Great Lakes Annual Precipitation, 1900-1986

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Abstract.

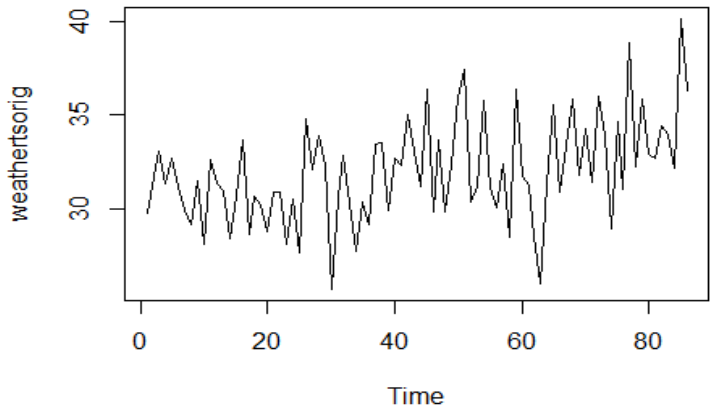
An analysis of the annual precipitation of rainfall in inches over the entire Great Lakes area, Michigan. The experiment uses the "Box-Jenkins Methodology" to fit an autoregressive integrated moving average model ARIMA(p,d,q) to the given time series, for the purpose of forecasting future levels of precipitation. The "Box-Jenkins Methodology" requires a stationary series that accounts for any seasonality present in the data. This approach identifies the best ARMA model(s) to select when constructing the model.

The "Great Lakes" data set is an example of a non-seasonal, non-stationary time series that experiences a slight upward linear trend. The series is differenced and transformed using "Box-Cox" in order to stabilize the mean and variance, correcting for stationarity. The best model fitted for the data was an ARIMA(4,1,0) found by observing the partial and auto correlation functions. The fit suggested the best estimates for the coefficients via the AIC. Verified as independent random variables, the residuals of the fitted model were tested for normality using the McLeod-Li, Ljung-Box, and Shapiro-Wilk test. The model proved to be an adequate representation of the data providing reasonable predictions for precipitation.

Introduction.

A study of the Annual rainfall in the surrounding Great Lakes area in Michigan. Data was collected from 1900-1986 and documented as a series of successive values courtesy of Prof. Rob Hyndman's Time Series Data Library. For the given series, I would like to capture a model to accurately predict the annual levels of precipitation and diagram the projected totals for the next 5 years. I am motivated in this particular set not only because it is a well documented area, but also to provide as a general starting point for studying variations in weather and changes in climate. I eventually want to be able to apply this skill to different aspects of weather, specifically forecasting natural disasters and other extreme systems.

Original Plot.



- **Trend:** There does seem to be a slight upward linear trend of the data. That seems to be potentially growing over time.
- **Seasonality**: Considering the data is recorded annually, and there is no apparent pattern visible throughout the data. we will conclude the data does not have a seasonal component, the plots being quite random in structure.
- **Behavior:** The overall senses of the data set seems to take a consistent extreme high/low behavior; but nothing that sticks out as inconsistent relative to the rest of the data set. Had the series been a bit more inconsistent with sharp behavior the "Box-Jenkins" approach may not not work here and the behavior could suggest just a "Random Walk" where the series is just completely random over time.

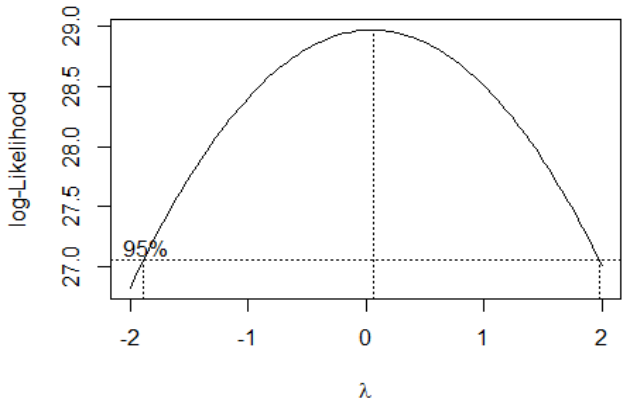
Box Cox Transformation.

