# Research and implementation of multi-dataset training for image classification with discrepant taxonomies

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# Abstract

Scientific documents often use LATEX for typesetting. While numerous packages and templates exist, it makes sense to create a new one. Just because.

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# I INTRODUCTION

In which the reasons for creating this package are laid bare for the whole world to see and we encounter some usage guidelines.

This package contains a minimal, modern template for writing your thesis. While originally meant to be used for a Ph. D. thesis, you can equally well use it for your honour thesis, bachelor thesis, and so on—some adjustments may be necessary, though.

# I.I WHY?

I was not satisfied with the available templates for LaTeX and wanted to heed the style advice given by people such as Robert Bringhurst or Edward R. Tufte. While there *are* some packages out there that attempt to emulate these styles, I found them to be either too bloated, too playful, or too constraining. This template attempts to produce a beautiful look without having to resort to any sort of hacks. I hope you like it.

# I.2 How?

The package tries to be easy to use. If you are satisfied with the default settings, just add

\documentclass{mimosis}

at the beginning of your document. This is sufficient to use the class. It is possible to build your document using either LETEX, XELETEX, or LuaLETEX. I personally prefer one of the latter two because they make it easier to select proper fonts.

# 1.3 Making this template yours

Prior to using this template, the first thing you want to do is probably a little bit of customisation. You can achieve quick changes in look and feel by picking your own fonts. With the fontspec package loaded and XŢEYEXor LuaEYTEXas your compiler, this is pretty simple:

```
\setmainfont{Your main font}
\setsansfont{Your sans-serif font}
\setmonofont{Your monospaced font}
```

Make sure to select nice combinations of that are pleasing to *your* eyes—this is your document and it should reflect your own style. Make sure to specify font names as they are provided by your system. For instance, you might want to use the following combination:

```
\setmainfont{Libre Baskerville}
\setsansfont[Scale=MatchLowercase]{IBM Plex Sans}
\setmonofont[Scale=MatchLowercase]{IBM Plex Mono}
```

If these fonts exist on your system, your normal text will look a little bit different from the other font used in this example PDF, while your sans-serif font will pair nicely with your monospaced font. You can also remove the Scale directive, but I find that most fonts pair better if they are adjusted in size a little bit. Experiment with it until you finds a combination that you enjoy.

XILTEX and Lual ITEX also offer you a way to change the font that is used for mathematical equations. If installed, the garamond-math package permits you to choose from different stylistic sets that slightly change how certain mathematical symbols look. For instance, the following command changes 'Fraktur' symbols:

```
\setmathfont{Garamond-Math.otf}[StylisticSet={6}]
```

# I.4 FEATURES

The template automatically imports numerous convenience packages that aid in your typesetting process. Table 1.1 lists the most important ones. Let's briefly discuss some examples below. Please refer to the source code for more demonstrations.

# I.4.I Typesetting mathematics

This template uses amsmath and amssymb, which are the de-facto standard for typesetting mathematics. Use numbered equations using the equation environment. If you want to show multiple equations and align them, use the align environment:

$$V \coloneqq \{1, 2, \dots\} \tag{1.1}$$

$$E := \{(u, v) \mid \operatorname{dist}(p_u, p_v) \le \epsilon\}$$
(1.2)

| Package          | Purpose  |
|------------------|--|
| amsmath          | Basic mathematical typography                          |
| amsthm           | Basic mathematical environments for proofs etc.        |
| babel            | Language settings                                      |
| booktabs         | Typographically light rules for tables                 |
| bookmarks        | Bookmarks in the resulting PDF                         |
| csquotes         | Language-specific quotation marks                      |
| dsfont           | Double-stroke font for mathematical concepts           |
| graphicx         | Graphics   |
| hyperref         | Hyperlinks   |
| multirow         | Permits table content to span multiple rows or columns |
| paralist         | Paragraph ('in-line') lists and compact enumerations   |
| scrlayer-scrpage | Page headings  |
| setspace         | Line spacing   |
| siunitx          | Proper typesetting of units                            |
| subcaption       | Proper sub-captions for figures                        |

Table 1.1: A list of the most relevant packages required (and automatically imported) by this template.

