CSCE 515 Midterm Exam Prep

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1 Checklist

- To Do: Exercise #9, 17, 18
- To Expand/Fix: Exercise #13, 20
- Not Sure: #1, 3
- Review Hard: #10
- Done: #2, 4, 5, 6, 7, 8, 11, 12, 14, 15, 16

2 Topics to Solidify

- Lighting
- Perspective

Contents

3 Study Guide Questions

3.1 Question 1: Confused

1. Both raster and vector displays use a refresh buffer. What is the difference between the two in terms of refresh buffer contents?

3.1.1 First Punt, Brad Burkman, 11/17 7:41pm

I have no idea. I do not have a vector display refresh buffer in my notes, and I can't find it online.

Here's what I do know.

Raster displays have a bitmap of the color and intensity of each pixel. Their refresh buffer holds the next screen to be displayed. It may have two layers so that the swap between the displays doesn't happen in the middle of an update. If it's a CRT monitor, it paints the pixels in rows, not following the contours of each object. Raster displays have a constant refresh rate.

Vector displays, like an oscilloscope, move a laser or electron beam over the screen following the contours of each object, drawing it smoothly. It starts redrawing the screen after it has finished drawing the screen.

3.2 Question 2: Done

2. For the midpoint line algorithm, how were floating point operations converted to integer operations for the main loop? (Be as specific as possible.)

3.2.1 First guess, Brad Burkman, 11/14 7:44pm

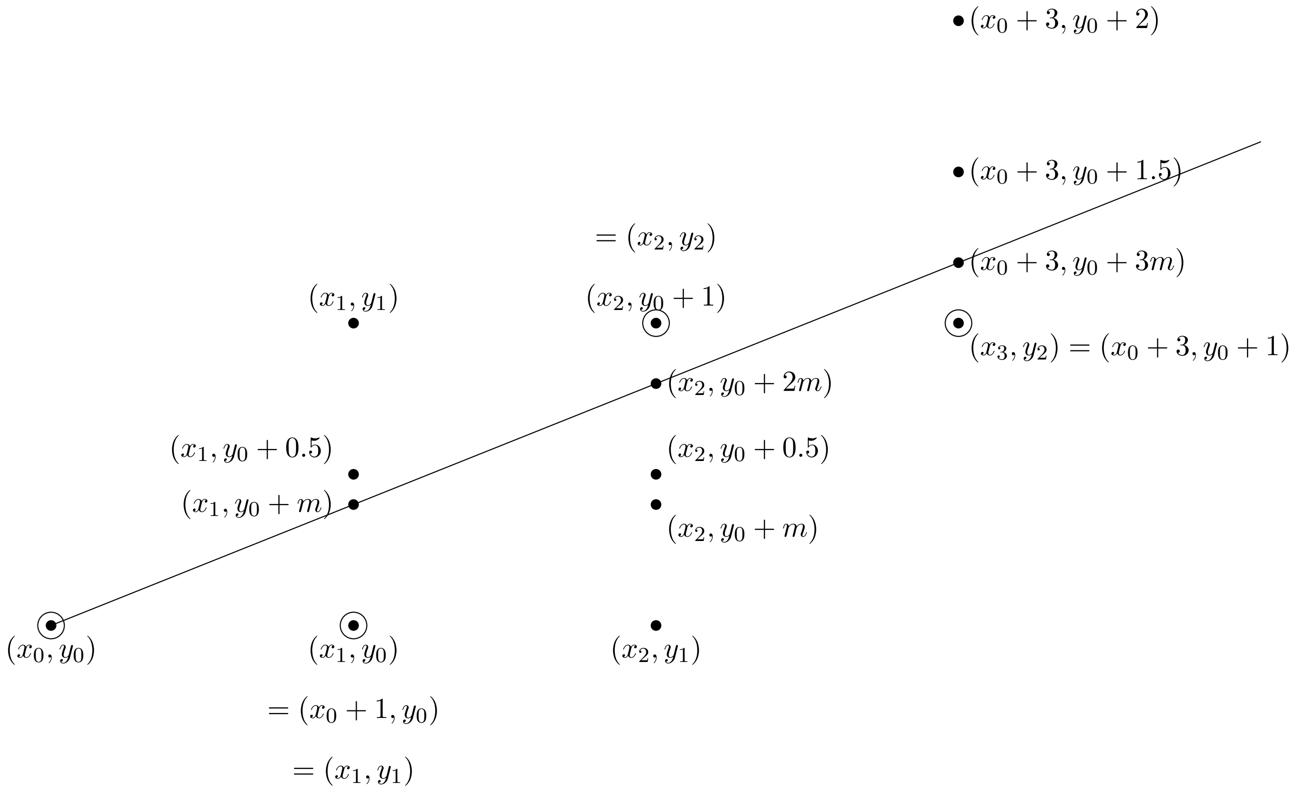
The floating point operations were multiplied by [something like] the number of pixels that each integer represented, then rounded, because we don't care about being between two pixels; we have to pick one.