Here we are interested in stabilizing variance and transforming the data to replicate a normal distribution. In order to do this we choose lambda that maximizes the likelihood for the data and apply the transformation to our series.

$$\lambda = \frac{X_t - 1}{\lambda}$$

$$\lambda = log(X_t)$$

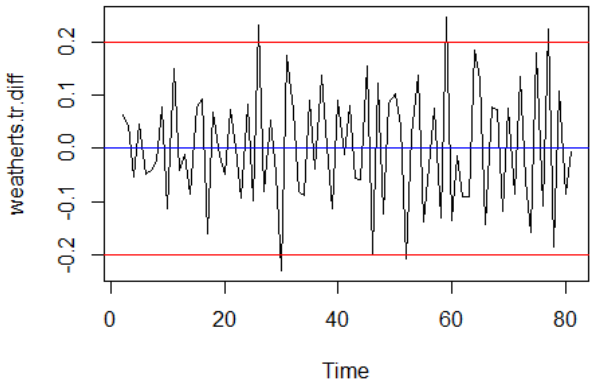
when $\lambda > 0$ or $lambda = 0$ respectively.



The lambda that will best transform our data is one that maximizes the likelihood. Notice in the graph above the lambda inside our 95% confidence interval that does is nearly zero. Thus we will use a log transformation to stabilize the variance within the series

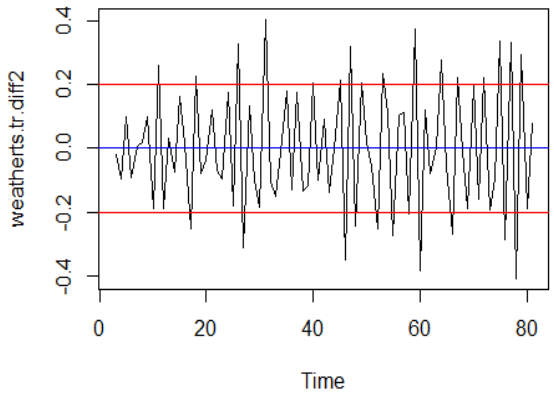
Differencing.

- **Removing Trend:** In order to remove trend we must difference the data by taking $X_t - X_{t-1}$. This removes the upward linear trend in the series and centers the mean around zero. Pictured in the graph above.



Over Differencing.

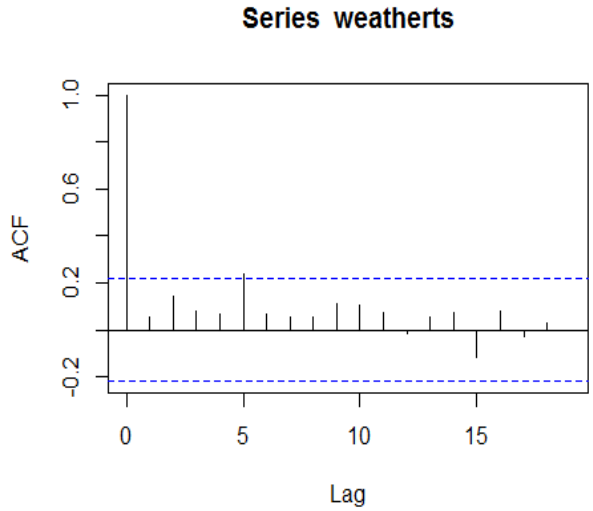
Sometimes it is necessary to difference a series twice to correct for trend within the data. This happens when the time series takes a quadratic form instead of a linear one. In the original plot of "Great Lakes" series the data seemed to take a possible increasing quadratic form.



- **Increasing Variance:** Notice at first glance, differencing the series twice the plot seemed to tighten up around zero for a seemingly better fit. Yet the problem lies with increasing variance. Comparing the approaches, differencing once had a Variance of 0.01258 , while differencing twice and a variance of 0.03896. The jump is a key sign of over differencing, and model represents more of a linear than quaratic form.

