interval over which the travel time records be utilized to calculate the average travel time.

We set departure time of the  $i$ th road link as the summation of departure time and the benchmark travel time of the  $(i-1)$ th road link.

$$D_i = D_{i-1} + B_{i-1}$$

The route utility can be obtained by a simple summation of all link utilities on that route. In practice, the preferred route is the one with lower utility value.

$$E(U_D^R) = \sum_{i \in \Gamma_R} E(U_{D_i}^i) \quad (10)$$

where  $U_D^R$  is the utility of route  $R$  at departure time  $D$ ;  $U_{D_i}^i$  is the utility of link  $i$  at departure time  $D_i$ ;  $\Gamma_R$  is the set of all links in the route  $R$ . One can see that the estimation of route utility may be different from that of link utility because each link utility estimation is taken separately over different time periods.

## 5. Case studies

In this section, we first calculate the link utility to find the travel time performance of certain links over different time periods. Then, we compare the utility of a route that consists of several continuous road links. The comparison evaluates the travel time performance of different routes over the same period. Finally we calculate all the unit-distance utility in our study area and present the travel time performance results over a certain period.

### 5.1. Link utility comparisons

Two road links with nearly the same length and similar geometry are selected. The first one is a 422 m road link on Jefferson Davis Hwy, and the second one is a 430 m road link on Dale Blvd. For each link, we calculate the link utility of each one-hour period from 8:00 a.m. to 17:00 p.m. The number of travel time observations on these two links is 1354 and 1093, respectively. The preference parameter of  $\beta, \gamma$  are set to be 1.5 and 2.5. The utility over time-of-day for two links on weekday is shown in Fig. 7.

Fig. 7 shows link utility values of two road links in different time of day. The utility of the travel time performance varies in different time periods. One can see that there is no clear time-of-day trend of utility values for both road links. This coincides with the observations in Section 3 that no typical peak hour pattern is observed for EVs for utility values. Both Fig. 7 (a) and (b) illustrates temporal link utility. In both cases, the link utility is lower than the robust benchmark travel time across different hours, except for that at 11:00 in Fig. 7(b). The exception may result from traffic incidents or other non-recurrent traffic congestions. But in most of the time, both links provide reliable travel time for EV operations. Through time-of-day comparisons, the overall performance of the first link is better. Also, in this case study, the preference parameters of delay late  $\beta$  and delay early  $\gamma$  are 1.5 and 2.5 respectively. These settings indicate the EV operators to value early arrivals much more than late arrivals. The parameters  $\beta$  and  $\gamma$  can be adjusted for different preferences. Table 3 shows how the utility value changes with  $\gamma$  for two links in the hour of 16:00. From Fig. 7, we see that mode of EV travel time is smaller than the benchmark. One can see that the utility values are sensitive to the changes of preference parameter but the ranks of the link utilities remain the same.

### 5.2. Route utility

Given a pair of origin and destination in Fig. 8, three competing routes, each of which consists more than 20 consecutive links, are selected to illustrate the route travel time performance. The time periods are set to be PM peak. The number of travel time observations for these routes is 35,819, 20,546 and 33,717 respectively. The preference parameters of  $\beta, \gamma$  are also set to be 1.5 and 2.5, respectively.