3.2.2 Second Atttempt, informed by notes, Brad Burkman, 11/14 7:58pm

Since this sample question is about a small piece of the Besenham algorithm, I presume the actual exam question could be anything about it.

Looking at the notes from 28 August, here's how the midpoint (Besenham) algorithm works.

3.2.3 Midpoint Line (Besenham) Algorithm



Initialize d at $m - 0.5$.

Since in this case d is negative, choose to move E rather than NE.

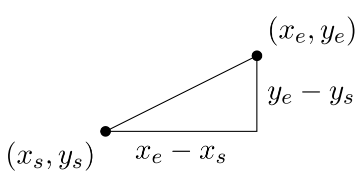
At the second step, because we moved E, $d = d + m$, so $d = 2m - 0.5$.

Since d is positive, move NE.

Now since we moved NE, $d = d + m - 1 = (2m - 0.5) + m - 1 = 3m - 1.5$

Since $d = 3m - 1.5$ is negative, move E.

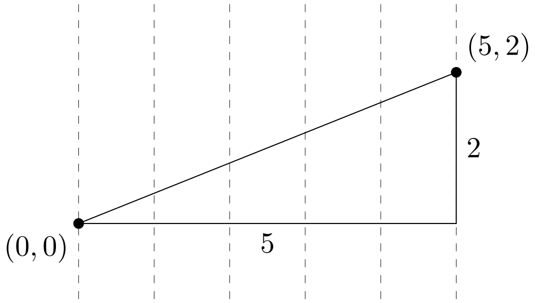
3.2.4 Making it Faster



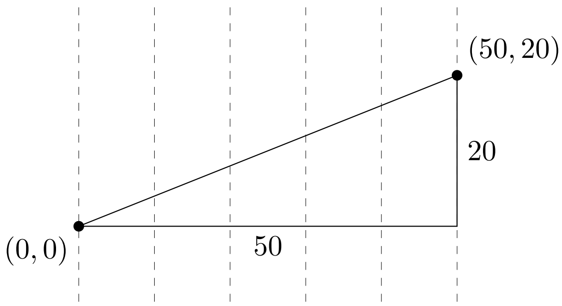
$$m = \frac{y_e - y_s}{x_e - x_s} = \frac{\Delta y}{\Delta x} \qquad d = m - 0.5$$

Scale everything by $2\Delta x$, so that $m = 2\Delta y$ and $d = 2\Delta y - \Delta x$.

Same example as above:



$$m = \frac{2}{5} = \frac{\Delta y}{\Delta x} \qquad d = m - 0.5$$



$$2m\Delta x = 4 = \Delta y$$

$$d = 4 - 5 = -1$$

Initialize d at $2\Delta y - \Delta x = 4 - 5 = -1$

Since in this case d is negative, choose to move E rather than NE.

At the second step, because we moved E, $d = d + 2\Delta y$, so $d = -1 + 4 = 3$.

Since d is positive, move NE.

Now since we moved NE, $d = d + 2\Delta y - 10 = 3 + 2(2) - 10 = -3$

Since $d = -3$ is negative, move E.

3.3 Question 3: Unsure

3. What is meant by each of the three terms geometry, topology, and attributes for a polygonal surface model?

3.3.1 Information from Class Notes, Brad Burkman, 17 November 10:11pm

- Vertex list gives the geometry, the shape.
- Index list gives the topology, the relationships between neighbors.
- Attributes are colors and normal vectors, perhaps the texture?