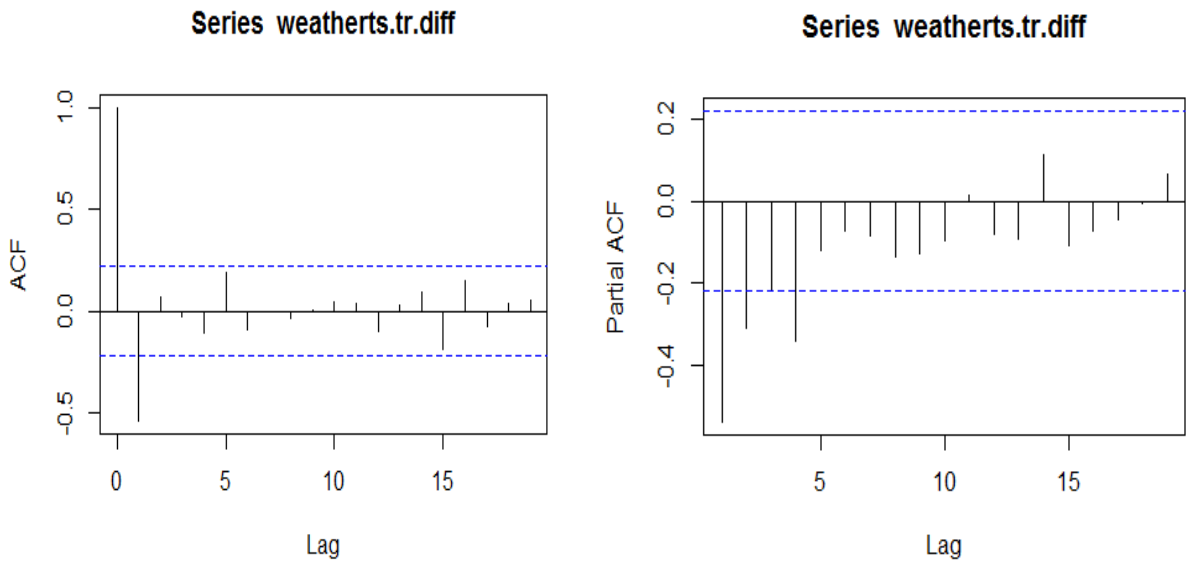
Identifying the model.

In order to identify and select the correct model we must look at the partial and auto correlation functions. Here is a look at the ACF and PACF of the original time series



The ACF of the original time series does not show much here, the series resembles a "random walk" which cannot be modeled or a possible Moving average MA(5). The original series is not stationary

Let's take at the ACF and PACF after transforming and differencing the series. After stabilizing the mean and variance, removing the trend in the data.



After transforming the data and removing the trend the ACF cuts off at after lag 1 and the PACF cuts off after lag 4. Since both the PACF and ACF experiences a quick drop off at specific lags this suggests a auto regressive moving average. specifically ARIMA(4,1,1). In order to test a variety of different models we will select a few other possibilities that could work.

- **Models** Arima(4,1,1), Arima(2,1,1), Arima(3,1,1)

Estimating Coefficients.

Now we fit the potential models to estimate the coefficients for the series.

```
## Call:
## arima(x = weatherts.tr, order = c(4, 1, 1), xreg = 1:length(weatherts.tr),
method = "ML")
##
## Coefficients:
##           ar1     ar2      ar3      ar4      ma1  1:length(weatherts.tr)
##       -0.0452  0.0296  -0.0272  -0.0493  -1.0000                  0.0012
## s.e.   0.1116  0.1115   0.1125   0.1119   0.0391                  0.0004
##
## sigma^2 estimated as 0.005813:  log likelihood = 90.1,  aic = -166.2




## Call:
## arima(x = weatherts.tr, order = c(2, 1, 1), xreg = 1:length(weatherts.tr),
method = "ML")
```

```
##
## Coefficients:
##           ar1     ar2      ma1  1:length(weatherts.tr)
##       -0.0431  0.0312  -1.0000                  0.0012
## s.e.   0.1119  0.1115   0.0377                  0.0004
##
## sigma^2 estimated as 0.005842:  log likelihood = 89.98,  aic = -169.96


## Call:
## arima(x = weatherts.tr, order = c(3, 1, 1), xreg = 1:length(weatherts.tr),
method = "ML")
##
## Coefficients:
##           ar1     ar2      ar3      ma1  1:length(weatherts.tr)
##       -0.0428  0.0294  -0.0233  -1.0000                  0.0012
## s.e.   0.1118  0.1117   0.1125   0.0381                  0.0004
##
## sigma^2 estimated as 0.005835:  log likelihood = 90,  aic = -168
```

Notice for our proposed model of Arima(4,1,1) does have the AIC closest to zero (AIC = -166.2) which suggests this is our best model. Yet one thing to pay attention to is the estimated coefficient of the Moving average component is exactly -1. This means the ARIMA(4,1,1) model is not stationary and falls out of the desired [-1,1] range after accounting for the standard deviation of 0.0391.