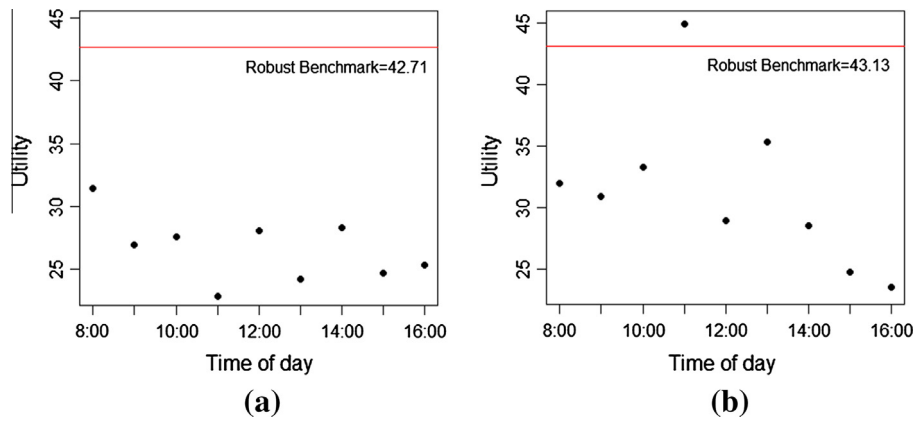
All the route information and the results of the utility function are shown in Table 4. For road links with no sufficient data, their robust benchmark travel times are used instead.

Route 1 shows the smallest utility, as compared to other two routes. Therefore, Route 1 is the most desirable route to be chosen for this EV trip. The proposed approach is promising for road networks containing a large number of signalized intersections. By comparing the utility of route travel time, one can distinctively determine the best route over different time periods. With longer observation periods, one can even study the hourly travel time reliability in different routes. It can be seen that the proposed utility model can be used in a decision support system (DSS) for route choice of EV response. It is worth mentioning that the unit utility (route utility per distance) of Route 1 is higher than that of Route 3, but the route length is relatively smaller. This aligns with common sense that routes with similar lengths do not guarantee the same travel time performance. This unit utility may be more directly related to the efficiency and performance analysis. However, as the EV relies highly on the travel time savings instead of efficiencies, route utility is more important for EV operators in making route choices.

### 5.3. Travel time reliability performance based on unit-distance utility

Emergency operation center may be interested in the travel time performance plot for all the road links over a certain period. We can normalize the utility of each link by dividing corresponding link length that is called unit-distance utility. This unit-distance utility is taken as an index of travel time reliability performance of the link because it diminished the effects of link length. A direct view of network reliability is shown in Fig. 9. For the purpose of display, we take the maximum of the utilities of a bi-directional link. The period is 3 h covering only PM peak, from 4:00 p.m. to 7:00 p.m. The number of the travel time observations is 72,989. From the map, EV operators can get useful information and direct knowledge of the travel time performance for different road links over a certain period.

From Fig. 9, we can compare the travel time reliability performance of different road links. Also, given sufficient travel time data collections, one can even compare the performance on the same link over different time periods. For EV operators, some longer links can cost more travel time but show higher reliability. It is possible that on some of the “long” links, the EV operators may be more likely to reach its destination within a preferred travel time (benchmark travel time). It is worth not-