3.4 Question 4: Done

~~4. Give an example of an operation that would normally be faster with pointers-to-a-vertex-list mesh representation than with an explicit representation. Justify your answer.~~

Replace with:

Triangle Strips Question

3.4.1 What kind of question could it be?

- Primitive restart index
- Vertices 0,1,2 define first triangle, 1,2,3 define second.
- First triangle in the strip determines winding order. Default is counterclockwise for the exterior, clockwise for the interior.
- Given a rotated surface with np number of points to be rotated and nm steps of rotation, how many vertices, cells, triangles, triangles, triangle strips, elements in the vertex list, elements in the index list ... are there?

3.5 Question 5: Done

5. Answer true or false for each item below.
   For transforms as discussed in class:

- Any two rotations are commutative.

  False. See example below.
- Any two translations are commutative.

  True.
- The inverse of a rotation matrix is its transpose.

  True
- The inverse of a translation matrix is its transpose.

  False
- For any composite sequence of translations and rotations, there exists a single rotation and translation pair that would have the same net effect. [Dr. Borst says it's true.]

3.5.1 Rotation Matrices

What do we know about rotation matrices?

- Each row, and each column, is a unit vector, because we're not changing scale.
- The determinant of the matrix is 1.
- They are orthogonal; `i.e.` $R^{-1} = R^T$.

3.5.2 Z-rotation matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.5.3 Y-rotation matrix

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.5.4 X-rotation matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let's multiply two of them together and see whether it's commutative.

$$YX = \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\beta & \sin\alpha\sin\beta & \cos\alpha\sin\beta & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ -\sin\beta & \sin\alpha\cos\beta & \cos\alpha\cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$XY = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ \sin\alpha\sin\beta & \cos\alpha & -\sin\alpha\cos\beta & 0 \\ -\cos\alpha\sin\beta & \sin\alpha & \cos\alpha\cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.6 Question 6: Done

6. Give a 4×4 homogeneous transform (sixteen numbers) for describing an object's local coordinate system with respect to the world coordinate system such that:

- The object's local origin is at world coordinate $(x, y, z) = (4, 0, 2)$,
- The object's local x-axis direction matches the world's z-axis direction, and
- The object's local z-axis direction matches the world's $-x$ (negative x) direction,

3.6.1 Brad's answer, 18 November 6:44 am

A $-90°$ rotation in the Y gives the correct orientation.

Do the rotation, then the translation.

$$A = T_{(4,0,2)}Y_{-90} = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-90) & 0 & \sin(-90) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-90) & 0 & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 & 4 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.7 Question 7: Done

Give the inverse of the transform.

$$A^{-1} = \left(T_{(4,0,2)}Y_{-90}\right)^{-1} = \left(Y_{-90}\right)^{-1}\left(T_{(4,0,2)}\right)^{-1} = Y_{90}T_{(-4,0,-2)}$$

$$\begin{bmatrix} \cos(90) & 0 & \sin(90) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(90) & 0 & \cos(90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & -2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Check.

$$AA^{-1} = \begin{bmatrix} 0 & 0 & -1 & 4 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & -2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \surd$$

3.8 Question 8: Done

8. The diagrams below show a house model. Give $\hat{\mathbf{n}}$, the unit normal vector for the right side of the roof.

3.8.1 First guess for #8, Brad Burkman, 7:58pm, 11 November

$$\hat{\mathbf{n}} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}$$

3.8.2 Dr. Borst's Discussion

Create a coordinate system, make a triangle, and use the cross product to find the normal.

Front left top corner of the house at $A(1, 1, 1)$,
Back left top corner of the house at $B(1, 1, -1)$,
Front peak of house at $C\,(0, 2, 1)$
Vector $\vec{u}$ from A to B: $\vec{u} = (0, 0, -2)$
Vector $\vec{v}$ from A to C: $\vec{v} = (-1, 1, 0)$

$$\vec{u} \times \vec{v} = \mathbf{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & -2 \\ -1 & 1 & 0 \end{vmatrix} = 2\vec{i} + 2\vec{j} + 0\vec{k}$$

$$\hat{\mathbf{n}} = \frac{\mathbf{n}}{|\mathbf{n}|} = \frac{2\vec{i} + 2\vec{j} + 0\vec{k}}{2\sqrt{2}} = \frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j} + 0\vec{k} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}$$

