STAT 621 Lecture Notes
The Sign Test

We will start our disucssion of classical nonparametric methods with a simple procedure for testing a hypothesis about the location, or center, of a population. While Normal-theory procedures use the mean as a measure of location (e.g., t-test), this procedure uses the median. This allows us to relax the assumption of Normality, and also to consider smaller sample sizes than we could if we needed to rely on results like the Central Limit Theorem. And the median is a more robust estimator of center for distributions that are skewed.

This procedure may be applied in the following situations.

- Paired data:

- One-Sample Data:

We'll focus on defining the procedure for paired data, and see that it translates straightforwardly to the one-sample case. First A little set-up and notation.

Data: Two measurements on each subject for $2n$ observations: $(X_1, Y_1), (X_2, Y_2), \ldots (X_n, Y_n)$. Define the differences between paired measurements as

$$Z_i = Y_i - X_i, \qquad i = 1, \ldots, n.$$

Assumptions:

1. $Z_1, \ldots, Z_n$ are mutually independent.

2. $Z_i \sim F_i$ with common median θ for all i.

What does θ represent?

Briefly discuss the assumptions.

Hypothesis: Here interest is in testing the null hypothesis

$$H_0 : \theta = 0.$$

For example, the median of the distribution of differences between post- and pre-treatment measurements is zero.

Test Statistic: The test statistic for the Sign Test is

$$B = \sum_{i=1}^{n} I(Zi > 0)$$

So B simply tells us the number of positive Z_i, i.e, the number of times Y_i is larger than X_i.

Rejection Rules and P-Values: To determine the rules by which we decide to reject the null hypothesis, we need to the the sampling distribution of our test statistic, under the assumption that the null is true. What is this sampling distribution?

So exact rejection rules and p-values should not be too difficult to find. For example,

Example: Samples of cream from 10 different dairies are each divided into two portions. One portion is sent to Laboratory A and the other to Laboratory B. The bacteria counts (1000/ml) reported from each laboratory are below.

Dairy	1	2	3	4	5	6	7	8	9	10
Lab A	11.7	12.1	13.3	15.1	15.9	15.3	11.9	16.2	15.1	13.6
Lab B	10.9	11.9	13.4	15.4	14.8	14.8	12.3	15.0	14.2	13.1

Consider using the sign test to test whether there is a difference in the counts from the two labs (more specifically whether the median difference is zero). State your hypotheses. Find the value of the test statistic. What is the null distribution?

The following R output has the upper and lower tail probabilities for null distribution of the test statistic, i.e., $\text{Bin}(10, 0.5)$. Suppose you'd like to have α no more than .05. What is your rejection region? What do you conclude? Find the p-value.

```
> b=0:10
> PgeqB=pbinom(b-1,10,.5,lower.tail=F)
> PleqB=pbinom(b,10,.5)
> cbind(b,PleqB,PgeqB)
       b        PleqB        PgeqB
 [1,]  0 0.0009765625 1.0000000000
 [2,]  1 0.0107421875 0.9990234375
 [3,]  2 0.0546875000 0.9892578125
 [4,]  3 0.1718750000 0.9453125000
 [5,]  4 0.3769531250 0.8281250000
 [6,]  5 0.6230468750 0.6230468750
 [7,]  6 0.8281250000 0.3769531250
 [8,]  7 0.9453125000 0.1718750000
 [9,]  8 0.9892578125 0.0546875000
[10,]  9 0.9990234375 0.0107421875
[11,] 10 1.0000000000 0.0009765625
```

The R package BSDA contains the function SIGN.test for computing the sign test. This is illustrated below. The function also reports a confidence interval for θ and some other information. I've omitted this for clarity.

```
> install.packages("BSDA")        # install and load the package first
> library(BSDA)

> labA=c(11.7, 12.1, 13.3, 15.1, 15.9, 15.3, 11.9, 16.2, 15.1, 13.6)
> labB=c(10.9, 11.9, 13.4, 15.4, 14.8, 14.8, 12.3, 15.0, 14.2, 13.1)

> SIGN.test(labA, labB, alt="two.sided")

      Dependent-samples Sign-Test

          data:  labA and labB
          S = 7, p-value = 0.3437
          alternative hypothesis: true median difference is not equal to 0

(Some stuff omitted)
```

Notes:

1. Variations for testing single samples and non-zero null values for θ.

2. A Large-Sample Approximation: If you have a large sample and still want to use this test, the following result can be used to derive an approximate procedure. For large n, the standardized variable
$$B^* = \frac{B - E(B)}{\sqrt{V(B)}}$$
is approximately $N(0, 1)$ if H_0 is true. Define an approximate testing procedure.