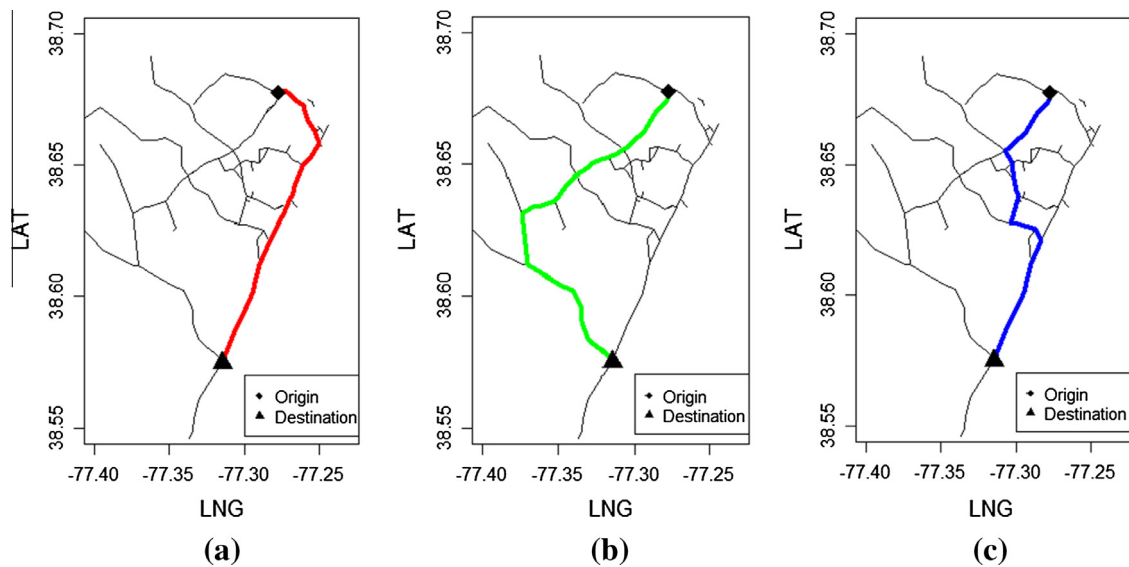


**Fig. 7.** Utility over time-of-day in (a) a link on Jefferson Davis Hwy, and (b) a link on Dale Blvd; the horizontal red line represents the robust benchmark travel time of the road link. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

**Table 3**

Sensitivity of preference parameters in the utility model.

Hour	$\beta$	$\gamma$	Utility of link on Jefferson Davis Hwy	Utility of link on Dale Blvd
16:00	1.5	2.5	25.33	23.49
16:00	1.5	3	16.63	13.67
16:00	1.5	3.5	−0.75	−5.97

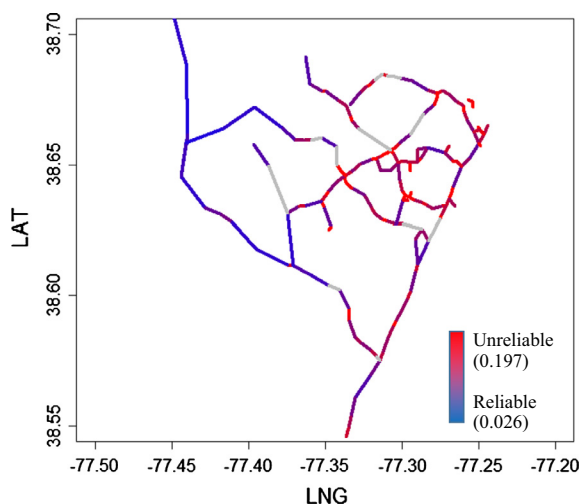


**Fig. 8.** Three routes and their origin and destination: (a) Route 1; (b) Route 2; (c) Route 3.

**Table 4**

The utility values of three routes.

	Route 1	Route 2	Route 3
Number of intersections	28	35	28
Length of the route (km)	13.55	19.29	14.26
Utility of the route	1237.0	1621.0	1260.8
Unit utility	0.091	0.084	0.088



**Fig. 9.** Normalized link utility (or unit-distance utility) representing EV link travel time performance in the network, gray lines represent links with no valid data.

ing that given more preemption records, one can further increase the resolution of the time periods and get more precise results.

## 6. Conclusions

This study developed a utility model to evaluate the performance and reliability of emergency vehicle (EV) travel time that draws very little attention in the previous literature.

Emergency vehicle preemption (EVP) records are employed to extract the EV travel time data. Link-based travel time is studied to reveal characteristics of EV travel time. Empirical results show that the mean unit travel time (travel time over the unit length of the road facility) of EVs is much smaller than the ones of transit and ordinary vehicles (OVs). Further, the median of EV travel time is much less than the mean, and this indicates a right-skewed distribution of EV travel time. The mean and standard deviation over different time-of-day does not show a clear time-of-day pattern, which is quite different from the previous observations in OVs. To evaluate the performance and reliability of travel time, a suitable utility-based model is proposed for EVs. The model incorporates the delay early and delay late, and also takes into account both the robust benchmark travel time and the distribution of standardized travel time of EVs.