We will try to correct this by excluding the moving average component in our proposed model and use ARIMA(4,1,0) or ARIMA(2,1,0) as our new potential model.

```
## Call:
## arima(x = weatherts.tr, order = c(4, 1, 0), xreg = 1:length(weatherts.tr),
## Coefficients:
##           ar1      ar2      ar3      ar4  1:length(weatherts.tr)
##       -0.8600  -0.6464  -0.4965  -0.3592                  0.0010
## s.e.   0.1044   0.1310   0.1290   0.1042                  0.0027
##
## sigma^2 estimated as 0.00652:  log likelihood = 87.17,  aic = -162.34
## Call:
## arima(x = weatherts.tr, order = c(2, 1, 0), xreg = 1:length(weatherts.tr),
method = "ML")
##
## Coefficients:
##           ar1      ar2  1:length(weatherts.tr)
##       -0.7044  -0.3134                   9e-04
## s.e.   0.1061   0.1057                   5e-03
## sigma^2 estimated as 0.007918:  log likelihood = 79.75,  aic = -151.51
```

Now we have two models where all the coefficients lie within the desired range, the AIC is lower, and now the proposed models are stationary.

We will narrow are attention to these two models. With Arima(2,1,0) ranking as our best model and ARIMA(4,1,0) as secondary.

- **Model:** ARIMA(p,d,q)

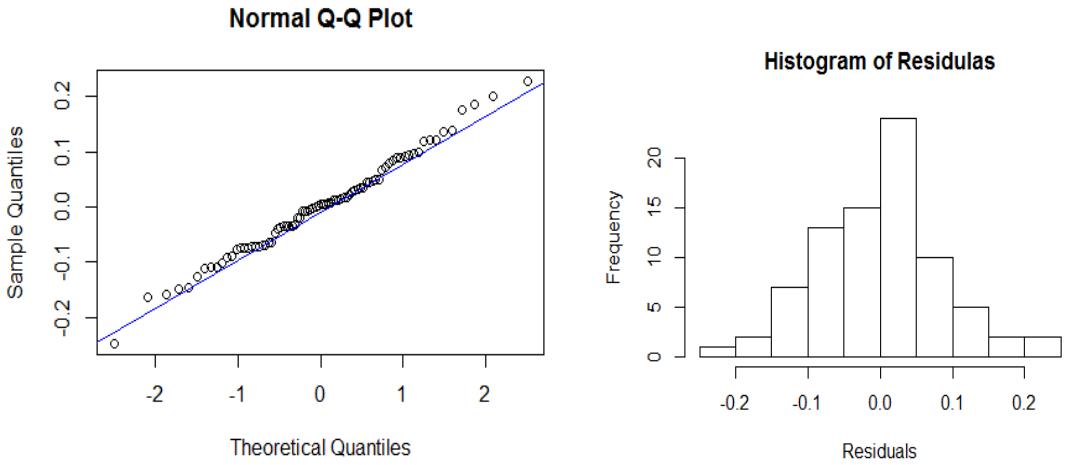
$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} - + \ldots + \phi_p X_{t-p} - \theta_1 a_{t-1} - \theta_2 a_{t-2} \ldots - \theta q a_{t-a}$$

Diagnostic Checking.

In order to use these model with any faith we must verify the model is adequate. We will do this by checking if the residuals are randomly distributed and follow a normal distribution.

1.Are the Residuals normally distributed?

- Check if the residuals follow the qqnorm() plot
- Do residuals resemble a normal curve



Looking at the results above the residuals relatively hug the linear line and the histogram has a resemblance to the normal distribution.

To verify results above we will use the Shapiro-Wilk test to check for normality.