Define new mathematical operators using \DeclareMathOperator. Some operators are already pre-defined by the template, such as the distance between two objects. Please see the template for some examples. Moreover, this template contains a correct differential operator. Use \diff to typeset the differential of integrals:

$$f(u) := \int_{v \in \mathbb{D}} \operatorname{dist}(u, v) \, \mathrm{d}v \tag{1.3}$$

You can see that, as a courtesy towards most mathematicians, this template gives you the possibility to refer to the real numbers  $\mathbb R$  and the domain  $\mathbb D$  of some function. Take a look at the source for more examples. By the way, the template comes with spacing fixes for the automated placement of brackets.

# I.4.2 Typesetting text

Along with the standard environments, this template offers paralist for lists within paragraphs. Here's a quick example: The American constitution speaks, among others, of (i) life (ii) liberty (iii) the pursuit of happiness. These should be added in equal measure to your own conduct. To typeset units correctly, use the siunitx package. For example, you might want to restrict your daily intake of liberty to 750 mg.

Likewise, as a small pet peeve of mine, I offer specific operators for *ordinals*. Use  $\t$ th to typeset things like July  $4^{th}$  correctly. Or, if you are referring to the  $2^{nd}$  edition of a book, please use  $\t$ nd. Likewise, if you came in  $3^{rd}$  in a marathon, use  $\t$ rd. This is my  $1^{st}$  rule.

If you want to write a text in German and use German hyphenation rules, set the language of your text to german using \selectlanguage{ngerman}, or add

\PassOptionsToPackage{spanish}{babel}

before the \documentclass command to load a specific language. The languages ngerman, french, and english are loaded by default, with english being selected.

Quotation marks can be typeset using the \enquote{...} command from the csquotes package, which is preloaded by latex-mimosis. Depending on the currently selected language, quotes will look like "this", "this", or « this ». One must never use "ASCII" quotation marks or even 'apostrophe' symbols.

# **I.5 CHANGING THINGS**

Since this class heavily relies on the scrbook class, you can use *their* styling commands in order to change the look of things. For example, if you want to change the text in sections to **bold** you can just use

\setkomafont{sectioning}{\normalfont\bfseries}

at the end of the document preamble—you don't have to modify the class file for this. Please consult the source code for more information.

# 2.I Universal Taxonomy

Our main goal is to create a universal taxonomy that connects multiple image classification datasets. This taxonomy maps every dataset class to a universal class, which allows us to analyse the relationships and shared concepts between datasets.

In the end, our taxonomy will allow us to train models that can classify images from multiple datasets at once, building a robust and flexible system that can quickly adapt to new domains.

## 2.I.I FORMAL DEFINITIONS

To formalise our algorithm for building a universal taxonomy, we first need to define some terms:

- **Dataset** D: A collection of images and labels written as  $D = \{(x_1, c_1), (x_2, c_2), ..., (x_n, c_n)\}$ , where  $x_i$  is an image and  $c_i$  is its label. Since we are dealing with multiple datasets, we number them as  $D_i = \{(x_1^i, c_1^i), (x_2^i, c_2^i), ..., (x_n^i, c_n^i)\}$ , where  $D_i$  is the dataset D with index i. In the same way, we denote the set of all classes in a dataset as  $C_i = \{c_1^i, c_2^i, ..., c_k^i\}$ .
- **Model** m: A neural network trained on a dataset  $D_I$  which maps an image  $x \in X$  to a class  $c_i^I \in C_I$ , denoted as  $m_I : X \mapsto C_I$ .
- Domain: Since both models and classes are dataset-specific, we define the term domain as
  the dataset D<sub>i</sub> and its classes C<sub>i</sub> that we are working with.
- Universal Classes: Our universal taxonomy will contain a set of classes that are not specific to any dataset. We denote these classes as  $C_U = \{c_1^U, c_2^U, ..., c_k^U\}$ . A universal class is a concept represented by a set of domain classes that share similar characteristics. We therefore define a function classes :  $C_U \mapsto \mathcal{P}(C)$ , where  $\mathcal{P}(C)$  is the power set of C, to represent the set of domain classes that belong to a universal class.
- **Graph**: We represent our taxonomy as a directed graph G = (V, E), where V is a set of vertices and E is a set of edges. Each vertex  $v_i$  represents a single class or universal class, which we define with class :  $V \mapsto C$ . Every edge  $e_{ij}$  between two vertices  $v_i$  and  $v_j$  indicates a relationship class  $(v_i) \to \text{class}(v_j)$ .