In our study, the robust benchmark unit travel time of EVs is represented by the mode of the distribution. This study shows that it also has a strong negative correlation with the link length. A power function is derived to describe the correlation between link length and benchmark travel time. Moreover, a robust benchmark travel time is defined as the maxima of modes among links with a similar length. The standardized travel time (STT) is derived from the travel time dataset. It is found that the distribution of STT does not change over time-of-day or in different links. According to these characteristics, the distribution of STT is further explored. Through Kolmogorov–Smirnov test, the Inv. Gaussian distribution is found to be the best distribution model for STT. Finally, both robust benchmark travel time and features of STT are incorporated into the utility model.

In the case studies, the proposed utility function is used to model and compare the travel time performance in different road links and different routes. This provides decision support for choosing EVs' response routes. Case studies on temporal link utility show random fluctuations without a clear time-of-day pattern, which coincides with the previous observations of EV travel time standard deviation. Both link level and route level studies show that the same length of route or link does not indicate the same travel time performance. To measure the link travel time reliability performance, a unit-distance utility is derived by dividing the link utility by link length. A map with the reliability of all links is provided in the entire network. One can see that the established travel time utility function for EV in this paper may help with network modeling and operations that require the consideration of travel time uncertainty.

Several extensions are worth noting for future research. This study models EV travel time performance using historical records. On this basis, there is a pressing need to explore the causes for unreliable travel times and their influential factors. One can further examine the probability of unreliable road links over a certain period under certain road conditions, and thus provide useful routing information for EV operators. It is also a good topic to predict the EV travel time, especially in a jammed road network. The prediction should take into consideration not only the link traffic condition but also the capacity of the approaching intersections and the emergency vehicle preemption. An accurate prediction of travel time is a crucial component for the emergency service system. Another possible extension is to model the long tails of a travel time distribution and devise strategies to handle these extreme cases.

## Appendix A

The derivation process from Eqs. (5) to (9) shows as follows:

Given Eq. (5),

$$E(U_D) = L \cdot \left[ B + \beta \int_{\frac{B_1 - \mu_D}{\sigma}}^{\frac{B_2 - \mu_D}{\sigma}} (\mu_D + X\sigma_D - B)f(X)dX + \gamma \int_{\frac{B_1 - \mu_D}{\sigma}}^{\frac{B_2 - \mu_D}{\sigma}} (\mu_D + X\sigma_D - B)f(X)dX \right]$$

Based on the approximation in Section 4.2, we replace  $B$  as  $\bar{B}$  to handle links with little historical data.

$$E(U_D) = L \cdot \left[ \bar{B} + \beta \int_{\frac{B_1 - \mu_D}{\sigma}}^{\frac{B_2 - \mu_D}{\sigma}} (\mu_D + X\sigma_D - \bar{B})f(X)dX + \gamma \int_{\frac{B_1 - \mu_D}{\sigma}}^{\frac{B_2 - \mu_D}{\sigma}} (\mu_D + X\sigma_D - \bar{B})f(X)dX \right]$$

$$E(U_D) = L \cdot \left[ \bar{B} + \beta \int_{\frac{B_1 - \mu_D}{\sigma}}^{\frac{B_2 - \mu_D}{\sigma}} (\mu_D - \bar{B})f(X)dX + \gamma \int_{\frac{B_1 - \mu_D}{\sigma}}^{\frac{B_2 - \mu_D}{\sigma}} (\mu_D - \bar{B})f(X)dX + \beta \int_{\frac{B_1 - \mu_D}{\sigma}}^{\frac{B_2 - \mu_D}{\sigma}} X\sigma_D f(X)dX + \gamma \int_{\frac{B_1 - \mu_D}{\sigma}}^{\frac{B_2 - \mu_D}{\sigma}} X\sigma_D f(X)dX \right]$$