3.9 Question 9: To Do

9. Recall the plane diagram above. Suppose plane orientation is stored as an $X - Z - Y$ fixed axis set (θ, ϕ, α), that is, three rotations about fixed world axes in the order X, Z, Y. In terms of the composite rotation resulting from these three components and for arbitrary plane orientation:

1) A small change in θ (X amount) will always cause rotation about:

local x_p local y_p local z_p world x world y world z NOTA

2) A small change in ϕ (Z amount) will always cause rotation about:

local x_p local y_p local z_p world x world y world z NOTA

3) A small change in α (Y amount) will always cause rotation about:

local x_p local y_p local z_p world x world y world z NOTA

4) If each component rotation is expressed as a matrix and then all three are composed into a single rotation matrix, the multiplication order for this representation would be:

$R_X R_Z R_Y$ $R_Y R_Z R_X$ $R_X R_Y R_Z$ $R_Z R_Y R_X$ NOTA

3.10 Question 10: Review

10. How could we convert an orientation expressed in a 3-angle set convention above to a single quaternion representing the same orientation? Give a concise but accurate conceptual description.

3.10.1 Dr. Borst's Answer

"Convert each of the three components into a quaternion, and multiply them." [Dr. Borst]

$X - Z - Y$ fixed axis set is multiplied as $R_Y R_Z R_X$.

3.11 Question 11: Done

11. The graph below represents the relationships between coordinate systems (objects, if you prefer) using notation from lecture. Suppose $\{D\}$ is the camera's local frame in an OpenGL application. When rendering an object having vertices described in frame $\{E\}$, what would the value of the modelview matrix be?

3.11.1 First Guess, Brad Burkman, 8:26pm 14 November

$$ {}^{D}_{C}T \cdot {}^{C}_{A}T \cdot {}^{A}_{B}T \cdot {}^{B}_{E}T = \left({}^{C}_{D}T\right)^{-1} \cdot \left({}^{A}_{C}T\right)^{-1} \cdot {}^{A}_{B}T \cdot {}^{B}_{E}T $$

3.12 Question 12: Done

12. Suppose, for the above diagram, we want to rotate frame $\{B\}$ by a rotation transform R that is to act as a rotation described with respect to the current $\{B\}$ frame. How exactly should R be applied to update one of the transforms in the graph?

3.12.1 Dr. Borst's Answer

${}^{A}_{B}T$ gets modified. Multiply by R on the right to be a local change to B.

3.13 Question 13: Expand and Review

13. Give any one of the perspective projection matrices, M_{per} from the lecture slides.

3.13.1 Orthographic Projection from Simple Example

$$M_{ort} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.13.2 Extended Orthographic Projection Matrix

$$M_{ort,norm} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \frac{2}{f-n} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -\frac{l+r}{2} \\ 0 & 1 & 0 & -\frac{b+t}{2} \\ 0 & 0 & 1 & \frac{n+f}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.13.3 Perspective Matrix

Basic perspective matrix.

$$\begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix} = M_{per} \cdot p = \begin{bmatrix} -n & 0 & 0 & 0 \\ 0 & -n & 0 & 0 \\ 0 & 0 & -n & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} -nx \\ -ny \\ -nz \\ z \end{bmatrix} = \begin{bmatrix} -nx/z & \\ -ny/z & \\ -n & 1 \end{bmatrix} = \begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix}$$

Generalized perspective matrix

$$M_{per} = \begin{bmatrix} -n & 0 & 0 & 0 \\ 0 & -n & 0 & 0 \\ 0 & 0 & -(n+f) & -nf \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Preserves some depth info (z coordinate)

Z-coordinate is unchanged at near and far boundaries

Transforms the frustum into an orthographic view volume.

3.14 Question 14: Done

14. What is meant by a normalizing transform and what two substeps can be used to build the normalizing transform for the orthographic projection case described in lecture? A good conceptual description is sufficient; exact transforms are not requested.

3.14.1 Dr. Borst's Answer

Normalizing transform takes a box in the view and transforms it to $[-1,1] \times [-1,1] \times [-1,1]$. Transform the center to the origin and scale.