• **Probability**: Every edge  $e_{ij}$  has a probability associated with it, which indicates the likelihood of classifying an image from class class( $v_i$ ) as class class( $v_j$ ). We denote this as a function probability :  $E \mapsto [0, 1]$ .

## 2.I.2 GRAPH CONSTRUCTION

Before building our universal taxonomy, we need to construct our initial graph:

- 1. **Foreign predictions:** For each dataset with its corresponding model, we run the model on all images from all other datasets. This gives us a set of predictions  $P_{ab} = \{(x_i^a, c_j^b)\}$ , where  $x_i^a$  is an image from dataset  $D_a$  and  $c_i^b$  is the class predicted by model  $m_b$  for that image.
- 2. **Prediction probabilities:** We count the number of times each class  $c_i^a$  was predicted as a foreign-domain class  $c_j^b$ . We denote this count in a matrix  $M_{ab} \in \mathbb{N}^{+|C_a| \times |C_b|}$ , where  $M_{ab}(i,j)$  is the number of times class  $c_i^a$  was predicted as class  $c_j^b$ . We then divide each entry in the matrix by its row sum to get the probability of classifying an image from class  $c_i^a$  as class  $c_i^b$ :

$$P_{ab}(i,j) = \frac{M_{ab}(i,j)}{\sum_{b=1}^{|C_a|} M_{ab}(i,k)}$$

This gives us a matrix  $P_{ab} \in [0, 1]^{|C_a| \times |C_b|}$ , where  $P_{ab}(i, j)$  is the probability of classifying an image from class  $c_i^a$  as class  $c_j^b$ .

- 3. **Graph construction:** We now create a directed graph that represents the relationships between classes and datasets by iterating over every dataset  $D_a$  with every dataset  $D_b$  where  $a \neq b$ :
  - a) We collect the indices of the per-row maximum values in the matrix  $P_{ab}$ :

$$I = \left\{ \operatorname{argmax}_{j \in \{1, \dots, |C_b|\}} P_{ab}(i, j) \mid i \in \{1, \dots, |C_a|\} \right\}$$

- b) For every  $i \in \{1, ..., |C_a|\}$  where  $P_{ab}(i, I_i) > 0$ :
  - i. We create the vertices  $v_k$  and  $v_l$  for classes  $c_i^a$  and  $c_l^b$  respectively if they do not already exist and add them to the graph (otherwise we find the existing vertices for these classes as  $v_k$  and  $v_l$ ).
  - ii. We create an edge  $e_{kl}$  between the vertices  $v_k$  and  $v_l$  and add it to the graph.
  - iii. We define probability  $(e_{kl}) = P_{ab}(i, I_i)$ .

# 2.I.3 TAXONOMY GENERATION

After constructing our initial graph structure, we now need to transform it into a universal taxonomy that merges classes from different datasets into universal classes where they share similar concepts.