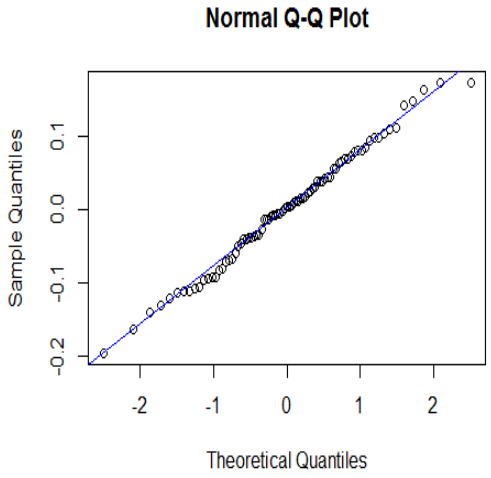
```
Shapiro-Wilk normality test
W = 0.99148, p-value = 0.8758
```

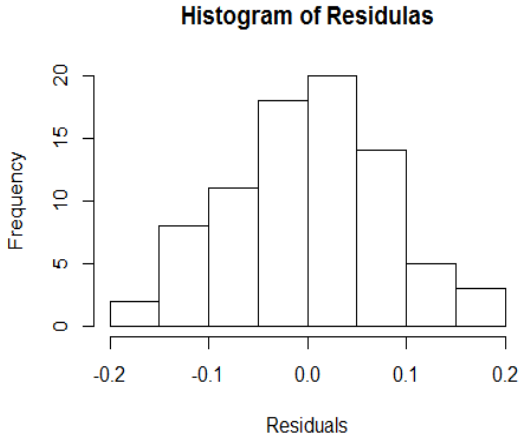
Our p-value is larger than our testing level of $ = 0.05 $ so we can conclude the data came from a normal distribution.

Finally we must check to see if the autocorrelations of the time series are different from zero. Which would imply the observed correlations in the data are just a result of the sampling process. To do this we will use the Ljung-Box, and McLeod-Li test: (Note: McLeod-Li test checks the Ljung-Box squares)

```
Box-Ljung test
X-squared = 0.43453, df = 1, p-value = 0.5098
Box-Ljung test
data:  fitNew$residuals^2
X-squared = 0.91425, df = 1, p-value = 0.339
```

Notice both p-values are larger than 0.05 and the data passes all the required tests. Thus the model is adequate.





The residuals closely follow the qqplot hugging the linear line nicely, and the histogram certainly resembles a normal distribution.

```
Box-Ljung test
X-squared = 0.43453, df = 1, p-value = 0.5098
Box-Ljung test
X-squared = 0.91425, df = 1, p-value = 0.339
Shapiro-Wilk normality test
W = 0.99148, p-value = 0.8758
```

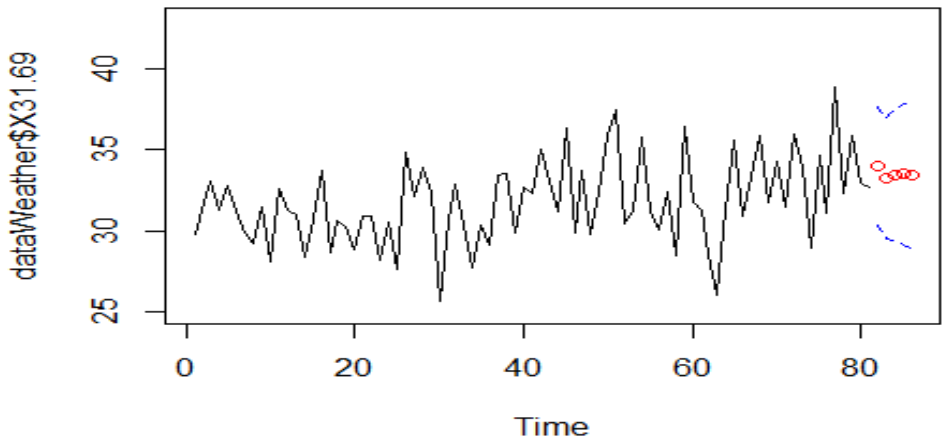
Our series of test all have a p-value larger than 0.05 and pass all pass. We can conclude the model is adequate.

Forecasting.

