PageRank

Ben Burns, Dan Magazu, Lucas Chagas,
Thomas Webster, Trung Do

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Motivation

- Problem: the internet has a *lot* of web pages
- A lot of the information out there either isn't relevant to us, or is inaccurate
- Motivation: we want a program that, when provided a phrase, returns webpages with information relevant to the input
- Intuitive solution: return back websites that either contain that phrase, or contain similar phrases
- This helps us find more *relevant* pages, but we can't know if we're getting the best information (much less *accurate* information) without manually going through each result
- We desire a stronger solution

PageRank

- Invented by Sergey Brin and Larry Page (1998)[1]
  - Publication marks them becoming co-founders of Google
- Idea: we want some way to numerically score each webpage based on how "important" it is
- Algorithm numerically scores each page p based on
  - How many other pages link to p (or "cite" it)
  - The "importance" of each of p's citations
- We then numerically order pages to rank them
- PageRank: the procedure for scoring each website
- Google: the database that indexes the PageRank of each website for search

[1] Many use the year of the original manuscript, 1996

Underlying Assumption

- Running our basic search engine gives us a collection of pages with information relevant to our query
- Assumption: More "important" and useful websites will be the ones with proportionally more inbound links
- Pages with very reliable, primary information are likely to be cited by lots of website authors, and therefore will have lots of "flow" into them

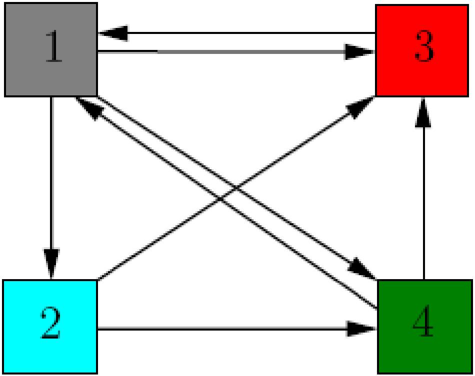
Finding Less Important Information

- A worry you may have is that we'll only find pages with lots of inbound links
- You can still find niche information by making your query more specific so that it won't match more general pages
- Searching "PageRank" will likely get you Wikipedia, but "Anatomy of PageRank architecture" gives you the original research literature.
- Your returned urls are still proportionally important results, your query just filtered out the numerically more "important", yet less relevant pages.

Formalizing the PageRank problem

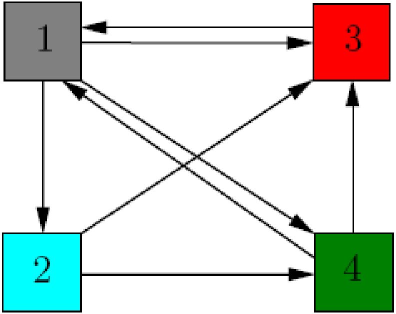
- We're going to construct a directed graph $G = (V, E)$
- For each website we consider, we construct a node $v_i \in V$
- For two distinct nodes $v_i,\ v_j \in V$, the *directed* edge $v_i v_j \in E$ iff there is a link on website i that goes to website j.
- If v_i and v_j are not distinct (a website is linking to itself), we ignore the link and do not construct a loop edge.
  - G is not a psuedograph
- Multiple hyperlinks on page i to page j are all represented by the single, directed edge
  - G is not a multigraph.

Visual Representation



- We have a set of 4 websites
- Each edge represents a hyperlink from the origin node to the destinationx node

Adjacency Matrix



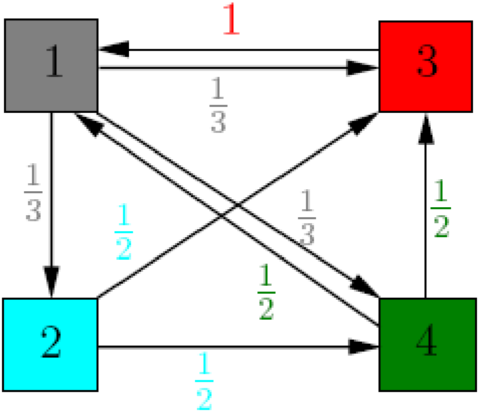
$$A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

- No self loops means the main diagonal is all zeros

Applying PageRank Values and Edge Weights

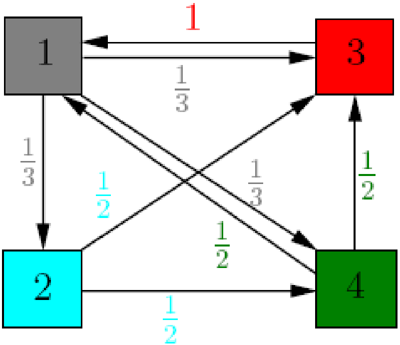
- At first, we assign each vertex with a weight of $\frac{1}{|V|}$
  - All vertices have equal weight, and our weights sum up to 1
- Consider the set of vertices that v_i has an edge to
  - $V_i = \{v_j | v_i v_j \in E\}$
- For vertex 1, $V_1 = \{2, 3, 4\}$. In other words, a user on page 1 can choose to click a link to traverse to either 2, 3, or 4.
-

Visual Representation



- In this case, all nodes have a PageRank value of 1.

Transition Matrix



$$T = \begin{pmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 1/2 & 1/2 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \end{pmatrix}$$

- All entries of the transition matrix are non-negative
- If v_i has at least one outgoing edge, the sum of the entries in row i is 1
  - Else, the sum is 0.
- This is a *row stochastic matrix*