#### TAXONOMY BUILDING RULES

We transform our initial graph of domain-to-domain relationships into a universal taxonomy through a loop that applies a set of rules until no more changes can be made. The rules are checked from first to last, starting from the first again after a rule is applied. The rules are as follows:

1. **Isolated Node Rule:** For any domain class A that has no relationships (neither incoming nor outgoing edges), create a new universal class B and add the relationship  $A \to B$ . We also define the probability of the relationship's edge as 1 and the classes of the universal class as  $\{A\}$ .

This ensures that all domain classes without relationships (which can be created by later rules) are still represented in the universal taxonomy.

- 2. **Bidirectional Relationship Rule:** When two classes have bidirectional relationships (A  $\rightarrow$  B and B  $\rightarrow$  A), they likely represent the same concept. We resolve this by creating a new universal class C and adding relationships  $A \rightarrow C$  and  $B \rightarrow C$  to the graph. The probability of the new relationships is set to the average of the bidirectional relationships and the classes of the universal class will be the two classes that were merged (or, if the two classes are universal classes themselves, the union of their classes).
- 3. **Transitive Cycle Rule:** If we have relationships  $A \to B \to C$  where A and C are in the same domain, we have a problem since classes within a domain are disjoint, which means that one of the relationships must be incorrect. We solve this by removing the relationship with the lower probability, thus breaking the cycle.
- 4. Unilateral Relationship Rule: A uniliteral relationship A → B indicates that the concepts of class A are a subset of the concepts of class B. We therefore create two new universal classes:
  - Class C, which contains both classes A and B and has incoming relationships from both classes with the probability of the uniliteral relationship. This universal class represents the union of the two classes.

Class D, which contains only class B and has a relationship from class B with a probability 1. This universal class represents the concepts of class B that are not in class A.

# 2.2 Synthetic Taxonomy Generation

# 2.2.1 The Need for a Controlled Ground Truth

To evaluate our taxonomy generation methods, we need a reliable ground truth with known relationships between datasets. This presents a challenge, as most existing image classification datasets lack clear inter-dataset relationships:

- ImageNet [1, 6] uses WordNet's [2] hierarchical structure to organize classes. However, this strict hierarchy doesn't match our use case where we need to connect datasets with different class structures and partial overlaps.
- Open Images [5] contains approximately 9 million images with multiple labels per image generated by Google's Cloud Vision API<sup>1</sup>. This multi-label approach makes it difficult to determine a single class for each image, which is required for our evaluation. Additionally, since most labels were automatically generated, it doesn't provide the verified ground truth we need.
- iNaturalist [3] offers a detailed taxonomy of plant and animal species, but its domainspecific nature makes it unsuitable for developing a general-purpose evaluation framework.

## 2.2.2 Our Approach: Building Synthetic Datasets

Instead of relying on existing taxonomies, we developed a method to generate synthetic datasets with controlled relationships. Our approach:

- 1. Define a set of "atomic concepts" that serve as building blocks for classes
- 2. Create multiple domains by sampling these concepts to form classes
- 3. Calculate inter-domain relationships based on shared concepts

This method allows us to precisely control the taxonomy structure while creating realistic relationships between domains. To generate images for these synthetic classes, we can leverage existing datasets by treating each original class as an atomic concept.

<sup>1</sup>https://cloud.google.com/vision

#### 2.2.3 FORMAL DEFINITIONS

We define our synthetic taxonomy framework on top of the definitions from subsection 2.1.1.

- Atomic Concepts  $\mathcal{U} = \{1, 2, ..., n\}$ : A set of atomic concepts will be a universe of concepts that make up the basis for our synthetic class generation.
- Synthetic Class: A class  $c_j^i$  will contain a subset of the atomic concepts from our universe:  $c_j^i \subseteq \mathcal{U}$ . To maintain disjoint class definitions, we ensure that  $c_j^i \cap c_k^j = \emptyset$  for all  $j \neq k$ .

## 2.2.4 RANDOMIZED DOMAIN GENERATION

To create realistic domains, we use normal distributions to sample the number of classes and concepts per class. This allows us to generate domains with varying sizes and complexities, mimicking different real-world datasets.