Now that we have gone through all of the steps of selecting models and verifying that they are adequate representation of the data. We will use them to forecast future levels of rain fall in the surrounding Great lakes area and check if the predictions are reasonable. Our 95% confidence intervals should capture the entirety of possibilities the data may take on in order to conclude positive results.

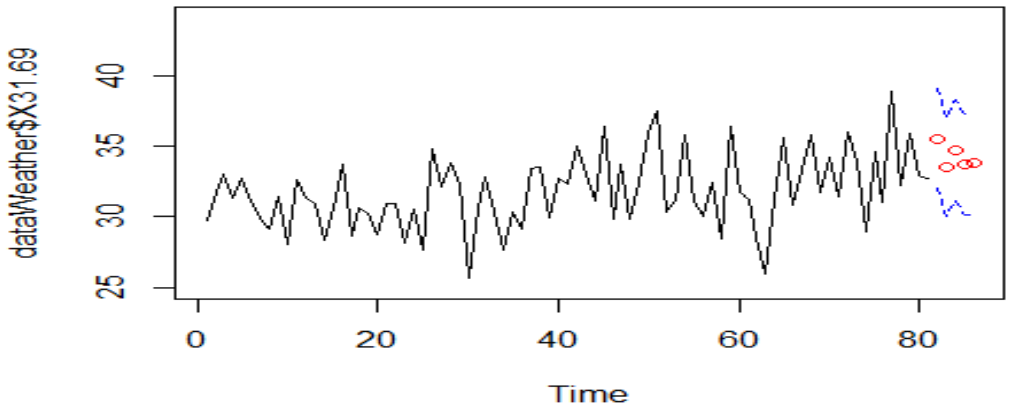
To do this I have removed the data entries for the years 1981-1986 from the original series and will check how accurate the 5 forecasted totals coincide with the actual true rainfall totals for the corresponding years.

ARIMA(2,1,0):



The model seems to accurately forecast reasonable levels of rainfall but the the 95% confidence interval doesnt capture the spike at 1985 where it jumps to 40inches of rainfall. The model may not be a good a good choice. Also the projected totals seem to lack any type of spikes in behavior and tends to follow more of a straight behavior. This goes against the grain of all the past totals through the series.

ARIMA(4,1,0):



The second model also seems to predict reasonable levels of precipitation and but more importantly seems to capture the extreme high/low pattern of the data set a lot better. The interval falls a little short of the extreme rainfall total at 1985 of 40 inches. This is a sharp edge in the series and might just lie outside of our 95% confidence interval by chance. I am not as concerned with this when using this model because again it does seem to capture the overall behavior well and the projected totals are arranged as such.

Conclusion.

The Arima(4,1,0) model seemed to provide accurate projections for annual levels of precipitation for the Great Lakes area. I am satisfied with the five projected totals and think the overall results using the Box Jenkins approach produced positive results. The series has an extreme nature and could quite possibly be modeled better using a different approach that better contains the sharp radical behavior for modeling.

Acknowledgments

Special thanks to Professor Feldman, Sergio Rodriguez for teaching, reviewing my project, and providing feedback on Time Series Modeling.

Thanks to R Software for the services.

References:

- https://datamarket.com/data/list/?q=provider:tsdl
- https://en.wikipedia.org/wiki/Ljung%E2%80%93Box_test
- http://support.sas.com/resources/papers/proceedings13/454-2013.pdf