$$E(U_D) = L \cdot \left[ \bar{B} + \beta(\mu_D - \bar{B})F(X)\Big|_{\frac{B_1 - \mu_D}{\sigma}}^{\frac{B_2 - \mu_D}{\sigma}} + \gamma(\mu_D - \bar{B})F(X)\Big|_{\frac{B_1 - \mu_D}{\sigma}}^{\frac{B_2 - \mu_D}{\sigma}} + \beta\sigma_D \int_{\frac{B_1 - \mu_D}{\sigma}}^{\frac{B_2 - \mu_D}{\sigma}} Xf(X)dX + \gamma\sigma_D \int_{\frac{B_1 - \mu_D}{\sigma}}^{\frac{B_2 - \mu_D}{\sigma}} Xf(X)dX \right]$$

$$E(U_D) = L \cdot \left[ \bar{B} + \beta(\mu_D - \bar{B})F(X)\Big|_{\frac{B_1 - \mu_D}{\sigma}}^{\frac{B_2 - \mu_D}{\sigma}} + \gamma(\mu_D - \bar{B})F(X)\Big|_{\frac{B_1 - \mu_D}{\sigma}}^{\frac{B_2 - \mu_D}{\sigma}} + \beta\sigma_D \int_{F\left(\frac{B_1 - \mu_D}{\sigma}\right)}^{F\left(\frac{B_2 - \mu_D}{\sigma}\right)} F^{-1}(X)dX + \gamma\sigma_D \int_{F\left(\frac{B_1 - \mu_D}{\sigma}\right)}^{F\left(\frac{B_2 - \mu_D}{\sigma}\right)} F^{-1}(X)dX \right].$$