#### PARAMETER SAMPLING

We sample the number of classes per domain and the number of concepts per class from truncated normal distributions to ensure realistic variation while maintaining control. Since normal distributions are unbounded, we use a truncated version:

$$f(x|\mu,\sigma,a,b) = \begin{cases} \frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{\sigma\left[\Phi\left(\frac{b-\mu}{\sigma}\right)-\Phi\left(\frac{a-\mu}{\sigma}\right)\right]} & \text{if } a \leq x \leq b\\ 0 & \text{otherwise} \end{cases}$$

Where:

- $\phi$  is the standard normal PDF
- $\Phi$  is the standard normal CDF
- *a* and *b* are lower and upper bounds

We implement this using SciPy's truncnorm module<sup>2</sup>, handling SciPy's standardization of bounds internally:

$$X \sim \text{TruncNorm}(\mu, \sigma^2, a, b)$$

<sup>&</sup>lt;sup>2</sup>https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.truncnorm.html

# Domain Generation Algorithm

To generate a domain  $C_i$ , we follow these steps:

1. **Sample set size:** Determine how many concepts l to use for the domain:

$$l \sim [\text{TruncNorm}(\mu_{\text{classes}}, \sigma_{\text{classes}}^2, 1, n)]$$

2. **Sample concept pool:** Randomly select l concepts from the universe  $\mathcal{U}$ :

 $P = \{a, b, c, ...\}$  where a, b, c, ... are sampled without replacement from  $\mathcal U$ 

- 3. Initialise domain:  $C_i = \{\}$
- 4. **Generate classes:** While concepts remain in the pool  $(P \neq \emptyset)$ :
  - a) Sample class size  $s_i$ :

$$s_i \sim [\text{TruncNorm}(\mu_{\text{class size}}, \sigma_{\text{class size}}^2, 1, |P|)]$$

- b) Form class  $c_i^i$  by selecting  $s_i$  concepts randomly from P
- c) Remove selected concepts:  $P = P \setminus c_i^i$
- d) Add class to domain:  $C_i = C_i \cup \{c_i^i\}$

This algorithm ensures that each concept is assigned to exactly one class within the domain, maintaining our disjointness constraint.

# 2.2.5 Modeling Cross-Domain Relationships

Once we've generated multiple domains, we need to model the relationships between them to create our ground truth.

## SIMULATING NEURAL NETWORK PREDICTIONS

Our taxonomy generation method assumes that neural network classifiers will predict related classes across domains with certain probabilities. To simulate this, we create "perfect" synthetic probabilities based on concept overlap.

# RELATIONSHIP CALCULATION

For any two domains  $C_A$  and  $C_B$ , we calculate the probability of classifying an instance of class  $c_i^A$  as class  $c_j^B$  using:

NaiveProbability
$$(i, j) = \frac{|c_i^A \cap c_j^B|}{|c_i^A|}$$

$$P_{i,j} = \text{NaiveProbability}(i, j) + \frac{1 - \text{NaiveProbability}(i, j)}{|C_B|}$$
(2.1)

Where:

- NaiveProbability(i, j) is the proportion of concepts in class  $c_i^A$  that also appear in class  $c_i^B$
- The second term distributes remaining probability mass evenly across all classes in domain
   C<sub>B</sub>, simulating the behavior of a neural network when encountering concepts it hasn't seen
   before

# A CONCRETE EXAMPLE

To illustrate this approach, consider two domains:

• Domain A: 
$$C_A = \{c_1^A = \{1, 2\}, c_2^A = \{3, 4\}\}$$

• Domain B: 
$$C_B = \{c_1^B = \{1, 2, 4\}, c_2^B = \{5, 6\}\}$$

For the relationship  $c_1^A \rightarrow c_1^B$ :