Appendix:

```
dataWeather = read.csv("C:\\Users\\Brock\\Desktop\\weather.orig.txt")
weatherts = ts(dataWeather$X31.6)

plot(weatherts)
var(weatherts)

acf(weatherts)
pacf(weatherts)
# Model: Arima(0,0,5)


#increasing variance box cox transformation
bctransform <- boxcox(dataWeather$X31.69 ~as.numeric(1:length(dataWeather$X31
.69)))
bctransform$x[which(bctransform$y == max(bctransform$y))]

#transformation
weatherts.tr = log(weatherts)
plot(weatherts.tr)

#differncing to remove upward trend
weatherts.tr.diff = diff(weatherts.tr)
plot(weatherts.tr.diff)
var(weatherts.tr.diff)

abline(h = 0.2, col = 'red')
abline(h = -0.2, col ='red')
abline(h = 0, col = 'blue')

#over differencing variance spiked de-stablizes
weatherts.tr.diff2 = diff(weatherts.tr,differences = 2)
plot(weatherts.tr.diff)
var(weatherts.tr.diff2)

abline(h = 0.2, col = 'red')
abline(h = -0.2, col ='red')
abline(h = 0, col = 'blue')


#acf pacf of new tr.diff
acf(weatherts.tr.diff)
pacf(weatherts.tr.diff)

# Model: Arima(4,1,1), Arima(2,1,1), Arima(3,1,1), Arima(4,1,0)

#suggested
arima(weatherts.tr, order = c(4,1,1), method = "CSS")
```

```r
# my fits, fit 2 and 3 are good, Fit2 is best.
fit1 = arima(weatherts.tr, order  = c(0,0,5), method = "ML",xreg=1 : length(w
eatherts.tr))
fit2 = arima(weatherts.tr, order  = c(4,1,0), method = "ML",xreg=1 : length(w
eatherts.tr))
fit4 = arima(weatherts.tr, order  = c(4,1,1), method = "ML",xreg=1 : length(w
eatherts.tr))
fit4 = arima(weatherts.tr, order  = c(4,1,1), method = "ML",xreg=1 : length(w
eatherts.tr))



#fit 2 diagnostics

qqnorm(fit2$residuals)
qqline(fit2$residuals, col ='blue')
hist(fit2$residuals, xlab = "Residuals" , main = "Histogram of Residulas")

resid.ts = (fit2$residuals)
plot(resid.ts)
abline(lm(resid.ts ~ as.numeric(1:length(resid.ts))))
abline(h=mean(resid.ts), col="red")

mean(resid.ts)
var(resid.ts)


Box.test(fit2$residuals, lag = 1, type = "Ljung-Box")
Box.test(fit2$residuals^2, lag = 1, type = "Ljung-Box")
shapiro.test(fit2$residuals)

#fit 3 diagnostics

qqnorm(fit4$residuals)
qqline(fit4$residuals, col ='blue')
hist(fit4$residuals, xlab = "Residuals" , main = "Histogram of Residulas")

resid.ts = (fit4$residuals)
plot(resid.ts)
abline(lm(resid.ts ~ as.numeric(1:length(resid.ts))))
abline(h=mean(resid.ts), col="red")

mean(resid.ts)
var(resid.ts)
```

```r
Box.test(fit4$residuals, lag = 1, type = "Ljung-Box")
Box.test(fit4$residuals^2, lag = 1, type = "Ljung-Box")
shapiro.test(fit4$residuals)


# predicting with fit 2
pred <- predict(fit2, n.ahead = 5, newxreg=(length(weatherts.tr)+1) : length(
weatherts.tr)+5)
pred.orig =  exp(pred$pred)
pred.se = exp(2*pred$pred*pred$se)

ts.plot(dataWeather$X31.69, xlim=c(1,length(dataWeather$X31.69)+5), ylim = c(
25,max(pred.orig+1.96*pred.se+5)))
points((length(dataWeather$X31.69)+1):(length(dataWeather$X31.69)+5),pred.ori
g, col="red")
lines((length(dataWeather$X31.69)+1):(length(dataWeather$X31.69)+5),pred.orig
+1.96*pred.se,lty=2, col="blue")
lines((length(dataWeather$X31.69)+1):(length(dataWeather$X31.69)+5),pred.orig
-1.96*pred.se,lty=2, col="blue")


#predicting with fit 3
pred <- predict(fit3, n.ahead = 5, newxreg=(length(weatherts.tr)+1) : length(
weatherts.tr)+5)
pred.orig =  exp(pred$pred)
pred.se = exp(2*pred$pred*pred$se)

ts.plot(dataWeather$X31.69, xlim=c(1,length(dataWeather$X31.69)+5), ylim = c(
25,max(pred.orig+1.96*pred.se+5)))
points((length(dataWeather$X31.69)+1):(length(dataWeather$X31.69)+5),pred.ori
g, col="red")
lines((length(dataWeather$X31.69)+1):(length(dataWeather$X31.69)+5),pred.orig
+1.96*pred.se,lty=2, col="blue")
lines((length(dataWeather$X31.69)+1):(length(dataWeather$X31.69)+5),pred.orig
-1.96*pred.se,lty=2, col="blue")
```