3.15 Question 15: Done

15. In the OpenGL pipeline, after the modelview matrix is applied to a vertex, the result is that the vertex is described with respect to which coordinate system?

ModelView Matrix is the composition of all of the transformations from the camera to the node being rendered.

Answer: Camera View

3.16 Question 16: Done

16. The six common parameters (near, far, left, ...) of a perspective frustum description are coordinates describing the viewing volume with respect to which coordinate system?

Answer: Camera View

3.16.1 First Guess, Brad Burkman, 9:03pm 14 November

Eye (camera) coordinate system.

3.17 Question 17: To Do

17. Below, circle all correct answers for questions about the illumination equation from lecture.

1) Which of the following vectors is (are) used in computing the ambient lighting term?

   Direction to light   Surface normal   Direction to viewer   None

2) Which of the following vectors is (are) used in computing the diffuse lighting term?

   Direction to light   Surface normal   Direction to viewer   None

3) Which of the following vectors is (are) used in computing the specular lighting term?

   Direction to light   Surface normal   Direction to viewer   None

3.18 Question 18: To Do

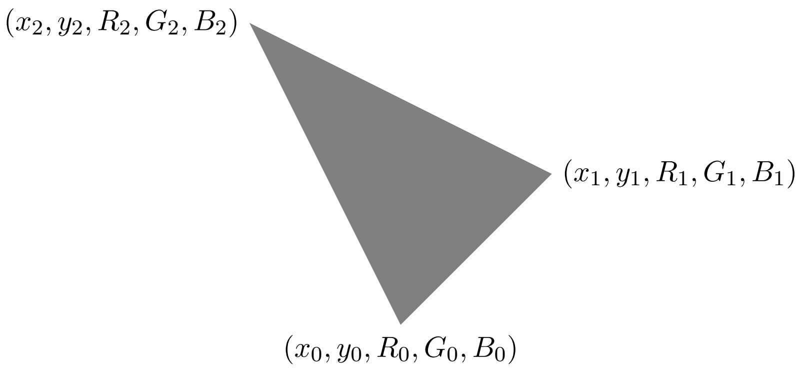
18. Describe why a triangle may look different with Phong shading than with Gouraud shading, and include a sketch of shaded triangles that illustrate the difference you discuss.

3.19 Question 19: Not on Exam

~~19. Sketch a scene with two or three polygons for which the painter's algorithm would be insufficient for visible surface determination.~~

3.20 Question 20: To Fix

20. Consider scan conversion of the illustrated triangle. Assume that scan conversion is done from the bottom up, that the triangle crosses several scan lines, and that the geometry matches the illustration.



1) In terms of the vertex coordinates and colors labeled on the diagram, give the coordinates and RGB color values for the first pixel to be colored.

2) In terms of the vertex coordinates and colors labeled on the diagram, give the coordinates and RGB values for the second pixel to be colored.

3.20.1 First Guess, Brad Burkman, 9:23pm 14 November

First pixel to be colored is the bottom.

$$(x_0, y_0, R_0, G_0, B_0)$$

Second pixel to be colored is above left of that.

$$m = \frac{y_2 - y_0}{x_2 - x_0} = \frac{\Delta y}{\Delta x}$$

Since we're going up one pixel, $\Delta y = 1$, so we're interested in Δx.

$$\Delta x = \frac{x_2 - x_0}{y_2 - y_0}$$

and we're going to round it down, because we're talking about pixel values, which are integers.

$$\Delta x = \left\lfloor \frac{x_2 - x_0}{y_2 - y_0} \right\rfloor$$

Change in colors works the same way.

$$\Delta R = \frac{R_2 - R_0}{y_2 - y_0}$$

So the coordinates and colors of the second point to be colored are:

$$\left(x_0 + \left\lfloor \frac{x_2 - x_0}{y_2 - y_0} \right\rfloor, y_0 + 1, R_0 + \frac{R_2 - R_0}{y_2 - y_0}, G_0 + \frac{G_2 - G_0}{y_2 - y_0}, B_0 + \frac{B_2 - B_0}{y_2 - y_0}\right)$$

3.20.2 Dr. Borst's Discussion

I realize I had forgotten about the offset between the line and the pixel.