## References

- Abdel-Aty, M.A., Kitamura, R., Jovanis, P.P., 1995. Investigating effect of travel time variability on route choice using repeated-measurement stated preference data. *Transp. Res. Rec.* (1493), 39–45.
- Asakura, Y., Kashiwadani, M., 1991. Road network reliability caused by daily fluctuation of traffic flow. Paper Presented to PTRC Summer Annual Meeting, 19th, 1991, University of Sussex, United Kingdom.
- Asensio, J., Matas, A., 2008. Commuters' valuation of travel time variability. *Transp. Res. Part E: Logist. Transp. Rev.* 44 (6), 1074–1085.
- Bates, J., Polak, J., Jones, P., Cook, A., 2001. The valuation of reliability for personal travel. *Transp. Res. Part E: Logist. Transp. Rev.* 37 (2), 191–229.
- Bogers, E., Van Zuylen, H., 2004. The importance of reliability in route choices in freight transport for various actors on various levels. Paper Presented to Proceedings of the European Transport Conference.
- Cervero, R., 1994. Rail transit and joint development: land market impacts in Washington, DC and Atlanta. *J. Am. Plan. Assoc.* 60 (1), 83–94.
- Chien, S.I.J., Kuchipudi, C.M., 2003. Dynamic travel time prediction with real-time and historic data. *J. Transp. Eng. – ASCE* 129 (6), 608–616.
- Emam, E.B., Ai-Deek, H., 2006. Using real-life dual-loop detector data to develop new methodology for estimating freeway travel time reliability. *Transp. Res. Rec.: J. Transp. Res. Board* 1959 (1), 140–150.
- Fosgerau, M., Karlström, A., 2010. The value of reliability. *Transp. Res. Part B: Methodol.* 44 (1), 38–49.
- Gifford, J., Pelletiere, D., Collura, J., 2001. Stakeholder requirements for traffic signal preemption and priority in Washington, DC, region. *Transp. Res. Rec.: J. Transp. Res. Board* (1748), 1–7.
- Gkritza, K., 2003. Analysis of the characteristics of emergency vehicle operations in the Washington DC Region.
- He, Q., Head, K.L., Ding, J., 2012. PAMSCOD: platoon-based arterial multi-modal signal control with online data. *Transp. Res. Part C: Emerg. Technol.* 20 (1), 164–184.
- He, Q., Head, K.L., Ding, J., 2014. Multi-modal traffic signal control with priority, signal actuation and coordination. *Transp. Res. Part C: Emerg. Technol.* 46, 65–82.
- Henchey, M.J., Batta, R., Blatt, A., Flanigan, M., Majka, K., 2014. A study of situationally aware routing for emergency responders. *J. Oper. Res. Soc.*
- Herman, R., Lam, T., 1974. Trip time characteristics of journeys to and from work. *Transp. Traffic Theory* 6, 57–86.
- Lin, W.-H., Kulkarni, A., Mirchandani, P., 2004. Short-term arterial travel time prediction for advanced traveler information systems. Paper Presented to Intelligent Transportation Systems.
- Lomax, T., Schrank, D., Turner, S., Margiotta, R., 2003. Selecting Travel Reliability Measures. Texas Transportation Institute Monograph (May 2003).
- McHale, G.M., Collura, J., 2003. Improving emergency vehicle traffic signal priority system assessment methodologies. Paper Presented to Transportation Research Board 82nd Annual Meeting.
- Nelson, E.J., Bullock, D., Trb, T.R.B., 2000. Impact of Emergency Vehicle Preemption on Signalized Corridor Operation – An Evaluation. Advanced Traffic Management Systems and Automated Highway Systems 2000: Highway Operations, Capacity, and Traffic Control, no. 1727, pp. 1–11.
- Obenberger, J., Collura, J., 2001. Transition strategies to exit preemption control: state-of-the-practice assessment. *Transp. Res. Rec.: J. Transp. Res. Board* (1748), 72–79.
- Park, B., Yun, I., Best, M., 2008. Evaluation of pre-emption and transition strategies for Northern Virginia smart traffic signal systems (NVSTSS).
- Polus, A., 1979. A study of travel time and reliability on arterial routes. *Transportation* 8 (2), 141–151.
- Richardson, A., Taylor, M., 1978. 'Travel time variability on commuter journeys'. *High Speed Ground Transp. J.* 12 (1).
- Senna, L., 1994. The influence of travel-time variability on the value of time. *Transportation* 21 (2), 203–228.
- Skabardonis, A., Geroliminis, N., 2005. Real-time estimation of travel times on signalized arterials.
- Small, Kenneth A., Winston, Clifford, Yan, Jia, 2005. Uncovering the distribution of motorists' preferences for travel time and reliability. *Econometrica* 73 (4), 1367–1382.
- Susilawati, S., Taylor, M.A.P., Somenahalli, S.V.C., 2013. Distributions of travel time variability on urban roads. *J. Adv. Transp.* 47 (8), 720–736.
- Taylor, M., 1982. Travel time variability – the case of two public modes. *Transp. Sci.* 16 (4), 507–521.
- Taylor, M.A.P., Susilawati, 2012. Modelling travel time reliability with the Burr distribution. In: Proceedings of Ewgt 2012 – 15th Meeting of the Euro Working Group on Transportation, vol. 54, pp. 75–83.

- Tu, H., Li, H., van Lint, H., van Zuylen, H., 2012. Modeling travel time reliability of freeways using risk assessment techniques. *Transp. Res. Part A: Policy Pract.* 46 (10), 1528–1540.
- van Lint, J.W.C., Hoogendoorn, S.P., van Zuylen, H.J., 2005. Accurate freeway travel time prediction with state-space neural networks under missing data. *Transp. Res. Part C: Emerg. Technol.* 13 (5–6), 347–369.
- Westgate, B.S., Woodard, D.B., Matteson, D.S., Henderson, S.G., 2013. Travel time estimation for ambulances using Bayesian data augmentation. *Ann. Appl. Stat.* 7 (2), 1139–1161.
- Zolotarev, V.M., 1986. One-dimensional stable distributions. *Transp. Res. Part B: Methodol.* 65. American Mathematical Soc.