• NaiveProbability(1, 1) = 
$$\frac{|\{1,2\} \cap \{1,2,4\}|}{|\{1,2\}|} = \frac{2}{2} = 1$$

• 
$$P_{1,1} = 1 + \frac{1-1}{2} = 1$$

For the relationship  $c_2^A \rightarrow c_1^B$ :

• NaiveProbability(2, 1) = 
$$\frac{|\{3,4\} \cap \{1,2,4\}|}{|\{3,4\}|} = \frac{1}{2} = 0.5$$

• 
$$P_{2,1} = 0.5 + \frac{1 - 0.5}{2} = 0.5 + 0.25 = 0.75$$

This example shows how our framework captures partial relationships between classes and how it simulates a perfect neural network classifier's behavior. The resulting probability prediction matrix between two domains can then be used to build a graph of relationships between classes, which can then be turned into a universal taxonomy using the methods described in subsection 2.1.2.

# No-Prediction Classes

Some datasets have a special class that indicates that the model could not classify the image. For these "no-prediction" classes, we need to adapt the relationship probability calculation: Instead of distributing the remaining probability mass evenly across all classes, we simply ignore it and therefore only have the probability of the overlapping concepts.

While this doesn't change the relationship that is picked as the most common foreign prediction, it does change the probability of the relationship, which is important for thresholding later on.

# 2.3 Universal Taxonomy Difference Metrics

We created our synthetic datasets to have a known ground truth of relationships between classes. Now, we need to find metrics to compare the predicted relationships from our taxonomy generation methods with the ground truth calculated from the synthetic datasets.

## 2.3.1 Universal Class Graph

We build our metrics to compare a ground truth synthetic taxonomy with a predicted taxonomy that uses the same domain classes (i.e. the same nodes in the graph).

In a universal taxonomy graph, every universal class  $c^U$  can be understood as an edge between the domain classes that have incoming edges to it. Every universal class has either one or two incoming edges, which will always be from different domain classes. Therefore, we can represent a universal taxonomy as a new graph, where every domain class is a vertex and every universal class is an edge between two vertices (or a self-loop if it has only one incoming edge).

We define a concatenation of all domain classes in a taxonomy as  $C_X = \bigcup_{i=1}^n C_i$ . We can now build an adjacency matrix  $A_X \in [0,1]^{|C_X| \times |C_X|}$  for the new universal class graph. A connection between two classes  $c_i^X$  and  $c_j^X$  with  $i \neq j$  exists if there is a universal class  $c_k^U$  that has incoming edges from both classes. A self-loop exists if there is a universal class  $c_k^U$  that has an incoming edge from class  $c_i^X$  without an incoming edge from any other class. The value of the connection is the average weight of the two (or one) incoming edges to the universal class  $c_k^U$ .

# 2.3.2 Edge Difference Ratio

Our first metric is the edge difference ratio (EDR) between two graphs  $G_1$  and  $G_2$ . The metric measures the difference in edges between two graphs, normalised by the edge weight sum of the union of the two graphs' edges (i.e. the weight of all edges without double counting). This nor-

malisation limits the metric to a range of [0, 1], where 0 indicates that the two graphs are identical and 1 indicates that they have no edges in common.

For two adjacency matrices  $A_1$  and  $A_2$  of graphs  $G_1$  and  $G_2$ , we define the edge difference ratio as follows:

EDR
$$(G_1, G_2) = \frac{\sum_{i \le j} |A_1(i, j) - A_2(i, j)|}{\sum_{i \le j} \max(A_1(i, j), A_2(i, j))}$$
 (2.2)

This definition only considers the upper triangular portion (including the diagonal) of the adjacency matrices to avoid double-counting symmetric edges, while still properly accounting for self-loops.

Our EDR metric is similar to the Jaccard index [4] as well as the Tanimoto coefficient [7] when we consider the adjacency matrices as sets of edges. In contrast to these metrics, however, our EDR metric supports weighted edges, which allows us to respect the probabilities of relationships between classes.

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